Hua Loo-Keng and Armand Wyler

Hua Loo-Keng

According to the book "Hua Loo-Keng" by Wang Yuan (translated by Peter Shiu) (Springer 1999):

"... **A mathematician has to be judged by his research acccomplishments and not by the number of university degrees earned. In Hua's case there are many of the former and none of the latter ... Teacher Hua only had a junior middle-school education** ...

In 1935, E. Cartan proved that, under analytic mapping, there are precisely six types of irreducible homogeneous symmetric bounded domains. Of these there are two types of exceptional domains with dimensions 16 and ...[27]... and the remaining four types are called classical domains ... R_I = ... m x n matrix ... R_II = ... symmetric matrices ... R_III = skew-symmetric matrix ... R_IV ...

Classical domains can thus be regarded as the generalisation of the unit disc in the plane to higher dimensions ...

[When I was at **Princeton 1959-63, William Feller said** that concrete examples were needed to understand abstract mathematical concepts, and **that his favorite concrete example to understand basic ideas was the harmonic functions on the unit disk**. - TS]

In 1943 Siegel published his famous paper on 'symplectic geometry' which dealt with the second type of classical doamins using matrices ... In 1944-1945, Hua published ... three papers .. which gave a direct and concrete method of calculating ... results in certain multi-faceted geometries.

There are both differences and duplications between Hua's and Siegel's research. ... Early in 1943, Hua Loo-Keng had already received an invitation to visit the Institute for Advanced Studies at Princeton ... Loo-Keng and former University of Gottingen Professor C. L. Siegel have independently developed the theory of automorphic functions ... in terms of age, achievement and reputation Siegel is well ahead of Loo-Keng ... the visit ... might involve sacrificing the achievement of independent discovery ... with the possibility that they would be recognised only as the consequence of the influence of Siegel [who was at the Institute for Advanced Studies from 1940 to 1951 when he returned to Gottingen] ...[so]... Hua Loo-Keng declined the invitation ...

[David B. Lowdenslager received his Ph.D. in mathematics in 1956 under Edward McShane at the University of Virginia, and in 1962 taught a graduate mathematics course, Math 535, at Princeton University where I audited it. Unfortunately, he died shortly after, but he had written while at the University of Illinois an important paper: Potential Theory In Bounded Symmetric Homogeneous Complex Domains (Ann. Math. 67 (May 1958, received May 27, 1957) 467-484), based on presentations to the Summer Institute on Differential Geometry, June 1956, and to the American Mathematical Society, December 1956. In that paper he said:

"... The purpose of this paper is to treat a generalization of the elementary potential theory of the unit circle ... L. K. Hua has determined kernels... for the domains considered here ... J.

> Mitchell ... showed me some mimeographed notes written about the summer of 1956 by L. K. Hua,and containing a very explicit calculation of a complete family of solutions of this equatin for the domains of Type I. Hua also indicated the computation of a kernel funcition which turned out to give the same formula, and how this can be used to get some of the most important results of the present paper for this case. Hua also stated that it is possible to get similar results for the other types of domains. ...
>
> Automorphism groups of the classical Cartan domains have been examined, without proofs, [see C. L. Siegel's Analytic Functions of Several Complex Variables (Notes by P. T. Bateman) Kendrick Press 2008, based on his 1948-1949 lectures at the Institute for Advanced Study in Princeton.], and proofs of some of Siegel's statements have appeared [see work of H. Klingen, who received his mathematics doctorate in 1955 at Gottingen under Siegel], but the actual subgroups used here may easily be shown to consist of automorphisms by examining the infinitesimal generator of each. ...".]

In 1944 ...[Hua]... determined the classification of complex symmetric and skew-symmmetric matrices under a unitary group ... In 1946 Hua determined the classification of Hermitian matrices under the orthogonal group ... In 1946 Hua published his paper on the specification of automorphisms in real symplectic groups which was the stepping stone for his research into the theory of classical groups ... Hua also generalized the notion of a bilinear (fractional linear) transformation in one complex variable to that for several complex variables ...

Hua visited the Soviet Union ...[in].... 1946 ... Hua's seminars in the Soviet Union were on matrix geometry, automorphic functions, and functions with several complex variables ...

in the Soviet Union ... A small university with 200 students would still have perhaps 600 of them studying mathematics ... Hua asked: "With all these students in mathematics, what will they do when they graduate?" ... Vinogradov ... said: "Mathematics is the mother of all the sciences. If a country does not excel in mathematics, then there is not much else to talk about". ...

[Later]... in ... 1946 ... Hua Loo-Keng went to the Institute for Advanced Study at Princeton ... In 1948, Hua characterised the automorphisms on symplectic groups over any field with characteristic not equal to 2. ... Hua ... wrote a supplement to Dieudonne's paper .. on automorphisms in classical group theory ... Hua ... wrote: "... Dieudonne's method becomes very clumsy for small n, and sometimes he is unable to solve the case for the least possible n ... On the other hand ...[Hua's]... method ... starts from the least possible n ...[and]... uses only the calculus of matrices ... Therefore the reader will have little difficulty in extending the special results of this paper to the general case by means of the inductive method used in 'Annals of Math. 49 (1948), 739-759' ...

In the spring of 1948, Hua Loo-Keng was appointed full professor at the University of Illinois in Urbana. ... At the University of Illinois, Hua Loo-Keng often went along with left-wing students ... It is clear that Hua Loo-Keng was supporting the Chinese Communist Party wholeheartedly ...

Hua's decision to return to China ... in 1950 ... was based on his belief ... that the Chinese Communist Party and the Chinese Government ... would ... support ... his wish ... for ... mathematics in China ... to arrive at international level ... Besides this, he saw the racial prejudice in the United States and the differences in cultural background. The isolationist policy being implemented against the Chinese Communist Party and all its work must also have had their effect on him. ...

[Hua's book]... **Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains** ... was published in 1957 by the Institute of Mathematics, Academica Sinica, as No. 4 in the A-series [Russian translation 1959; English translation 1963] ...[It]... was the continuation of his research work on the theory of automorphic functions. ...

associated rings, Lie rings and Jordan rings - all have matrix forms. Linear groups, orthogonal groups, simplectic groups and Lorentz groups are all matrix groups. In geometry there are matrix representations for linear geometry, circle geometry and Grassmann geometry. Even the study of several complex variables in classical domains cannot escape from matrix representations. ...

In the past, Hua had obtained the matrix representation for the group of motions and also the Bergman kernels in the four types of classical domains. ... each complex variable corresponds to two real variables.

Now let R be a bounded and simply connected domain whose points ... is made up of n complex variables, so that the corresponding Euclidean space has dimension 2n. Suppose that L is a part of the boundary for R satisfying the following condition: Every analytic function in R takes its maximum modulus on L and that, for each point x in L, there is an analytic function f(z) in R which takes its maximum modulus at x. We then call L the characteristic manifold of R [also known as the Shilov Boundary of R], and it is a uniquely determined compact manifold.

Generally speaking, L is only a part of the boundary for R and its dimension is much smaller than 2n - 1 . ...

The following three kernels can be computed: ... Bergman kernel ... Cauchy kernel ... Poisson kernel ...

Using the Cauchy kernel, analytic functions in R can now be represented by functions with values in such characteristic manifolds L, namely the Cauchy formula. The positive orientation for the Poisson kernel can now be used to define the harmonic functions. ...

Applying the Poisson kernel Hua Loo-Keng and Lu Qikeng founded the theory of harmonic functions in classical domains and solved the Dirichlet problem associated with the Laplace-Beltrami equation. They discovered ...[that]... if a function satisfies a differential equation, then it must satisfy a system of differential equations. Another phenomenon is that, once the functional values have been assigned to the characteristic manifold for a classical domain, the corresponding Dirichlet problem is completely specified and solved. Here Hua Loo-Keng discovered a set of harmonic operators that share similar properties to those possessed by differential operators, and these operators are now called 'Hua operators' Lu Qikeng gave a detailed study of the geometric structure, boundary and extremal properties associated with classical domains ...[see Analysis and Geometry on Complex Homaogeneous Domains, by Faraut, Kaneyuki, Koranyi, Lu, and Roos (Birkhauser 2000) based on lectures at the CIMPA Autumn School in Beijing, September 15-30, 1997]...

Hua ... seemed to have been the only person working on the subject ... but the work ... influenced ... the work on the theory of functions of several complex variables by the Russian mathematician I. I. Pyateskij-Shapiro ...

In 1956 Hua Loo-Keng initiated the mathematics competition movement in China ... his lecture was ...Starting from the Yang Hui Triangle ... By 1962 ... his competition lecture was Starting from Zu Chongshi's Calculations for the Circle ... the process was ... forced to a halt by ... the 'Cultural Revolution' ... in ... 1966 ... On 9 September 1976, Mao Zedong died. On 6 October 1976, the 'Gang of Four' were arrested.

... The arrest of the 'Gang of Four' signalled the conclusion of the 'Cultural Revolution' and the mathematics competition returned in 1978 ...

Hua Loo-Keng was cleverer than most people, but he would never refer to this as 'talent'. For him, 'hard work' and 'persistence' were far more important ...

By 1983, Springer had published four of his books ... about two thousand pages ... The original plan for the Selecta was to include ... Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains, but the copyright of the English edition... belongs to the American Mathematical Society, and it was not willing to relinquish such rights. ...

Great text remains a thousand years,

Glorious knight lasts but a lifetime

... the Nationalist Government were criminally incompetent, especially during the period just after victory over Japan. All one could see was dishonesty, chicanery and corruption with greedy people bringing about inflation by trying to become rich using any means ... inevitably, many of the intellectual elite became disillusioned and abandoned the Nationalist Party, with many siding with the Communist Party ... Hua too, became one of those intellectuals ...

...[In 1950]... Hua Loo-Keng wrote ...

"... It is obvious that there are two camps in the current world.

In one camp the common good is sought for the masses,

while the other camp specialises in keeping most of the benefits for the small number of people belonging to the ruling classs.

The former group occupies the moral high ground

while the latter is full of contradictions. ...".

Armand Wyler

During the 1966-1976 Cultural Revolution, Hua Loo-Keng was unable to support research related to his work on functions of several complex variables

so when **Heinz Hopf's 1967 Ph.D. student at ETH Zurich, Armand Wyler, applied Hua's work** including the paper "On the theory of functions of several complex variables. II A complex orthonormal system in the hyperbolic space of the Lie-hypersphere" (Acat math. Sinica 5 (1955), 1-25) **to theoretical physics, Hua was not able to support Wyler's efforts**.

In his paper "On the Coformal Groups in the Theory of Relativity and their Unitary Representations" (Arch. Ration. Mech. Anal. 31 (1968) 35-50), Wyler said:

> "... The aim of this paper is to study the most extensive invariance group of the Maxwell equations; it will be shown that this group can be imbedded in a group interpreted in the sphere geometry of Lie, in line with the ideas of Klein ... The symmetric domains D constructed ...[here].. are of the form
>
> SO(p,2) / SO(p) x SO(2)
>
> ... the set H(D) ... of holomophic functions in D is a Hilbert space ... a complete orthogonal basis of H(D) has been given explicitly by Hua for all Hermitian symmetric domains. ...".

In his paper "L'espace symetrique du groupe des equations de Maxwell" (C. R. Acad. Sc. Paris, t. 269 (20 octobre 1969) 743-745), Wyler said:

> "... le calcul des noyaux de Poisson su les espces symetriques mene a une interpretation geometrique de la constante e^2 / hc , pour laqueslle cette theorie donne la valeur 1 / 137,637 ... Soit M^n l'espace de Minkowski dote de la metrique des signature (1,n-1); le groupe conforme C(M^n), groupe d'invariance des cones isotropes de M^n, is isomorphe au groupe SO(n,2) de la forme quadratique de signature (n,2) ... **l'espace hermitien symetrique SO(n,2)/SO(n)xSO(2) ... est donnee ... La realisation du type disque unite ... D^n ...[with]... Q^n ... etant la variete caracteristique [Shilov Boundary] de D^n , le noyau de Poisson P_n(z,x) est defini sur D^n x Q^n ... on obtient le coefficient**
>
> **(9/8) pi^(-4) (V(D^5))^)1/4) = 1 / 137,037**
>
> **Le calcul du volume euclidien V(D^5) du domaine D^5 et la formule donant le noyau de Poisson sont dus a Hua** ...".

In his paper "Les groupes des potentiels de Coulomb et de Yukawa" (C. R. Acad. Sc. Paris, t. 271 (11 janvier 1971) 186-188), Wyler carried the same fine structure constant calculation to more decimal places, getting

1 / 137,03608

and also, using Clifford algebra for spinors and Green functions for the Shilov Boundaries Q^5 and Q^4, calculated the proton / electron mass ratio to be 6 pi^5 = 1836,118 ...".

Robert Gilmore (author of "Lie Groups, Lie Algebras, and Some of Their Applications" (Wiley 1974)) saw Wyler's papers. In a 1983 letter to me, he said:

> "... While in Europe in the summer of 1971, I [Robert Gilmore] manufactured the opportunity to visit Armand Wyler in Zurich. My visit to him had one purpose: to determine what he had done. It so happened that Professor George Mackey (Mathematic's Harvard) was visiting ETH, and we both converged on Wyler with this same purpose. After a rather gruelling ordeal it

> became apparent that there was no theory on which his calculations were based. He said that if he piqued the interest of the physics community, there might be more study of his favorite subject: the various components of the boundaries of complex domains associated with Lie groups. He was right in a very restricted sense: I was one of the few physicists who studied this material to find out what was going on.
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> Shortly afterward, Freeman Dyson invited Wyler to the Institute for Advanced Study to find out what he did. Dyson found out what I found out - that there was no theory underlying his results. ...".

The August 1971 issue of Physics Today carried an article (by GBL) saying in part:

> "... Armand Wyler ... got his PhD from the Federal Institute of Technology [ETH Zurich under Heinz Hopf] in 1966. After a year as an instructor at Stanford and a year at MIT, he is at the Mathematics Institute in Zurich. He will spend the academic year 1971-72 as a member of the Institute for Advanced Study. ...
>
> [Wyler's]... astonishing agreement of theory with experiment has recently been widely discussed among theorists, many of whom say that they have difficulty understanding the theory. .. What connection do the groups Wyler uses have with physics? That is a hard question raised by many critics of the work ... many physicists have suspected that the conformal group should play a greater role in the future of physics than it has in the past. Wyler's is the first attempt to use the conformal group in a radically new way.
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> One critic said that even if Wyler turns out to be correct, there would be no sequel to the work because it has, at least now, no conceivable connection with other concepts in physics. ...
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> A noted theorist said that just because he does not understand the Wyler theory, he does not think one can with good conscience disregard it. ...
>
> Although some theorists believe that the fine-structure constant may have something to do with a geometrical calculation, most would argue against a geometrical interpretation of the proton-to-electron mass ratio. Most theorists would argue that the ratio would certainly involve strong-interaction dynamics. ...".

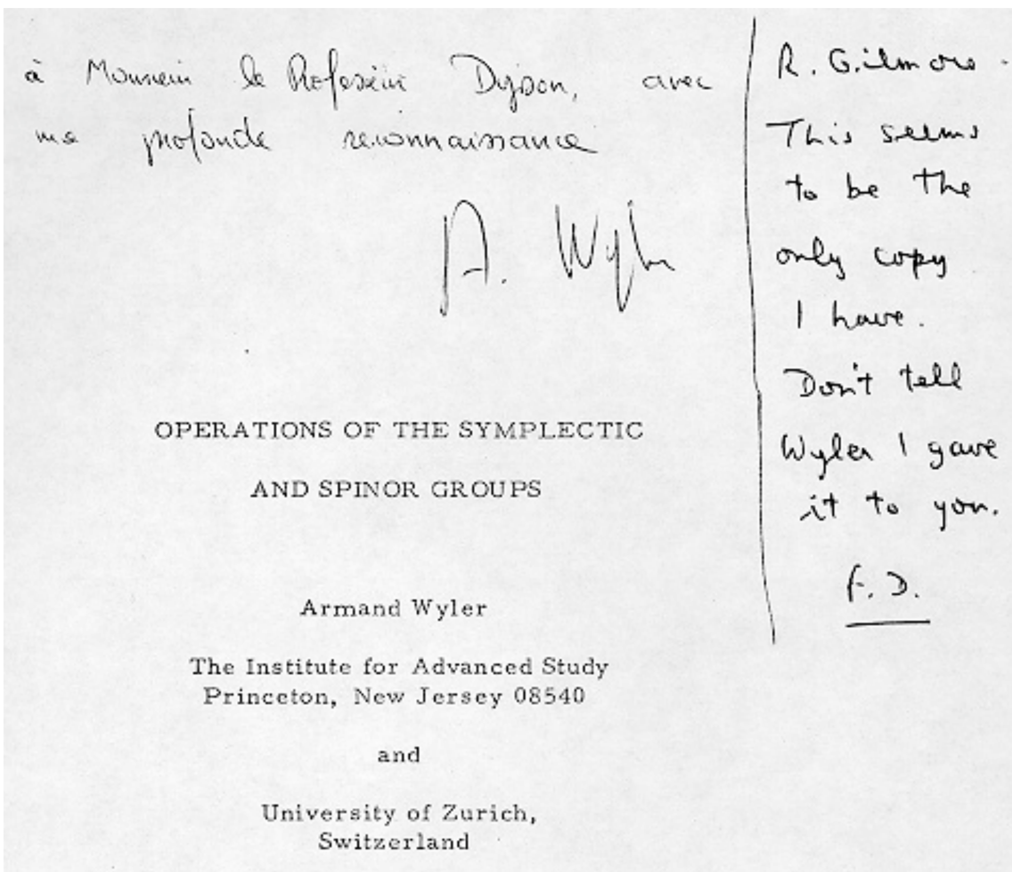
In the November 1971 issue of Physics Today, the editor discussed many other formulas for the fine structure constant using integers, pi, etc, such as 4 pi^3 + pi^2 + p1 = 137.03630, and quoted Asher Peres's conclusion "... we cannot discard the possibility that Wyler's result is a mere numerical coincidence ...". Wyler is quoted as saying that he "... feels that the difference is that is formula is derived from a theoretical formalism which is related to the physical world - the conformal group O(4,2) which is the natural invariance group of Maxwell equations.".

Also in November 1971, Baldwin Robertson (in Phys. Rev. Lett. 27 (29 November 1971) 1545-1547) criticised Wylers' work on the basis that, in calculating the volumes of spaces, "There is no known reason for setting the radius equal to one.".

Robertson's objection was answered by Robert Gilmore (in Phys. Rev. Lett. 28 (14 February 1972) 462-464) who said

> "... the reason for setting r = 1 in ... Wyler's expression for the fine-structure constant ...[is that]... Only for the choice r = 1 are all elements in the bounded domain D^n also coset representatives and therefore group operations. ... Wyler's work has pointed out that it si possible to map an unbounded physical domain - the interior of the forward light cone - onto the interior of a bounded domain on which there also exists a complex structure. This mapping should prove of immense calculational value in the future. This transformation from unbounded to bounded complex domains is mathematically rigorous ...
>
> unfortunately, Wyler does not clearly specify how the Bergman density V^(-1)(D^5) enters the calculation. This is, in fact, still an unanswered question. There is as yet no rigorous mathematical basis for the appearance of the factor [V(D^5)]^(1/4). ... ".

Near the end of the 1971-72 academic year, Wyler had not explained to anyone at the Institute for Advanced Studies about the Bergman density or the factor [V(D^5)]^(1/4). His personality was such that he did not really interact with anyone (not even Dyson), but just stayed in his cubicle (then called a cubby-hole) and worked alone, not even giving any talks. Dyson was unhappy at not seeing anything done, and told Wyler to write up whatever he had done. So, Wyler wrote two papers and gave them to Dyson in June 1972. Dyson gave the papers to Robert Gilmore, who in turn gave them to me, and I scanned them and put them on the web at www.tony5m17h.net/WylerIAS.pdf . Note the handwritten dedication by Wyler (when he gave them to Dyson)



à Monsieur le Professeur Dyson, avec ma profonde reconnaissance

A. Wyler

R. Gilmore.
This seems to be the only copy I have. Don't tell Wyler I gave it to you.
F. D.

OPERATIONS OF THE SYMPLECTIC AND SPINOR GROUPS

Armand Wyler

The Institute for Advanced Study
Princeton, New Jersey 08540

and

University of Zurich,
Switzerland

and the handwritten note by Dyson (when he gave them to Robert Gilmore).

A very sad aspect of all this was found by my friend Ark Jadczyk who "… learned … directly from a fellow

Swiss physicist who was in Geneve at the time .. that Wyler was locked away in an instituion for the insane … and … that Wyler had "lost it" while AT Princeton, and was sent home and institutionalized …".

My guess is that the trigger event whereby Wyler "lost it" was Dyson's inabiilty to understand, and Wyler's inability to explain in a way that Dyson could understand, the physical basis for his work.

What if there had been no Cultural Revolution, and Hua had been free to work with Wyler?

Could Hua have made Wyler's work understandable to Dyson?

If so, would the Standard Model have been developed in the mid-1970s with Calculated Force Strengths and Particle Masses?