Physics of F4, E6, and E8

Frank Dodd (Tony) Smith, Jr., December 2007 (based on Physics Forum discussion with Garrett Lisi, Mitchell Porter, et al, and Steven Weinberg on how to build a physics Lagrangian and some material about an E8 7-grading based on an spr post by Thomas Larsson and some Clifford/Geometric Algebra ideas of David Hestenes and some Octonion ideas of Ian Porteous and some Division Algebra ideas of Geoffrey Dixon and some comments on E8 and helicity and some comments on E8 and Spin-Statistics and Signatures and Pin and Spin and some comments on the history of my model and some comments on E8 and The Golden Compass and an E8 Root Vector movie and some comments on E8 geometry.)

Here is how I see Physics Models based on F4, E6, and E8:

F4

The exceptional Lie algebra f4 =

- so(8) 28 gauge bosons of adjoint of so(8)
- + 8 vectors of vector of so(8)
- + 8 +half-spinors of so(8)
- + 8 -half-spinors of so(8) (mirror image of +half-spinors)

Therefore, you can build a natural Lagrangian from f4 as

- 8 vector = base manifold = 8-dim Kaluza-Klien 4+4 dim spacetime
- fermion term using 8 +half-spinors as left-handed first-generation particles and the 8 -half-spinors as right-handed first-generation antiparticles.
- a normal (for 8-dim spacetime) bivector gauge boson curvature term using the 28 gauge bosons of so(8).

If you let the second and third fermion generations be composites of the first, i.e., if

- the 8 first-generation particles/antiparticles are identified with octonion basis elements denoted by O,
- and you let the second generation be pairs OxO
- and the third generation be triples OxOxO
- and if you let the opposite-handed states of fermions not be fundamental, but come in dynamically when they get mass,

then

f4 looks pretty good IF you can get gravity and the standard model from the 28 so(8) gauge bosons.

If you want to make gravity from 15-dim Conformal Lie algebra so(2,4) by a generalized McDowell-Mansouri mechanism

then you have 28-15 = 13 so(8) generators left over, which are enough to make the 12-dim SM,

BUT

the 15-dim Conformal Gravity and 12-dim Standard Model are not both-at-the-same-time either

- Group-type subroups of Spin(8)
- or Algebra-type Lie algebra subalgebras of so(8)
- or factors of the Weyl group of so(8), since
    - the Weyl group of so(8) is of order 2^3 4! = 8 x 24 = 192

  - the Weyl group of so(2,4) is of order 2^2 3! = 4 x 6 = 24
  - the Weyl group of su(3) is of order 3! = 6
  - the Weyl group of su(2) is of order 2! = 2
  - the Weyl group of u(1) is of order 1! = 1

  Not only does the Weyl group of so(8) have only one factor of 3 while the Conformal Group and Standard Model have two factors of 3, but the total order of the Weyl groups of the Conformal Group and Standard Model is 24 x 6 x 2 x 1 = 288 which is larger than the order 192 of the Weyl group of so(8).

So, if you try to get both the 15 CG and 12 SM to fit inside the 28 so(8),

- you see that they do not fit as Lie Group subgroups
- and you see that they do not fit as Lie algebra subalgebras
- and you see that they do not fit as Weyl group factors

so

what I have done is to look at them as root vectors, where the so(8) root vector polytope has 24 vertices of a 24-cell

- and the Conformal Gravity so(2,4) root vector polytope has 12 vertices of a cuboctahedron
- and the remaining 24-12 = 12 vertices can be projected in a way that gives the 12-dim SM.

My root vector decomposition (using only one so(8) or D4) is one of the things that causes Garrett Lisi to say that I have "... a lot of really weird ideas which ...[he, Garrett]... can't endorse ...".

So, from a conservative point of view, that you must use group or Lie algebra decompositions (not even considering a somewhat unconventional Weyl group factor approach, for which the f4 approach also will not work) ,

f4 will not work because one copy of D4 so(8) is not big enough for gravity and the SM.

Also, f4 has another problem for my approach: f4 has basically real structures, while I use complex-bounded-domain geometry ideas based on ideas of Armand Wyler to calculate force strengths and particle masses.

So, although f4 gives you a nice natural idea of how to build a Lagrangian as

- integral over vector base manifold
- of curvature gauge boson term from adjoint so(8)
- and spinor fermion terms from half-spinors of so(8)

f4 has two problems:

- 1 - no complex bounded domain structure for Wyler stuff (a problem for me)
- 2 - only one D4 (no problem for me, but a problem for more conventional folks).

So, look at bigger exceptional Lie algebra:

E6

e6 is nice, and has complex structure for my Armand Wyler-based calculation of force strengths and particle masses, so e6 solves my problem 1 with f4 and I can and have constructed an e6 model,

but e6 still has only one D4, so e6 is still problematic from the conventional view, as e6 does not solve the conventional problem 2 with f4.

So, do what Garrett Lisi did, and go to the largest exceptional Lie algebra, e8:

E8

If you look at e8 in terms of E8(8) EVIII = Spin(16) + half-spinor of Spin(16)

you see two copies of D4 inside the Spin(16) (Jacques Distler mentioned that) which are enough to describe gravity and the SM.

I think that Garrett's use of e8 is brilliant, and have written up a paper about e8 (and a lot of other stuff) HERE .

There is a link to a pdf version, and there is a misprint on page 2 where I said EVII instead of EVIII, and probably there are more misprints, but as I said in the paper"... Any errors in this paper are not Garrett Lisi's fault. ...".

My view of e8 differs in some details from Garretts:

- I don't use triality for fermion generations, since my second and third generations are composites of the first, as described above in talking about f4
- and I use a different assignment of root vectors to particles etc, which can be seen in an animated rotation using Carl Brannen's root vector java applet see my .mov file HERE . In the movie:
  - There are D4+D4+64 = 24+24+64 = 112 root vectors of Spin(16) :
    - 24 yellow points for one D4 in the Spin(16) in E8
    - 24 purple points for the other D4 in the Spin(16) in E8
    - 64 blue points for the 8 vectors times 8 Dirac gammas in the Spin(16) in E8
  - There are 64+64 = 128 root vectors of a half-spinor of Spin(16) :
    - 64 red points for the 8 first-generation fermion particles times 8 Dirac gammas
    - 64 green points for the 8 first-generation fermion antiparticles time 8 Dirac gamma

---

Steven Weinberg on How to Build a Physics Lagrangian

Given E8 = adjoint Spin(16) + half-spinor Spin(16) and physical interpretation

- There are D4+D4+64 = 24+24+64 = 112 root vectors of Spin(16) :
  - 24 yellow points for one D4 in the Spin(16) in E8 which D4 gives MacDowell-Mansouri Gravity
  - 24 purple points for the other D4 in the Spin(16) in E8 which D4 gives the Standard Model gauge bosons
  - 64 blue points for the 8 vectors times 8 Dirac gammas in the Spin(16) in E8 which vectors give 8-dim Kaluza-Klein spacetime

- There are 64+64 = 128 root vectors of a half-spinor of Spin(16) :
    - 64 red points for the 8 first-generation fermion particles times 8 Dirac gammas
    - 64 green points for the 8 first-generation fermion antiparticles time 8 Dirac gamma

is it natural to put them together to form the Lagrangian of my E8 physics model ?

In the 1986 Dirac Memorial Lectures published in the book Elementary particles and the Laws of Physics (Cambriddge 1987)

Steven Weinberg said (in the following I sometimes substitute the word "fermion" for "electron" and the words "gauge bosons" for "photon" and the words "the equation" for "(1)" referring to equation (1), and I sometimes insert my comments indented and enclosed by brakcets [ ]):

"... Let's examine the following equation:

L =

- PSIbar (gamma^mu d/dx_mu + m) PSI

- (1/4) (d/dx_mu A_nu - d/dx_nu A_mu)^2

+ i e A_mu PSIbar gamma^mu PSI

- MU (d/dx_mu A_nu - d/dx_nu A_mu) PSIbar sigma^mu nu PSI

- G PSIbar PSI PSIbar PSI

+ ...

L stands for Lagrangian density; roughly speaking you can think of it as the density of energy.

Energy is the quantity that determines how the state vector rotates with time, so this is the role that the Lagrangian density plays; it tells us how the system evolves.

L ...[is]... written as a sum of products of fields and their rates of change.

PSI is the field of the fermion (a function of the spacdetime position x), and m is the mass of the fermion.

d/dx_mu means the rate of change of the field with position. ...

the gamma^mu matrices are called Dirac matrices.

A_mu is the field of the gauge bosons ...

Each term has an independent constant, called the coupling constant, that mutiplies it. These are the quantities e , MU , G , ... in the equation. The coupling constant gives the strength with which the term affects the dynamics.

No coupling constant appears in the first two terms simply because I have chosen t absorb them into definition of the two fields PSI and A_mu. ...

Experimentally we know that the formula consisting of just the first three terms, with all higher terms neglected, is adequate to describe electrons and photons to a fantastic level of accuracy. This theory is known as quantum electrodynamics or QED. ...

[An] argument ... why the behaviour of electrons and photons is described by just the first three terms in the equation ... goes back to work by Heisenberg in the 1930s ... The argument is based on dimensional analysis ... I will work in a system of units called physical units, in which Planck's constant and the speed of light are set equal to one. With these choices, mass is the only remaining unit; we can express the dimensions of any quantity as a power of mass.

For example, a distance or time can be expressed as so many inverse grammes. A cross-sction ... is given in terms of som many inverse grammes squared. ...

Now suppose that all interactions have coupling constants that are pure numbers, like the constant e in the third term of the equation ... Then itr would be very easy to figure out what contribution an observable gets from its cloud of virtual gauge bosons and fermion-antfermion pairs at very high energy E.

Lets suppose an observable O has dimensions [mass]^(-a) where a is positive. ... Now, at very high virtual-particle energy, E , much higher than any mass, or any energy of a particle in the initial or final state, there is nothing to fix a unit of energy. The contribution of high energy virtual particles to the observable O must then be given an integral like [the following expression (3)]

O = INTEGRAL(to oo) 1 / E^(a+1) dE

because this is the only quantity wihcih has the right dimensions, the right units, to give the observable O. ... The lower bound in the integral is some finite energy that marks the dividing line between what we call high and low energy. ... This argument only works because there are no other quantities in the theory that have the units of mass or energy. ...

On the other hand, suppose that there are other constants around that have units of mass raised to a negative power. Then if you have an expression involving a constant C_1 with units [mass]^(-b_1) , and another constant C_2 with units [mass]^(-b_2) nd so on, then ... we get a sum of terms of the form [of the following expression (4)]

O = C_1 C_2 ... INTEGRAL(to oo) E^(b_1 + b_2 + ...) / E^(a+1) dE

again because these are the only quantities tha have the right units for the observable O. ...

Expression (3) is perfectly well-defined, the integral converges ... as long as the number a is greater than zero.

However, if b_1 + b_2 + ... is greater than a , then (4) will not be well-defined, because the numerator will have more powers of energy than the denominator and so the integral will diverge.

The point is that no matter how many powers of energy you have in the denominator, i.e. no matter how large a is , (4) eventually will diverge when you get up to sufficiently high order in the coupling constants, C_1 , C_2 , etc., that have dimensionls of negative powers of mass, because if you have enough of these constants, then eventually b+1 + ... is greater than a .

Looking at the Lagrangian density in the equation, we can easily work out what the units of the constant e , MU , G , etc., are.

[In 4-dimensional physical spacetime]... All terms in the Lagrangian density must have units [mass]^4 , because length and time have units of inverse mass and trhe Lagrangian density integrated over spacetime must have no units.

From the m PSIbar PSI term, we see that the fermion field must have units [mass]^(3/2) , because ... [t]he derivative operator (the rate of change operator) has units of [mass]^1 ...[and] 3/2 + 3/2 + 1 = 4 .

> [In an e-dimensional spacetime, the fermion field must have units [mass]^(7/2) , because 7/2 + 7/2 + 1 = 8 .]

The derivative operator (the rate of change operator) has units of [mass]^1 , and so the gauge boson field also has units of [mass]^1.

Now we can work out what the units of the coupling constants are. ...

the electric charge ... e ... turns out to be a pure nuber, to have no units.

But then as you add more and more powers of fields, more and more derivatives, you are adding more and more quantities that have units of positive powers of mass, and since the Lagrangian density [in 4-dimensional physical spacetime]... has to have fixed units of [mass]^4 , therefore the mass dimensions of the associated coupling constants must get lower and lower, until eventually you come to constants like MU and G which have negative units of mass. ... Specifically, MU has the units of [mass]^(-1) , while G has the unts [mass]^(-2) ... Such terms in the equation would completely spoil the agreement between theory and experiment ... so experimentally we can say that they are not there to a fantrastic order of precision and ... it seems that this could be explained by saying that such terms must be excluded because they would give infinite results, as in (4).

... that is exactly waht we are lookign for: a theoretical framework based on quantum mechanics, and a few symmetry principles, in which the specific dynamical principle, the Lagrangian, is only mathematically consistent if it takes one particular form.

At the end of the day, we ... have the feeling that "it could not have been any other way". ...

I described to you the success quantum electrodynamics has had in the theory of photons and electrons ...

In the 1960s these ideas were applied to the weak interactions of the nuclear particles, with a success that became increasingly apparent experimentally during the 1970s.

In the 1970s, the same ideas were applied to the strong interactions of the elementary particles, with results that ... have been increasingly experimentally verified since then.

Today we have a theory based on just such a Lagrangian as given in the equation. In fact,

if you put in some indices on the fields so that there are many fields of each type, then the first three terms of the equation give just the so-called standard model ...

It is a theory that seems to be capable of describing all the physics that is accessible using today's accelerators. ... The standard model works so well because all the terms which could make it look different are naturally extremely small. A lot of work has been done by experimentalists trying to find effects of these tiny terms ... but so far nothing has been discovered.

> [Neutrino masses have been discovered since Weinberg gave his talk in1986, but they can be considered to be part of the lepton sector of the Standard Model.]

So far, no effect except for gravity itself has been discovered coming down to us from the highest energy scale where we think the real truth resides. ... ".

> Some of the ... omissions in the above quote indicate that Weinberg's views stated above reflect his thinking "... until about five or six years ..." before he gave the talk in 1986, and the rest of the talk indicates that his thinking as of 1986 was "... that the ultimate constituents of nature, when you look at nature on a scale of 10^15 - 10^19 GeV , are not particles or fields but strings ...".
>
> I prefer to see string theory in terms of my E6 bosonic string model, with fermions coming from orbifolding and strings being physically interpreted as world-lines of particles, which model is consistent with my E8 physics model which is consistent with the Standard Model plus MacDowell-Mansouri gravity from the Conformal Group which gives a Dark Energy : Dark Matter : Ordinary Matter ratio that is consistent with observations. My E6 and E8 models allow calculation of what Weinberg describes as "... the ... fairly large number ... of free parameters ... that have to be chosen "just so" in order to make the [standard model] theory agree with experiment ...".

E8 graded structure

In a post to the spr thread Re: Structures preserved by e_8 Thomas Larsson says:

"... e_8 also seems to admit a 7-grading,

g = g_-3 + g_-2 + g_-1 + g_0 + g_1 + g_2 + g_3,

of the form

e_8 = 8 + 28* + 56 + (sl(8) + 1) + 56* + 28 + 8* .

Kaneyuki does not mention anything about this, because from his point of view 3- and 5-gradings are more interesting. Incidentally, this grading refutes my claim that mb(3|8) is deeper than anything seen in string theory, since now e_8 also admits a grading of depth 3 and I learned about it in an M theory paper: P West, E_11 and M theory, hep-th/0104081, eqs (3.2) - (3.8). OTOH, the above god-given 7-grading of e_8 is not really useful in M theory, because g_-3 is identified with spacetime translations and one would therefore get that spacetime has 8 dimensions rather than 11. ...".

If you see (sl(8) + 1) as 64 = 8v x 8g, and if you regard the 8v as the basis of an 8-dimensional Kaluza-Klein spacetime and the 8g as its 8 Dirac gammas then you get

e_8 = 8 + 28* + 56 + 8v x 8g + 56* + 28 + 8*

and the even part of the 7-grading has 120 elements

e_8_even = 28* + 8v x 8g + 28 = D4* + 8v x 8g + D4

If you see the 8 Dirac gammas of 8g as corresponding to the octonion basis elements {1,i,j,k,e,ie,je,ke} and denote by 7g those corresponding to the 7 octonion imaginary basis elements {i,j,k,e,ie,je,ke} and denote by 1g the one corresponding to the octonion real basis element {1}, then you get

e_8 = 8 x 1g + 28* + 8 x 7g + 8v x 8g + 8* x 7g + 28 + 8* x 1g

so that if you let the 8 (now denote it by 8s') correspond to the 8 fundamental first-generation fermion particles and the 8* (now denote it 8s") correspond to the 8 fundamental first-generation fermion antiparticles, you get for the odd part of the 7-grading the 128 elements

e_8_odd = 8s' x 1g + 8s' x 7g + 8s" x 7g + 8s" x 1g = 8s' x (1g + 7g) + 8s" x (1g + 7g) =

= 8s' x 8g + 8s" x 8g

This is consistent with the structure of my version of E8 physics in which, as Thomas Larsson says "... spacetime has 8 dimensions ...".

David Hestenes - Left and Right Ideals of Clifford/Geometric Algebra

In Clifford Algebras and Their Applications in Mathematical Physics (Proceedings of the NATO and SERC Workshop, 15-27

September 1985, ed. by J. S. R. Chisholm and A. K. Common (Reidel 1986) at pages 9-10, 23, 327-328), David Hestenes said:

> "... Clifford Algebras ... become vastly richer when given geometrical and/or physical interpretations. When a geometric interpretation is attached to a Clifford Algebra, I prefer to call it a Geometric Algebra, which is the name originally suggested by Clifford himself. ...
>
> the theory of geometric representations should be extended to embrace Lie groups and Lie algebras. A start has been made in ... D. Hestenes and G. Sobczyk, Clifford Algebra to Geometric Calculus, Reidel Publ. Co., Dordrecht/Boston (1984) ... I conjectured there that every Lie algebra is isomorphic to a bivector algebra, that is, an algebra of bivectors under the commutator product. Lawyer-physicist Tony Smith has proved that this conjecture is true by pointing to results already in the literature. ...
>
> the columns of a matrix are minimal left ideals in a matrix algebra, because columns are not mixed by matrix multiplication from the left. The Dirac matrix algebra C(4) has four linearly independent minimal left ideals, because each matrix has four column. The Dirac spinor for an electron or some other fermion can be represented in C(4) as a matrix with nonvanishing elements only in one column, like so

```
PSI_1     0     0     0

PSI_2     0     0     0

PSI_3     0     0     0

PSI_4     0     0     0
```

> where the PSI_i are complex scalars. The question arises: Is there a physical basis for distinguishing between different columns?
>
> The question looks more promising when we replace C(4) by the isomorphic geometric algebra R_4,1 in which every element has a clear geometric meaning. Then the question becomes: Is there a physical basis for distinguishing between different ideals?
>
> The Dirac theory clearly shows that a single ideal (or column if you will) provides a suitable representation for a single fermion. This suggests that each ideal should represent a different kind of fermion, so the space of ideals is seen as a kind of fermion isospace. I developed this idea at length in my dissertation, classifying leptons and baryons in families of four ...".

In the same Workshop proceedings I said I(at pages 377-379, 381-383):

> "... The 16-dimensional spinor representation of Spin(8) reduces to two irreducible 8-dimensional half-spinor representations that can correspond to the 8 fundamental fermion lepton and quark first-generation particle and to their 8 antiparticles ...
>
> Numerical values for force strengths and ratios of particle masses to the electron mass are given. ... Armand Wyler ... (1971), C. R. Acad. Sci. Paris A272, 186 ... wrote a paper in which he purported to calculate the fine structure constant to be a = 1 / 137.03608 ... from the volumes of homogeneous symmetric spaces. ... Joseph Wolf ... (1965), J. Math. Mech. 14, 1033 ... wrote a paper in which he classified the 4-dimensional Riemannian symmetric spaces with quaterniuonic structure. There are just 4 equivalence classes, with the following representatives:
>
> - T4 = U(1)^4
> - S2 x S2 = SU(2) / U(1) x SU(2) / U(1)
> - CP2 = SU(3) / S(U(2) x U(1))
> - S4 = Spin(5) / Spin(4)
>
> ... Final Force Strength Calculation ...

- fine structure constant for electromagnetism = 1 / 137.03608
- weak Fermi constant times proton mass squared = 1.03 x 10^(-5)
- color force constant (at about 10^(-13) cm.) = 0.6286
- gravitational constant times proton mass squared = 3.4 - 8.8 x 10^(-39).

> ... PARTICLE MASSES ...

- the electron mass ...[is assumed to be given at its experimentally observed value]...
- electron-neutrino mass = 0 ... [Note that this is only a tree-level value.] ...
- down quark constituent mass = 312.8 Mev ...
- up qaurk constituent mass = 312.8 Mev ...
- muon mass = 104.8 Mev ...
- muon-neutrino mass = 0 ... [Note that this is only a tree-level value.] ...
- strange quark constituent mass = 523 Mev ...
- charm quark constituent mass = 1.99 Gev ...
- tauon mass = 1.88 Gev ...
- tauon-neutrino mass = 0 ... [Note that this is only a tree-level value.] ...
- beauty quark constituent mass = 5.63 Gev ...
- truth quark constituent mass = 130 Gev ...

> CERN has announced that the truth quark mass is about 45 Gev (Rubbia ... (1984), talk at A.P.S. D.P.F. annual meeting at Santa Fe ... but I think that the phenomena observed by CERN at 45 Gev are weak force phenomena that are poorly explained ... As of the summer of 1985, CERN has been uable to confirm its identification of the truth quark in the 45 Gev events, as the UA1 experimenters have found a lot of events clustering about the charged ... W mass and the UA2 experimenters have not found anything convincing. (Miller ... (1985), Nature 317, 110 ... I think that the clustering of UA1 events near the charged ... W mass indicates that the events observed are ... weak force phenomena. ...".

Since I have been critical of CERN for its error in truth quark obersvations, I should state that my paper in that 1985 Workshop also contained errors, the most conspicuous of which may have been my statement that "... there should be three generations of weak bosons ...".

Mathematical Structure of the 64-dimensional things of the form 8 x 8

Combining the David Hestenes idea of left ideals representing fermions with 8-dimensional D4 half-spinors and an 8-dimensional D4 vector Kaluza-Klein spacetime and 8-dimensional Clifford/Geometric Algebra Dirac gammas gives physical meaning to the three 64-dimensional structures

- 8v x 8g
- 8s' x 8g
- 8s" x 8g

of my version of an E8 physics model. It is useful to study the mathematical structure of such 64-dimensional spaces of the form 8 x 8.

Ian Porteous

Ian Porteous, in his book Clifford Algebras and the Classical Groups (Cambridge 1995) says(page 180-182):

> "... The existence of the Cayley algebra [octonions] depends on the fact that the [64-dimensional] matrix algebra R(8) [of 8 x 8 real matrices] may be regarded as a ... Clifford algebra for the [7-dimensional] positive-definite orthogonal space R7 in such a way that conjugation of the Clifford algebra corresponds to transposition in R(8). For then ... the images of R and R7 in R(8) together span an eight-dimensional linear subspace, passing through ...[the 8-dimensional

unit]... 1 , such that each of its elements, other than zero, is invertible. This eight-dimensional subspace of R(8) will be denoted Y.

Proposition 19.3 Let [the 8-dimensional real space] R8 -> Y be a linear isomorphism. Then the map

R8 x R8 -> R8 ; (a,b) -> a b = (mu(a))(b)

is a bilnear product on R8 such that, for all a,b in R8 , a b = 0 if and only if a = 0 or b = 0 . Moreover, any non-zero element e in R8 can be made the unit element for such a product by choosing mu to be the inverse of the isomorphism

Y -> R8 ; y -> y e .

The division algebra with unit element introduced in Proposition 19.3 is called the Cayley algebra on R8 with unit element e. ... We shall ... speak simply of the Cayley algebra, denoting it by O (for octoniions) ... it is advantageous to select an element of length 1 in R8 ... we seelect e_0 , the zeroth element of the standard basis for R8. ... we have implicitly assigned to R8 its standard positive-definite structure ... The space Y also has an orthogonal structure ... The Cayley algebra O inherits both ... the choice of e as an element of length 1 guarantees that these two structures coincide. ... though the product on R(8) is associative, the product on O need not be. ... The Cayley algebra O is alternative ...".

Geoffrey Dixon

Geoffrey Dixon in hep-th/9303039 says:

"... multiplication tables for ... O are constructable from the following elegant rules: ...

- Imaginary Units ... e_a , a = 1, ..., 7 ,
- Anticommutators ... e_a e_b + e_b e_a = 2 delta_ab ,
- Cyclic Rules ... e_a e_a+1 = e_a-2 = e_a+5 ,
- Index Doubling ... e_a e_b = e_c => e_(2a) e_(2b) = e_(2c) , ...

The octonion algebra is generally considered ill-suited to Clifford algebra theory becauseO is nonassociative, and Clifford algebras are associative. This problem disappears once we identify O as the spinor space of OL , the adjoint algebra of actions of O on itself from the left. OL is associative. ... a complete basis for OL consists of the elements

1 , e_La , e_Lab , e_Labc ,

Therefore OL is 1 + 7 + 21 + 35 = 64-dimensional, and OL ...[is isomorphic to the real 8 x 8 matrix algebra]... R(8) . ... OL is iksomorphic to the Clifford algebra ...[Cl(0,6)]... of the space R^(0,6) , the spinor space of which is 8-dimensional over R . In the case the spinor space is O itself, the object space of OL . ... the algebra OR of right adjoint actions of O on itself is the same algebra as OL . Every action in OR can be written as an action in OL .

A 1-vector basis for OL , playing the role of the Clifford algebra ...[Cl(0,6)]... of R^(0,6) is { e_Lp , p = 1, ..., 6 } .

The resulting 2-vector basis is then { e_Lpq , p,q = 1, ..., 6 , p =/= q } . This subspace is 15-dimensional, closes under the commutator product, and is in that case isomorphic to so(6). The interesection of this Lie algebra with the Lie algebra of the automorphism group of O , G2 , is su(3) , with a basis

su(3) -> { e_Lpq - e_Lrs , p,q,r,s distinct, and from 1 to 6 } .

... SU(3) is the stability group of e_7 , hence the index doubling automorphism of O is an SU(3) rotation ...".

Geoffrey Dixon, in his book Division Algebras, Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics (Kluwer 1986), says (pages 43-45, 141-142, 191-192, 197, 209-211, 215-216) (in the following quote I have changed some notation from l to j and have particularized some division algebra notation from the general division algebra K to the octonion division algebra O) :

> "... An algebraic idempotent, A, is by definition a nonzero element satisfying : A^2 = A . A is nontrivial if A =?= 1 ... [and]...
>
> A (1- A) = A - A^2 = A - A = 0 and (1-A)^2 = 1 - 2A + A^2 = 1 - 2A + A = 1 - A .
>
> So ... 1 - A is aslo an idempotent, and ... A and 1-A are orthogonal. ... nontrivial idempotents are divisors of zero, hence the identity is the sole idempotent of any division algebra ... This ...[does not apply]... to OL = OR = R(8), which is not a division algebra.
>
> Certain elements of OL are diagonal in the adjoint representation. A basis for these consists of the identity, 1_L , together with the e_Labc satisfying e_Labc(1) = e_a (e_b e_c) = 1 ... In particular, define I_a = e_L(3+a)(6+a)(5+a) (indices from 1 to 7 , modulo 7), and let I_0 be the identity. Their adjoint representations are

```
I_0 = 1_L   -> diag(++++++++)
I_1 = e_476 -> diag(+---+-++)
I_2 = e_517 -> diag(++---+-+)
I_3 = e_621 -> diag(+++---+-)
I_4 = e_732 -> diag(+-++---+)
I_5 = e_143 -> diag(++-++---)
I_6 = e_254 -> diag(+-+-++--)
I_7 = e_365 -> diag(+--+-++-)
```

> ... Being diagonal, the I_a clearly commute. They also satisfy I_a I_a+1 = I_a+3 , a in {1, ..., 7} (had e_a e_a+1 = e_a+3 been chosen as the multiplication for O , then ... I_a I_a+1 = I_a+5, so these choices are in this manner dual to each other ...) ...
>
> the identity of OL can be elegantly resolved into orthogonal primitive idempotents using the I_a. A primitive idempotent can not be expressed as the sum of two other idempotents ... orthogonal primitive idempotents resolving the identity of OL ... are ...

```
P_0 = (1/8) ( 1 + e_L476 + e_L517 + e_L621 + e_L732 + e_L143 + e_L254 + e
+L365 ) ,
P_1 = (1/8) ( 1 - e_L476 + e_L517 + e_L621 - e_L732 + e_L143 - e_L254 - e
+L365 ) ,
P_2 = (1/8) ( 1 - e_L476 - e_L517 + e_L621 + e_L732 - e_L143 + e_L254 - e
+L365 ) ,
P_3 = (1/8) ( 1 - e_L476 - e_L517 - e_L621 + e_L732 + e_L143 - e_L254 + e
+L365 ) ,
P_4 = (1/8) ( 1 + e_L476 - e_L517 - e_L621 - e_L732 + e_L143 + e_L254 - e
+L365 ) ,
P_5 = (1/8) ( 1 - e_L476 + e_L517 - e_L621 - e_L732 - e_L143 + e_L254 + e
+L365 ) ,
P_6 = (1/8) ( 1 + e_L476 - e_L517 + e_L621 - e_L732 - e_L143 - e_L254 + e
+L365 ) ,
P_7 = (1/8) ( 1 + e_L476 + e_L517 - e_L621 + e_L732 - e_L143 - e_L254 - e
+L365 ) ,
```

> ... These satisfy SUM(a=0 to 7) P_a = 1 , and P_a P_b = delta_ab P_b . ... They are related as follows (a in {0, 1, ..., 7}):
>
> - P_a = e_La P_0 e*_La ;

- P_a e_La = e_La P_0 ;
- e_La P_a = P_0 e_La ;

if e_a e_b = e_c (a,b,c in {1, ..., 7}, then e_La P_0 e_Lb = - e_Lb P_c e_La ...

for example ... (P_0 + P_1 + P_2 + P_6) is an idempotent projecting from O a subalgebra isomorphic to Q:

(P_0 + P_1 + P_2 + P_6) O = Q Likewise ... (P_0 + P_1) O = C ... and ... P_0 O = R . ...

The matematical context upon which the model building ... rests relied heavily on treating the ... division algebras as spinor spaces of their left adjoint algebras (identified as Clifford algebras), of tensoring those adjoint algebras with ... [the 2 x 2 real matrix algebra]... R(2) (doubling the size of the spinor space) ... These ... same methods will be employed here to generate bases for the Lie algebras of the groups of a version of the magic square. Each will be derived from a tensor product of two division algebras ...

The foundation upon which the method rests is R(2) . In R(2) define ...

$$E = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \qquad A = \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} \qquad B = \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \qquad W = \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$$

Let O(x)O be the tensor product of two ...[copies of the octonion division algebra O]...

Let c_k ... denote ...[a basis]... for the pure hypercomplex part...[of O]... In ... O(x)O (2) the elements

W , c_Lk A , c_LljB

anticommute (and associate) and form the basis for the 1-vector generators of a ... Clifford algebra with negative definite Euclidean metric. Under commutation they generate the 2-vectors

c_Lk B , c_Lj A , c_Lk1k2 E , c_Lj1j2 E , c_Lk c_Lk W ,

Together ...[those]... elements form a basis for a representation of the Lie algebra so(dimO + dimO). I'll call this External_OO and call it the external subalgebra.

To this collection we now add the spinors of O(x)O(2) , namely the elements of (O(x)O)^2 , without yet specifying a commutator product on this linear space. I'll denote this Spinor_OO . ...

The total resulting linear space will be denoted MS_OO , MS for magic square ...

Let e_La and e'_Lb be distinct and mustually commuting bases for the hypercomplex octonions.

External_OO is spanned by

W , e_La A , e'_La B (1-vector basis for ...[the Clifford algebra Cl(0,15)]...) and

e_Lab B , e'_Lab A , e_La e'_Lb W , e_Lab E , e'_Lab E (2-vectors)

...[with dimension 1 + 7 + 7 + 21 + 21 + 21 + 21 + 21 = 1+14 +105 = 120]...

External_OO = so(16) .

Spinor_OO is 128-dimensional, and ... because OL = OR ...[there is no Internal_OO]...

That's 120 + 128 = 248 elements altogether, and we make the identification:

MS_OO = LE8 . ...

Getting LE8 from O(x)J3(O) ...[where J3(O) is the 27-dimensional exceptional Jordan algebra]... is slightly trickier. In this case there are two distinct copies of O commuting with each other (denote them O1 and O2) ...

We begin .. with the 28 so(8) generators ...[that]... are elements of LF4... and the 3 so(3) generators ... [that]... account for 3 of the 52 dimensions of LF4 ... Together ...[they]... account... for 3 + 28 = 31 of the 52-dimensional LF4. ...

in ...[this]... O1(x)J3(O2) case we expand so(3) to LF4 , the Lie algebra of the sutomorphism of J3(O1) . That gives us 28 elements from so(8 , and 52 elements from LF4 (which contains another distinct so(8)). Of the 52 generators of theis new LF4 , 28 are diagonal ... and 24 are off-diagonal. Commutators of the 28 diagonal generators (the so(8) of O1) with the so(8) of O2 yield nothing new, but each of the 24 off-diagonal generators gives rise to a 7-dimensional space of new generators. That yields,

28 + 52 + 168 = 248

generators all together, and the set closes here on LE8 ...".

Note that 168 is the order of PSL(2,7) = SL(3,2) which can be thought of as the group of linear fractional transformations of the vertices of a heptagon and is so related to octonion multiplication rules, and that SL(2,7) of order 336 double covers the Klein Quartic.

E8 Physics and Helicity

In E8 physics,

- the 8 first-generation fermion particles and 8 Dirac gammas are represented by 8x8 = 64 of the 128 half-spinor Spin(16) elements of E8 and
- the 8 first-generation fermion antiparticles and 8 Dirac gammas are represented by the other 8x8 = 64 of the 128 half-spinor Spin (16) elements of E8.

Since the all belong to one half-spinor representation of Spin(16), they all have the same helicity. Let that helicity correspond to left-handed fermion particles.

Since antiparticles are effectively particles travelling backward in time, the corresponding helicity for fermion antiparticles is right-handed.

Therefore, in E8 physics, fermion particles are fundamentally left-handed and fermion antiparticles are fundamentally right-handed.

Opposite handedness arises dynamically, and can be seen in experiments involving massive fermions moving at much less than the speed of light.

L. B. Okun, in his book Leptons and Quarks (North-Holland (2nd printing 1984) page 11) said:

"... a particle with spin in the direction opposite to that of its momentum ...[is]... said to possess left-handed helicity, or left-handed polarization. A particle is said to possess right-handed helicity, or polarization, if its spin is directed along its momentum. The concept of helicity is not Lorentz invariant if the particle mass is non-zero. The helicity of such a particle depends oupon the motion of the observer's frame of reference. For example, it will change sign if we try to catch up with the particle at a speed above its velocity. Overtaking a particle is the more difficult, the higher its velocity, so that helicity becomes a better quantum number as velocity increases. It is an exact quantum number for massless particles ...

The above space-time structure ... means ... that at ...[v -> speed of light]... particles have only left-handed helicity, and antparticles only right-handed helicity. ...".

E8 and Spin-Statistics and Signatures and Pin and Spin

Soji Kaneyuki has written a chapter entitled Graded Lie Algebras, Related Geometric Structures, and Pseudo-hermitian Symmetric Spaces, as Part II of the book Analysis and Geometry on Complex Homogeneous Domains, by Jacques Faraut, Soji Kaneyuki, Adam Koranyi, Qi-keng Lu, and Guy Roos (Birkhauser 2000). Kaneyuki lists a Table of Exceptional Simple Graded Lie Algebras of the Second Kind including

e(17) for which g = E8(8)

- g(+2) = 14
- g(+1) = 64 = 8 fermion particles x 8 Dirac gammas
- g(0) = so(7,7) + R
- g(-1) = 64 = 8 fermion antiparticlex x 8 Dirac gammas
- g(-2) = 14

Kaneyuki also considers the even part of such algebras

g(ev) = g(-2) + g(0) + g(2)

= 14 + so(7,7)+R + 14 = 14 + 92 + 14 = 120 = so(8,8) = so(7,1) + 64 + so(1,7)

- The step immediately above is by real Clifford periodicity Cl(16) = Cl(8) (x) Cl(8) and
- preserving the (7,7) substructure by adding (0,1) and (1,0) to it to get so(7,1) + so(1,7)

= so(7,1) + so(1,7) + 8-dim Kaluza-Klein spacetime x 8 Dirac gammas

If all 120 g(ev) generators are physically bosonic and if all 128 generators of the odd g(-1) and g(+1) are physically fermionic then under E8

- fermion times fermion = boson
- boson times boson = boson
- boson times fermion = fermion times boson = fermion

so Spin-Statistics is satisfied.

As to signature (diagram from Spinors and Calibrations, by F. Reese Harvey (Academic 1990)):

$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$	$M_{64}(\mathbf{C})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R}) \oplus M_{128}(\mathbf{R})$	$M_{256}(\mathbf{R})$
$M_{8}(\mathbf{C})$	$M_{8}(\mathbf{H})$	$M_{8}(\mathbf{H}) \oplus M_{8}(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{R}) \oplus M_{64}(\mathbf{R})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{C})$
$M_{4}(\mathbf{H})$	$M_{4}(\mathbf{H}) \oplus M_{4}(\mathbf{H})$	$M_{8}(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{C})$	$M_{64}(\mathbf{H})$
$M_{2}(\mathbf{H}) \oplus M_{2}(\mathbf{H})$	$M_{4}(\mathbf{H})$	$M_{8}(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{C})$	$M_{32}(\mathbf{H})$	$M_{32}(\mathbf{H}) \oplus M_{32}(\mathbf{H})$
$M_{2}(\mathbf{H})$	$M_{4}(\mathbf{C})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{R}) \oplus M_{8}(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$
$M_{2}(\mathbf{C})$	$M_{4}(\mathbf{R})$	$M_{4}(\mathbf{R}) \oplus M_{4}(\mathbf{R})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{C})$	$M_{8}(\mathbf{H})$	$M_{8}(\mathbf{H}) \oplus M_{8}(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$
$M_{2}(\mathbf{R})$	$M_{2}(\mathbf{R}) \oplus M_{2}(\mathbf{R})$	$M_{4}(\mathbf{R})$	$M_{4}(\mathbf{C})$	$M_{4}(\mathbf{H})$	$M_{4}(\mathbf{H}) \oplus M_{4}(\mathbf{H})$	$M_{8}(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$
$\mathbf{R} \oplus \mathbf{R}$	$M_{2}(\mathbf{R})$	$M_{2}(\mathbf{C})$	$M_{2}(\mathbf{H})$	$M_{2}(\mathbf{H}) \oplus M_{2}(\mathbf{H})$	$M_{4}(\mathbf{H})$	$M_{8}(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H} \oplus \mathbf{H}$	$M_{2}(\mathbf{H})$	$M_{4}(\mathbf{C})$	$M_{8}(\mathbf{R})$	$M_{8}(\mathbf{R}) \oplus M_{8}(\mathbf{R})$	$M_{16}(\mathbf{R})$

Cl(7,1) is the 8x8 Quaternion Matrix Algebra M(Q,8)

Cl(1,7) = Cl(0,8) is the 16x16 Real Matrix Algebra M(R,16) which has effective Octonionic structure.

If a preferred Quaternionic subspace is frozen out of the Octonionic spacetime of Cl(1,7), then its 8-dimensional (1,7) vector spacetime undergoes dimensional reduction to

- 4-dimensional (1,3) associative physical spacetime plus
- 4-dimensional (0,4) coassociative CP2 internal symmetry space

and Cl(1,7) is transformed into quaternionic Cl(2,6) = M(Q,8).

After dimensional reduction, since Cl(1,7) = Cl(2,6) = M(Q,8) you effectively have two copies of Cl(2,6) = M(Q,8).

Note that some might object that Spin(p,q) does not come directly from Cl(p,q) but rather comes from its even subalgebra, so that sometimes when I write Spin(p,q) I should be writing Pin(p,q), where, as Ian Porteous says in his book Clifford Algebras and the Classical Groups (Cambridge 1995):

> "... the Pin and Spin groups doubly cover the relevant orthogonal and special orthogonal groups.
>
> Proposition 16.14 Let X be a non-degenerate quadratic space of positive finite dimension. Then the maps
>
> PinX -> O(X) ... and SpinX -> SO(X) ...
>
> are surjective, the kernel in each case being isomorphic to S0 [the zero-sphere { -1, +1 }]...
>
> When X = R(p,q) the standard notations for [the Clifford group] GAMMA(X) ...[and for] ... GAMMA0((X) , PinX and SpinX will be GAMMA(p,q) , GAMMA0(p,q) , Pin(p,q) and Spin(p,q).
>
> Since ...[the even Clifford subalgebra Cle(p,q) is isomorphic to the even Clifford subalgebra Cle(q,p)]...
>
> - GAMMA0(q,p) is isomorphic to GAMMA0(p,q) and
> - Spin(q,p) is isomorphic to Spin(p,q).

Finally, GAMMA0(0,n) is often abbreviated to GAMMA0(n) and Spin(0,n) to Spin(n). ...".

Further, Pertti Lounesto says in his book Clifford Algebras and Spinors (Second Edition Cambridge 2001):

"... 17.2 The Lipschitz grooups and the spin groups The Lipschitz group GAMMA(p,q) , also called the Clifford group although invented by Lipschitz 1880/86 , could be defined as the subgroup in Cl(p,q) generated by invertible vectors x in R(p,q) ...

The Lipshitz group has a normalized subgroup Pin(p,q) ... The group Pin(p,q) has an even subgroup Spin(p,q) ...".

Further, in Spinors and Calibrations (Academic 1990) F. Reese Harvey says:

"... The Grassmannians and Reflections ... G(r,V) ...[is]... the grassmannian of all unit, oriented, nondegenerate r-planes through the origin in V ...[G(r,V)]... consists of all simple vectors in /\r(V) that are of unit length. That is,
u is in G(r,V) if u = u_1 /\ ... /\ u_r with u_1 , ... , u_r in V and |u| = 1 (||u|| = +/-1) .

... Given u in G(r,V) , reflection along u , denoted R_u , is defined by

R_u(x) = -x if x is in span(u) and R_u(x) = x if s is ...[orthogonal to]... span(u) ...

... Remark 10.20 ... each reflection R_u in O(V) along a subspace span u of V is replaced in the double cover Pin(V) of O (V) be either of the two elements +/-u in G(r,V) in /\r(v) in Cl(V) in the Clifford algebra. ...

By definition, the group Pin is generated by the element G(1,V) in Pin. ... the definition of Spin ... suffers from the defect that the generators u in G(1,V) are not in Spin. This defect can be corrected .. if e is any unit vector and S(n-1) denotes the unit sphere in V , then e.S(n-1) generates Spin ... In addition ... Proposition 10.21 (n = dim(V) >= 3). The group Spin is the subgroup of Cl*(V) ... of invertible elements in ... Cl(V) ... generated by G(2,V) ...".

What is the physical difference between Pin and Spin?

Roughly, Pin has reflections and so can map fermion particles into fermion antiparticles. In the example of Cl(8) = M(R,16) , Pin(8) sees spinors as 1x 16 columns like

```
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 = x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
x 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0   x
```

while Spin has no reflections, so Spin(8) sees spinors as two mirror-image sets of 8 +half-spinor particles and 8 -half-spinor antiparticles like

```
x 0 0 0 0 0 0 0                   x
```

```
x 0 0 0 0 0 0 0                   x
x 0 0 0 0 0 0 0                   x
x 0 0 0 0 0 0 0                   x
x 0 0 0 0 0 0 0                   x
x 0 0 0 0 0 0 0                   x
x 0 0 0 0 0 0 0                   x
x 0 0 0 0 0 0 0                   x
                                =   +
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
                x 0 0 0 0 0 0 0       x
```

In the quaternionic example of Cl(2,6) = M(Q,8) , Pin(2,6) sees spinors as

```
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0 = X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
```

while Spin(2,6) sees spinors as

```
X 0 0 0           X
X 0 0 0           X
X 0 0 0           X
X 0 0 0           X
                =   +
        X 0 0 0       X
        X 0 0 0       X
        X 0 0 0       X
        X 0 0 0       X
```

In the quaternionic example of Cl(2,4) = Cl(6,0) = M(Q,4) , Pin sees spinors as

```
X 0 0 0   X
X 0 0 0 = X
X 0 0 0   X
X 0 0 0   X
```

while Spin(2,4) (in my view where the even Cle(2,4) is taken to be Cl(1,4) = M(Q,2)+M(Q,2) instead of Cl(2,3) = M(C,4)) sees spinors as

```
X 0       X
X 0       X
        =   +
```

```
X 0       X
X 0       X
```

As to Cl(2,3) = M(C,4), my view is that its even Cle(2,3) is taken to be Cl(1,3) = M(Q,2) instead of Cl(2,2) = M(R(4).

As to Cl(1,4) = M(Q,2)+M(Q,2), my view is that even Cle(1,4) is taken to be Cl(1,3) = M(Q,2) = Cl(0,4).

In short, for E8 physics I form even subalgebras from Cl(2,6) on down to Cl(1,3) so that quaternionic structure is maintained.

I think that Pin is more fundamental than Spin because the overall symmetry should include reflections that can transform between particles and antiparticles, even though the particle-antiparticle distinction is useful in setting up the structure of the E8 model and its Lagrangian. However, Spin is more widely known than Pin, so sometimes (particularly in exposition) I write Spin when Pin would be technically more nearly correct.

Some History of my Physics Model

In the 1960s-early 1970s Armand Wyler wrote a calculation of the fine structure constant using geometry of bounded complex domains. It was publicized briefly (almost as much as Garrett Lisi's E8 model is publicized now) but Wyler never showed convincing physical motivation for his interpretation of the math structures, and it was severely ridiculed and ignored (with sad personal consequences for Wyler).

Also in the 1960s, Joseph Wolf classified 4-dim spaces with quatenionic structure:

- (I) Euclidean 4-space [the 4-torus T4];
- (II) SU(2) / S(U(1)xU(1)) x SU(2) / S(U(1)xU(1)), … [S2 x S2] …;
- (III) SU(3) / S(U(2)xU(1)), … [CP2] …; and
- (IV) Sp(2) / Sp(1)xSp(1) … [= Spin(5) / Spin(4) = S4] …,

and the noncompact duals of II, III, and IV

and I noticed that they corresponded to

- U(1) electromagnetism,
- SU(2) weak force,
- SU(3) color force, and
- Sp(2) MacDowell-Mansouri gravity

so

I thought that it might possibly be useful to apply Wyler's approach to the geometries of those 4-dim quaternionic structures.

It was only in the 1980s that I was able to cut back on the time devoted to my law practice to try to learn enough math/physics to try to work out the application of Wyler's stuff to Wolf's classification, and I did so by spending a lot of time at Georgia Tech auditing seminars etc of David Finkelstein (who was tolerant enough to allow me to do so). I had learned some Lie group / Lie algebra math while an undergrad at Princeton (1959-63), but I did not know Clifford algebras very well until studying under David Finkelstein.

Then (early 1980s) N=8 supergravity was popular, so I looked at SO(8) and its cover Spin(8), and noticed that:

- Adjoint Spin(8) had 28 gauge bosons enough to do MacDowell-Mansouri gravity plus the Standard Model, but not if they were included as conventional subgroups;
- Vector Spin(8) looked like 8-dim spacetime;
- +half-spinor Spin(8) looked like 8 left-handed first-generation fermions;
- -half-spinor Spin(8) looked like 8 right-handed first-generation fermions.

To break the 8-dim spacetime into a 4-dim physical spacetime plus a 4-dim internal symmetry space I used the geometric methods that had been developed by Meinhard Mayer (working with Andrzej Trautman) around 1981.

A consequence of that dimensional reduction was second and third generations of fermions as composites (pairs and triples) of states corresponding to the first-generation fermions.

When I played with the Wyler-type geometry stuff, I got particle masses that looked roughly realistic, and a (then) prediction-calculation of the Tquark mass as around 130 GeV (tree-level, so give or take 10% or so).

When in 1984 CERN announced at APS DPF Santa Fe that they had seen the Tquark at 45 GeV, I gave a talk there (not nearly as well-attended as Carlo Rubbia's) saying that CERN was wrong and the Tquark was more massive (I will not here go into subsequent history of Dalitz, Goldstein, Sliwa, CDF, etc except to say that I still feel that experimental data supports the Tquark having a low-mass state around 130-145 GeV, and that the politics related to my position may have something to do with my current outcast status with the USA physics establishment.)

Since Spin(8) is bivector Clifford algebra of the real Clifford algebra Cl(8), and since real Clifford algebra 8-periodicity means that any very large real Clifford algebra can be factored into tensor products of Cl(8), it can be a building block of a nice big algebraic QFT (a real version of the complex hyperfinite II1 von Neumann factor).

Since the Adjoint, Vector, and two half-Spinor reps of Spin(8) combine to form the exceptional Lie algebra F4, I tried to use it as a unifying Lie algebra,

but I eventually saw that the real structure of F4 was incompatible with the complex bounded domain structures of the Wyler approach, so I went to E6, which is roughly a complexification of F4, and used E6 to construct a substantially realistic version of 26-dim bosonic string theory (fermions coming from orbifolding). Since by then I was blacklisted by the Cornell arXiv, I put that up on the CERN website as CERN-CDS-EXT-2004-031

As of then, the major conventional objection to my model was how I got 16 generators for a MacDowell-Mansouri gravity U(2,2) and 12 generators for the Standard Model from the 28 generators of Spin(8) (I used root vector patterns, because they do not consistently fit as subgroups and subalgebras).

Now, Garrett Lisi's E8 model has two copies of the D4 Spin(8) Lie algebra, so I can use it to be more conventional and get MacDowell-Mansouri gravity from one D4 and the Standard Model from the other one, so I wrote that up in my 82-page pdf paper at

http://tony5m17h.net/GLE8Cl8TSxtnd.pdf

Note that now I am not only blacklisted by arXiv, but pressure forced CERN to terminate its external preprint service, so I cannot even put it there as I was able to do in 2004 with my E6 string model.

All the gory details of calculations are set out in my 82-page paper, so I won't go into any more detail here.

I apologize for, in trying to be brief, leaving out a lot of people who helped me learn stuff, including but not limited to people at the University of Alabama and Robert Gilmore at Drexel and others.

PS - I should add that while at Georgia Tech in the late 1980s -early 1990s I enrolled in the physics PhD program, but that ended when I encountered the comprehensive exam (a 3-day closed book test) which I could not pass (my then 50-year-old memory had trouble recalling formulas), so I am in that sense a failure without official PhD qualification.
