Frank Dodd (Tony) Smith, Jr. - March 2012:

Neutrino Mixing

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry space,
lying entirely therein. According to the D4-D5-E6-E7-E8 VoDou
Physics Model the mass of nu_3 is zero at tree-level
but it picks up a first-order correction propagating
entirely through internal symmetry space by
merging with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at both beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that
M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
= 7.35 / 137 = 5.4 x 10^(-2) eV.
Note that the neutrino-plus-electron loop can be anchored
by weak force action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.
According to the D4-D5-E6-E7-E8 VoDou Physics Model the mass

of nu_2 is zero at tree-level
but it picks up a first-order correction at only one (but not both)
of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so
the first-order corrected mass of nu_2 is

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M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
       = 9 x 10^(-3)eV.
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The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski spacetime.
thus having only one anchoring to CP2 interna symmetry space.
According to E8 Physics the mass of nu_1 is zero at tree-level
but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3 first-order corrections
or the 6 different possible anchorings for nu_2 first-order corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so
the first-order corrected mass of nu_1 is

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M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
       = 1.5 x 10^(-3)eV.
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Therefore:

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the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
                                     = ( 2916 - 81 ) x 10^(-6) eV^2 =
                                     = 2.8 x 10^(-3) eV^2
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and

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the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
                                     = ( 81 - 2 ) x 10^(-6) eV^2 =
                                     = 7.9 x 10^(-5) eV^2
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The 3 × 3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

$$
U = \begin{pmatrix}
c12\ c13 & s12\ c13 & s13\ e{-}id \\
-\ s12\ c23 - c12\ s23\ s13\ eid & c12\ c23 - s12\ s23\ s13\ eid & s23\ c13 \\
s12\ s23 - c12\ c23\ s13\ eid & -\ c12\ s23 - s12\ c23\ s13\ eid & c23\ c13
\end{pmatrix}
$$

where cij = cos(theta_ij) , sij = sin(theta_ij)

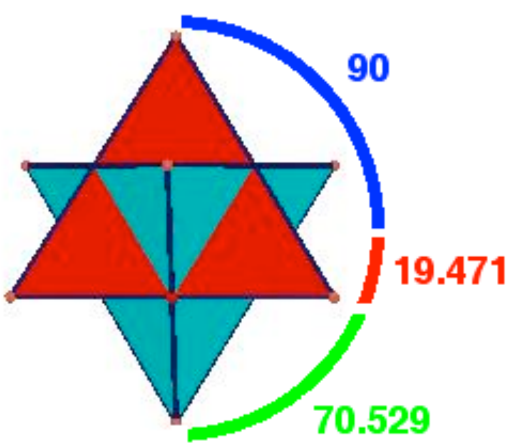
The angles are

theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees
The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson

	nu_1	nu_2	nu_3
nu_e	0.85	0.53	0
nu_m	-0.37	0.60	0.71
nu_t	0.37	-0.60	0.71

if the matrix is modified by taking into account
the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.