Force Strengths, CP2, and Kaluza-Klein

My D4-D5-E6-E7-E8 VoDou Physics model seems to be generally consistent with the ideas on this web page:

- At the Planck scale critical temperature there is a transition from 8 large dimensions and 4 large dimensions with 4 small compactified CP2 dimensions;
- The spinor fermion particles and antiparticles are not put in by hand, but are inherited from the high-energy 8-large-dimension structure so that
  - at current experimental energy levels there are 3 generations and
  - an inherited subtle supersymmetry gives ultraviolet finiteness and, along with Segal Conformal Gravity in 4-dimensional physical spacetime, gives a realistic picture of the dark energy cosmological "constant" and its relationship to dark matter and ordinary matter.

A paper of Carlos Castro led me to consider comparing my D4-D5-E6-E7-E8 VoDou Physics model with Kaluza-Klein models. Although I have not worked out the torsion details for the Batakis CP2 model, I am optimistic that it can be worked out in detail and that the Batakis CP2 model will turn out to be substantially equivalent to my D4-D5-E6-E7-E8 VoDou Physics model .

In March 2005, **Carlos Castro** has written a paper

ON GEOMETRIC PROBABILITY, HOLOGRAPHY, SHILOV BOUNDARIES AND THE FOUR PHYSICAL COUPLING CONSTANTS OF NATURE

which paper deals with geometric techniques that have been used in calculations of particle masses and force strength constants:

- by Armand Wyler in C. R. Acad. Sci. Paris A 269, (1969) 743. C. R. Acad. Sci. Paris A 272 (1971) 186;
- by G. Gonzalez-Martin in a book entitled Physical Geometry University of Simon Bolivar, publishers, Caracas, June 2000, and a paper in physics/0009052 entitled The proton/electron Geometric Mass Ratio", and in a paper in physics/0009051 entitled "The fine structure constant from Relativistic Groups";
- by Walter Smilga in a paper hep-th/0304137 entitled Higher order terms in the contraction of SO (3,2), and

- by me

as well as

- by Christian Beck in Spatio-Temporal Vacuum fluctuations of Quantized Fields (World Scientific, Singapore 2002);
- the work of Pierre Noyes in his book Bit-Strings Physics : a Discrete and Finite Approach to Natural Philosophy , Series in Knots in Physics, vol. 27 (Singapore, World Scientific, 2001) using Mersenne Primes and
- the work of Matti Pitkanen using Mersenne Primes and p-adic numbers.

My understanding of some history of the work of Armand Wyler is:

- **1970s -** Wyler calculates value of 1 / 137.03608 and the physics community (see Physics Today articles) decides it hates him and his work;
- **1986-87 -** fine structure constant experimentally = 1 / 137.0360 (+/- 1 in last digit) listed in 1987 (3rd) edition of Introduction to High Energy Physics by Perkins in good agreement with Wyler's calculation, but there was an Adjustment of the Fundamental Physical Constants in Rev. Mod. Phys. 59 (1987) 1121 which resulted in a new (as of 1986-87) value of the fine structure constant as measured by experiment of 1 / 137.0359895 (+/- 61 in last two digits) so that the 1986-87 Adjustment moved away from Wyler's value;
- **1989-90 -** David Gross (armored with the 1986-87 Adjustment) renews in Physics Today attacks on Wyler's work;
- **1998 -** a new Adjustment (called the 1998 recommended values) is made, giving value of the fine structure constant as measured by experiment of 1 / 137.03599976 (+/- 50 in last two digits)
- **2001 -** March 2001 issue of Physics Today has an article by Mohr and Taylor entitled Adjusting the Values of the Fundamental Constants (pages 29-34), which article says in part: "... the numbers form a tightly linked set - very few of the values are independent of the others. ... For example ... Now the electron charge is determined indirectly from other constants. ... The basic approach to finding a self-consistent set of values of these constants is to identify the critical experiments, determine the theoretical expressions as functions of the fundamental constants that make predictions for the measured quantities, and adjust the values of the constants to achieve the best match between theory and experiment. ...".

The values of the fine structure constant etc that you see on the web site of the Particle Data Group are only ONE POSSIBLE "self-consistent set of values" which set was obtained by "adjust[ing] ... to achieve the best match". It is in my opinion not only possible, but likely, that the 1986-87 Adjustments were adjusted to achieve a mismatch between the value of the fine structure constant and Wyler's hated-by-the-physics-community calculations and that there very likely exists even at the present time another "self-consistent set of values" containing a fine structure constant value that is in good agreement with Wyler's calculations. I note that individuals (including me) do not have sufficient time and resources to construct in detail such a "self-consistent set of values", and it is obvious that the physics establishment would be quite hostile to expending money, time, and effort in a project to study the total landscape of possible "self-consistent set[s] of values". My point here is that you should in my opinion not take too seriously any disagreement between the current consensus value of the fine structure constant and Wyler's calculations. Nature is NOT governed by the rule of the majority of voting humans, and a human consensus is NOT always right.

Also, the stated error bars in the current recommended values of the fine structure constant are (in my opinion) probably grossly wrong and subject to serious misinterpretation. For instance:

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1 / Wyler's value = 137.03608
1 / 1998 Adjusted recommended experimental value =
                  = 137.03599976 (+/- 50 in last two digits)
Difference = 137.03608 - 137.03600 (rounded) = 0.00008
Error bar =                                  = 0.0000005
Number of error bars in difference = 800 / 5 = 160
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could be misinterpreted as showing that Wyler's value (which is also my calculated value) is wrong by 160 standard deviations, and therefore clearly ruled out by experiment.

However, in my opinion (even disregarding the possibility of rigging the 1986-87 and subsequent adjustments of physical "constant" values to disagree with Wyler, such a way of thinking is just an illustration of the progression, in increasing order of badness: Lies - Damn Lies - Statistics. I think that this is one of many instances in which stated physics error bars used are ridiculously unrealistic.

In his paper described above, **Carlos Castro** says, in part [**I have extensively modified it by editing out the parts of it with which I disagree, which changes are substantial, so any errors you see in reading this edited version are probably my fault. For example, I have omitted Carlos Castro's use of a factor 2V(S2) which he used as volumes of light-cone boundaries at past and future infinities with respect to virtual emission/absorption of gauge boson force carriers** (I think that all such bosons MUST come from past infinity and go to future infinity, so that any factor should be 1x1 = 1 = unity, and not anything involving 2V(S2).) **Therefore, what you see here may NOT be what Carlos Castro thinks, and he should not be blamed for any errors that my editing has created.** That having been said, **I also want to say that overall I think that Carlos Castro's paper is brilliant, and I especially like his idea linking the Wyler-type models to generalized Kaluza-Klein models such as described by Batakis. I have designated material that I omitted by ... and have enclosed my additions in [ ]** .] :

> "... Geometric Probability ... is the study of the probabilities involved in geometric problems, e.g., the distributions of length, area, volume, etc. for geometric objects under stated conditions. One of the most famous problem is the Buffon's Needle Problem of finding the probability that a needle of length l will land on a line, given a floor with equally spaced parallel lines a distance d apart. ... the Geometric Probability is essentially

the ratio of the areas of a rectangle of length 2d, and width l and the area of a circle of radius d . For l > d, the solution is slightly more complicated. ... The Buffon needle problem provides with a numerical experiment that determines the value of pi empirically. Geometric Probability is a vast field with profound connections to Stochastic Geometry.

Feynman long ago speculated that the fine structure constant may be related to pi . This is the case as Wyler found long ago [1960s-70s]... We will based our derivation of the fine structure constant based on Feynman's physical interpretation of the electron's charge as the probability amplitude that an electron emits (or absorbs) a photon. The clue to evaluate this probability within the context of Geometric Probability theory is provided by the electron self-energy diagram. Using Feynman's rules, the self-energy (p) as a function of the electron's incoming (outgoing) energy-momentum p mu is given by ...[an]... integral involving the photon and electron propagator along the internal lines ... The integral is taken with respect to the values of the photon's energy-momentum k mu ... one can see that the electron self-energy is proportional to t**he fine structure constant alpha_EM = e^2**, the square of the probability amplitude (in natural units of hbar = c = 1) and **physically represents the electron's emission of a virtual photon (off-shell ...) of energy-momentum k at a given moment, followed by an absorption of this virtual photon at a later moment**.

Based on this physical picture of the electron self-energy graph, we will evaluate t**he Geometric Probability that ...[a particle]... emits a ...[gauge boson]... at ... (infinite past) and re-absorbs it at a much later time ... (infinite future)**. The off-shell (virtual) ...[gauge boson]... associated with the ...[particle]... self-energy diagram ... can remain off-shell ... k^2 =/= 0 ....[all the time]... between the moments of emission and absorption by the ...[particle]...**[as it moves along its path in the]... conformally compactified real Minkowski spacetime Q4 = Mbar4 = S3 x S1/Z2 = S3 x RP1 ... [which has]... precisely the same as the topology of the Shilov boundary Q4 of the 4 complex-dimensional ... [D4 = Spin(4, 2) / Spin(4) x U(1) = SU(2,2) / S(U(2)xU(2))]**...

> [Shilov boundaries often describe the physically relevant part of larger domains, as in the case in which compactified Minkowski spacetime is the Shilov boundary Q4 of the complex domain D4.] ... the (Shilov) boundaries are essential mathematical features to understand the geometric derivation of all the coupling constants. ... we will recur to Penrose's ideas ... of conformal compactifications of Minkowski spacetime ... The conformal group [Spin(4,2) = SU(2,2)] leaves the light-cone ... [invariant]... and ... does not alter the causal properties of spacetime ... The action of the discrete group Z2 [in Q4 = S3 x S1/Z2] amounts to an antipodal identification of ... the past timelike infinity ... with the future timelike infinity ...
>
> Shilov boundaries of homogeneous (symmetric spaces) complex domains,

> G/K ... are not the same as the ordinary topological boundaries (except in some special cases). ... Shilov boundaries are the minimal subspaces of the ordinary topological boundaries ...[for which]... the holomorphic data in the interior (bulk) of the domain is fully determined by the holomorphic data on the Shilov boundary. The latter has the property that the maximum modulus of any holomorphic function defined on a domain is attained at the Shilov boundary.

... The action of the discrete group Z2 amounts to an antipodal identification of t\... the past timelike infinity ... with the future timelike infinity ... ... **D4 of 4 complex dimensions is an 8 real-dim Hyperboloid of constant negative scalar curvature that can be identified with the conformal relativistic curved phase space associated with the electron (a particle) moving in a 4D Anti de Sitter space AdS4.** [It]... is a Hermitian symmetric homogeneous coset space associated with the 4D conformal group SO(4, 2) since D4 = SO(4, 2) / SO(4)xSO(2). ...

[The most basic force is **gravity, which holds space-time together by emitting/ absorbing virtual gravitons with probability 1**. In that sense, the Geometric Probability strength of gravity is the strongest of any of the four forces, and should be the denominator in the ratio describing the fundamental strength of each force. Therefore, **the first calculation should be of the Geometric Probability strength of gravity**.]

... **the MacDowell-Mansouri-Chamseddine-West formulation of Gravity [is] based on the ...[anti-deSitter]... group SO(3, 2)** which has the same number of 10 generators as the 4D Poincare group. ...

[**Since the largest local isotropy group of]... D4 = SO(4, 2) / SO(4)xSO(2) ...[is SO(4), and since the Euclidean version of the Anti de Sitter Group is SO(5), neither D4 nor its Shilov boundary] ... space Q4 = S3 x RP1 is ... large enough to implement the action of ... the Anti de Sitter Group SO(3, 2) [that]... is required in the MacDowell-Mansouri-Chamseddine-West formulation of Gravity**. ...

[A sufficiently large space with SO(5) local isotropy group is]... 5 complex-dimensional ... D5 = SO(5, 2) / SO(5)x SO(2) ... the 10 real-dim Hyperboloid ... corresponding to the conformal relativistic curved phase space of a particle moving in 5D Anti de Sitter Space AdS5 ...

[Its Shilov boundary]... space Q5 = S4 x RP1 is large enough to implement the action of SO(5) via the ... space S4 = SO(5)/SO(4). This justifies the embedding procedure of D4 -> D5 ... and Q4 -> Q5 ...

Hence, the Geometric Probability ratio [for the force strength of gravity] becomes ...
[proportional to the measure mu[Q4] of the entire compactified Minkowski Mbar4 through which the virtual graviton can propagate, and inversely proportional to the measure mubar[Q4] of Q4 as embedded in Q5, so that

GeomStrength_gravity = mu[Q4] / mubar[Q4]

and in terms of the larger embedding Shilov boundary Q5, using the embedding-adjustment space S4 = SO(5)/SO(4), we have

GeomStrength_gravity = mu[Q4] / mubar[Q4] = V(S4) mu[Q5] / mu_N[Q5]

The]... standard normalized measure mu_N[Q5] [is] based on the Poisson kernel and involv[es] a normalization factor of 1/V(Q5) [and] is not invariant under the full group SO (5, 2) . It is only invariant under the isotropy group of the origin SO(5)xSO(2). In order to construct an invariant measure on Q5 under the full group SO(5, 2) one requires to introduce a crucial factor related to the Jacobian measure involving the action of the conformal group SO(5, 2) on the full bulk domain D5. ...[so]... we turn to the Hermitian metric on D5 constructed by Hua ... which is SO(5, 2)-invariant and is based on the Bergmann kernel ... involving a crucial normalization factor of 1/V D5).

As explained by ...[G. Gonzalez-Martin]... one has :

mu_N[Q5] / mu[Q5] = (1 / V(Q5)) (1 / ||Jac_C||) = (1 / V(Q5)) sqrt(1 / ||Jac_C (Jac_C*)||) = (1 / V(Q5)) sqrt(1 / ||Jac_R||) = (1 / V(Q5)) sqrt(1 / sqrt(| det(g) |) = (1 / V(Q5)) (| det(g) |^(-(1/4))) = **V(D5)^(1/4) / V(Q5)**

... the z dependence of the complex Jacobian is no longer explicit because the determinant of the SO(5, 2) matrices is unity. This explains very clearly the origins of the factor [V (D5)]^(1/4) in Wyler's formula for the fine structure constant ... This reduction factor of V (Q5) is in this case given by V (D5)1/4. As we shall see below, the power of 1/4 is related to the inverse of the dim(S4) = 4. This summarizes, briefly, the role of Bergmann kernel ... in the construction by Hua ... and adopted by Wyler ... of the Hermitian metric of a bounded homogenous (symmetric) complex domain. To sum up, we must perform the reduction from V (Q5) -> V (Q5) / V (D5)^(1/4) in the construction of the normalized measure mu_N[Q5] .

[Comment by Tony Smith - With respect to the factor V(D5)^(1/4), I think that with the Jacobian language used by Gonzalez-Martin to get the factor V(D5)^1/4 is substantially equivalent to my statement about my model "... Vol(Dforce)^(1 / mforce) stands for a dimensional normalization factor (to reconcile the dimensionality of the Internal Symmetry Space of the target vertex with the dimensionality of the link from the origin to the target vertex) ..." and to Smilga's statement about his model"... If we integrate over Dbar5 using this new parameter set, each si will be responsible for a contribution of V(D5)^(1/4) to the volume of Dbar5. Three of these parameters can be mapped onto the transferred momentum q. The fourth parameter s4 ... corresponds to a momentum transfer within each of the particle momenta, without any momentum transfer between the particles. such transitionscontribute to the volume of C5. We can perform the integration over s4 and obtain a correcting factor to the aready calculated volume V(C5) of V(D5)^(1/4). ...".

Taking the V(D5)^(1/4) factor into account, we have

GeomStrength_gravity = mu[Q4] / mubar[Q4] = V(S4) mu[Q5] / mu_N[Q5] **=**

= V(S4) V(Q5) / V(D5)^(1/4)

where the volumes are]...

- V(D5) = pi^5 / (2^4 x 5!)
- V(Q5) = 8 pi^3 / 3
- V(S4) = 8 pi^2 / 3

[Define all the force strengths in terms of GeomStrength_G, the geometric force strength of gravity, which is the strongest of all geometric force strengths. Therefore,

alpha_G = GeomStrength_gravity / GeomStrength_gravity **= 1**

and, for the other forces we have alpha_force = GeomStrength_force / GeomStrength_gravity

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...[For **alpha_C, the color force strength**, use... **the internal symmetry space ... CP2 = SU(3) / U(2) = SU(3) / SU(2) x U(1)** ... associated with the SU(3) color group ... the volume V (CP2) = V (S4) = 8 pi^2 / 3 ... V(S5) = 4 pi^3 ... V(B6) = pi^3 /6 ... we have

alpha_C = (V(CP2) V(S5) / V(B6)^(1/4)) / GeomStrength_gravity **= 0.6286**

]... that [value] corresponds to the strong coupling constant at an energy related to the pion masses ... 241 MeV ...[As Batakis says in his paper about CP2 Kaluza-Klein, "... [Color SU(3)] has been introduced at the level of the metric via the Kaluza ansatz ...[and] has a role for the coset space ...[CP2]... analogous to that of the Lorentz (or Poincare) group for ordinary spacetime ...".]...

...[**For alpha_W, the weak force strength**, use ... the CP1 = S2 subspace of the internal symmetry space CP2, since the weak force SU(2) acts globally on S2 = CP1 = SU(2) / U (1) = Spin(3) / Spin(2)

Since it takes two copies of S2 to cover all 4 dimensions of covariant polarization for weak bosons, and since the Shilov boundary of the interior of S2xS2 is S2 + S2 (two copies of S2) ... V(S2) = 4 pi ... V(Q3) = 4 pi^2 ... V(D3) = pi^3 / 24 ... we have

alpha_W = (2V(S2) V(Q3) / V(D3)^(1/2)) / GeomStrength_gravity **= 0.2536**

]... that [value] corresponds to the weak geometric coupling constant alpha_W at an energy of the order of ... 146 GeV ... the Fermi coupling GFermi ....[in terms of the proton mass is about]... 1.04 x 10^(-5) ... [As Batakis says in his paper about CP2 Kaluza-Klein, "... [SU(2) x U(1)] symmetry will break ... with the introduction of torsion. The mixed [torsion] components of the ...[SU(2) x U(1) symmetry.].. will be associated with the spin-1 field ... with a (possibly broken) SU(2) x U(l) gauge symmetry. ...".]...

...[**For alpha_E, the electromagnetic force strength**, use ... an S1 subspace of the internal symmetry space CP2 = C2 u CP1 = C2 u S2 located in the C2 independent of the weak force S2 = CP1, since the electromagnetic force U(1) acts globally on S1 = U(1) . Note that for each of the two S2 placement choices described for alpha_W, there are two C s in the C2 = CxC part of CP2 = C2 u S2, and each of those C s contains a natural S1 , so that in all there are 4 S1 placement choices for alpha_E.

Since it takes four copies of S1 to cover all 4 dimensions of covariant polarization for electromagnetic photons, and since the Shilov boundary of the interior of S1xS1xS1xS1 = T4 is S1 + S1 + S1 + S1 (four copies of S1) ... V(S1) = 2 pi ... V(Q2) = 2 pi / 2 since Q2 = RP1 = S1/Z2 ... V(D2) = pi since D2 is a circle of radius 1 whose circumference is S1 only half of which circumference is Q2 ... we have

alpha_E = (4 V(S1) V(Q2) / V(D2)^(1/1)) / GeomStrength_gravity **= 1 / 137.03608**

]... that [value] corresponds to ... the scale of the Bohr radius ... The Bohr radius is associated with the ground (most stable) state of the Hydrogen atom ... The spectrum generating group of the Hydrogen atom is well known to be the conformal group SO(4, 2) due to the fact that there are two conserved vectors, the angular momentum and the Runge-Lentz vector. After quantization, one has two commuting SU(2) copies SO(4) = SU(2)xSU (2). Thus, it makes physical sense why the Bohr-scale should appear in this construction. ... The Bohr radius corresponds to an energy of 137.036 x 2 x 13.6 eV = 3.72 x 10^3 eV . It is well known that the Rydberg scale, the Bohr radius, the Compton wavelength of electron, and the classical electron radius are all related to each other by a successive scaling in products of EM. ... [As Batakis says in his paper about CP2 Kaluza-Klein, "... [SU(2) x U(1)] symmetry will break ... with the introduction of torsion. The mixed [torsion] components of the ...[SU(2) x U(1) symmetry.].. will be associated with the spin-1 field ... with a (possibly broken) SU(2) x U(l) gauge symmetry. ...".]...

Concluding, the Geometric Probability that an electron emits a photon ... and absorbs it ... is given by the ratio of the ratios of measures, and it agrees with Wheeler's ideas that one must normalize the couplings with respect to the geometric coupling strength of Gravity ...

[The above results are]... consistent with the Kaluza-Klein compactification procedure of obtaining 4D EM from pure Gravity in 5D ...[and its generalization to M4 x CP2 by Batakis]. ...".

In his paper

Extra gauge field structure uncovered in the Kaluza-Klein framework,

Class. Quantum Grav. 3 (1986) L99-L105, **N. A. Batakis** says:

"... In a standard Kaluza-Klein framework,

M4 x CP2 allows the classical unified description of an SU(3) gauge field with gravity.

However,

the possibility of an additional SU(2) x U(l) gauge field structure is uncovered.

... As a result, M4 x CP2 could conceivably accommodate the classical limit of a fully unified theory for the fundamental interactions and matter fields. ... [There are]... two generic possibilities ... for the enlargement of Einstein's framework, namely

- ... increase the number of spacelike dimensions ...[which]... is mainly exploited in the ...[ordinary]... Kaluza-Klein programme ...[in which]... the extra dimensions form a vertical 'internal' compact space of very small ... volume ... and ...
- ... allow the presence of torsion without upsetting the metricity of connections ... [which involves not only]... a torsion (totally within M4) ... in the context of the Einstein-Cartan theory ...[but also]... the ... mixed components of a torsion in the total space, namely components which are neither totally vertical nor completely horizontal. ... such a torsion creates a new and non-trivial possibility for the accommodation of unified theories in the KK framework ...[in a]... way in which an eight-dimensional manifold, locally of the form M4 x CP2, could ... accommodate the classical limit of a fully unified theory for the fundamental interactions and matter fields ...

...**[In]... M4 x CP2 ... the groups G1 and G2 are SU(3) and SU(2) x U(1)**, respectively.

The ... **G1 [SU(3)] ... results from the well known identification of CP2 with the coset space SU(3)/U(2)**.

... the **G2 ... SU(2) x U(1)** ...[has a]... Killing form is not zero but ... is degenerate, namely (- 1, - 1, - 1, 0). However, in view of the U(1) factor ... a non-degenerate metric (- 1, - 1, - 1, - 1) can be (and often is) defined on SU(2) x U(1). This possibility makes SU(2) x U(1) a perfectly acceptable G2 ...The metric is given by ...

$$g_{MN} = \left(\begin{array}{c|c} \overset{0}{g}_{\mu\nu} + \kappa_1^2 \xi^a_P \xi^b_Q A^{(1)P}{}_\mu A^{(1)Q}{}_\nu \overset{0}{g}_{ab} & \kappa_1 \xi^a_Q A^{(1)Q}{}_\mu \overset{0}{g}_{an} \\ \hline \kappa_1 \xi^a_Q A^{(1)Q}{}_\nu \overset{0}{g}_{am} & \overset{0}{g}_{mn} \end{array} \right)$$

... and the connection 1-form w^M_N is defined as w^M_N = w^0M_N + K^M_n with w^0M_N the Riemannian connection of the metric [shown immediately above]... and the contorsion K^M_N defined ... in terms of the torsion ... T^m_ab = k2 theta^m_I F^(2) I*_ab where * denotes the M4 dual, k2 is a constant and theta^m_I is a vielbein (employed to change the group index I to the [CP2]... index m) such that g_mn theta^m_I theta^n_J = -g_IJ ... The Riemann scalar curvature is then given by an equation similar to ...

$$R^{(5)} = \overset{1}{R}{}^{(5)} - 2K^{MN}{}_{M;N} - K^{MN}{}_N K_{ME}{}^E + K_{MNE} K^{ENM} \quad ...$$

$$R^{(5)} = \overset{1}{R}{}^{(5)} - \tfrac{1}{4}\kappa_2^2(F^{(2)})^2 + \text{surface terms}$$ ...

$$R^{(5)} = R^{(4)} - \tfrac{1}{4}\kappa_1^2(F^{(1)})^2 - \tfrac{1}{4}\kappa_2^2(F^{(2)})^2 + \text{surface terms.}$$

... and the Bianchi identities hold ... **The resulting Einstein-Hilbert action ... when expressed totally within M4 will, besides gravity (with a cosmological constant), contain the two gauge fields [SU(3) and SU(2) x U(1)], with the relative scales between the three parts set by ... constants k1 and k2 as in ...**

the Einstein-Hilbert action reduces in four dimensions (with κ_0 a constant) to

$$I = \kappa_0^2 \int (-\overset{0}{g}{}^{(4)})^{1/2}\, d^4x\, (-R^{(4)} + \tfrac{1}{4}\kappa_1^2(F^{(1)})^2 + \tfrac{1}{4}\kappa_2^2(F^{(2)})^2). \qquad (17)$$

... We recall

- that **A(1) [of SU(3)] has been introduced at the level of the metric via the Kaluza ansatz ...[and] has a role for the coset space ...[CP2]... analogous to that of the Lorentz (or Poincare) group for ordinary spacetime**,
- while A(2) has been introduced directly through a field strength F(2) at the level of the connection. In view of the Bianchi identities for the manifold [M4 x CP2] ... **F (2) will have a well defined and conserved energy-momentum tensor. However, its gauge group structure is apparently not mandatory. What our construction has shown is that the geometry allows a maximal gauge group structure ... The corresponding [G2 = SU(2) x U(1)] gauge symmetry is apparently unprotected, in contrast to the G1 gauge symmetry**.

These results are obviously desirable in view of the ... association we seek for the two gauge fields with the strong and electroweak interactions. We also observe that we have exhausted the generic possibilities for the introduction of interaction fields into the geometry: besides the metric and a general metric connection, there is no other independent intrinsic geometric structure available within our framework. Thus, the following geometric picture seems to be emerging.

- The gravitational and SU(3) gauge field potentials must be considered as more fundamental and they completely specify the metric - essentially they are the metric of M4 x CP2. If no torsion exists, a symmetric metric connection is uniquely defined from this metric and M4 x CP2 would then exhibit a complete left-right symmetry.
- However, ... [SU(2) x U(1)] symmetry will break ... with the introduction of torsion. The mixed [torsion] components of the ...[SU(2) x U(1) symmetry.].. will

be associated with the spin-1 field F(2) with a (possibly broken) SU(2) x U(l) gauge symmetry.

- ...[The torsion]... components totally within M4 or CP2 could accommodate matter fields in the form of, respectively, spin density and energy-momentum density condensates, with mechanisms analogous to those already known ...".

An interesting thing about the 1986 paper of Batakis is that it provides a constructive counterexample to a well-known paper by Edward Witten entitled Search for a Realistic Kaluza-Klein Theory, published in Nuclear Physics B (1981) 412-428, in which **Witten said**:

> "...**seven dimensions is in fact the minimum dimensionality of a manifold with SU(3) xSU(2)xU(1) symmetry ... If, therefore, we wish to construct a theory in which SU(3) xSU(2)xU(1) gauge fields arise as components of the gravitational field in more than four dimensions, we must have at least seven extra dimensions**. ...".

It is sad that Witten's brilliant understanding of higher mathematics is accompanied by such a lack of physics intuition.

In a 1983-84 paper

Calculation of Gauge Couplings and Compact Circumferences from Self-Consistent Dimensional Reduction

by Candelas and Weinberg in Nuclear Physics B237 (1984) 397-441 (reprinted in a book

Modern Kaluza-Klein Theories

edited by Applequist, Chodos, and Freund (Addison-Wesley 1987), **Candelas and Weinberg** say:

> "... we wish to show how fine-structure constants in general can be calculated in certain theories, in which the gauge fields arise from the metric of a higher-dimensional space. ... There are more general (4+N)-dimensional models ... in which N dimensions form a compact manifold, and a massless gauge field appears in four dimensions for each Killing vector of this manifold. A general prescription has ... been given for calculating the

various gauge couplings in such models in terms of the ratio of 2 pi (16 pi G)^(1/2) and various r.m.s. circumferences. ... In this paper we consider dynamical compactification ... The (4+N)-dimensional space is again suposed to break up into a 4-dimensional Minkowski space and a curved compact N-dimensional manifold, with the curvature governed by Einstein's field equations. ... the energy-momentum tensor on the right-hand side on the right-hand side of these equations is ... supposed to arise ... from the one-loop fluctuations in various matter fields. ... the energy-momentum tensor is balanced by the curvature, ans solutions are possible without mass parameters in the lagrangian, and with the scale of the compact manifold set by the gravitational constant ... For an N-dimensional compact manifold whose linear dimensions are of order rho, the one-loop energy density of f light matter fields is of order f rho^(-4-N) . The (4+N)-dimensional gravitational constant Gbar is or order G rho^N , and the Einstein tensor is of order rho^(-2) . Hence ... rho^2 = G f . The L-loop gravitational corrections to the one-loop matter energy density ... are less for L >= 2 than the one-loop matter terms by a factor ... (1 / f)^(L-1) . Also, the L-loop purely gravitational contributions to the energy density ... are less for L >= 1 than the one-loop matter terms by a factor ... (1 / f)^L . ... for f sufficiently large the scale of the compact manifold is of order sqrt(G f) ... For manifolds ... [including]... spheres, CPN, and manifolds of simple groups ... we can normalize the one free parameter rho^2 in the metric ...

We now make the further assumptions that the matter fields are massless in 4+N dimensions ... When a (4+N)-dimensional space breaks up into a 4-dimensional Minkowski space and a compact manifold, the perturbations of this metric appear in 4 dimensions in part as a set of massless fields: the Yang-Mills fields A^mu_a(x) and the gravitations field g_mu nu(x) .

... the classical Einstein-Hilbert action of pure gravity in 4+N dimensions yields in 4 dimensions an action .. where ... F^mu nu_e are the Yang-Mills curls of those gauge fields A^mu_e that correspond to closed Killing curves of the compact manifold ...[if there were not many species of matter fields]... then G_0 could be identified as the Newton gravitational coupling constant G, and the normalization condition for the Yang-Mills fields would yield ... g_e = (2 pi (16 pi G)^(1/2) / N_e s_e) ...[where]... s_e is the r.m.s. circumference of the manifold along these curves ... However ... assuming ... many species of matter fields ... radiative corrections generate induced ... terms ...[involving]... new dimensionless coefficients ... D_N_e adn E_N ... we see that [for one-parameter manifolds ... of odd dimensionality]... the true Newton constant G is given by (1 / 16 pi G) = (1 / 16 pi G_0) + (E_N / rho^2) ...

Even a manifold that is stable against all deformations will become unstable if the temperature is raised above a critical value T_c ... This suggests that there is a dramatic phase transition at T = T_c , in which the compactified dimensions explode outward. One wonders ... whether our universe started with equal circumferences in all 3+N spatial directions, and became tightly contracted in N of those dimensions

only when the temperature fell below the critical temperature T_c . ...".

Note that the critical temperature for dimensional reduction is consistent with the cosmology, high-temperature physics, and current-experiment-energy-level physics of the D4-D5-E6-E7-E8 VoDou Physics model.

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