Pure Spinors to Associative Triples to Zero-Divisors

Frank Dodd (Tony) Smith, Jr. - 2012

Abstract:

Both Clifford Algebras and Cayley-Dickson Algebras can be used
to construct Physics Models.
Clifford and Cayley-Dickson Algebras have in common
Real Numbers, Complex Numbers, and Quaternions,
but in higher dimensions Clifford and Cayley-Dickson diverge.
This paper is an attempt to explore the relationship between
higher-dimensional Clifford and Cayley-Dickson Algebras
by comparing Projective Pure Spinors of Clifford Algebras
with Zero-Divisor structures of Cayley-Dickson Algebras.

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Robert de Marrais and Guillermo Moreno,
pioneers in studying Zero-Divisors, unfortunately have passed (2011 and 2006).

Pure Spinors to Associative Triples

Pure spinors are those spinors that can be represented by simple exterior wedge products of vectors. For Cl(2n) they can be described in terms of bivectors Spin(2n) and Spin(n) based on the twistor space Spin(2n)/U(n) = Spin(n) x Spin(n). Since (1/2)((1/2)(2n)(2n-1)-n^2) = (1/2)(2n^2-n-n^2) = = (1/2)(n(n-1)) Penrose and Rindler (Spinors and Spacetime v.2) describe Cl(2n) projective pure half-spinors as Spin(n) so that the Cl(2n) full space of pure half-spinors has dimension dim(Spin(n)) + 1.

dim	Half-Spinors	Projective Pure Half-Spinors		Associative Triples
Cl(n) = 2^n bivector = = Spin(n)	2^(2^(n-1)-1)	(1/2)2^(n-1)(2^(n-1)-1) Spin(n-1)		SUM(PPHSp) (2^N-1)(2^N-2)/3!
2^1 = 2	2^0 = 1	Spin(0)	0	0
2^2 = 4	2^1 = 2	Spin(2)	1	1
2^3 = 8	2^3 = 8	Spin(4)	6	7
2^4 = 16	2^7 = 128	Spin(8)	28	35
2^5 = 32	2^15 = 32,768	Spin(16)	120	155
2^6 = 64	2^31 = 2,147,483,648	Spin(32)	496	651
2^7 = 128	2^63 = 9.2 x 10^18	Spin(64)	2,016	2,667
2^8 = 256	2^127 = 1.7 x 10^38	Spin(128)	8,128	10,795

The number of Cl(n) Associative Triples
is
the Sum for all k ≤ n
of the number of Cl(k) Projective Pure Half-Spinors

Associative Triples to Loops

Raul Cawagas in Discussiones Mathematicae General Algebra and Applications 24 (2004) 251-265 and in arxiv 0907.2047 said:
"... The ... sedenion ... multiplication rule can ... be expressed ... by means of 35 associative triples (or cycles).
These are listed below in two sets:
octonion triplets [7] and sedenion triplets [28].

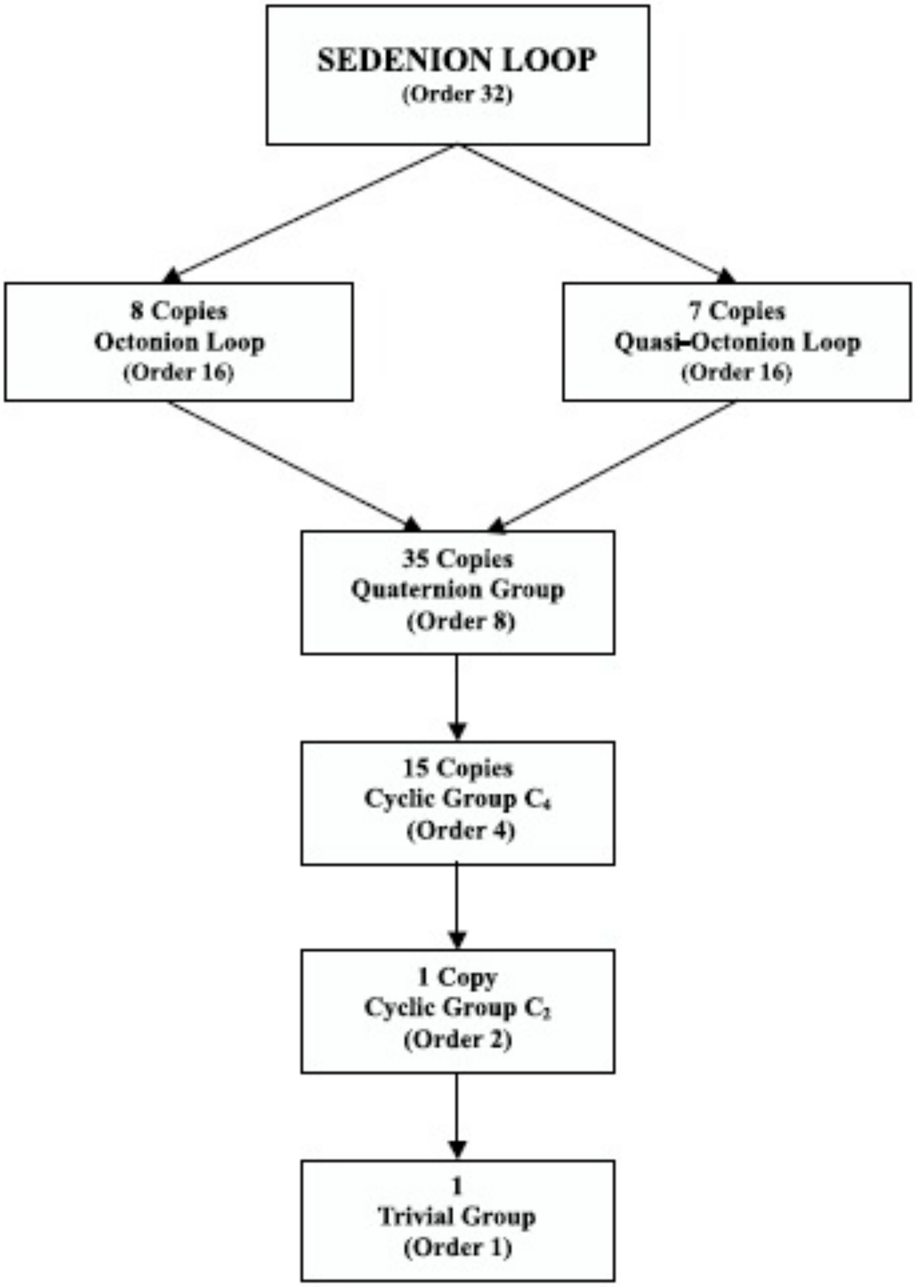
OCTONION TRIPLETS:

(1,2,3), (1,4,5), (1,7,6), (2,4,6), (2,5,7), (3,4,7), (3,6,5)

SEDENION TRIPLETS:

(1,8,9), (1,11,10), (1,13,12), (1,14,15)
(2,8,10), (2,9,11), (2,14,12), (2,15,13)
(3,8,11), (3,10,9), (3,15,12), (3,13,14)
(4,8,12), (4,9,13), (4,10,14), (4,11,15)
(5,8,13), (5,12,9), (5,10,15), (5,14,11)
(6,8,14), (6,15,9), (6,12,10), (6,11,13)
(7,8,15), (7,9,14), (7,13,10), (7,12,11)

...
the set E16 = {ei | i = 0, 1, ... , 15}
of 16 sedenion base elements generates a set
SL = { +/- ei | i = 0, 1, ... , 15} of order 32;
where e0 is the identity element,
that forms a non-commutative loop under sedenion multiplication
...indices i=0,1,...,7 correspond to the octonion base elements
... i=8,...,15 correspond to the pure sedenion base elements.
This loop SL,
which we shall call the Cayley-Dickson sedenion loop,
is embedded ... in the sedenion space and its subloops determine
the basic subalgebras of S ...
SL has ... 66 ... non-trivial and normal ... subloops
...
There are 15 ... maximal subloops ... of order 16 ...
8 ... NAFILs [non-associative finite invertible loops]...
isomorphic to the octonion loop OL ...
7 NAFILs ... isomorphic to ... the quasi-octonion loop O~L
...
There are 35 subloops of order 8 ... isomorphic to the
quaternion group Q of order 8
...
There are 15 subloops of order 4 ... isomorphic to the cyclic
group C4 of order 4
...
There is only 1 subloop of order 2 ... isomorphic to the cyclic
group C2 of order 2 and is the center of SL
...


SEDENION LOOP
(Order 32)
8 Copies
Octonion Loop
(Order 16)
7 Copies
Quasi-Octonion Loop
(Order 16)
35 Copies
Quaternion Group
(Order 8)
15 Copies
Cyclic Group C4
(Order 4)
1 Copy
Cyclic Group C2
(Order 2)
1
Trivial Group
(Order 1)

. . .

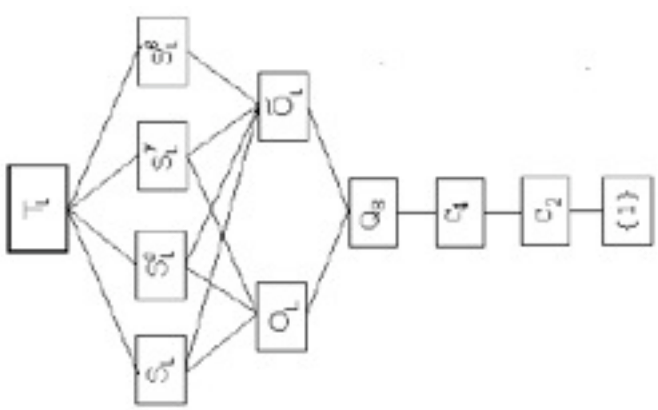
The trigintaduonion algebra T of dimension D = 32 ...
contains an embedded NAFIL loop TL of order 64....
The maximal subloops of TL are the 31 subloops of order 32
(called sedenion-type loops) ... One of these is the "standard"
sedenion loop SL generated by the basis of the sedenion algebra
S. In addition to SL three more of these 31 sedenion-type loops
of order 32 have been identified as distinct (non-isomorphic).
In terms of the basis elements of T, we have:

SL(#2) = {0,1,2,3, 4, 5, 6, 7, 8, 9,10,11,12,13,14,15}
(std sedenion loop)
SaL(#7) = {0,1,2,3, 8, 9,10,11,20,21,22,23,28,29,30,31}
(a-sedenion loop)
SbL(#10) = {0,1,2,3,12,13,14,15,20,21,22,23,24,25,26,27}
(b-sedenion loop)
ScL(#4) = {0,1,2,3, 4, 5, 6, 7,24,25,26,27,28,29,30,31}
(c-sedenion loop)
... these four distinct subloops represent exactly four
isomorphy classes:
SL(#2) ... 16 subloops
SaL(#7) ... 7 subloops
SbL(#10) ... 7 subloops
ScL(#4) ... 1 subloop...

TL has 155 subloops of order 16. These are octonion-type NAFIL
loops that form exactly two isomorphy classes:

C{OL} of octonion loops ... +/-{0;1;2;3;4;5;6;7} ...
= 50 loops
C{O~L} of quasi-octonion loops ...
+/{0;1;2;3;12;13;14;15} ... = 105 loops...

all of the 155 subloops of order m = 8 are groups isomorphic to
the quaternion group Q ...
the 31 subloops of order m = 4 are groups isomorphic to the
cyclic group C4...
the lone subloop of order m = 2 is a group isomorphic to the
cyclic group C2. ...



...

[Loops to Zero-Divisors]

R. P. C. de Marrais has determined ... exactly 84 pairs of ... Sedenion ... zero divisors ...
The known zero divisors of the sedenion algebra S are all confined to copies in S of the quasi-octonion algebra O~
...

Table 5. List of sedenion *zero divisor pairs*. Source: Robert de Marrais, http://arXiv.org/abs/math.GM/0011260. As in Table 1, the numerals are the indices of the base elements, that is, $i \equiv e_i$.

GoTo#1	Based on Octonion Triplet(1,2,3)–Automorpheme:(1,2,3,12,13,14,15)			
	(1+13)(2–14)	(1+14)(2+13)	(1–12)(2–15)	(1–15)(2+12)
	(2–14)(3+15)	(2+13)(3–12)	(2–15)(3–14)	(2+12)(3+13)
	(3+15)(1–13)	(3–12)(1–14)	(3–14)(1+12)	(3+13)(1+15)
GoTo#2	Based on Octonion Triplet(1,4,5)–Automorpheme:(1,4,5,10,11,14,15)			
	(1+14)(4–11)	(1+11)(4+14)	(1–15)(4–10)	(1–10)(4+15)
	(4–11)(5+10)	(4+14)(5–15)	(4–10)(5–11)	(4+15)(5+14)
	(5+10)(1–14)	(5–15)(1–11)	(5–11)(1+15)	(5+14)(1+10)
GoTo#3	Based on Octonion Triplet(1,7,6)–Automorpheme:(1,7,6,10,11,12,13)			
	(1+11)(7–13)	(1+13)(7+11)	(1–10)(7–12)	(1–12)(7+10)
	(7–13)(6+12)	(7+11)(6–10)	(7–12)(6–13)	(7+10)(6+11)
	(6+12)(1–11)	(6–10)(1–13)	(6–13)(1+10)	(6+11)(1+12)
GoTo#4	Based on Octonion Triplet(2,4,6)–Automorpheme:(2,4,6,9,11,13,15)			
	(2+15)(4–9)	(2+9)(4+15)	(2–13)(4–11)	(2–11)(4+13)
	(4–9)(6+11)	(4+15)(6–13)	(4–11)(6– 9)	(4+13)(6+15)
	(6+11)(2–15)	(6–13)(2–9)	(6–9)(2+13)	(6+15)(2+11)
GoTo#5	Based on Octonion Triplet(2,5,7)–Automorpheme:(2,5,7,9,11,12,14)			
	(2+9)(5–14)	(2+14)(5+9)	(2–11)(5–12)	(2–12)(5+11)
	(5–14)(7+12)	(5+9)(7–11)	(5–12)(7–14)	(5+11)(7+ 9)
	(7+12)(2–9)	(7–11)(2–14)	(7–14)(2+11)	(7+9)(2+12)
GoTo#6	Based on Octonion Triplet(3,4,7)–Automorpheme:(3,4,7,9,10,13,14)			
	(3+13)(4-10)	(3+10)(4+13)	(3–14)(4– 9)	(3–9)(4+14)
	(4–10)(7+9)	(4+13)(7–14)	(4–9)(7–10)	(4+14)(7+13)
	(7+ 9)(3–13)	(7–14)(3–10)	(7–10)(3+14)	(7+13)(3+ 9)
GoTo#7	Based on Octonion Triplet(3,6,5)–Automorpheme:(3,6,5,9,10,12,15)			
	(3+10)(6–15)	(3+15)(6+10)	(3–9)(6–12)	(3–12)(6+ 9)
	(6–15)(5+12)	(6+10)(5–9)	(6–12)(5–15)	(6+9)(5+10)
	(5+12)(3–10)	(5–9)(3–15)	(5–15)(3+9)	(5+10)(3+12)

... Each zero divisor in the pair consists of two base elements of the form (o +/- s), where o is an octonion base element (belonging to an octonion triplet), while s is a pure sedenion base element.
...
All 16-dimensional subalgebras of the trigintaduonions T (like S; S ; S ;and S have zero divisors ...".

As to larger algebras,
Jenya Kirshtein in arxiv 1102.5151 and in 6 Nov 2011 slides for a Bob Liebler Algebraic Combinatorics Conference said:
"... The Cayley-Dickson loop [Cn] is the multiplicative closure of basic elements of the corresponding Cayley-Dickson algebra. ... Size of Cn is 2^(n+1) ...
The first few examples of the Cayley-Dickson loops are
the real group R2(abelian);
the complex group C4 (abelian);
the quaternion group H8 (not abelian);
the octonion loop O16 (Moufang);
the sedenion loop S32 (not Moufang);
the ...[trigintaduonion]... loop T64
...
all subloops of Cn of size 16 fall into two isomorphism classes ... either O16, the octonion loop,
or O~16, the quasi-octonion loop ...
the automorphism groups of the first five Cayley-Dickson loops are

|Aut (C4)| = 2,
|Aut (H8)| = 24 = 6 x 4,
|Aut (O16)| = 1344 = 14 x 12 x 8, [14x12=168=PSL(2,7)=SL(3,2)]
|Aut (S32|| = 2688 = 2 x (14 x 12 x 8),
|Aut (T64)| = 5376 = 2 x 2 x (14 x 12 x 8)

... |Aut(Qn)| = 1344 x 2^(n-3) (Aut(Q(n-1)) x Z2)...

Isomorphism Classes of Maximal Subloops

Size (Qn)	Max subloops	Isomorphism classes	Representatives
C4	1 [= PG(0,2)]	1	R2
H8	3 [= PG(1,2)]	1	C4
O16	7 [= PG(2,2)]	1	H8
S32	15 [= PG(3,2)]	2	O16 and ˜O16
T64	31 [= PG(4,2)]	4	S32,˜S132,˜S232,˜S332
Q128	63 [= PG(5,2)]	8	
Q256	127 [= PG(6,2)]	at most 16	

...[
H. S. M. Coxeter in Projective Geometry said: "... the number of points in **PG(n,q) is ... (q^(n+1) - 1) / (q-1) ...**"
Steven H. Cullinane in Solomon's Cube (2003) said:
"... Klein's quartic, like all non-singular quartic curves, has 28 bitangents ... Both the **28 bitangents and the 27 lines may be represented within the 63-point space PG(5,2) ...**".
Patrick Du Val in On the Directrices of a Set of Points in a Plane (1931) said: "...
the **27 lines** on a cubic surface can be put in 1-1 correspondence with the vertices of a [**E6**]... polytope in **...** **6-dim** space **...**
the **28 bitangents** of the general plane quartic ...[have]... similar relation ...[to an **E7**]... polytope in **7-dim ...**
[120] tritangent planes of the canonical curve of genus 4 ...
[have] similar relation ...[to an **E8**]... polytope in **8-dim ...**".
Compare these Del Pezzo Surfaces F(N,2):

N	Lines	Polytope	Dim	Symmetry Group
3	27	Gosset 2_21	6	E6
2	56=28+28	Gosset 3_21	7	E7
1	240=120+12	Gosset 4_21 (Witting)	8	E8

]...
the automorphism groups of C4, H8 and O16 are as big as they possibly can be ...
On the contrary, the automorphism groups of S32 and T64 are only double the size of the preceeding ones ...
all the subloops of index 2 of .. all Cayley-Dickson loops .. [are]... one of three types ...

C(n-1) ...
D + De ...
D + (C(n-1)\D)e ...

[where] D [is] a subloop of C(n-1) of index 2
...

Starting at S32, any subloop of Cn of the third type is not a Cayley-Dickson loop

...".

Since the study of Cayley-Dickson subloops fails to be inclusive of all subloops for Cayley-Dickson algebras as large as C32 = T32, other techniques must be used.

What New Phenomena Emerge at C32 = T32 = Pathions?

Robert de Marrais in math.GM/0011260 and math.RA/0207003 and NKS 2004 and NKS 2006 and arxiv 0704.0112 and arxiv 0804.3416 said:
"... the automorphism group of
the ... 2^4-ion ... Sedenions' ZD's has order 14x1x6 = 84 ...
Let's expand our horizons ... we'll call these
    Pathions... 2^5-ions ... for the "32 Paths" of Kabbalah ...
    Chingons... 2^6-ions ... for the ... I Ching ...
    Routons... 2^7-ions ... for ... Route 128 ...
    Voudons... 2^8-ions ... for the 256 deities of ... Ifa ...

for 2^n-ions, n > 3, the ZD-pairings formula, up to Voudons,
gives 6x(2^(n-1)-4) = 24, 72, 168, 360, 744
...[However, ZD-pairings are not the only Zero Divisors]...
for ... 2^5-ions ... Pathions ... Moreno's ... total count of
"irreducible" zero-divisors ... should have ... for n > 4 ...
automorphism group ... G2 x (n-3) x S3 = (5-3) x 84 = 168
[where S3 = symmetric group of order 3x2x1 = 6] ... But ...
the Pathions have 15x7 = 105 "16-less" [associative] triplets of
form (O,S,S') ... which yield 210 ... index-pairs...Assessors...
in addition ... 42 of the (P,P') pairings are ZD's as well,
which makes 252 x 2 diagonals in each = 504 "irreducible" ZD's
specific to the Pathions ... Add in the 84 first showing in the
Sedenions, and we have 588 in all:
3 1/2 times the count ...168... Moreno's formula would allow ...
when ... extended to 2^6-ions, we get not 3x84,
but 4x84 = 336 ZD-diagonals, the maximal symmetry of ...
Klein's ... tessellation by triangles of the hyperbolic plane
[**double cover of PSL(2,7) = SL(,32) related to singularities**
744=3x248 = constant term of the elliptic modular invariant]...
We know that surprises will keep on coming at least up to the
2^8-ions ... Voudons ... due to the 8-cyclical structure of ...
[Real]... Clifford Algebras ...
The 8-cycle implies an iterable ... pattern ...
cranking out of numbers which are 2^Nth roots of 0,
approaching as a limit-case an ...
infinitude of roots form[ing] a "loop" of some sort
...
the total number of trips [associative triples], which ...
in a given set of 2^N-ions, is just (2^N -1)(2^N -2)/3! ...

155 for the 32-D Pathions, where the signature of “scale-free” behavior ... implicit “fractality” ... is first revealed.
...
to understand what happens in 32-D ...[N = 5]...

Box-Kite “Explosions” in 32D:
Two Types, Triptych Triples, 4-Fold Spandrels

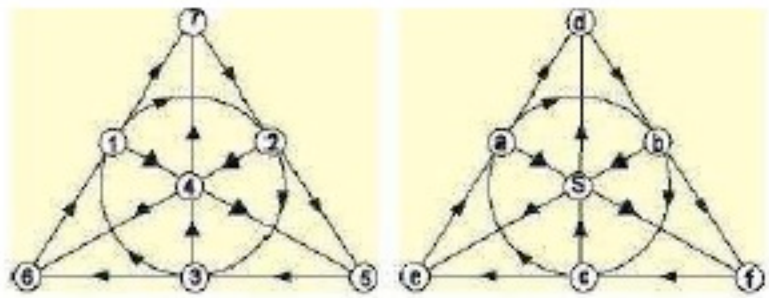


Figure 3: The 7-point, 7-line finite projective group, a.k.a the Fano Plane, hosts the labels for Octonion units, and shows their triplets' orientations. The same layout can be used to shorthand Box-Kite structures: the Zigzag and Trefoil L-trips sit at $(\mathbf{a},\mathbf{b},\mathbf{c})$, and $(\mathbf{a},d,e)$; $(d,\mathbf{b},f)$; $(e,f,\mathbf{c})$. The Strut Constant S, meanwhile, sits in the middle.

... ...

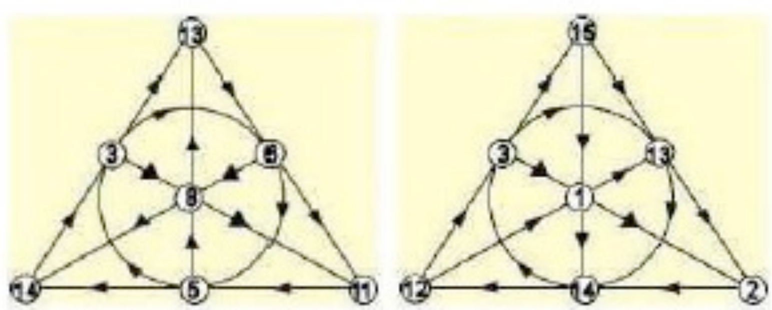
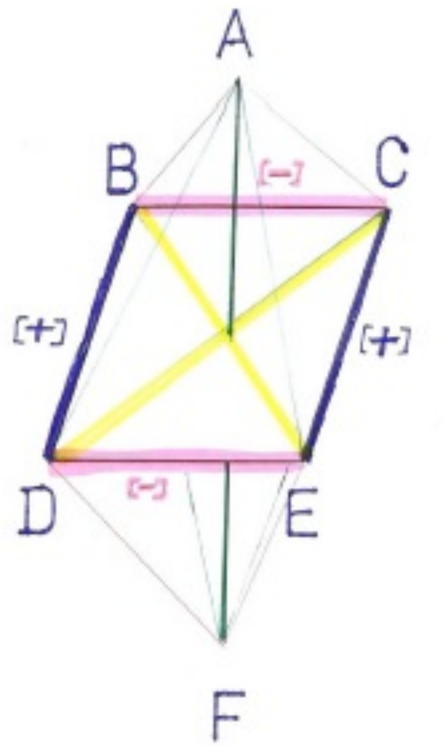


Figure 4: Pathion box-kites: Left, a normal (“Type 1”) with S = 8, and “Rule 0” Z-trip $(3,6,5)$ at (a,b,c) – itself the Z-trip for the S = 1 Sedenion box-kite. Right, a “Type 2” with S = 1, with Z-trip the “Rule 2” left side of the S = 8 “Type 1.”

... we must first explain ZD’s workings in the sedenions ...
one could pick any Octonion (7 choices) and match it with any of the 6 suitable Sedenions with index > 8,
making for 42 planes or Assessors whose diagonal line-pairs would contain all and only ZDs.
But these lines do not all mutually zero-divide with each other; those which do, though, can have their behavior summarized in 7

geometrically identical diagrams, the octahedral wireframe figures called box-kites ...



...

Label the 3 vertices of some triangle among the octahedral grid's 8 with the letters A, B, C,
and those of the opposite face F, E, D,
so that these are at opposite ends of lines through the center - AF, BE, CD - which we call struts.
Assume each vertex represents a plane whose two units are indicated by the same letter, in upper or lower case depending on whether the index is greater or less than G - U and L indices respectively ... G [being] the Generator of the 2^N-ions ...
Call the seventh Octonion index not found on a vertex the strut constant, henceforth S, and use it to distinguish the 7 box-kites, each of which contains but 6 of the 42 Sedenion Assessors.
For any chosen S, there will be 3 pairs of Octonions forming trips with it, and the indices forming such pairs are placed on strut-opposite vertices (i.e., at ends of the same strut ...)
Neither diagonal at one end of a strut ... A+/-a ...
will mutually zero-divide with either at the other ... F+/-f ...

Either diagonal, however, at any Assessor, will produce zero when multiplied by exactly one of the Assessor diagonals at the other end of a shared edge.
...

the Sail, an (algebraically closed) triad of Assessors representable by a triangle on the box-kite ... there is exactly one Sail per box-kite with all three edges marked “[-]”. This is the Zigzag [associative triple]
...
all 7 Sedenion box-kites can be envisioned ... by “embroidering” just one [as a superposition] ... Such a 7-in-1 representation, or brocade, is of great efficacy in high dimensions, where the box-kite count grows rapidly with N, and the types (including the “hidden” ones) are more numerous
...
if diagonals at A and B mutually zero-divide, each also does so with a diagonal of C ... we say A and B emanate C ... Emanation Tables or ETs ... for a given G and S will fill a square spreadsheet whose edge has length 2^(N-1)-2 ... For Sedenions, we get a 6x6 table, 12 of whose cells ... are empty ... includ[ing] the label lines ...and ... their ... mirror-reversed copies ... increases the edge-size of the ET box to 2^(N-1) ...

The Simplest (Sedenion) Emanation Tables

	2	**4**	**6**	**7**	**5**	**3**
2		6	-4	5	-7	
4	6		-2	3		-7
6	-4	-2			3	5
7	5	3			-2	-4
5	-7		3	-2		6
3		-7	5	-4	6	

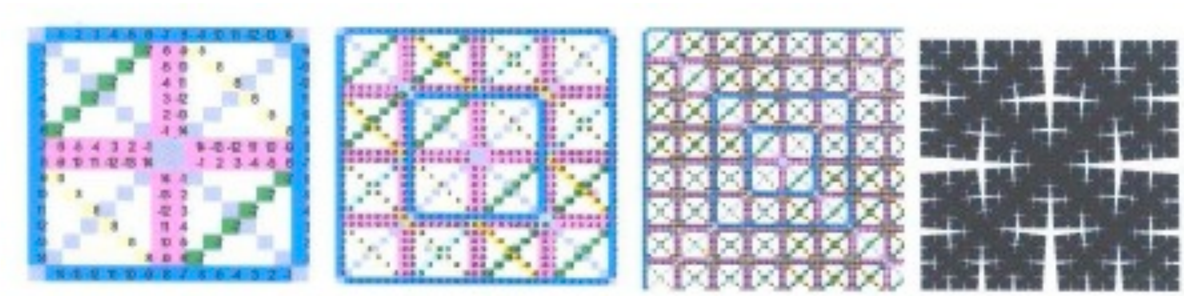
For ***S*=1** Box-Kite, put L-indices of the 6 vertices as labels of Rows and Columns of a ZD "multiplication table," entering them in left-right (top-down) order, with smallest first, and its strut-opposite in the mirror-opposite slot: 2 *xor* 3 = 4 *xor* 5 = 6 *xor* 7 = 1 = ***S***.

If R and C don't mutually zero-divide, leave cell (R,C) ***blank***.

Otherwise, enter the L-index of their emanation (the 3rd Assessor in their common Sail). (Oh, yeah: ignore the minus signs.)

... box-kites are “all or nothing” structures: either all edges support ZD-currents, or none do. These latter “hidden box-kites” (HBKs ...) were the sources of the off-diagonal empty

cells in the 2^(N−1) − 2 cells-per-edge square ETs ... emanation tables ... for fixed-S 2^N-ions ...
as N grew indefinitely large for fixed S, such tables' empty space approached a "fractal limit."
...
high-N 2^N-ions, beginning with the Pathions,
have surprising patterns of empty cells ...
an ET's empty "spreadsheet cells" - emerging in any and all ET's for N >4, and S>8 and not a power of 2 - mapped to pixels in a planar fractal. ...
the Zigzag-like HBKs each house their own ZD-free copy of the Octonions - and hence, the basis for the recursive CDP spawning of "parallel universes" of 2^N-ion index sets, suggesting a nonlinear kind of "superposition" among indefinite numbers of such ...



ETs for S=15, N=5,6,7 (nested skyboxes in blue)· · · and "fractal limit."

... increasing *N* from 5 to 6 to 7 approaches the white-space complement of one of the simplest (and least efficient) plane-filling fractals, the Cesaro double sweep ...
for any strut constant greater than 8 and not a power of 2,
one generates indefinitely extensible ... ET's ...
in a never-ending ... sequence of nested skyboxes, the empty spaces of which approach a fractal ...
while a box-kite's edges are turned off by augmenting its S with a new leftmost bit ... performing a second such augmenting results in a box-kite which is once again "turned on."
We have ... a process of "hide/fill involution":
its repeated application produces spandrels from proper box-kites;
quartets of higher-N proper box-kites from each HBK in each such spandrel;
quartets of higher-N spandrels from each of these;
and so on, ad infinitum.
And, we also have a unique link between any proper 2^N-ion box-kite and "loading zones" we call cowbird's nests of 8-D, Octonion-copy spaces completely free of ZD's

(one per 2^(N+1)-ion spandrel).
These provide the basis for a sort of “storage space” or memory,
to be searched and accessed by ZD-navigating protocols.
...

the Fundamental Theorem of Zero-Divisor Algebra

...
Any integer K > 8 not a power of 2 can uniquely be associated
with a Strut Constant S of ZD ensembles, whose inner skybox
resides in the 2^N-ions with 2^(N−2) < K < 2^(N−1).
The bitstring representation of S completely determines
an infinite-dimensional analog of a standard plane-confined
fractal, with each of the latter’s points associated with an
empty cell in the infinite Emanation Table, with all non-empty
cells comprised wholly of mutually orthogonal primitive
zerodivisors, one line of same per cell
...
any full meta-fractal requires the use of an infinite G, which
sits atop an endless cascade of singleton leftmost bits,
determining for any given S an indefinite tower of ZDs ...
[Given] an associative triplet (a,b, c) ... written in CPO
(short for “cyclically positive order” ... three more such
associative triplets (henceforth, trips) can be generated by
adding G to two of the three, then switching their resultants’
places in the CPO scheme ... [This] allow[s] one to move up and
down towers of values and give[s] us a natural basis for
generating and tracking unique IDs with which to “tag” and
“unpack” data ... with “storage” provided free of charge by the
empty spaces of our meta-fractals
...
Rodrigo Obando's ongoing work ...[uses]... 168 4-variable
monotone functions, and the same number of complements, ... to
determin[e] the classification of [elementary Cellular Automata]
behaviors ... His basic tactic (and its clear “resonance” with
ZD setups) goes like this.
Split the bit representation of a rule into two “primitives.”
One string contains bits indicative of an initial input
combination that has the central cell's value = 0.
The other contains those telling what happens
if said cell value = 1.
For the (r=2) situation that clearly obtains with the Trip-Sync
linkup of Pathions to Sedenions,
the possible rules equal 2 raised to the (2^5 =) 32nd power,
which is more than 4 billion rules altogether ...
[some of]... which ... are

Class 1 (boringly homogenous outcomes) ...
Class 2 (evolving into simply separated periodic structures) ...
Class 3 ... yield[ing] up chaotic aperiodic patterns ...
Class 4 ... generat[ing] complex "Class 4" patterns ...

Obando's work strongly suggests ... that we can... predict what sort of behavior a rule will display,
and even say which will be Class 4.
First, partition a rule into "primitives," then see if their bit representations are Boolean monotones. Because the "primitives" have half the string each, this means we're dealing with the much more manageable count of 2^16 logical expressions,
or 65,536 possibilities, that each "half-rule" can display.
If the primitives are properly chosen from the two "monotonous" sets of 168 ... we'll get Class 4 complexity.
But meanwhile,
we have two sets of 168 ZD "primitives," on opposite sides of the Sedenion/Pathion "infobarrier" ...
each Box-Kite "edge current," if encoded as a string of 0's and 1's, can be uniquely specified by precisely 16 bits ...
any "successful" ZD product entails the involvement
of 2 Assessors, which for the Sedenion "target" (and isomorphically for Pathion "broadcaster") means 4 bits for each S index,3 for each o, and 2 extra bits "left over."
Suppose we say these latter determine the diagonals ... as specified at the two ends of the connecting edge ... we get two distinct strings for each "edge-current," depending upon which Assessor we write out first ...
if the first Assessor is "less than" the second ... then assume they both employ the "/ " diagonal.
If the first written down is the greater, however, assume " \ ".
Next,
use the first of our leftovers for a "switch bit" ...
if "0," do nothing; if "1," switch the diagonal used by the second in sequence ...
one last "extra" ...[is]... interpret[ed as]... turning the "edge-current" on or off
... while the "168" in the ZD universe are ... not Boolean monotone functions - their fundamental workings ... are defined by XORing, which is not an allowable operation in monotone string-building ... there ... is ... good reason to expect a rich harvest of transformations between the two mathematical languages. ...".

Rodrigo Obando in NKS 2003 said:
"... elementary cellular automata are the simplest of the spaces that produce interesting behavior. This rule space is one-dimensional with k = 2 and r = 1. Given these parameters there can be up to 256 elementary rules. ... Even though these rules are simple and deterministic, there has been no way to know the class of behavior from the rule itself until it is evolved. ...

p_0 ($b_5b_4b_1b_0$)

p_1 ($b_7b_6b_3b_2$)

r=1	c_0	0	1				2						3				4
c_1	p_1\p_0	0	4	8	2	1	9	6	3	5	12	10	14	11	7	13	15
0	15	204	220	236	206	205	237	222	207	221	252	238	254	239	223	253	255
1	13	196	212	228	198	197	229	214	199	213	244	230	246	231	215	245	247
	14	200	216	232	202	201	233	218	203	217	248	234	250	235	219	249	251
	11	140	156	172	142	141	173	158	143	157	188	174	190	175	159	189	191
	7	76	92	108	78	77	109	94	79	93	124	110	126	111	95	125	127
2	6	72	88	104	74	73	105	90	75	89	120	106	122	107	91	121	123
	9	132	148	164	134	133	165	150	135	149	180	166	182	167	151	181	183
	3	12	28	44	14	13	45	30	15	29	60	46	62	47	31	61	63
	5	68	84	100	70	69	101	86	71	85	116	102	118	103	87	117	119
	12	192	208	224	194	193	225	210	195	209	240	226	242	227	211	241	243
	10	136	152	168	138	137	169	154	139	153	184	170	186	171	155	185	187
3	8	128	144	160	130	129	161	146	131	145	176	162	178	163	147	177	179
	2	8	24	40	10	9	41	26	11	25	56	42	58	43	27	57	59
	1	4	20	36	6	5	37	22	7	21	52	38	54	39	23	53	55
	4	64	80	96	66	65	97	82	67	81	112	98	114	99	83	113	115
4	0	0	16	32	2	1	33	18	3	17	48	34	50	35	19	49	51

Wolfram Classes
1
2
3
4

Figure 4. Partition of Rule Space with Wolfram Classes identified.

... The problem arises when the neighborhood is expanded such as for r = 2 where the number of rules becomes 2^32 ...".

Zero-Divisor Annihilator Geometry

Daniel Biss, Daniel Christensen, Daniel Dugger, & Daniel Isaksen in Large Annihilators in Cayley-Dickson Algebras I (2008) and II said:
"... We study zero-divisors in ... Cayley-Dickson ... algebras.
... Although a complete description of zero-divisors seems to be out of reach, we can describe precisely the elements whose annihilators have dimension 2^n - 4n + 4 ...
Each algebra An is constructed from the previous one A(n-1) by a doubling procedure ... for all n ≥ 4, the algebra An admits non-trivial zero-divisors ... as n grows the locus of zero-divisors in An becomes more complicated ... If x belongs to An,
then the annihilator of x is Ann x = { y in An | xy = 0 }
...
A theorem of Moreno says that the (real) dimension of Ann x is always a multiple of 4 ...
If x belongs to An, them dim(Ann x) ≤ 2^n - 4n + 4
... if d is any multiple of 4 such that 0 ≤ d ≤ 2^n - 4n + 4 ,
then there exist elements of An [with dim(Ann x) = d

for x in An dim(Ann x) ≤ 2^n - 4n + 4 (and is multiple of 4)

A4	dim(Ann in A4) = 0, 4
A5	dim(Ann in A5) = 0, 4, 8, 12, 16
A6	dim(Ann in A6) = 0, 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44
A7	dim(Ann in A7) = 0, 4, ... , 104
A8	dim(Ann in A8) = 0, 4, ... , 228

]... when n is large, one has zero-divisors whose annihilator is 'almost' the whole algebra ...
The space of zero-divisors in An is closed under scalar multiplication, and so it forms a cone in the real vector space underlying An. It is therefore natural to focus on the norm 1 elements and look at the space

ZD(An) = { x in An | ||x|| = 1, Ann x =/= 0 }

... to understand the topological properties of this space ...
look at the subspaces

ZD_k(An) = { x in An | ||x|| = 1, dim(Ann x) =/= k }

These strata are ... complicated, and unknown even in ... A5.
...
When n ≥ 4 the space ZD_(2^n-4n+4)(An) is homeomorphic to a disjoint union of 2^(n-4) copies of the Stiefel variety V2(R7)
... the space of ordered pairs of orthonormal vectors in R7.
...

Every Cayley-Dickson algebra An contains
a distinguished element in ,
and the 2-dimensional subspace { 1 , in } is a subalgebra
isomorphic to [the complex numbers] C that we denote by Cn
... multiplication in An behaves well with respect to Cn ...
For every a in An that is orthogonal to Cn ...
(a , +/- in a) is a zero-divisor in A(n+1) ... and ...
it has dimension 2^n - 4 + dim(Ann a)
...
Let ≥ 2. An element of An is a top-dimensional zero-divisor
if its annihilator has dimension 2^n - 4n + 4 ...
Let Tn be the space of top-dimensional zero-divisors in An
that have norm 1 ... Tn ... is ... ZD_(2^n-4n+4)(An) ...
T2 and T3 are homeomorphic to S3 and S7 [3-sphere and 7-sphere
...

```
                 Top-Dimensional Zero-Divisors Tn

       ZD_2^n-4n+4(An) = 2^(n-4) copies of V2(R7) = Tn

              ZD_4(A4) =          V2(R7) =  11-dim
              ZD_16(A5) =     2 x V2(R7) =  22-dim
              ZD_44(A6) =     4 x V2(R7) =  44-dim
              ZD_104(A7) =    8 x V2(R7) =  88-dim
              ZD_228(A8) =   16 x V2(R7) = 176-dim

ZD_4(A4) = V2(R7) = space of ordered pairs of orthonormal vectors in R7
                  = 11-dim Stiefel Manifold O(7)/O(7-2) = O(7)/O(5)
                  = Spin(7)/Spin(5) = G2xS7 / S3xS3xS4 = G2/ S3 = SU(3)xS3

ZD = space of all pairs of unit vectors whose product is zero.

ZD(A4) = G2 = 14-dim automorphism group of Octonions
            = Spin(7)/ S7 = SU(3) x S3 x S3
            = S3 x V2(R7)

G2 = SU(3) x S6 = SU(3) x S3 X S3 = V2(R7) x S3

LieSphere7 = Spin(7)/Spin(5)xSpin(2) = G2xS7 / S4xS3xS3xS1 = G2/S3xS1 =
           = SU(3)xS3xS3 / S3xS1 = SU(3)xS2

LieSphere6 = Spin(6)/Spin(4)xSpin(2) = SU(4)/S3xS3xS1
           = SU(3)xS1xCP3 / S3xS3xS1   because CP3 = SU(4)/SU(3)xS1
           = SU(3) x CP3/S3xS3 = SU(3) x CP3/S6

G2 = SU(3) x S6 = LieSphere6 x  CP3

( In the parts of the above that are my comments,
  x may denote Cartesian, Tensor, or Fibre Bundle product. ) ]...".
```