Bell’s Theorem
on Quantum Correlations is based on the Hopf Fibration \( \text{RP}1 \to \text{S}1 \to \text{S}0 = \{-1,+1\} \).

Joy Christian has shown that it is more realistic
to base Quantum Correlations on the Hopf Fibrations
\( \text{S}1 \to \text{S}3 \to \text{S}2 = \text{CP}1 \) and \( \text{S}3 \to \text{S}7 \to \text{S}4 = \text{QP}1 \) and \( \text{S}7 \to \text{S}15 \to \text{S}8 = \text{OP}1 \)
where R, C, Q, and O are Real, Complex, Quaternion, and Octonion Division Algebras.

In his book “Disproof of Bell’s Theorem” (BrownWalker Press, 2nd ed, 2014)
Joy Christian said:
“... Every quantum mechanical correlation can be understood as
a classical, local-realistic correlation among a set of points of a parallelized 7-sphere
... physical space ... respects the symmetries and topologies of a parallelized 7-sphere
... because 7-sphere ...[is]... homeomorphic to the ...[Octonion]... division algebra ... it is the property of division that ....[is]... responsible for ... local causality in the world
... To understand this reasoning better, recall that, just as a parallelized 3-sphere is a 2-sphere worth of 1-spheres but with a twist in the manifold \( \text{S}3 (=/= \text{S}2x\text{S}1) \), a parallelized 7-sphere is a 4-sphere worth of 3-spheres but with a twist in the manifold \( \text{S}7 (=/= \text{S}4x\text{S}3) \)
... just as \( \text{S}3 \) is
a nontrivial fiber bundle over \( \text{S}2 \) with Clifford parallels \( \text{S}1 \) as its linked fibers,
\( \text{S}7 \) is also
a nontrivial fiber bundle ... over \( \text{S}4 \) ... with ... spheres \( \text{S}3 \) as its linked fibers.
... it is the twist in the bundle \( \text{S}3 \) that forces one
to forgo the commutativity of complex numbers (corresponding to the circles \( \text{S}1 \)) in favor of the non-commutativity of quaternions.
In other words, a 3-sphere is not parallelizable by the commuting complex numbers but only by the non-commuting quaternions. And it is this noncommutativity that gives rise to the non-vanishing of the torsion in our physical space.

In a similar vein, the twist in the bundle \( \text{S}7 (=/= \text{S}4x\text{S}3) \) forces one to forgo the associativity of quaternions (corresponding to the fibers) in favor of the non-associativity of octonions.
In other words, a 7-sphere is not parallelizable by the associative quaternions but only by the non-associative octonions.
... it can be parallelized ... because its tangent bundle happens to be trivial: Once parallelized by a set of unit octonions, both the 7-sphere and each of its 3-spherical fibers remain closed under multiplication. This, in turn, means that the factorizability or locality condition of Bell is ... satisfied within a parallelized 7-sphere. The lack of associativity of octonions, however, entails that, unlike the unit 3-sphere [which is homeomorphic to the ... group SU(2)], a 7-sphere is not a group manifold ... the torsion within the 7-sphere ... varies from one point to another of the manifold. It is this variability of the parallelizing torsion within that is ultimately responsible for the diversity and non-linearity of the quantum correlations we observe in nature ...”.

The 7-sphere S7 is the unit sphere in 8-dim space. S7 is not a Lie algebra, but is a Malcev algebra and is naturally embedded in the D4 Lie algebra Spin(8) which is topologically composed of ( but /= the simple product S7 x S7 x G2 ) 2 copies of S7 and 14-dim Lie Algebra G2 of the Octonion Automorphism Group.

28-dim D4 Lie algebra Spin(8) can be represented by 8x8 antisymmetric real matrices It is a subalgebra of 63-dim A7 Lie Algebra SL(8,R) of all 8x8 real matrices with det = 1.

Unimodular SL(8,R) is the non-compact Lie algebra corresponding to SU(8). SL(8,R) effectively describes the 8-dim SpaceTime of E8 Physics as a generalized checkerboard of SpaceTime HyperVolume Elements. Anderson and Finkelstein in Am . J. Phys. 39 (1971) 901-904 said: "... Unimodular relativity ... expresses the existence of a fundamental element of spacetime hypervolume at every point. ...". From the Real Clifford Algebra Cl(16) and 8-Periodicity 64-dim R+SL(8,R) appears from factoring Cl(16) = tensor product Cl(8)xCl(8) as the tensor product of the 8-dim vector spaces 8v of each of the Cl(8) factors so that 64-dim R+SL(8,R) = 8v x 8v If you regard the two Cl(8) as Fourier duals then one 8v describes 8-dim Spacetime Position and the other 8v describes its Momentum.

David Brown, in May 2012 comments on scottaaronson.com blog, said: “... Where did Bell go wrong? Bell used quantum SU(1) states whereas Christian correctly uses quantum SU(8) states ...[from]... Christian’s parallelized 7-sphere model. ... Every quantum mechanical Christian SU(8) correlation can be understood as a realistic, non-local Christian SU(8) correlations among a set of points of a parallelized 7-sphere ... More importantly, if Christian’s theory of local realism is true then SU(8) should be the gauge group for physical reality ...”. SU(8) is the compact version of SL(8,R), so it seems to me that it is David Brown’s idea, possibly motivated by SU(8) and SL(8,R) in E7 of D = 4 N = 8 supergravity models, that Joy Christian’s S7 Quantum Correlations have fundamental SL(8,R) structure.
Rutwig Campoamor-Stursberg in Acta Physica Polonica B 41 (2010) 53-77, “Contractions of Exceptional Lie Algebras and SemiDirect Products”, showed that SL(8, R) appears in the E8 Maximal Contraction = semi-direct product H92 x SL(8, R) where H92 is (8 + 28 + 56 + 1 + 56 + 28 + 8)-dim Heisenberg Creation/Annihilation Algebra

so that H92 x SL(8, R) has 7-graded structure:

grade -3 = Creation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components for a total of 8 fermion component creators (related to SpaceTime by Triality)

grade -2 = Creation of 8 + 3 + 1 = 12 Bosons for Standard Model and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity for a total of 28 Boson creators

grade -1 = Creation of 7 massive Dirac fermion
each with 8 SpaceTime Components for a total of 56 fermion component creators

grade 0 = 1 + SL(8) = 1 + 63 = 64-dim representing 8-dim SpaceTime of HyperVolume Elements

grade 1 = Annihilation of 7 massive Dirac fermions
each with 8 SpaceTime Components for a total of 56 fermion component annihilators

grade 2 = Annihilation of 8 + 3 + 1 = 12 Bosons for Standard Model and 16 Conformal U(2,2) Bosons for MacDowell-Mansouri Gravity for a total of 28 Boson annihilators

grade 3 = Annihilation of 1 fermion (tree-level massless neutrino)
with 8 SpaceTime Components for a total of 8 fermion component annihilators (related to SpaceTime by Triality)

Here is how Physics Structures expand from Joy Christian’s S7 to E8 Physics:

7-dim S7 - Lie Algebra -> 28-dim Spin(8)

28-dim Spin(8) - Full 8x8 Matrix -> 63-dim SL(8, R)

63-dim SL(8, R) - Creation/Annihilation -> 248-dim H92xSL(8, R)

248-dim H92xSL(8, R) - Expansion -> 248-dim E8
The E8 expansion of H92 x SL(8,R) has physical interpretation leading to a Local Classical Lagrangian with Base Manifold Spacetime, Gravity + Standard Model Gauge Boson terms, and Fermion terms for 8-dim spacetime and First-Generation Fermions (with 4+4 dim Kaluza-Klein and Second and Third Fermion Generations emerging with Octonionic Symmetry being broken to Quaternionic):

248-dim E8 = 120-dim D8 + 128-dim half-spinors of D8

In Symmetric Space terms:

E8 / D8 = (64+64)-dim (OxO)P2 Octo-Octonionic Projective Plane
64 = 8 components of 8 fermion particles
64 = 8 components of 8 fermion antiparticles

D8 / D4xD4 = 64-dim = 8 position coordinates x 8 momentum coordinates

one D4 = 28 = 12 Standard Model Ghosts + 16 Conformal Gravity Gauge Bosons
(4 of the 16 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

other D4 = 28 = 16 ConformalGravity Ghosts + 12 Standard Model Gauge Bosons
(4 of the 12 are not in the 240 E8 root vectors, but are in its 8-dim Cartan subalgebra)

My E8 Physics model (viXra 1405.0030 vG) was initially inspired back in the 1980s by D = 4, N = 8 supergravity models.

Yoshiaki Tanii in his book “Introduction to Supergravity” (Springer 2014) said:
... Poincare supergravity constructed in the highest spacetime dimension is D = 11, N = 1 theory ... the low energy effective theory of M theory ...
D = 11 supergravity has AdS4 x S7 spacetime ...
This ... corresponds to the AdS4 solution of D = 4, N = 8 gauged supergravity ...
D = 4, N = 8 gauged supergravity is ... related to a compactification of D = 11 supergravity ... by a seven-dimensional sphere S7 ...

N = 8 supergravity ... the maximal supergravity ...[has]... multiplets ...

1   8  28  56  70  56  28   8   1
... D = 4, N = 8 Supergravity ... has global E7(+7) and local SU(8) symmetries. ...

Supergravity itself did not quite work for me. In hindsight,
D = 4, N = 8 maximal global symmetry is only E7 with maximal compact SU(8)
(noncompact version of SU(8) is SL(8,R) which is only part of the maximal contraction of E8)
and the supergravity with maximal global symmetry E8 with maximal compact D8
is   D = 3, N = 8 whose spacetime is only 3-dimensional. (Samtleben, arXiv 0808.4076).

The S7 led me to work with Spin(8) which is the bivector Lie algebra of the Real Clifford Algebra Cl(8) with graded structure 1  8 28 56 70 56 28 8 1
When Spin(8) seemed too small, I went to F4 which contained
Spin(8) for Gauge Bosons, Spin(9) / Spin(8) for 8-dim SpaceTime,
and F4 / Spin(9) for 8 fermion particles + 8 fermion antiparticles.
When F4 failed to have desired complex structure, I went to E6.
When E6 failed to have all the necessary fermion components and gauge boson ghosts,
I went to E8 and found the E8 Physics model that as of now seems to be realistic.
How does Bell-Christian-Brown SL(8,R) Quantum Theory fit with the Bohm Quantum Potential of E8 Physics (http://vixra.org/pdf/1405.0030vG.pdf)?

Comparison of Bohm's Quantum Potential hidden variable "lambdas" with Bell's "lambdas" and Joy Christian's (arxiv 0904.4259)"lambdas":

Peter Holland, in his book "The Quantum Theory of Motion, an Account of the de Broglie - Bohm Causal Interpretation of Quantum Mechanics" (Cambridge 1993) said:

"... 11.5.1 Bell's Inequality ... In discussing the EPR spin experiment Bell supposed that the results of the two spin measurements are determined completely by a set of hidden variables lambda and made two assumptions which he claimed should be satisfied by a local hidden-variables theory:
(i) The result A of measuring sigma1 . a on particle 1 is determined solely by a and lambda, and the result B of measuring sigma2 . b on particle 2 is determined solely by b and lambda , where a and b are unit vectors with a . b = cos(delta).
Thus A = A( a , lambda ) = +/- 1 and B = B( b , lambda ) = +/- 1
Possibilities such as A = A( a , b , lambda ) and B = B( a , b , lambda ) are excluded.
(ii) The normalized probability distribution of the hidden variables depends only on lambda : rho = rho( lambda ).
Possibilities such as rho = rho( lambda , a , b ) are excluded.

... We now consider to what extent assumptions (i) and (ii) are valid in the causal [Bohm Potential] interpretation ... The hidden variables are then the particle positions x1 , x2 (the internal orientation spin vectors s1 , s2 along the trajectories are determined by the positions and the wavefunction ...) ... the eventual results ... for each of sz1 and sz2 is determined by the intial positions of both particles and by delta , i.e., A = A( x1 , x2 , a . b ) , B = B( x1 , x2 , a . b ) Thus assumption (i) is not valid ...
Neither is assumption (ii) satisfied. ...
In reproducing ... the quantum mechanical correlation function ...
Ppsi( a , b ) = ... = - cos( delta ) ... the causal [Bohm Potential] interpretation disobeys both of Bell's basic assumptions. ...".

So, Bell's "lambdas" obey (i) and (ii) and so obey Bell's inequality and
Bohm's "lambdas" violate (i) and (ii) and so violate Bell's Inequality but obey the quantum experimentally observed correlation function.
Joy Christian (see arxiv 0904.4259) explicitly violates (i) by replacing
\[ A = A( a , \lambda ) = \pm 1 \text{ and } B = B( b , \lambda ) = \pm 1 \]
with
\[ A = A( a , \lambda ) \text{ in } S^2 \text{ and } B = B( b , \lambda ) \text{ in } S^2. \]
However, Joy does not violate (ii). Joy says: "... once the state \( \lambda \) is specified and the two particles have separated, measurements of \( A \) can depend only on \( \lambda \) and \( a \), but not \( b \), and likewise measurements of \( B \) can depend only on \( \lambda \) and \( b \), but not \( a \)...[ compare the (ii)-violation by Bohm's \( \lambda \) as stated above ]... Assuming ... that the distribution \( \rho( \lambda ) \) is normalized on the space \( \Lambda \), we finally arrive at the inequalities ... exactly what is predicted by quantum mechanics ... we have been able to derive these results without specifying what the complete state \( \lambda \) is or the distribution \( \rho( \lambda ) \) is, and without employing any averaging procedure ... the correlations [ for the examples of 0904.4259 ] ... are simply the local, realistic, and deterministic correlations among certain points of ... \( S^3 \) and \( S^7 \) ... This implies that the violations of Bell inequalities ... have nothing to do with quantum mechanics per se ...".

So, even though Joy's \( \lambda \) do not violate (ii), when Joy "... derive[s] ... the exact quantum mechanical expectation value ... - a . b " Joy's result is consistent with that of Bohm's "\( \lambda \)".

Joy's "\( \lambda \)" are classical and local (in Joy's sense).

Bohm's "\( \lambda \)" are quantum and, since Joy does not change Bell's (ii), nonlocal (in Joy's sense).

Joy's "\( \lambda \)" and Bohm's "\( \lambda \)" are consistent with each other with respect to their calculated quantum expectation values.
Could Joy's "lambdas" be considered as a Classical Limit of Bohm's "lambdas"?

Consider again Peter Holland's book in which he says:
"... 6.9 Remarks on the path integral approach ... Feynman['s] ... route to quantum mechanics ... rests on the trajectory concept and so may be expected to have some connection with the causal [Bohm Potential] formulation. ... Feynman provides a technique for computing ... the transition amplitude (Green function or propagator) ... from the classical Lagrangian ... One considers all the paths ... and associates with each an amplitude ... These tracks are ... called 'classical paths' ... one sums (integrates) over all the paths ... the solution .. is given by ... Huygens' principle ... of all the paths ... one of them will be the actual trajectory pursued by the quantum particle according to the [Bohm Potential] guidance formula ... We shall refer to ... it ... as the 'quantum path' ... For an infinitesimal time interval ... the propagator is just the classical wavefunction ... a finite path may be decomposed into many such infinitesimal steps, the net propagator being obtained by successive applications of Huygens' construction ... We may view the Feynman procedure as a method of obtaining the quantum action from the set of all classical actions. ...".

If Joy Christian's classical "lambdas" are identified with Feynman path Lagrangian / Green function propagators, and if their Huygens' sums can be seen to produce the Bohm "lambdas",
then Joy's work will show a nice smooth classical limit (as opposed to Bell's discordant classical limit) for the Bohm Quantum Potential.

If the Bohm Quantum Potential can then be used as a basis for a construction of a realistic AQFT (Algebraic Quantum Field Theory)
then maybe Joy Christian's work will help show a useful connection (and philosphical reconciliation) between
the Classical Lagrangian physics so useful in detailed understanding of the Standard Model
and
of AQFT along the lines of
generalization of the Hyperfinite II1 von Neumann factor algebra.