

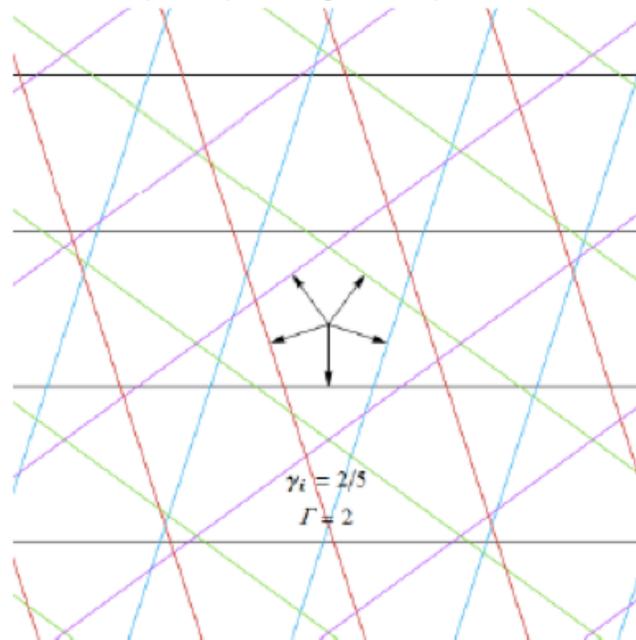
Penrose Tiling and Hydrogen Spectrum

Frank Dodd (Tony) Smith, Jr. - 2016

Abstract

This paper is my attempt at understanding Klee Irwin's idea that Penrose Tiling can encode the Hydrogen Spectrum.

Klee Irwin (quantumgravityresearch.org) has been discussing with me his idea that Penrose Tiling can encode the Hydrogen Spectrum, such as the Lyman series. At first I was very skeptical but now I think that Klee Irwin's idea is correct. Alan H. Schoen's Infinite Tillings web page says: "... thin and thick rhombs (the so-called "golden" rhombs) serve as prototiles for ... Roger Penrose's quasiperiodic tilings ... Nicolas G. de Bruijn described an algebraic theory of Penrose tilings. He introduced the concept of pentagrids ... five superimposed grids of parallel lines unit distance apart ...



... For any pair of intersecting grid lines, the smaller angle of intersection is either 36° or 72° . The sides of a rhomb dual to a vertex are orthogonal to the two grid lines that intersect at the vertex ... the tiling by rhombs that is dual to a given pentagrid is a Penrose tiling ... the Penrose tiling... [for the pentagrid shown above]... is called STAR ...

The ... STAR Penrose tiling... arise[s] from a small central core via a recursive sequence of (a) radially outward reflections in the enclosing necklaces (pentagonal rings of Conway mirror worms) of successively larger central regions, followed by (b) lateral reflections of the images produced in (a) to fill the empty triangular gaps left by (a). ...

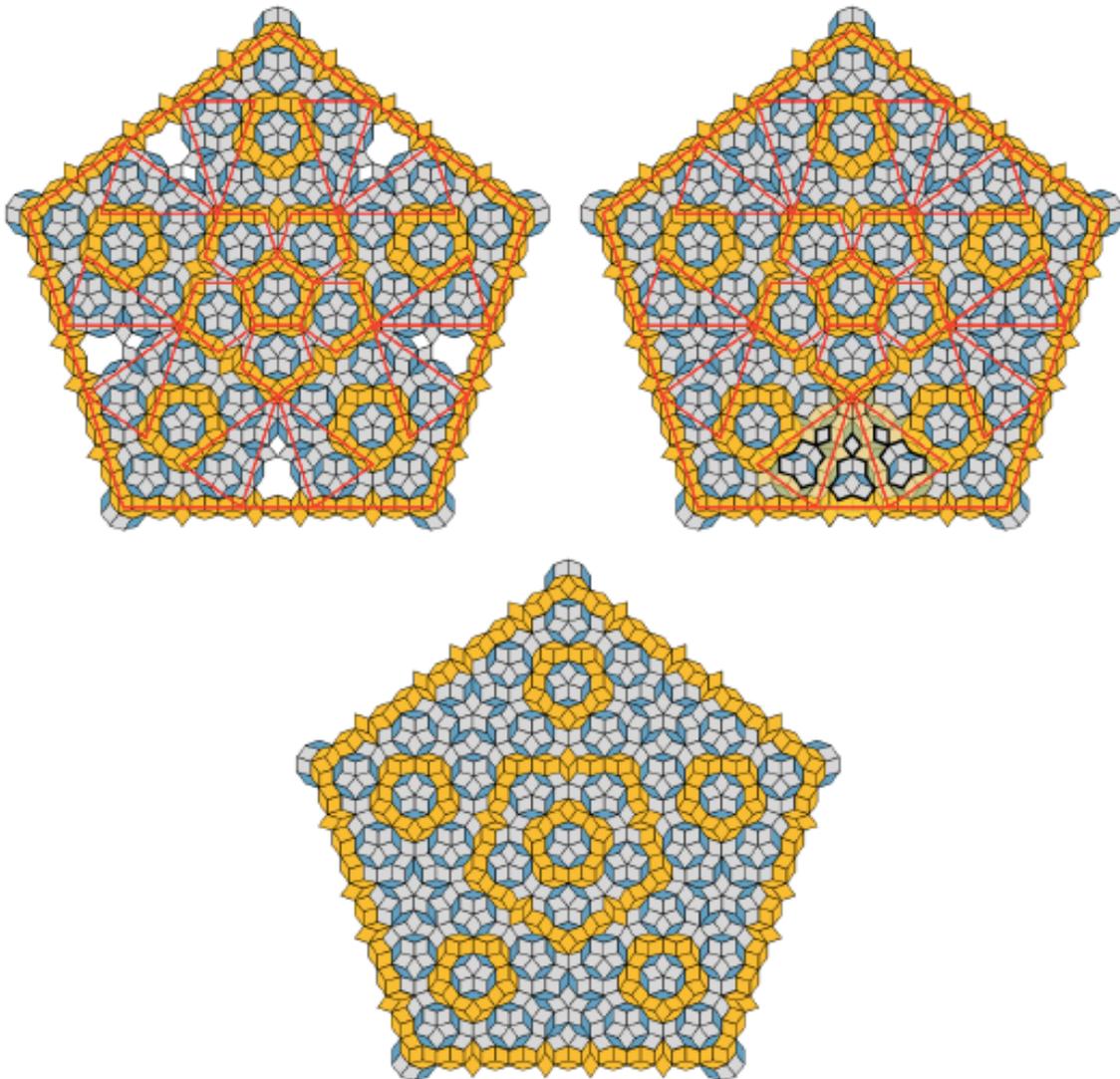


Fig. C1.4
Penrose STAR
three stages
d5

... At each stage of recursion, the area in the interior of the pentagonal boundary mirror of the ... STAR increases by the factor

$$f5^2 = (1 + 2 \cos \pi/5)^2 \approx 6.85.$$

Since $2 \cos \pi/5 = (1+\sqrt{5})/2 = \phi$, the golden ratio (≈ 1.618),
the total tiled area in successive steps is proportional to

$$\begin{aligned} &1 + 0\phi \\ &2 + 3\phi \\ &13 + 21\phi \\ &89 + 144\phi \\ &\dots \end{aligned}$$

Let $S1 = \{1, 2, 13, 89, \dots\}$ and $S2 = \{0, 3, 21, 144, \dots\}$.

Since the Fibonacci sequence is $\{.., 1, 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, \dots\}$,
where

$$\begin{aligned} F1 &= 1, \\ F2 &= 0, \\ F3 &= 1, \\ F4 &= 1, \\ F5 &= 2, \\ F6 &= 3, \\ &\text{etc.}, \end{aligned}$$

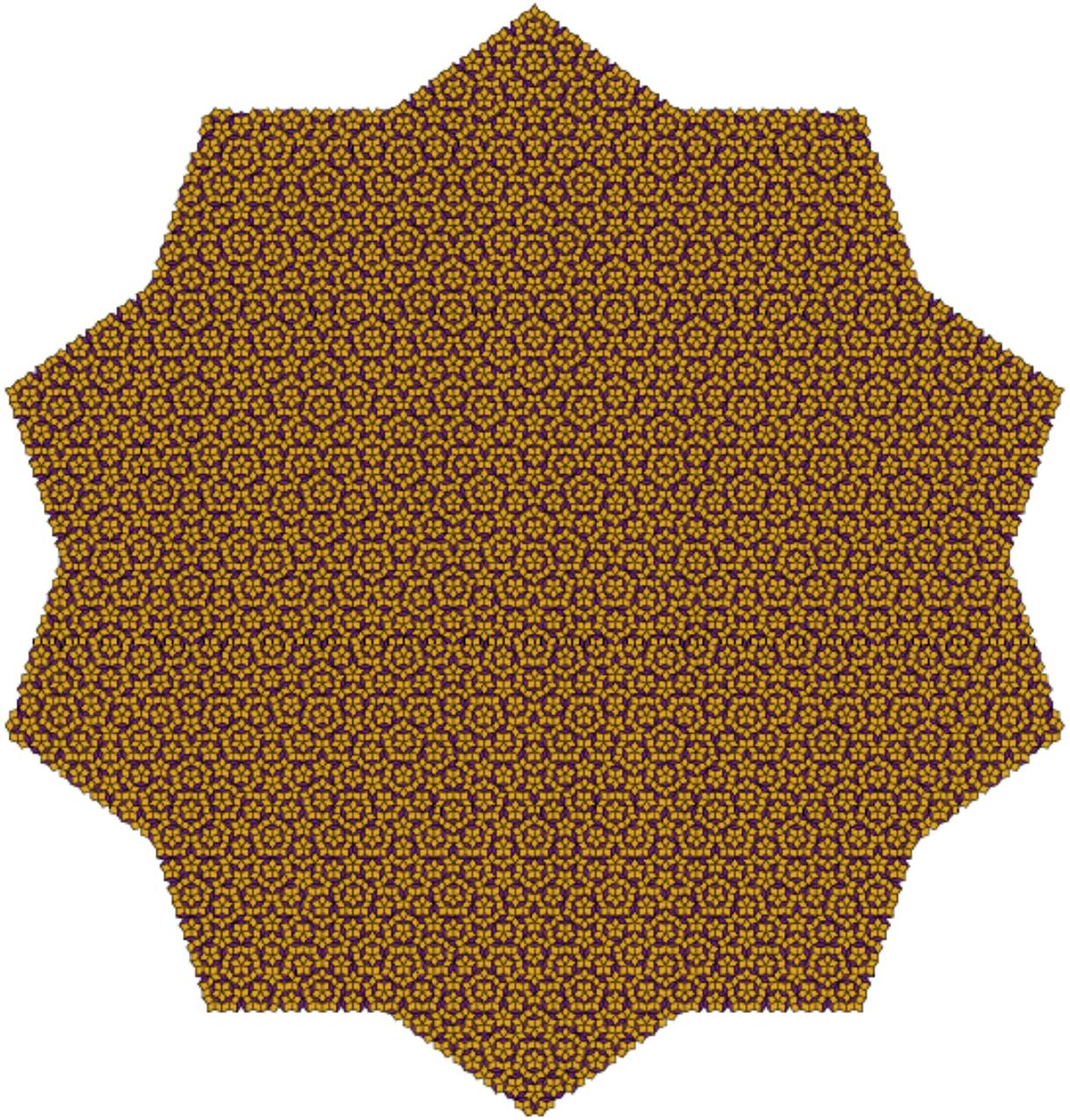
then

$$\begin{aligned} S1 &= \{F1, F5, F9, F13, \dots\} \text{ and} \\ S2 &= \{F2, F6, F10, F14, \dots\}. \end{aligned}$$

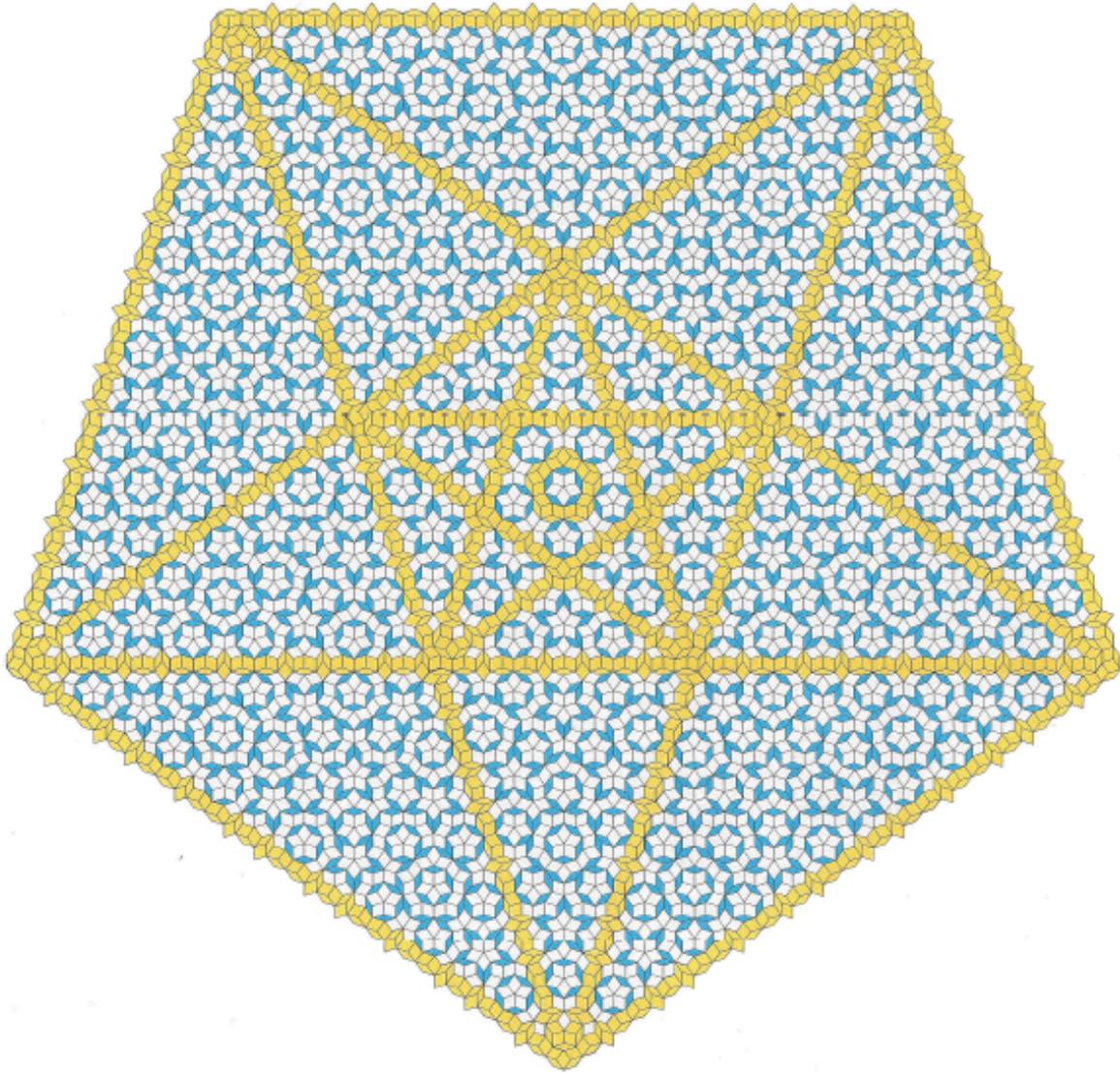
... [Here are]

two differently shaded images of ... the d5-symmetric Penrose tiling ... STAR.

...



In the second image ... concentric pentagonal rings of Conway mirror-worms are distinguished by color ...

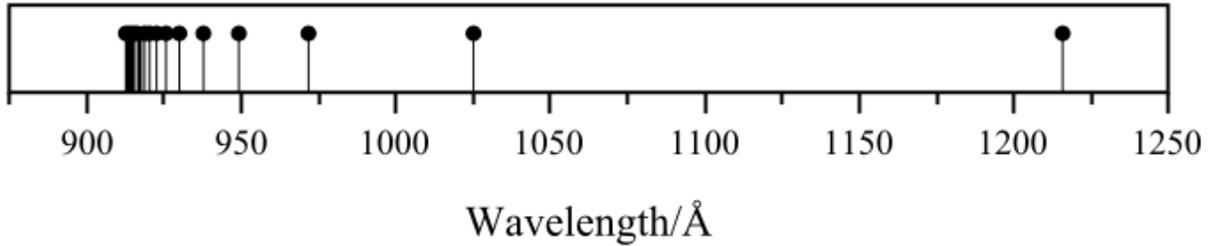


... ”

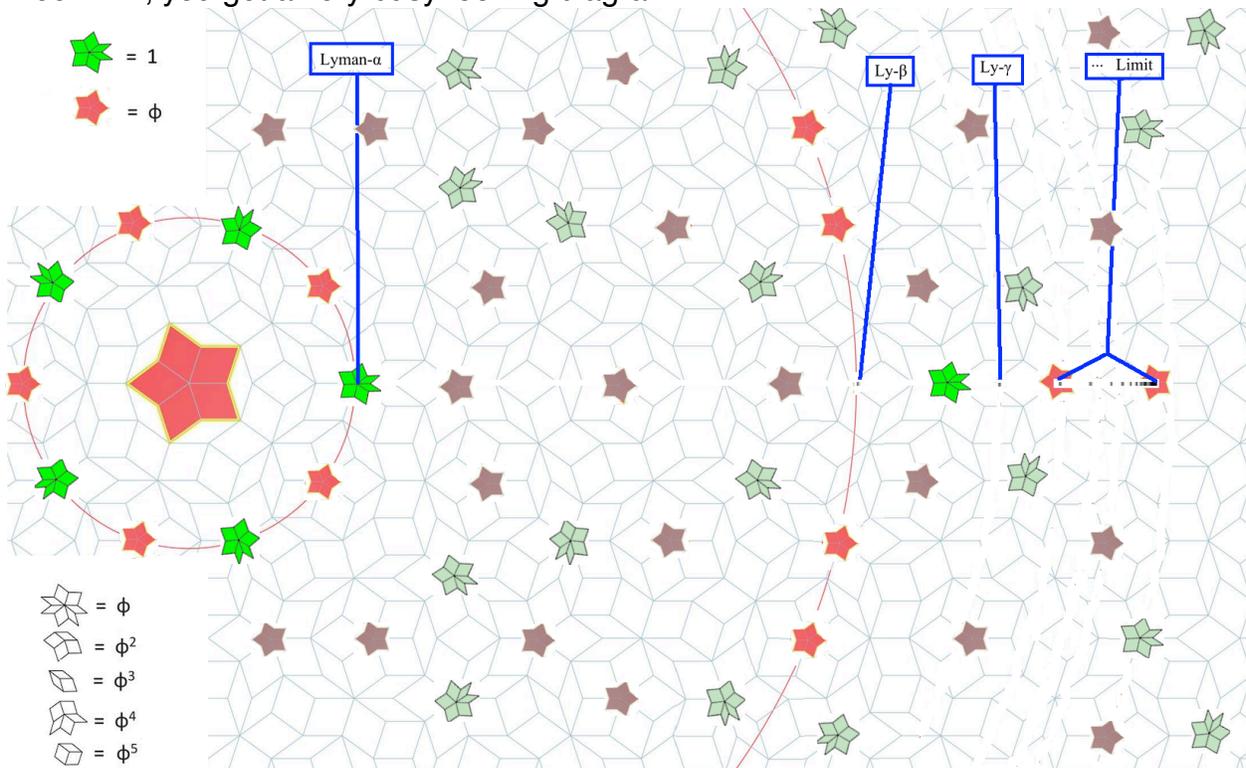
Klee Irwin (quantumgravityresearch.org) uses the STAR Penrose Tiling to describe his idea that Penrose Tiling can encode the Hydrogen Spectrum.

Wikipedia shows the Lyman series as

Limit	...	Ly- γ	Ly- β	Lyman- α
912 Å		972 Å	1026 Å	1216 Å

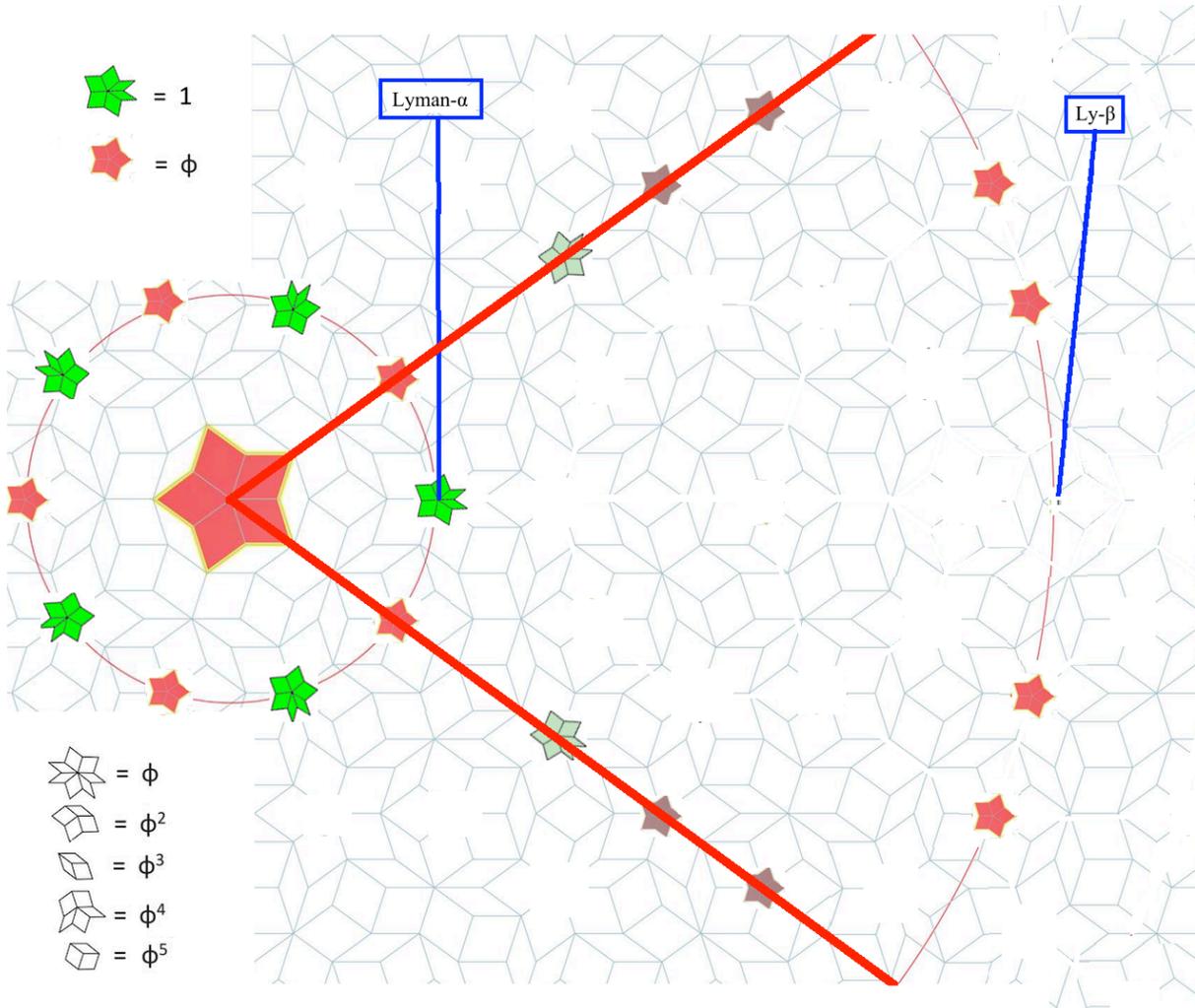


When you graphically superimpose the Lyman points on the Penrose Tiling used by Klee Irwin, you get a very busy-looking diagram.



The diagram has 5 similar pie-slice segments, so study of one segment is sufficient.

Lyman-alpha and Lyman-beta lie on two circles concentric around the central star:

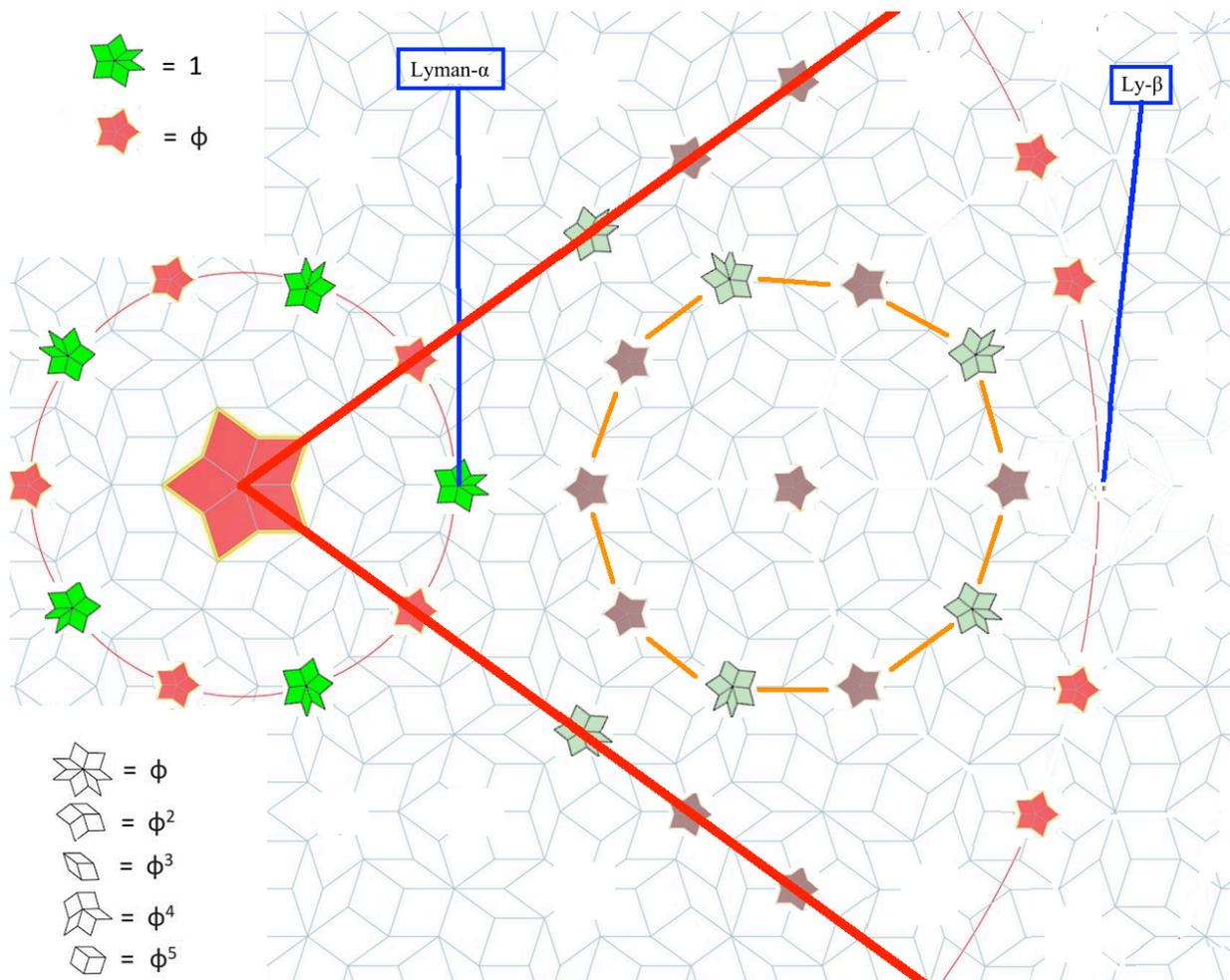


The Lyman-alpha inner circle is obviously special, the minimal decagon of 5-element stars (5 of them) and 6-element things (5 of them). Two adjacent 5-element stars mark the boundary of each of the 5 pie-slice segments, one of which (right-side horizontal) is shown in detail.

At this initial stage it is not clear why the Lyman-beta circle is more special than other circles through vertices of the Penrose Tiling.

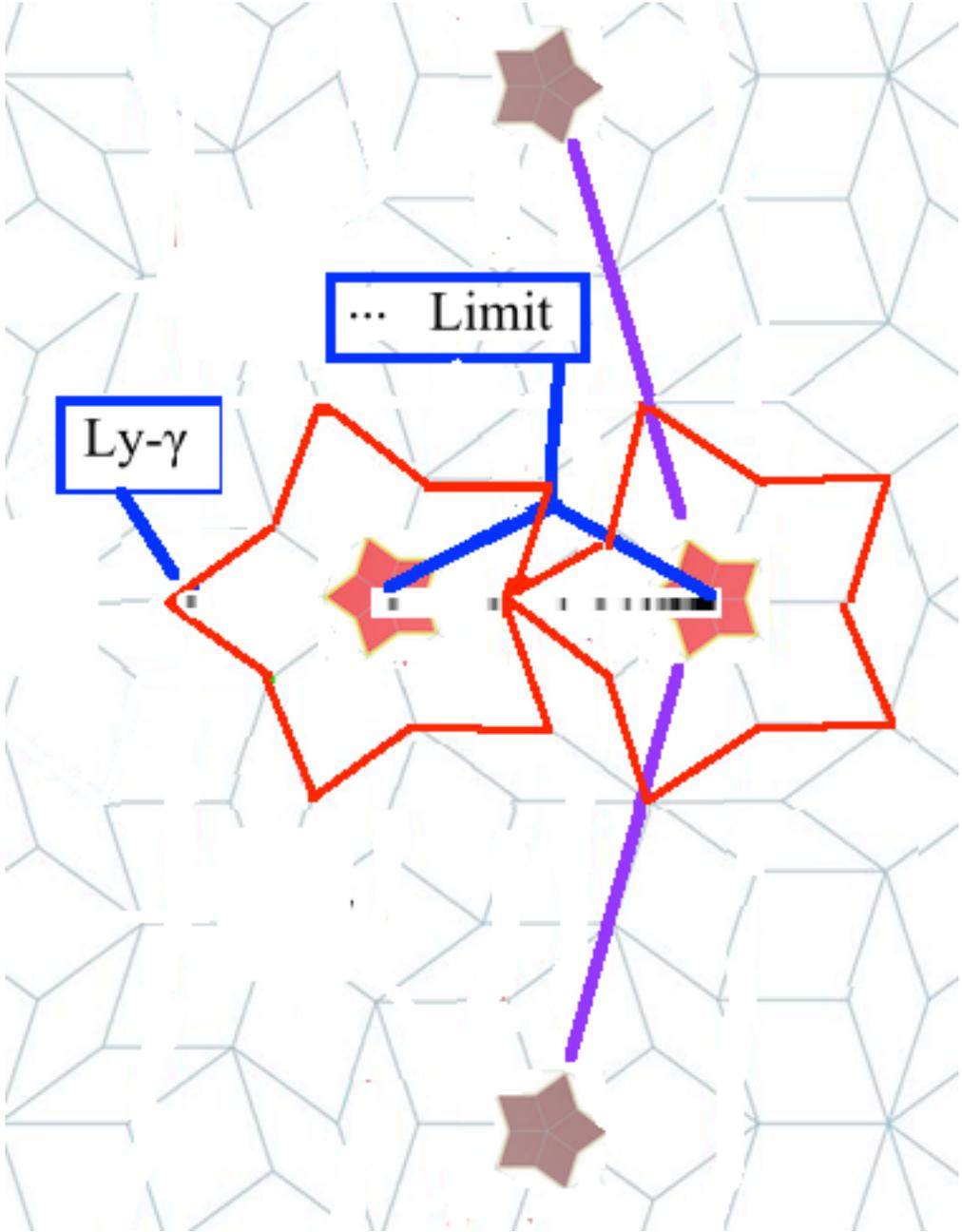
However (see page 7) further analysis of the Penrose Tiling pattern will show

the existence of a decagon between the Lyman-alpha circle and the Lyman-beta circle

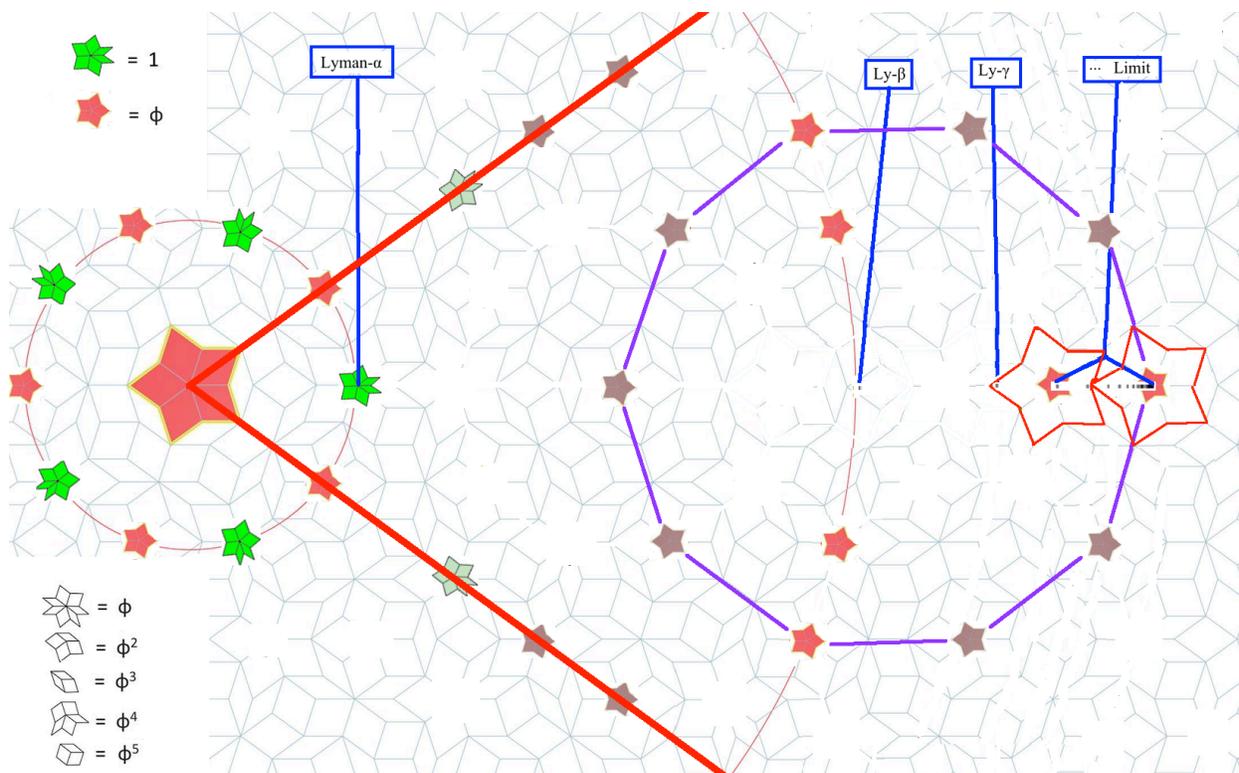


such that the Lyman-beta circle is the minimal circle through stars that lies beyond the intermediate decagon.

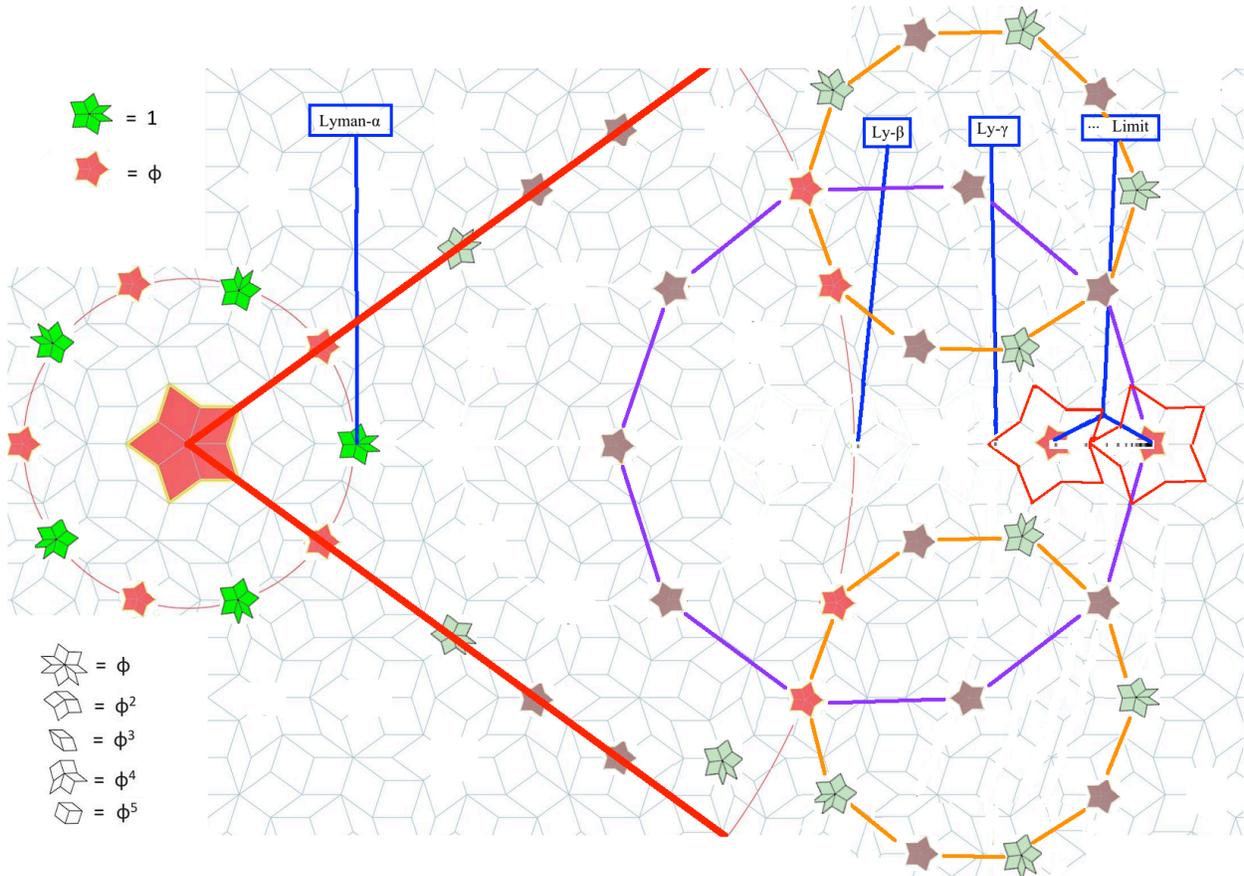
Lyman-gamma to Lyman-Limit lie inside two outer stars:



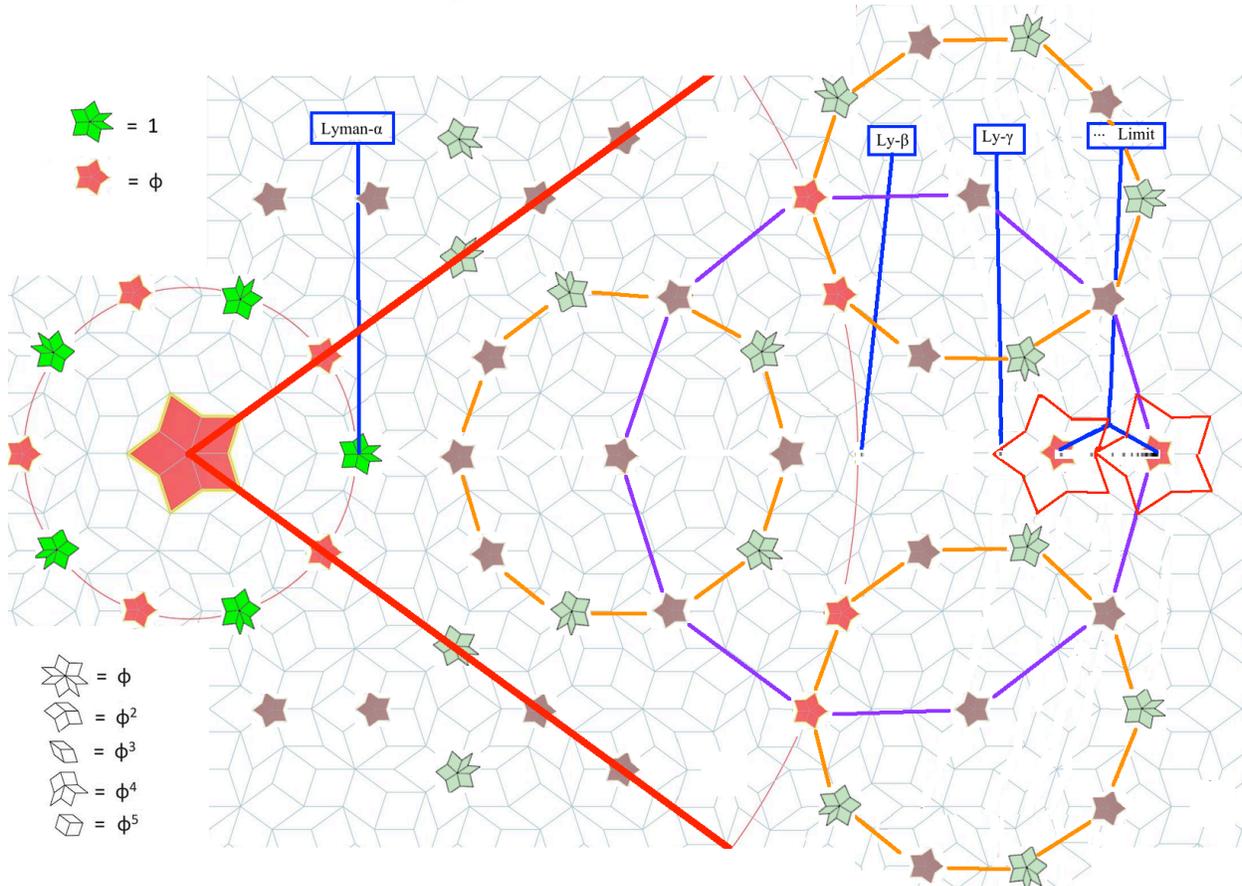
The outer Lyman-Limit star lies on a decagon sharing two stars with the Lyman-beta circle:



Four stars of the Lyman-Limit decagon and two more stars of the Lyman-beta circle are shared with two small decagons (including their center points) that are above and below the two Lyman-gamma to Lyman-Limit stars:

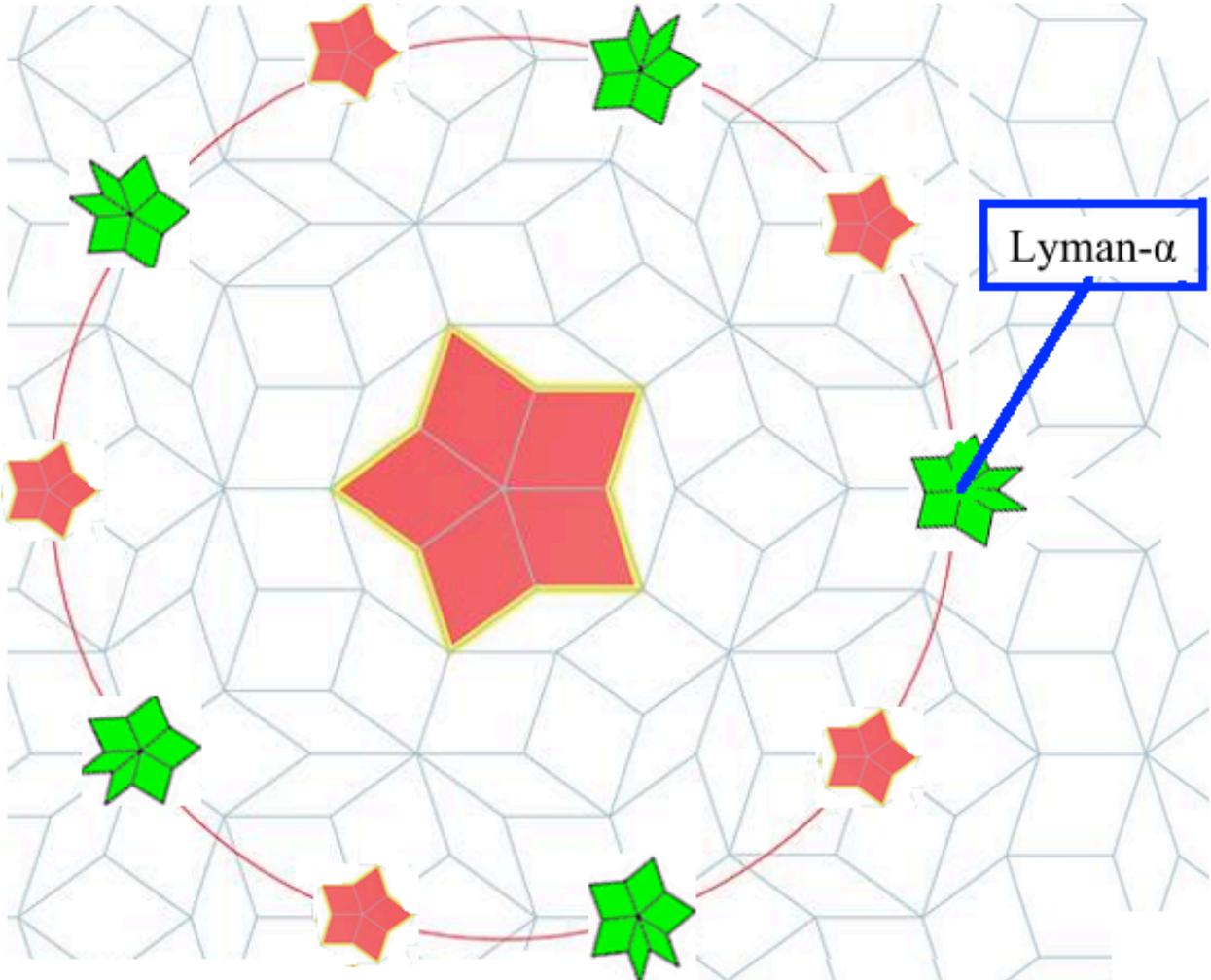


Two stars of the Lyman-Limit decagon are shared with a third small decagon lying between the Lyman-alpha and Lyman-beta circles. The Lyman-Limit decagon also has 3 stars located at the centers of the three small decagons so the Lyman-Limit decagon, which contains the Lyman-Limit star, is linked to the three small decagons.



James Gleick, in his book “Chaos: Making a New Science”, said:
 “... James Yorke ... in his “Period Three Implies Chaos” ... 1975 ... paper. ... proved that in any one-dimensional system, if a regular cycle of period three ever appears, then the same system will also display regular cycles of every other length, as well as completely chaotic cycles. ...
 Even with the simplest equation, the region of chaos in a bifurcation diagram proves to have an intricate structure ... First, the bifurcations produce periods of 2, 4, 8, 16.... Then chaos begins, with no regular periods. But then, as the system is driven harder, windows appear with odd periods. A stable period 3 appears, and then the period-doubling begins again 6, 12, 24.... The structure is infinitely deep. When portions are magnified, they turn out to resemble the whole diagram. ...”.

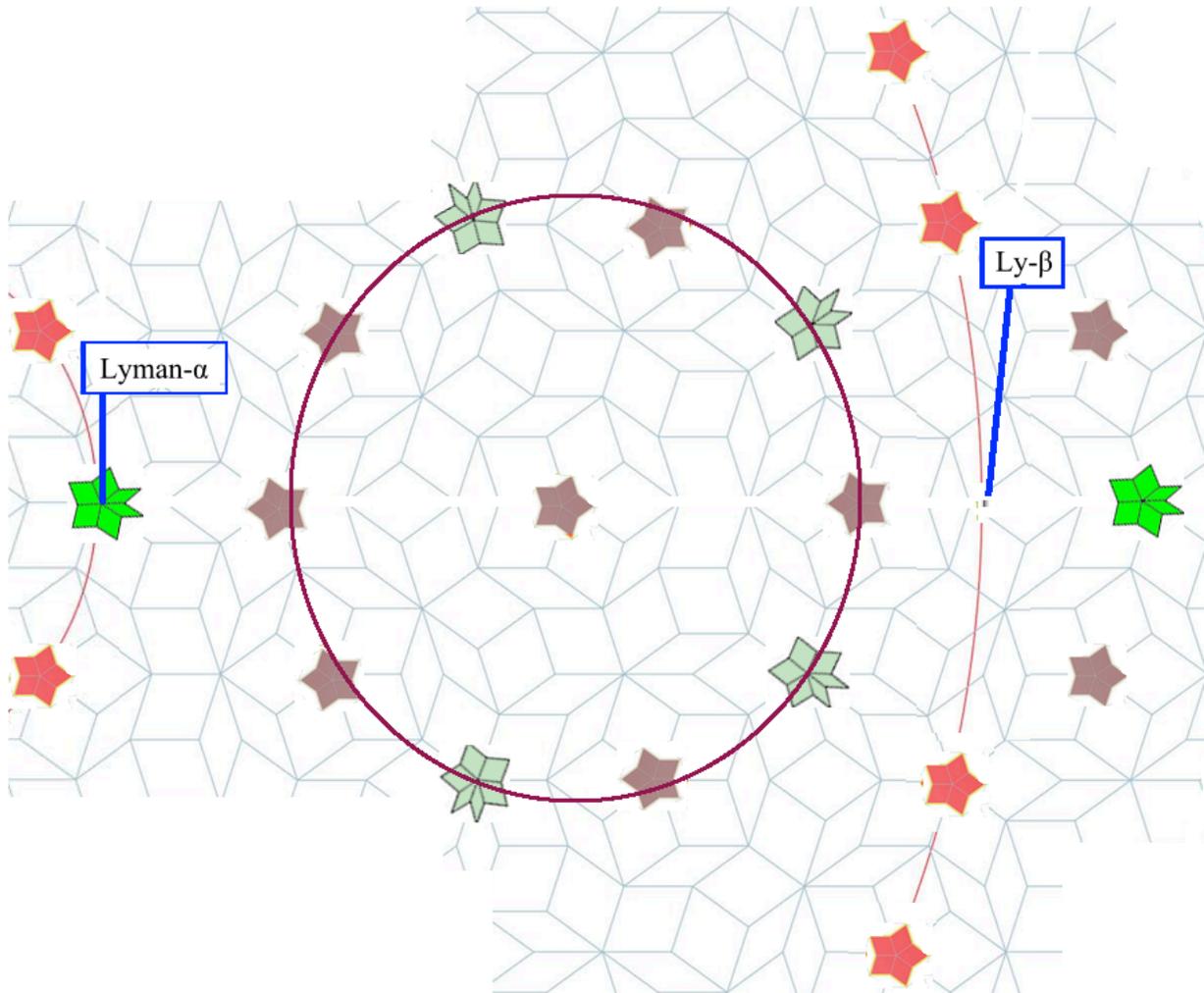
Lyman-alpha



This is a closed circle of 5 stars surrounding a central star with Lyman point on the circle between two stars.

It is simple and symmetrical and represents the ground state.

Lyman-beta

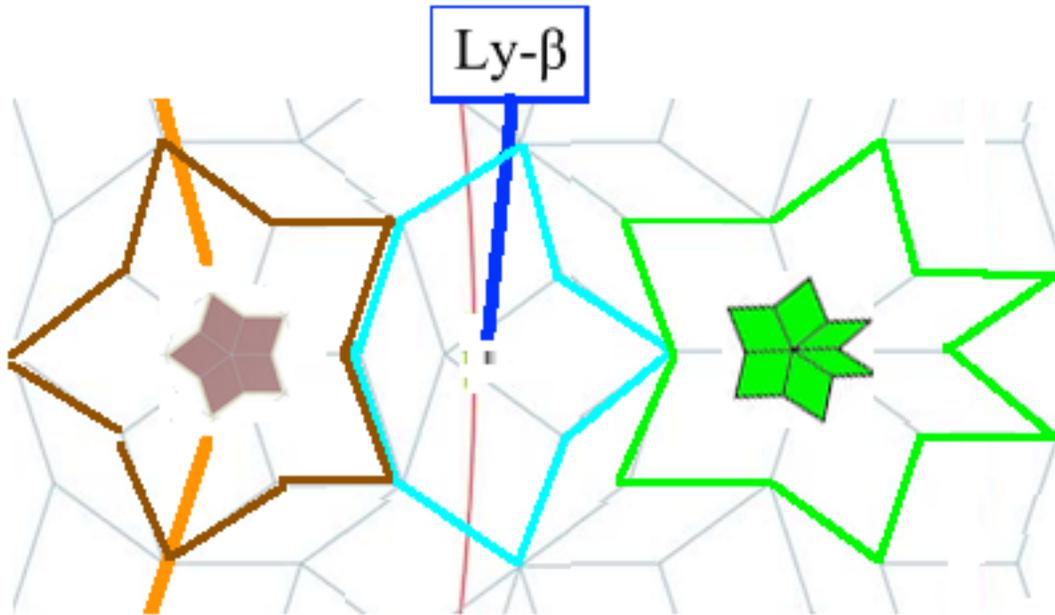


This is a loop containing 6 stars around a central star that is on the radial line from the original configuration center through the first Lyman point.

Its Lyman point is on the radial line but is outside the loop, lying on a circle concentric with the original configuration center and between 2 sets of stars lying on that circle.

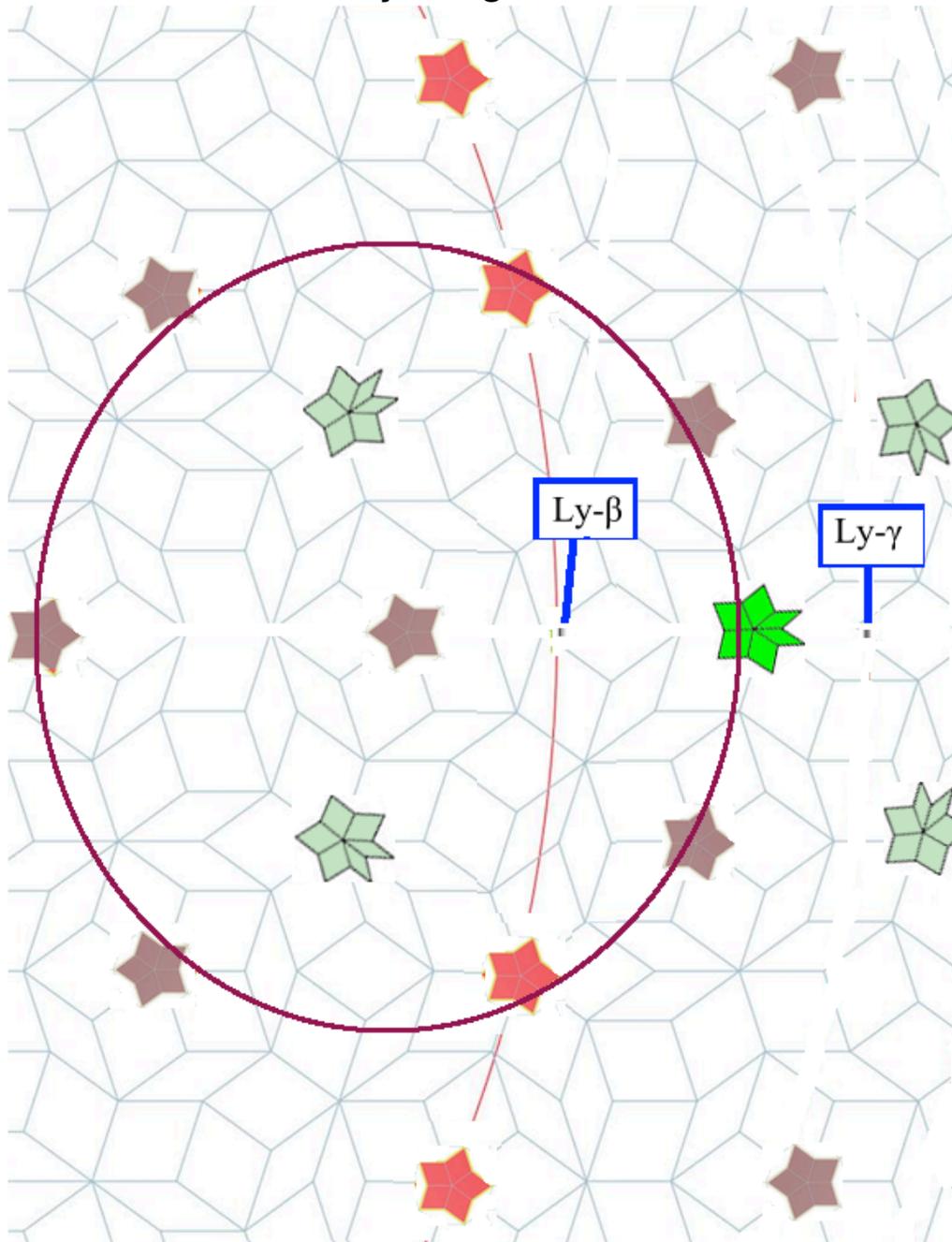
The loop is asymmetric, with the outer 3 stars more widely separated than the inner 3. It represents an excited state beyond the ground state.

Here is a more detailed diagram of the location of the Lyman-beta point:



The LYman-beta point is near the center of the cyan 4-element configuration that connects the outermost brown 5-element star of the first small decagon loop with the green 6-element configuration that is between the second and third small decagon loops.

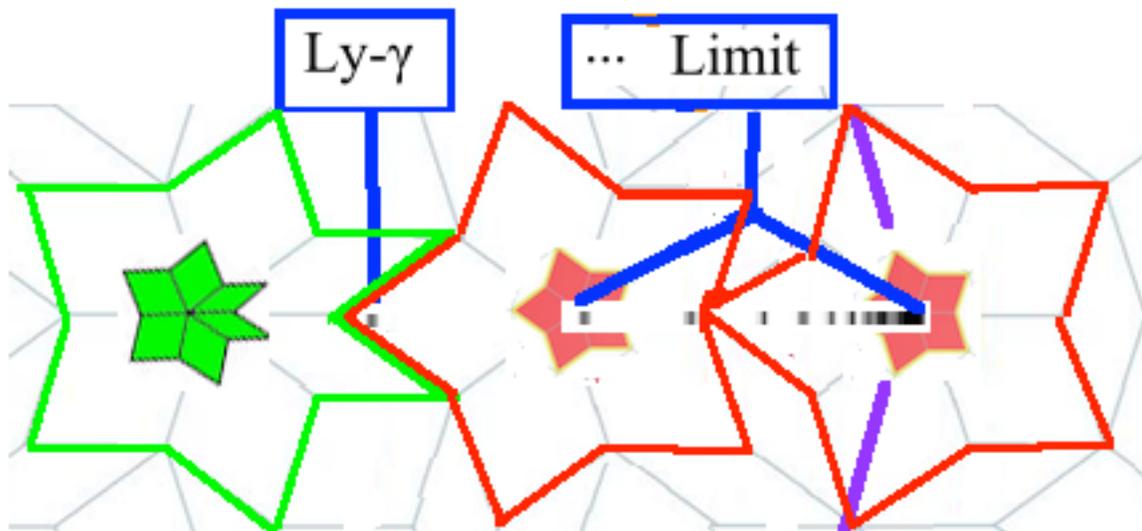
Lyman-gamma



This is a loop containing 7 stars around a central star that is on the radial line from the original configuration center through the first Lyman point. The loop is asymmetric, with the outer part having 4 stars and the inner part 3. It represents an excited state beyond the ground state.

Its Lyman-gamma point is on the radial line but is outside the loop.

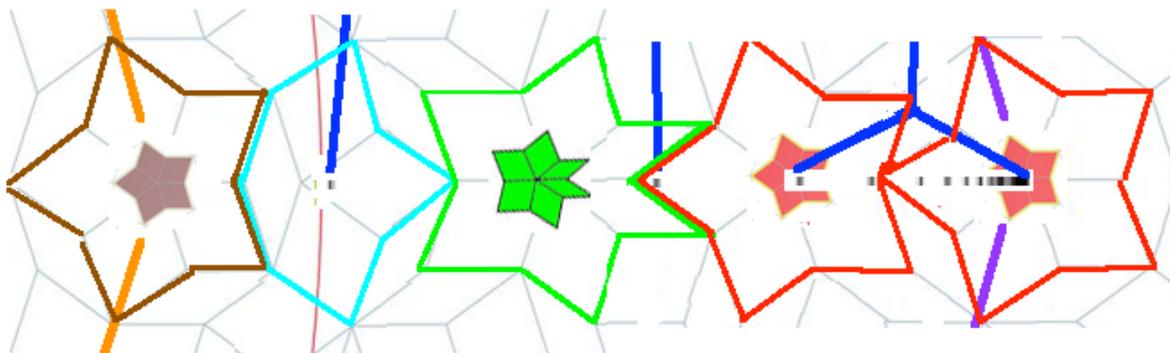
Here is a more detailed diagram of the location of the Lyman-gamma point:



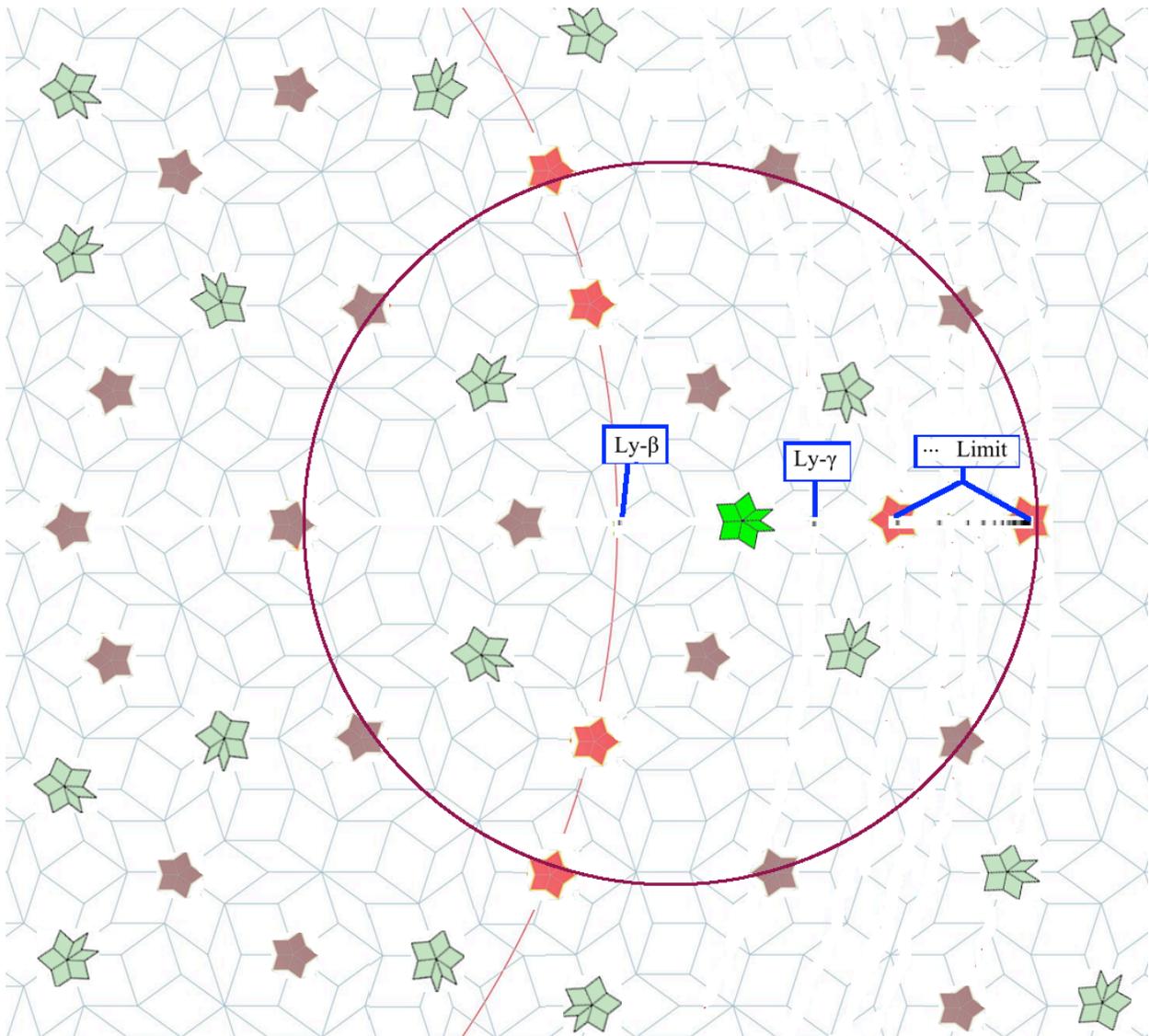
The Lyman-gamma point is near the innermost point of the red 2-star configuration that includes the outermost red 5-element star of the large decagon loop which point is near the adjoining green 6-element configuration.

Lyman-Limit

The overall configuration of the Lyman-beta, Lyman-gamma, and Lyman-Limit points is



Here is a wider view, showing them with respect to the large decagon loop:

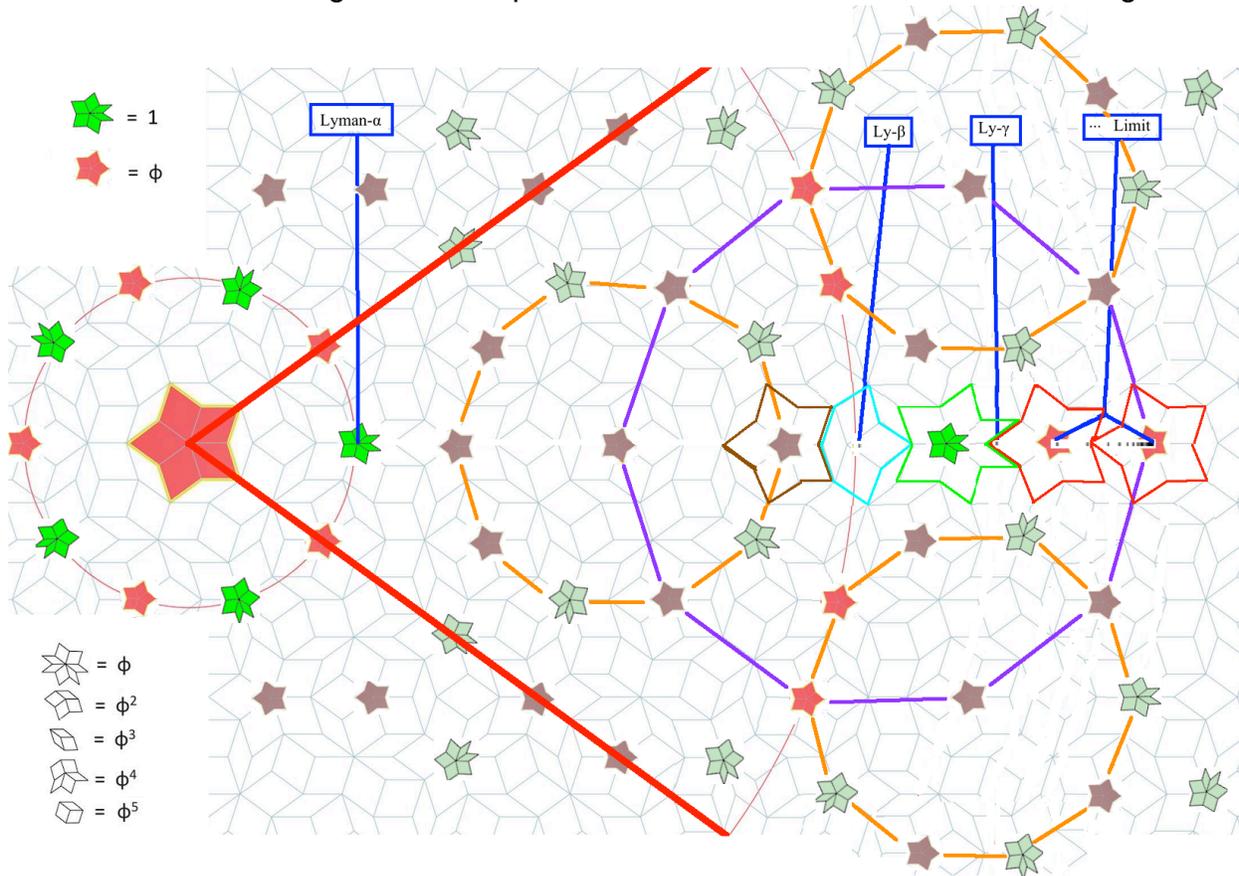


This is a loop containing 10 stars around the second Lyman point.

The Lyman Limit point is at the 10th star on the loop.

The loop is a symmetric circle with respect to the 10 stars but is not symmetric around the second Lyman point because the Lyman Limit point star on the circle is linked to a star inside the loop by all the Lyman points between Lyman-gamma and the Limit. This asymmetric linking represents relatively high energy levels.

The Lyman Limit loop is a large decagon that is linked with the Lyman-beta circle and with 3 other small decagons made up of 6 5-element stars and 4 6-element things each:



As of now, I have not done any numerical calculations of energy levels, and am only in the initial stages of trying to relate tiling asymmetries etc with energies of energy levels.