IFA, Adinkra, Llull, E8 and Physics

Frank Dodd (Tony) Smith, Jr. - 2011 - [My comments within quoted material are in red.]

IFA

At least as far back as 12,000 years ago, Africans had developed IFA Oracle
divination based on the square of 16 = 16x16 = 256 = 2^8
corresponding to the vertices of an 8-dimensional hypercube
and to the binary 2-choice Clifford algebra Cl(8) and
so to related ones such as Cl(8)xCl(8) = Cl(16).
One IFA way to choose among the 2^8 possibilities is to cast an Opele Chain

(image from Folkcuba.com web site)

of 8 shells that when cast can land either up or down.

Since the number of sub-hypercubes in an 8-dimensional hypercube
is 6,561 = 81x81 = 3^8,
the IFA Oracle has N=8 ternary 3-structure as well as binary 2-structure:

N	2^N	3^N
0	1	
1	2	3
2	4 =2x2	9 = 3x3
3	8	27
4	16 = 4x4	81 = 9x9
5	32	243
6	64 = 8x8	729 = 27x27
7	128	2187
8	256 = 16x16	6561 = 81x81

As ancient African games such as Owari show, binary 2- structure corresponds to
static states and ternary 3- structure corresponds to dynamic states.
Mathematically, using binary 2-choice static states to define dynamics on 3 ternary
neighbor states produces the 256 elements of Elementary Cellular Automata.

“... **Adinkra** ...

Adinkra are visual forms that ... integrate striking aesthetic power, evocative mathematical structures, and philosophical conceptions ... against the background of the cosmos. This cosmic framework is suggested by the Adinkra symbol Gye Nyame ... meaning ... “This great panorama of creation dates back to time immemorial, no one lives who saw its beginning and no one will live to see its end, except Nyame [= God]. ...”...
similarities between Classical Andinkra and the mathematical technology ... are ... developed by James Gates and Michael Faux ... to model relationships between fundamental physical structures of the universe. ...”. (quote from The Oxford Encyclopedia of African Thought, Vol. 1, by F. Abiola Irele and Biodun Jeyifo)

The African Adinkra character Gye Nyame  looks like an Opele Chain

and also

 looks like the Chinese character meaning Law 

The Chinese character’s pronunciation FA was probably introduced into Mandarin after 1000 AD. It is possible that the similarity of the Chinese pronunciation FA and the African FA is a result of African influence on China through contact by Chinese ocean voyages around 1000 AD, and the role of IFA as Divination Law.

Llull

Near the end of the 13th century, Ramon Llull of Mallorca studied the 16 possibilities of IFA (in the Arabic form of the Ilm al Raml) and realized that they had a Fundamental Organizational Principle that he summarized in a Wheel Diagram with 16 vertices connected to each other by 120 lines.

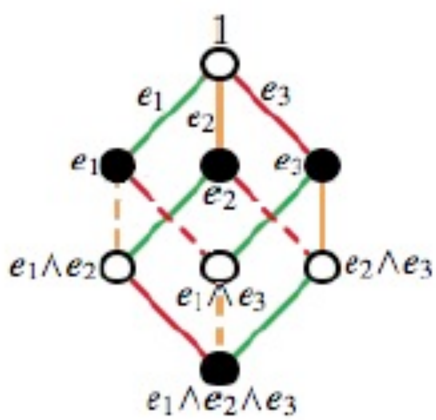
If the 16 vertices represented a 16-dimensional vector space, then the 120 lines connecting pairs of vectors represented 120-dimensional D8 bivectors of rotations in that 16-dimensional vector space. That total geometry is described by the Real Clifford Algebra Cl(16) with 16-dim vector space.
Some Llullian ideas seem to be encoded in Tarot, which was developed roughly contemporaneously with Llull, and whose cards correspond to the 78-dimensional E6 Lie algebra.

E8

The 120 D8 bivectors combine with 128 Cl(16) half-spinors to produce E8.
In 2007, Garrett Lisi used the 248-dim E8 Lie algebraas the basis for a Physics Model, but his model has been shown to have technical flaws.
However,
it is possible (as outlined in this paper) to construct a realistic E8 Physics Model which is free of such technical flaws.

Physics

Sylvia Naples in a May 2009 Bard College Senior Project at
http://math.bard.edu/student/pdfs/sylvia-naples.pdf
said: "... In a supersymmetric theory, every ... fermion ... corresponds to a partner particle, or superpartner, ... boson ... A supersymmetry transformation turns a fermion, a particle with half integer spin, into a boson with integer spin. ... supersymmetry must be spontaneously broken; if there were superpartners with the same mass as known particles, we would have observed them. If supersymmetry is broken, superpartners would have greater masses, allowing us to observe them only at high energies. ...
Adinkra is indigenous to Western Africa ...[African IFA divination is directly related to cubic 2^N structures with 2^8 as fundamental.]...
In [hep-th/0408004]... Faux and Gates define Adinkra graphs [the most fundamental of which seem to me to correspond to African IFA Divination]... that visually represent off-shell N-extended one-dimensional supersymmetry ... N-extended supersymmetries ... are combined to form more intricate structures with additional supersymmetries ... the representation of supersymmetry in multiple dimensions is encoded in the representation theory of one-dimensional supersymmetry algebras ...
In this paper, our main concern is the classification of Adinkra graphs. ...
The free Adinkra is a graph that gives a description of the exterior algebra ...
[Clifford algebra adds spinor structure to the graded framework of the exterior algebra.] ...
The vertices of the free Adinkra correspond to elements of the basis of the exterior algebra, and the edges of the free Adinkra correspond to the wedge operation ...



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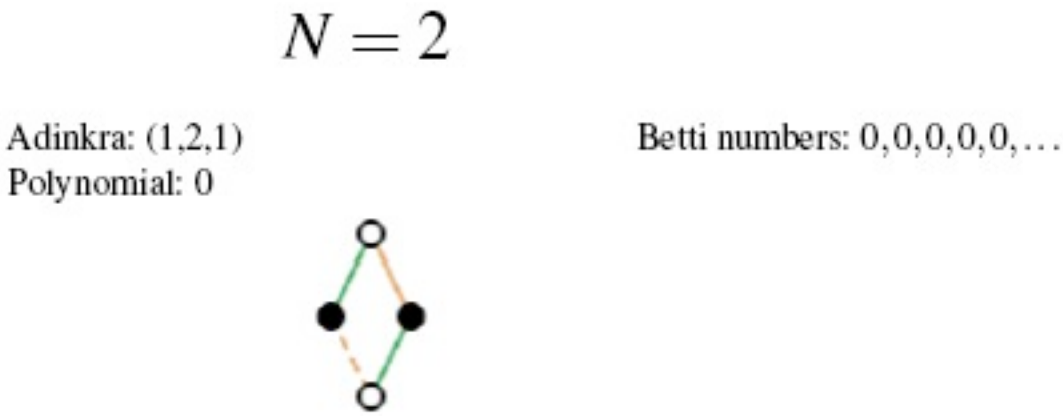

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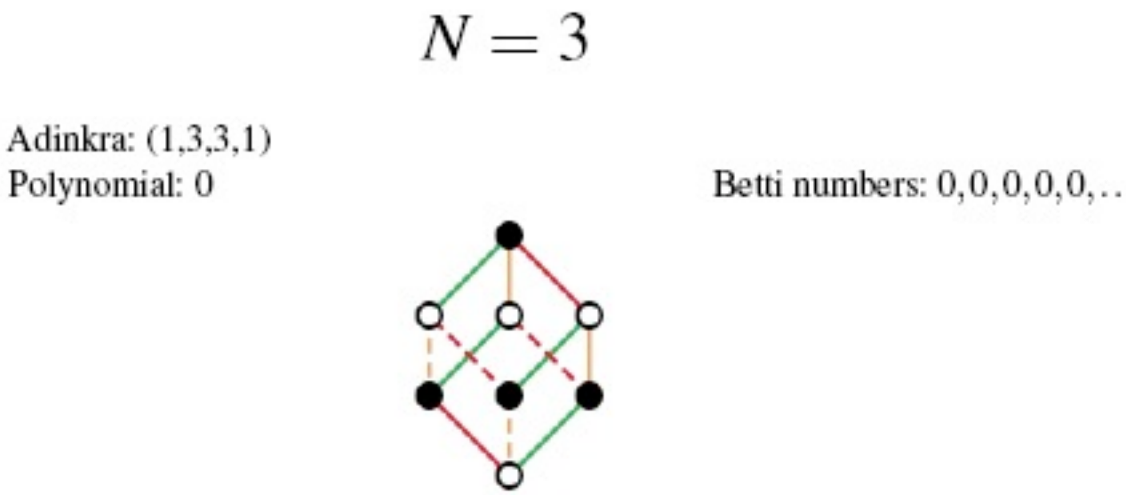

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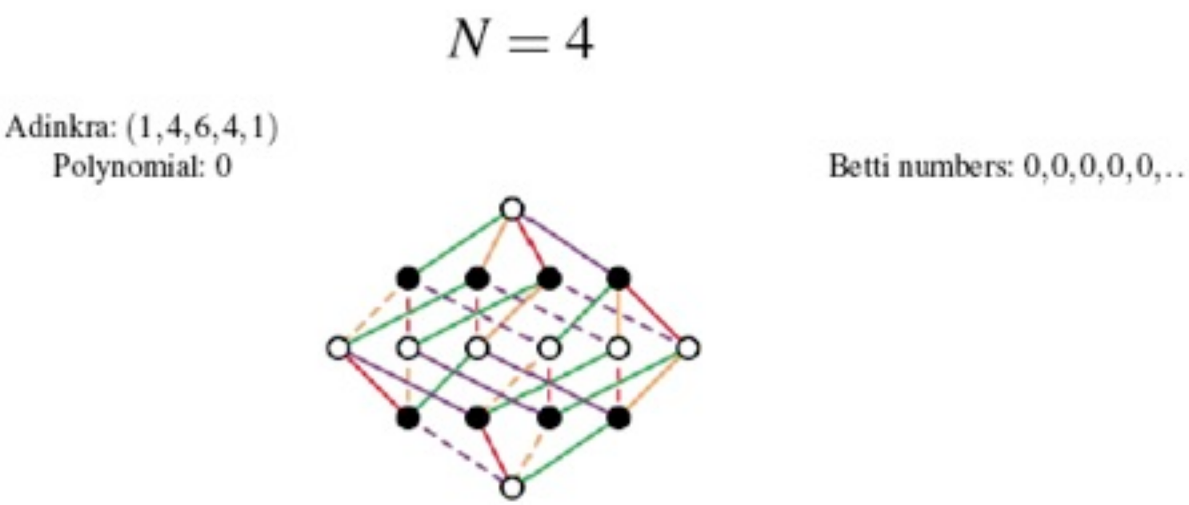
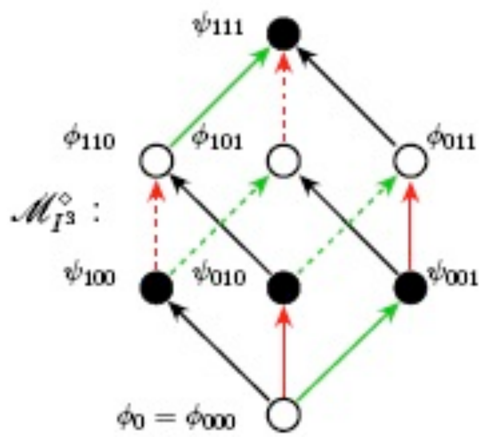


...". [As Clifford Algebras, (1,2,1) corresponds to Cl(2) = Q = Quaternions, (1,3,3,1) to Cl(3) = Q + Q, and (1,4,6,4,1) to Cl(4) = M(2,Q) = 2x2 Q matrices.]

Gates and co-authors Doran, Faux, Hubsch, Iga, Landweber, and Miller in arxiv 0806.0050 [hep-th] said:
“... Adinkras are directed graphs with various colorings and other markings on vertices and edges ... The fundamental example of an Adinkra topology is that of the N-cube, I^N = [0; 1]N. It has 2^N vertices and N x 2^(N-1) edges.
We may embed it in R^N by locating the vertices at the points
p = (p1, ... , pN) in R^N, where pI = 0 or 1 in all 2^N possible combinations. ...
For every vertex, p, the weight of p, written wt(p), equals the number of J in {1, ... ,N} for which pJ = 1. [Weight corresponds to Clifford Algebra grade.]
...
As the weights of the vertices are either odd or even, we color them either black or white, respectively ... black for fermions ... white for bosons ... [In Clifford Algebra, even corresponds to the Even Subalgebra of Cl(N) and odd to the Odd Part of Cl(N). In Thomas Larsson’s E8 7-grading

8 + 28 + 56 + 64 + 56 + 28 + 8

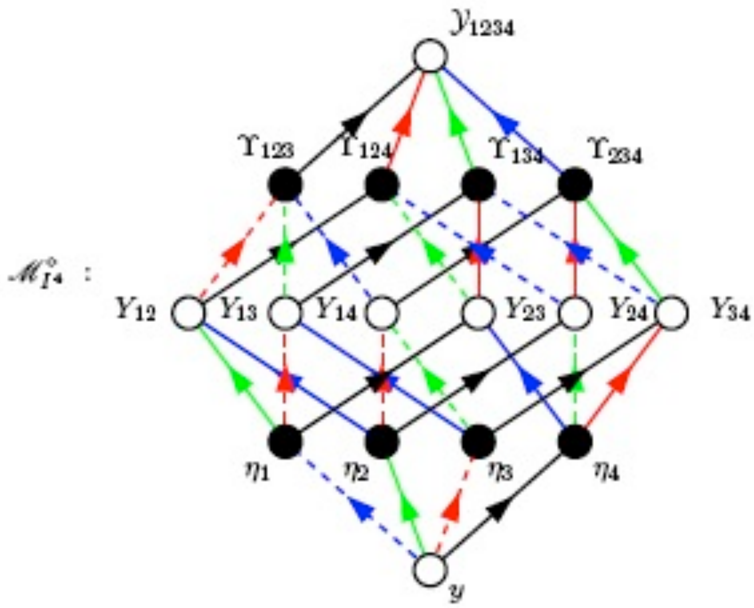
even corresponds to the 120-dim adjoint of Spin(16) D8 and
odd corresponds to a 128-dim half-spinor of Spin(16) D8.] ...
We associate the numbers from 1 to N with N different colors ... The result is called the colored N-cube.
...
We now construct ... MI^Ncubic ... Adinkras ... called Top Clifford Algebra superfields ... let N = 3 ...



...”.

[In the MI^Ncubic picture interpreted as a Clifford Algebra,
the 3 grade-1 elements correspond to 3-dim space,
the 3 grade-2 elements to spin(3) = su(2) covering of the rotations in 3-dim space,
and the spinors to Quaternions Q.]

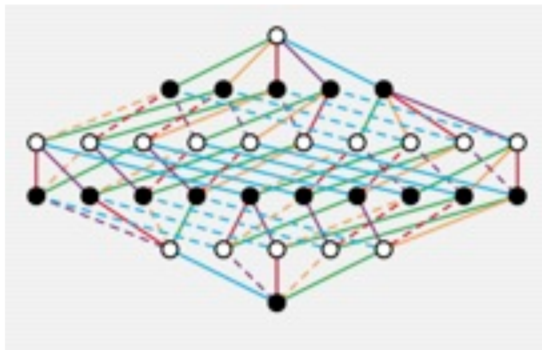
Gates and co-authors Doran, Faux, Hubsch, Iga, Landweber, and Miller in arxiv 0811.3410 [hep-th] said:
"... Herein, we relate Adinkras to Clifford algebras ...
The extreme example is when the Adinkra is one-hooked], that is, there is only one vertex v of lowest engineering dimension [grade], and that this is the unique vertex having all its adjacent edges oriented away from it ...
In the one-hooked case ... there is only one Adinkra possible, with only one Adinkra topology, and with only one doubly even code capable of describing it. ...
Here is an example of a hypercube I^4 topology ...



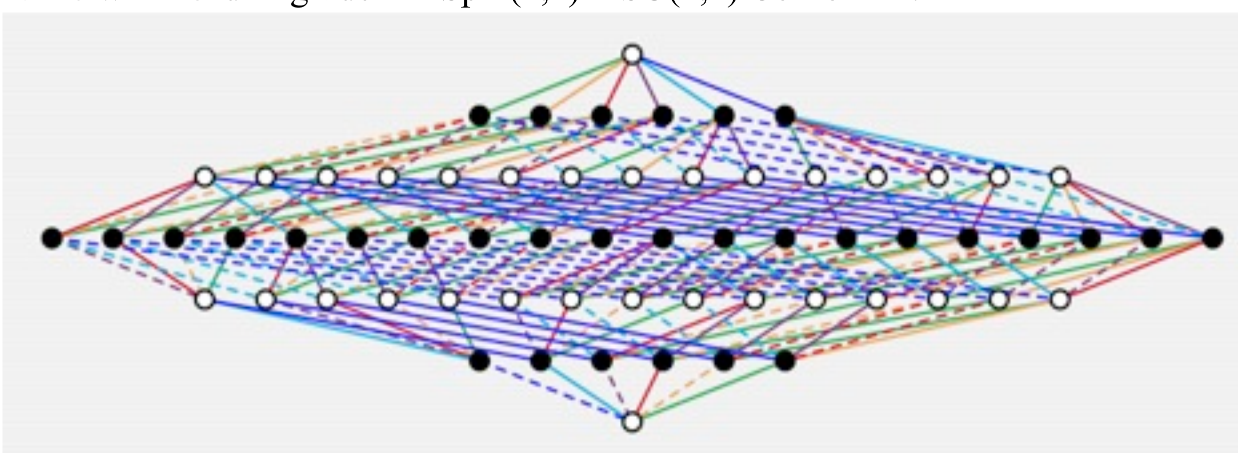
[In the MI^Ncubic picture interpreted as a Clifford Algebra,
the 4 grade-1 elements correspond to 4-dim spacetime with signature (1,3),
the 6 grade-2 elements to spin(1,3) Lorentz transformations,
and the spinors to pairs of Quaternions (Q,Q),
with (Q,0) being + half-spinors and (0,Q) being mirror image -half-spinors.]...".

G. D. Landweber's 2006 program Adinkramat at http://www.cohomology.com/ produces Adinkra graphs of MI^Ncubic, such as for N = 5, 6, 7, and 8:

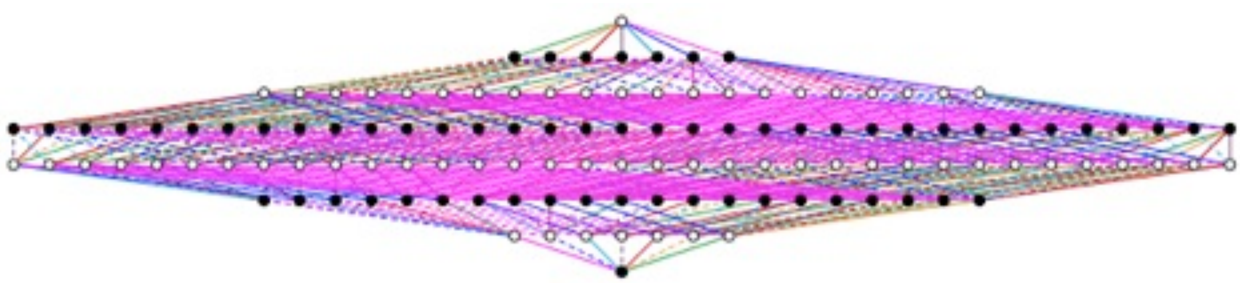
N = 5 with 10-dim grade 2 = Spin(2,3) DeSitter:



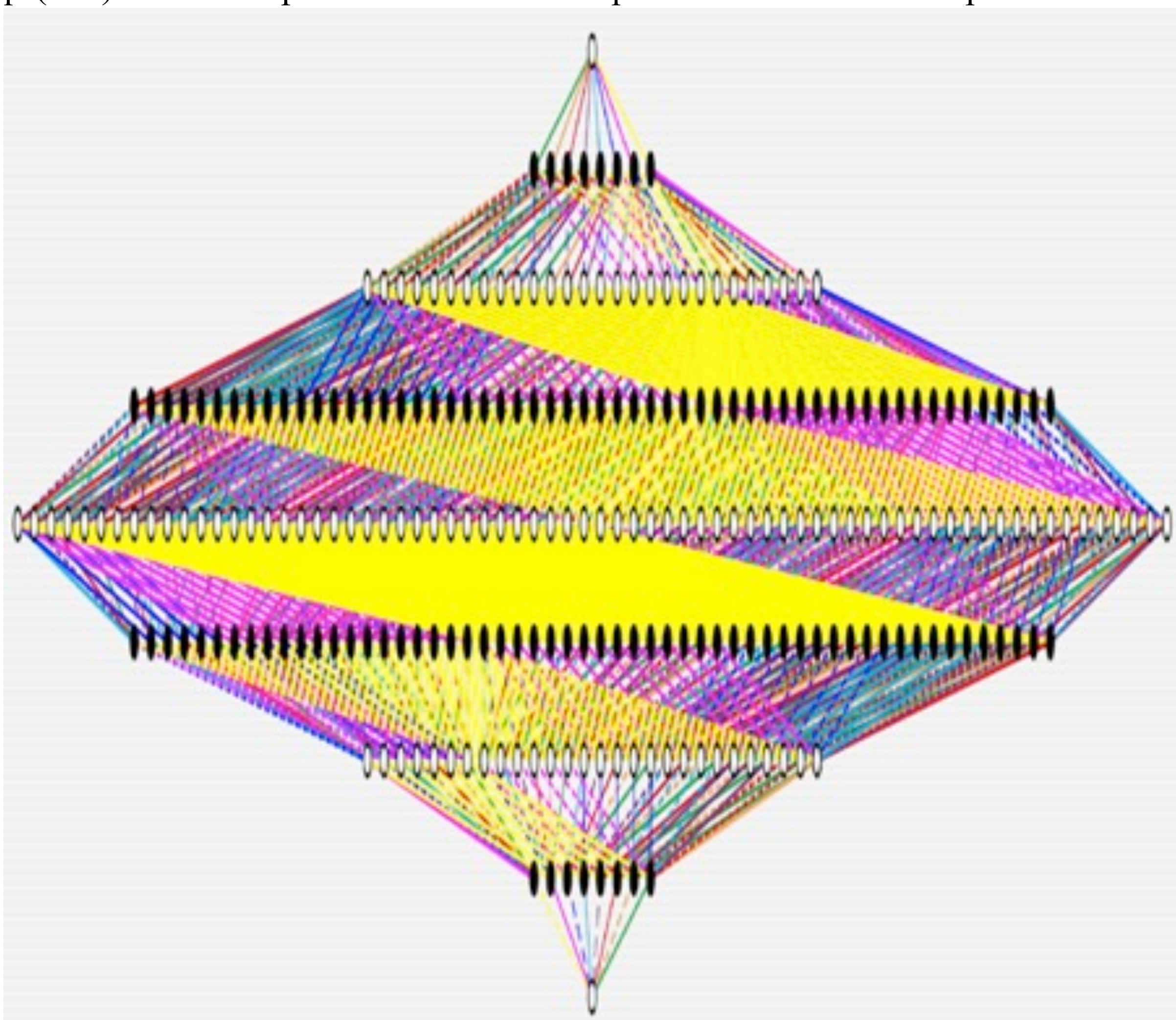
N = 6 with 15-dim grade 2 = Spin(2,4) = SU(2,2) Conformal:



N = 7 with 21-dim grade 2 = Spin(7):



N = 8 of real Clifford Algebra Cl(8) with 28-dim grade 2 = Spin(8)
and graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
with 2^8 = 256 elements and
sqrt(256) = 16-dim spinors = 8-dim +half-spinors and 8-dim -half-spinors



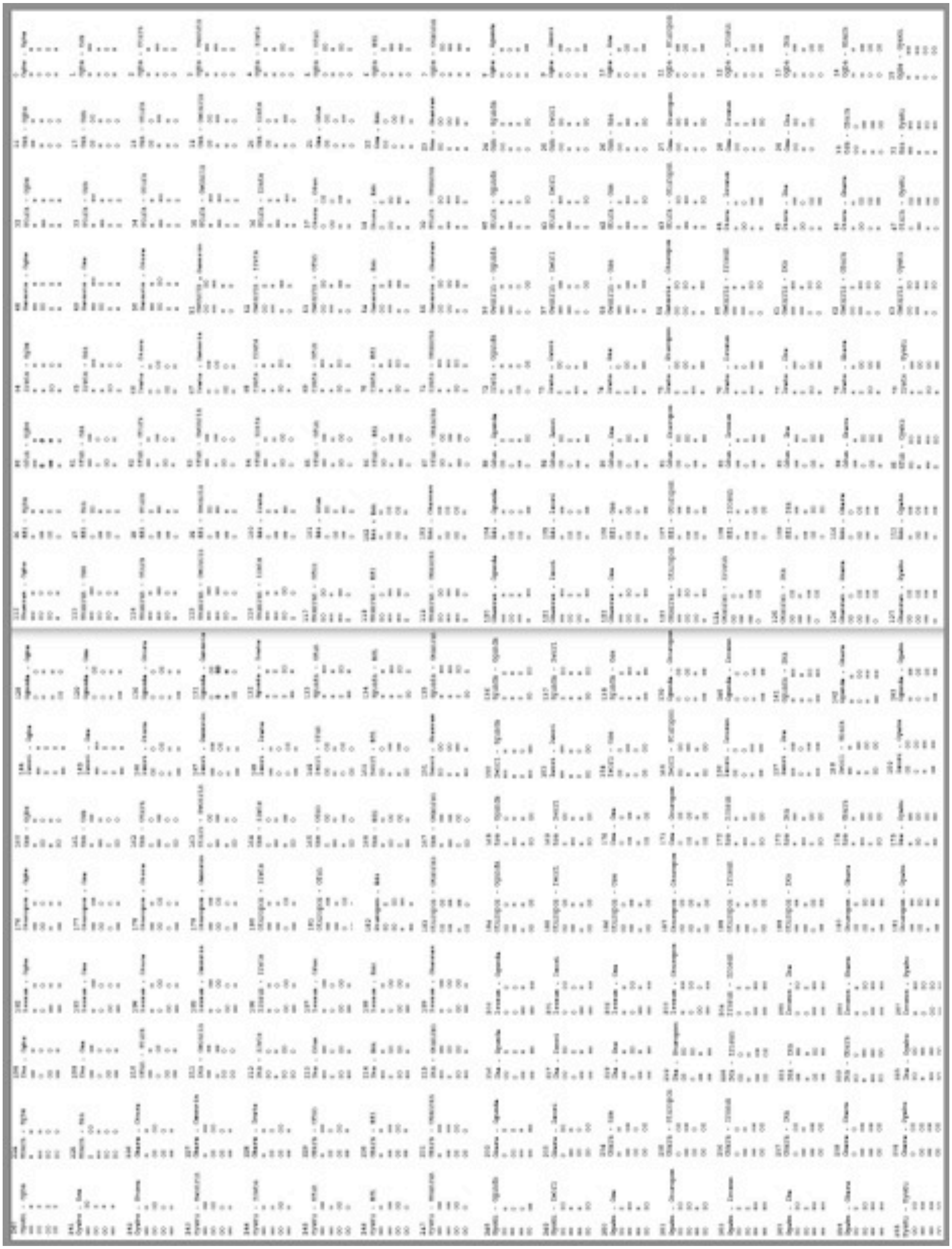
The 2^8 = 16x16 = 156 elements of Cl(8) correspond directly to:
the 256 vertices of an 8-dim hypercube;
the 256 Odu of IFA Divination;
the 256 fundamental Cellular Automata.

248 of the 256 correspond directly to the 248 generators of the E8 Lie algebra
and the physical elements of my E8 physics model.

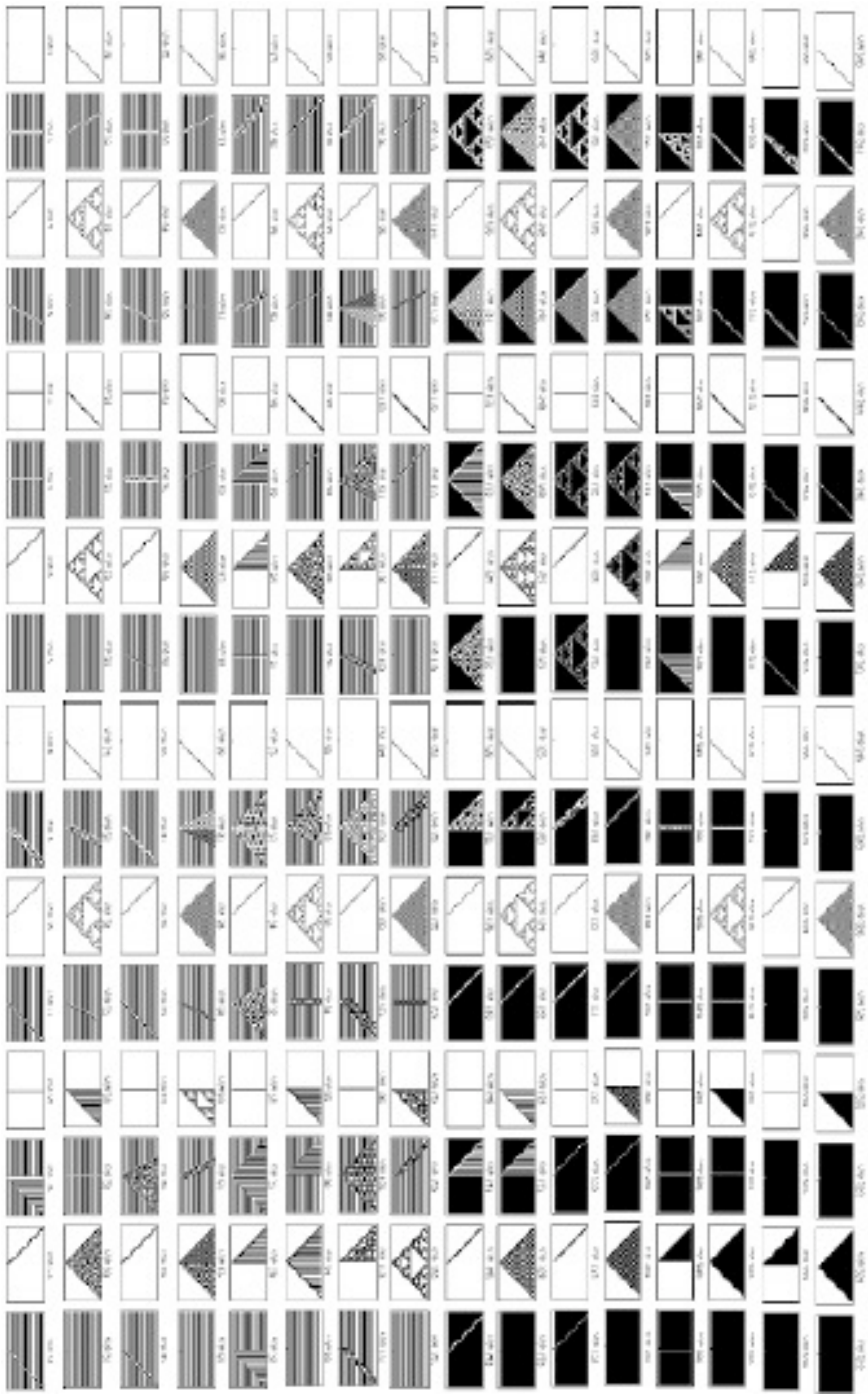
240 of the 256 correspond directly to the 240 Root Vectors of E8.

Here are some images showing some details of those correspondences:

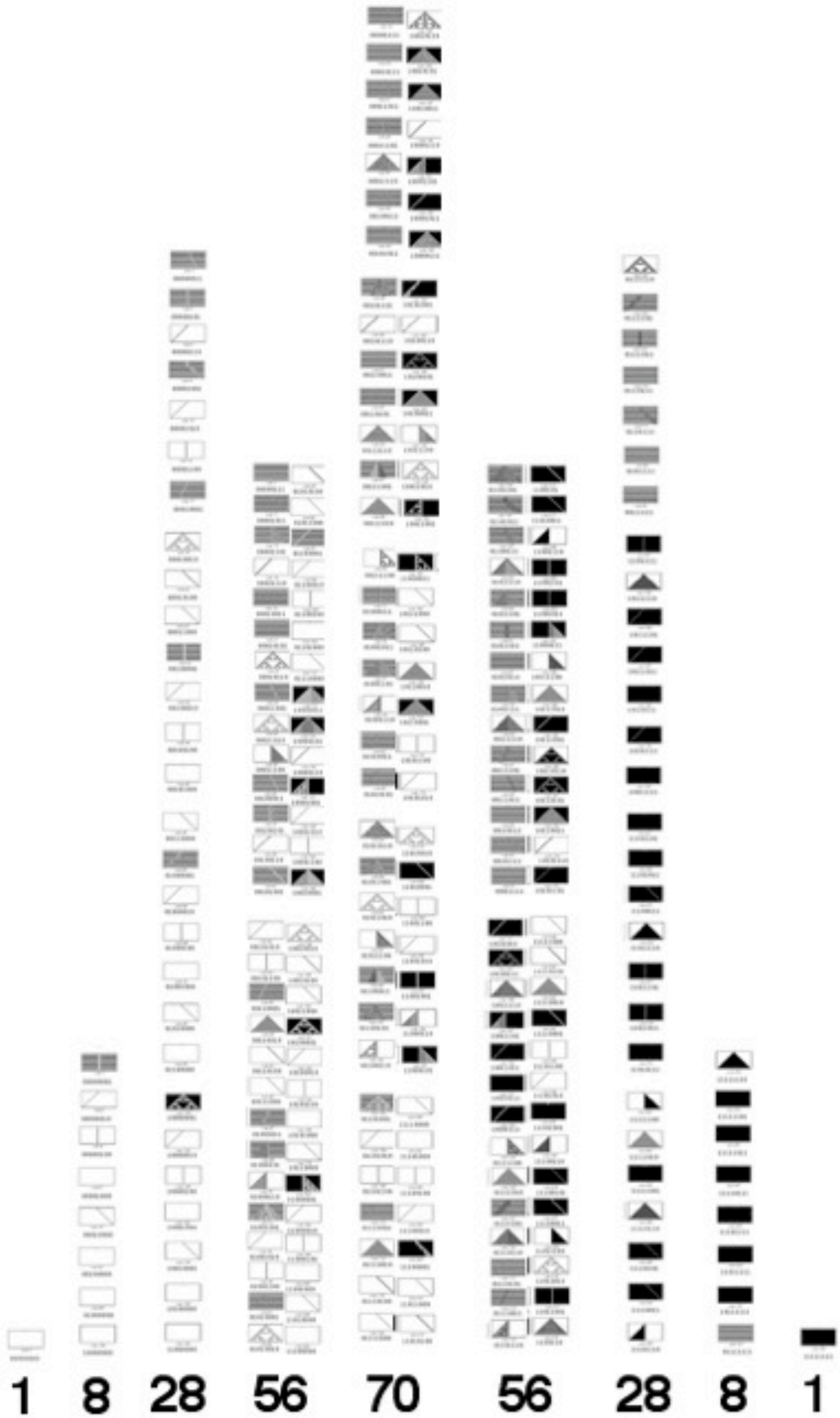
The 256 Odu of IFA:



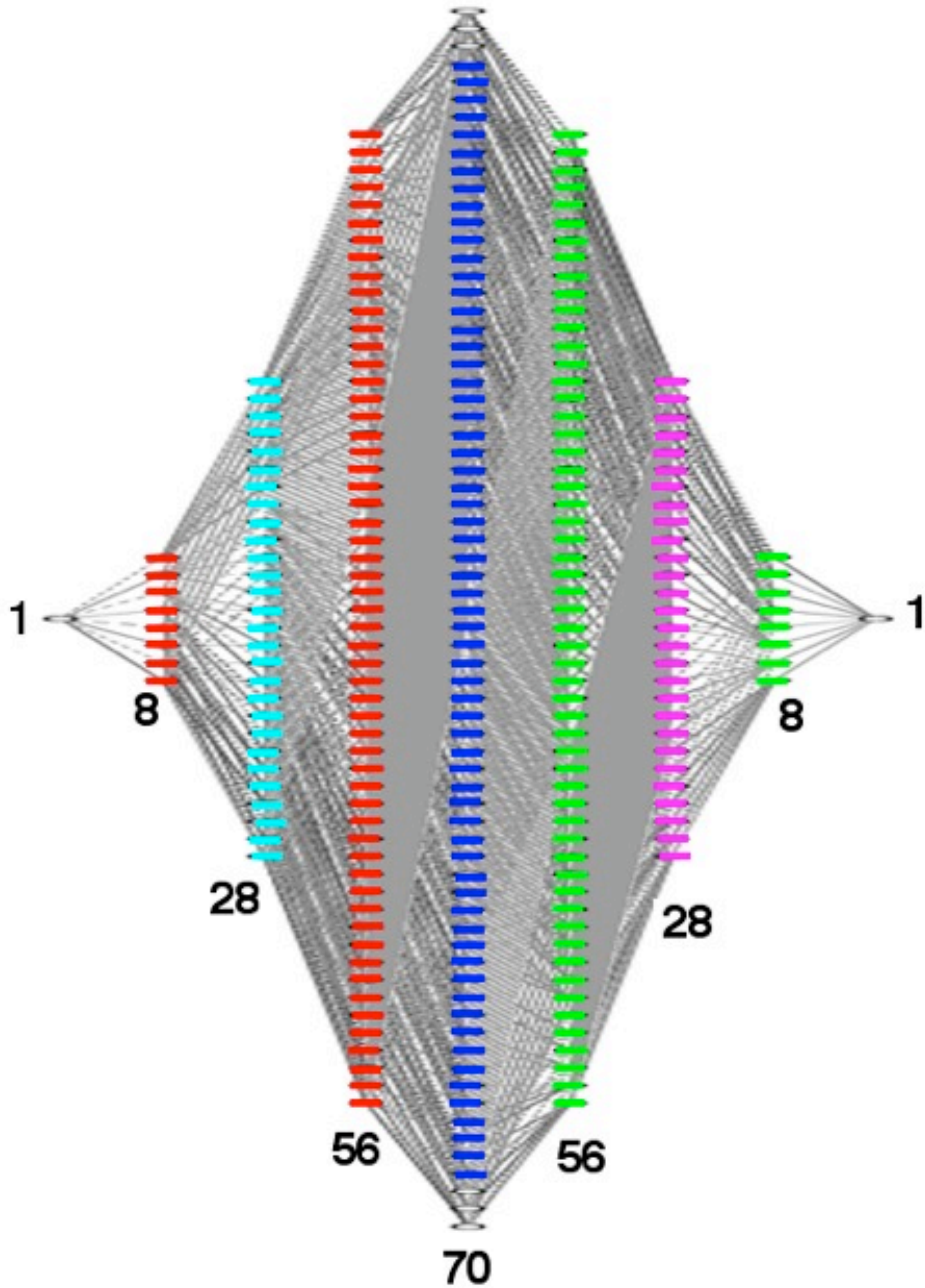
The 256 Fundamental Cellular Automata



have the same graded structure as the Cl(8) real Clifford algebra
and the 8-dimensional hypercube and the N = 8 MI^Ncubic Adinkra graph:



1
8
28
56
70
56
28
8
1


1
8
28
56
70
56
28
8
1

248 E8 = 120 D8 ⊕ 128 D8 Half Spinor

28 D4

D4 contains U(2,2)
U(2,2) = Spin(2,4)
Conformal Gravity

28 D4`

D4` contains U(4)
U(4) contains SU(3)

CP2 = SU(3) / U(2)
U(2) = SU(2) x U(1)

(8v , 8v)

M4 x CP2

(8s , 8s)

{ nu, ruq, guq, buq,
e, rdq, gdq, bdq }

(8c , 8c)

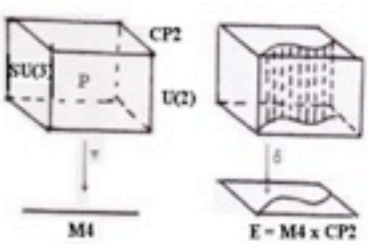
{ $\overline{\text{nu}}$, $\overline{\text{ruq}}$, $\overline{\text{guq}}$, $\overline{\text{buq}}$,
$\overline{\text{e}}$, $\overline{\text{rdq}}$, $\overline{\text{gdq}}$, $\overline{\text{bdq}}$ }

Generations:	Propagator Origin		Propagator Destination
1	M4	⟶	M4
2	M4	⟶	CP2
	CP2	⟶	M4
3	CP2	⟶	CP2

Lagrangian:

$$\int_{\text{KKspacetime}} \text{gauge term} + \text{fermion term}$$

Higgs-Mayer:



Kobayashi-Nomizu:

THEOREM 11.7. *Assume in Theorem 11.5 that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ *(direct sum) and* $\mathrm{ad}\,(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}\,(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then*

(1) *There is a 1:1 correspondence between the set of K-invariant connections in P and the set of linear mappings* $\Lambda_{\mathfrak{m}}: \mathfrak{m} \to \mathfrak{g}$ *such that*

$$\Lambda_{\mathfrak{m}}(\mathrm{ad}\,(j)(X)) = \mathrm{ad}\,(\lambda(j))(\Lambda_{\mathfrak{m}}(X)) \qquad \text{for } X \in \mathfrak{m} \quad \text{and} \quad j \in J;$$

the correspondence is given via Theorem 11.5 by

$$\Lambda(X) = \begin{cases} \lambda(X) & \text{if } X \in \mathfrak{j}, \\ \Lambda_{\mathfrak{m}}(X) & \text{if } X \in \mathfrak{m}. \end{cases}$$

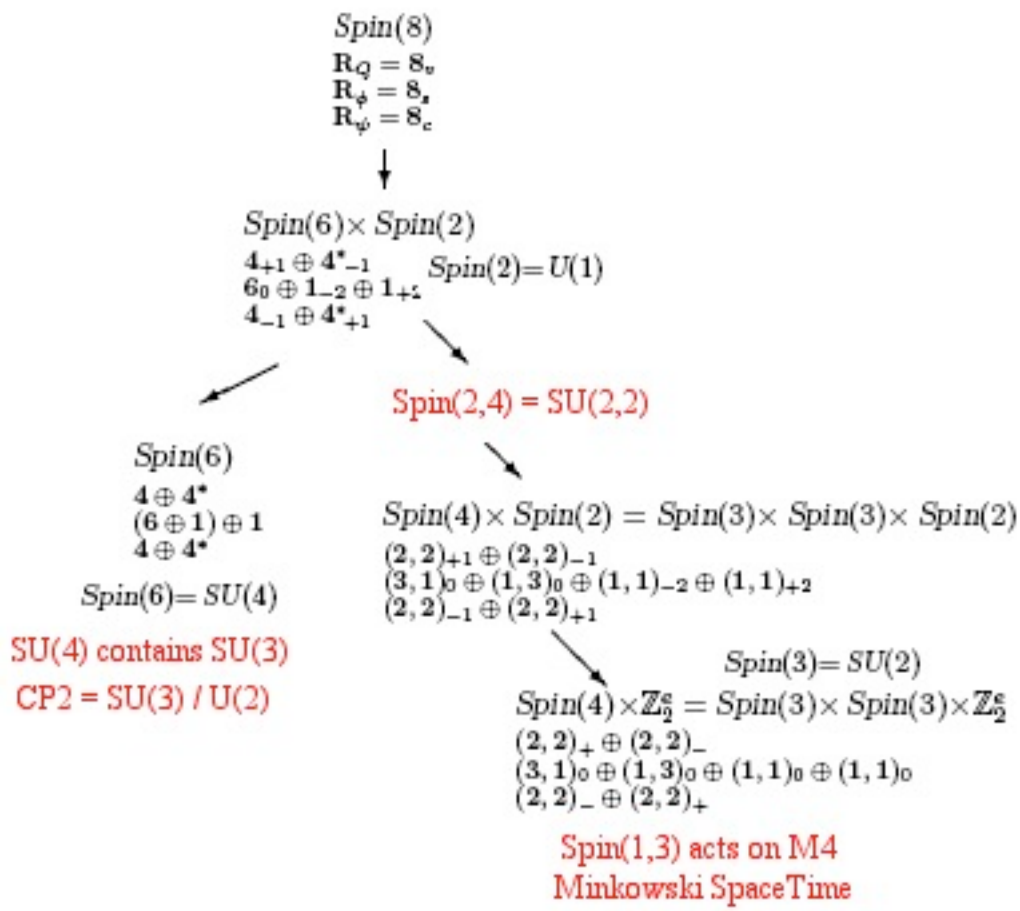
(2) *The curvature form* Ω *of the K-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \qquad \text{for } X, Y \in \mathfrak{m},$$

The Higgs and the T-quark form a system in which the Higgs is effectively a T-quark condensate.

Gates and co-authors Faux and Hubsch in arxiv 0904.4719 [hep-th] said:
"... A two-dimensional on-shell model under a compactification produces a one-dimensional off-shell model. ... An off-shell model in one bosonic dimension identifiable as the worldline, with N = 8 supersymmetry ... is the lowest N-extended supersymmetry where the minimal supermultiplet is ... unique in having eight bosons and eight fermions, transformed into each other by ...
Spin(8), and ... its Z2-extension, Pin(8) ...
the Pin(8) / Spin(8) = Z2 reflections swap the two spinors of Spin(8): 8s <--> 8c, and are the Z2 part of the ... unique triality ... S3 outer automorphism of Spin(8). ...
[acts on]... 8v ... 8s ... 8c ...

... relevant subgroup chains of Spin(8) ...[include]...



... The E8 Algebra ... has a Spin(16) maximal, regular subalgebra.
With respect to this, the E8 adjoint representation decomposes as
248 --> 120 + 128 ...”.

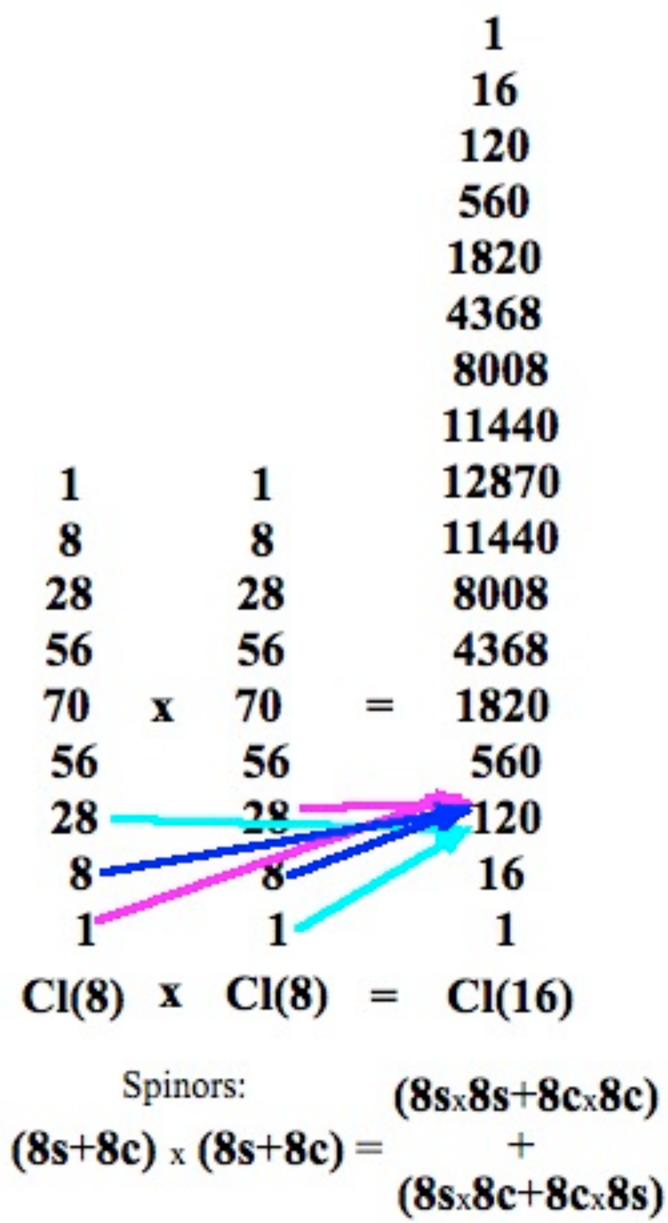
Since Spin(16) is the bivector Lie algebra of the Cl(16) real Clifford algebra, you need to look at the N = 16 Adinkra to construct E8. However, even with the Adinkramat program, it is hard to make a nice visual Adinkra diagram with all the 2^16 = 65,536 vertices for N = 16 and Cl(16),
so here I will just list the dimensions of the graded structure

1
16
120
560
1820
4368
8008
11440
12870
11440
8008
4368
1820
560
120
16
1

for N = 16 of real Clifford Algebra Cl(16) with 120-dim grade 2 = Spin(16)
with 2^16 =65,536 elements and
sqrt(65,536) = 256-dim spinors =
= 128-dim +half-spinors and 128-dim -half-spinors.

An easy way to understand the E8 building block parts of Cl(16), that is,
the 120-dim Spin(16) D8 bivectors and the 128-dim Spin(16) D8 half-spinors,

is to use the 8-periodicity property of real Clifford Algebras
to factor Cl(16) into the tensor product Cl(8) x Cl(8):



By that factorization:

the 256-dim Cl(16) full spinors are the sum of these tensor products:

Cl(8) half-spinor 8s x Cl(8) half-spinor 8s
Cl(8) half-spinor 8s x Cl(8) half-spinor 8c
Cl(8) half-spinor 8c x Cl(8) half-spinor 8s
Cl(8) half-spinor 8c x Cl(8) half-spinor 8c

so the 128-dim Cl(16) half-spinor = 8s x 8s + 8c x 8c = 64ss + 64cc
has E8 triality transformation among 64vv and 64ss and 64cc
consistently inherited from Cl(8) Spin(8) D4 triality among 8v and 8s and 8c.

and

the 120-dim Cl(16) grade-2 bivectors are the sum of these tensor products:

Cl(8) grade-0 x Cl(8) grade-2 = 1 x 28 = 28
Cl(8) grade-1 x Cl(8) grade-1 = 8v x 8v = 64vv
Cl(8) grade-2 x Cl(8) grade-0 = 28 x 1 = 28

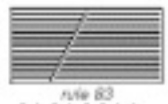
so that 120-dim Cl(16) grade-2 = 28 + 64vv + 28
which is the even part of the E8 7-grading 8 + 28 + 56 + 64 + 56 + 28 + 8
(Note that 64vv = U(8) contains the SU(8) subgroup of D8 Spin(16)
and that D8 / U(8) is a rank 4 symmetric space of 28+28 = 56 dimensions.)
and
is contained in the even part 1 + 28 + 70 + 28 + 1
of the Cl(8) grading 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1

The 8 elements of the Cl(8) grading that are not directly included in E8 are:

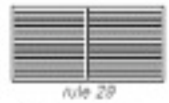
1 of grade-0 + 1

00000000

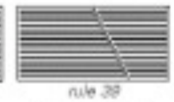
3 in grade-4 - e_5712 - e_1345 - e_6123



01010011



00011101



00100111

3 in grade-4 + e_6728 + e_3468 + e_4578 to

11100010

10101100

11011000

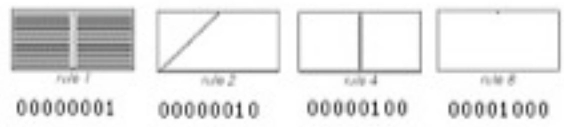
1 of grade-8 + e_12345678

11111111

They can be interpreted as 8 of the 16 components of
the Cl(8) primitive idempotent f =
= (1/2)(1 + e_1248) (1/2)(1 + e_2358) (1/2)(1 + e_3468) (1/2)(1 + e_4578) =
= (1/16)(1 - e_5712 - e_1345 - e_6123
- e_4671 - e_7234 - e_2456 - e_3567
+ e_2358 + e_1248 + e_5618 + e_7138
+ e_6728 + e_3468 + e_4578 + e_J)

which 8 components correspond to the 4-dim M4 physical spacetime part
of the 8-dim M4 x CP2 Kaluza-Klein spacetime of E8 physics.

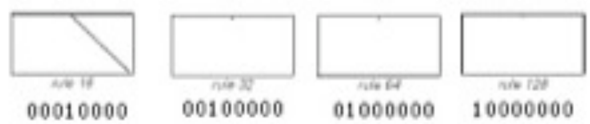
The 8-dim M4 x CP2 Kaluza-Klein spacetime of E8 physics corresponds to the 8 grade-1 elements:



which represent the T X Y Z basis elements of M4 physical spacetime and which, by Triality, correspond to the fundamental fermion particles:

neutrino, red down quark, green down quark, blue down quark

and



which represent the R Q P C basis elements of CP2 internal symmetry space and which, by Triality, correspond to the fundamental fermion particles:

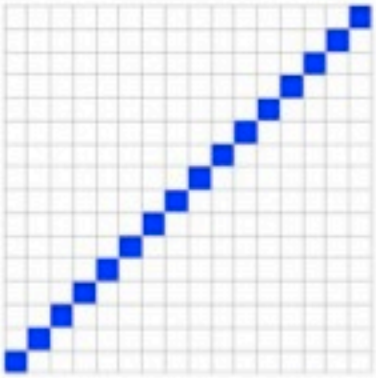
blue up quark, green up quark, red up quark, electron.

Since the Cl(8) Clifford Algebra is the algebra of 16x16 real matrices, each of its 256 elements can be written as such matrices.

If the 8 grade-1 elements are written explicitly as real 16x16 matrices, the rest of the 256 Clifford Algebra element matrices can be calculated from the 8.

Based on an internet discussion with Greg Moxness who has very interesting material and beautiful graphics on his web site at http://theoryofeverything.org/ here is an example of explicit real 16x16 matrices for the 8 grade-1 elements:

Note that
the 16x16 algebra matrices of the Cl(8) Clifford Algebra
are not the same as
the 16x16 matrices showing binary order structure of its 256 elements.



Cl(0,8) Matrices

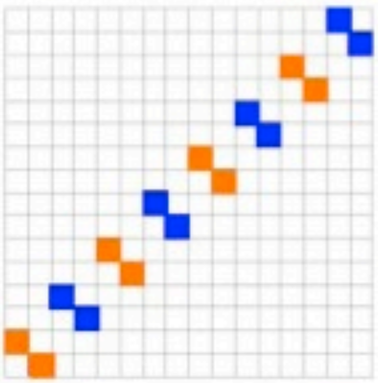
1

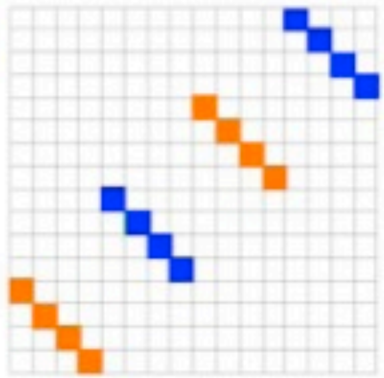
2 3 4

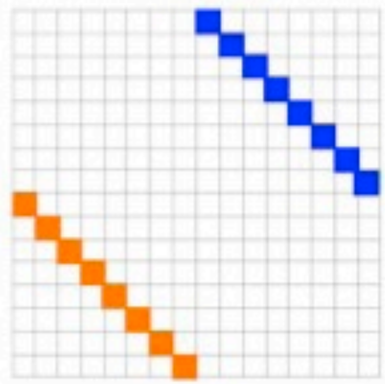
5 6 7

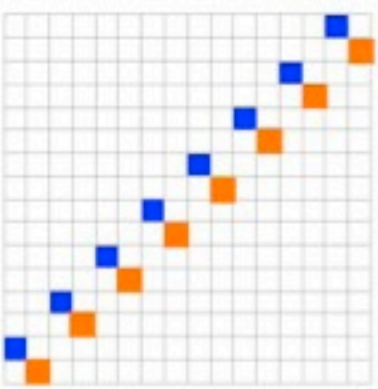
8

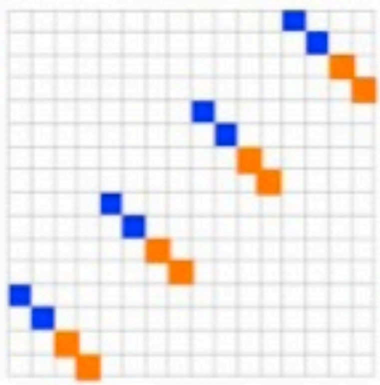
-1 ■ | 1 ■

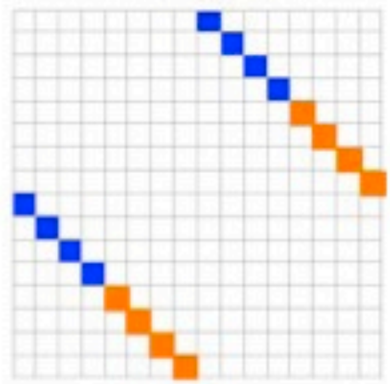












For odd dimensions, the additional generator (a generalization of γ^5) is

$$\Gamma_{2\nu+1} = (-i)^\nu \prod_{a=1}^{2\nu} \Gamma_a .$$

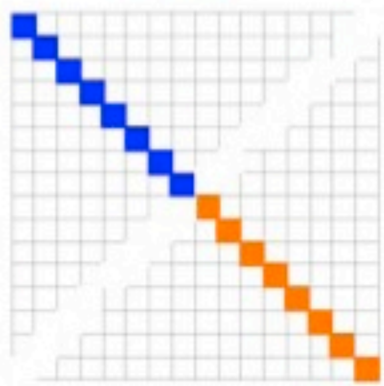
The Dirac matrices satisfy canonical anti-commutation

relation $\Gamma_i \Gamma_j + \Gamma_j \Gamma_i = 2\delta_{i,j}$.

The above definition corresponds to the so-called "chiral basis," where Dirac matrices are block anti-diagonal.

Other bases are possible, and are related to the chiral basis by rotations.

The Dirac matrices generate Euclidean Clifford algebra.

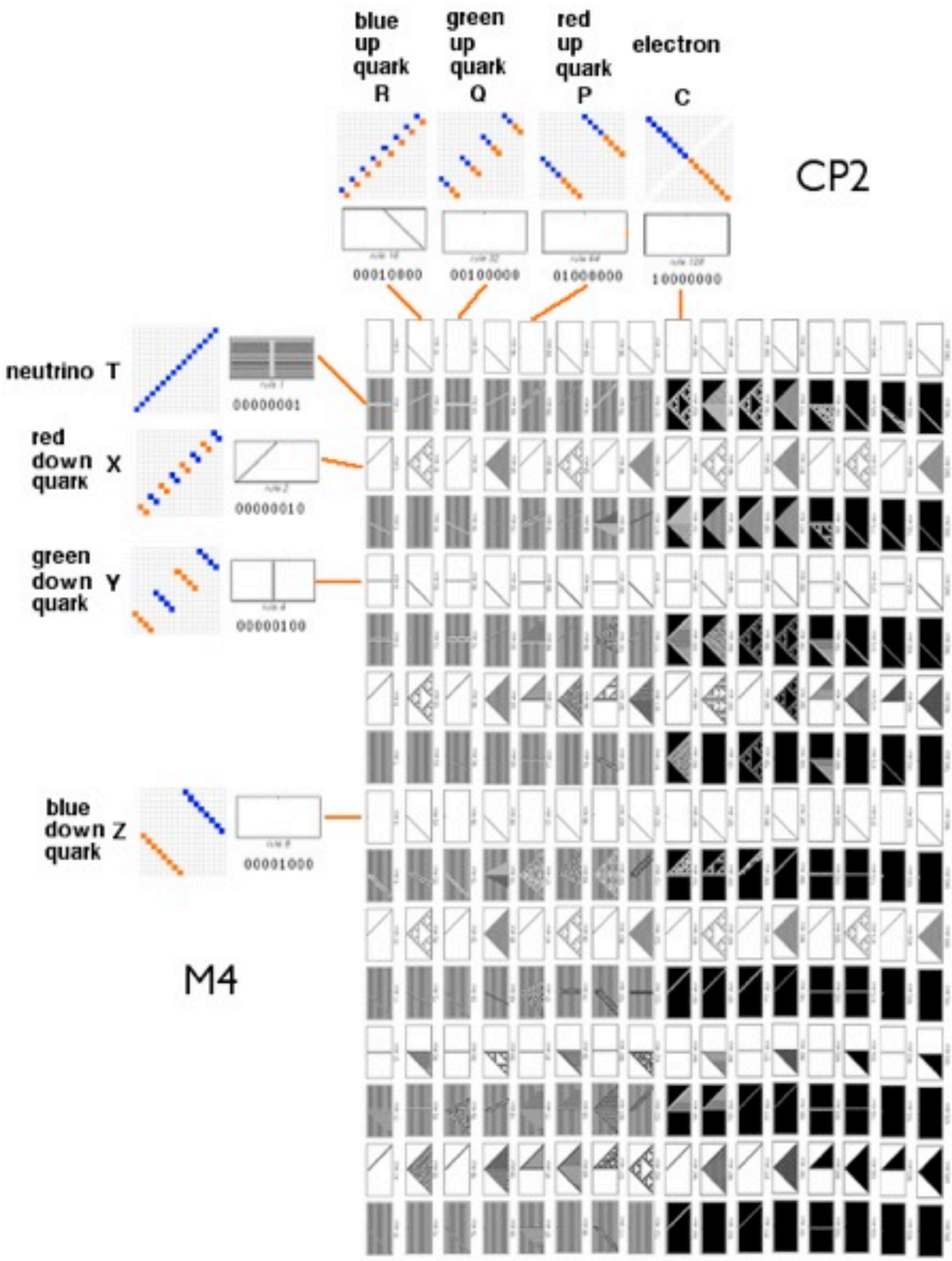


modified from:

Dirac Matrices in Higher Dimensions" from The Wolfram Demonstrations Project

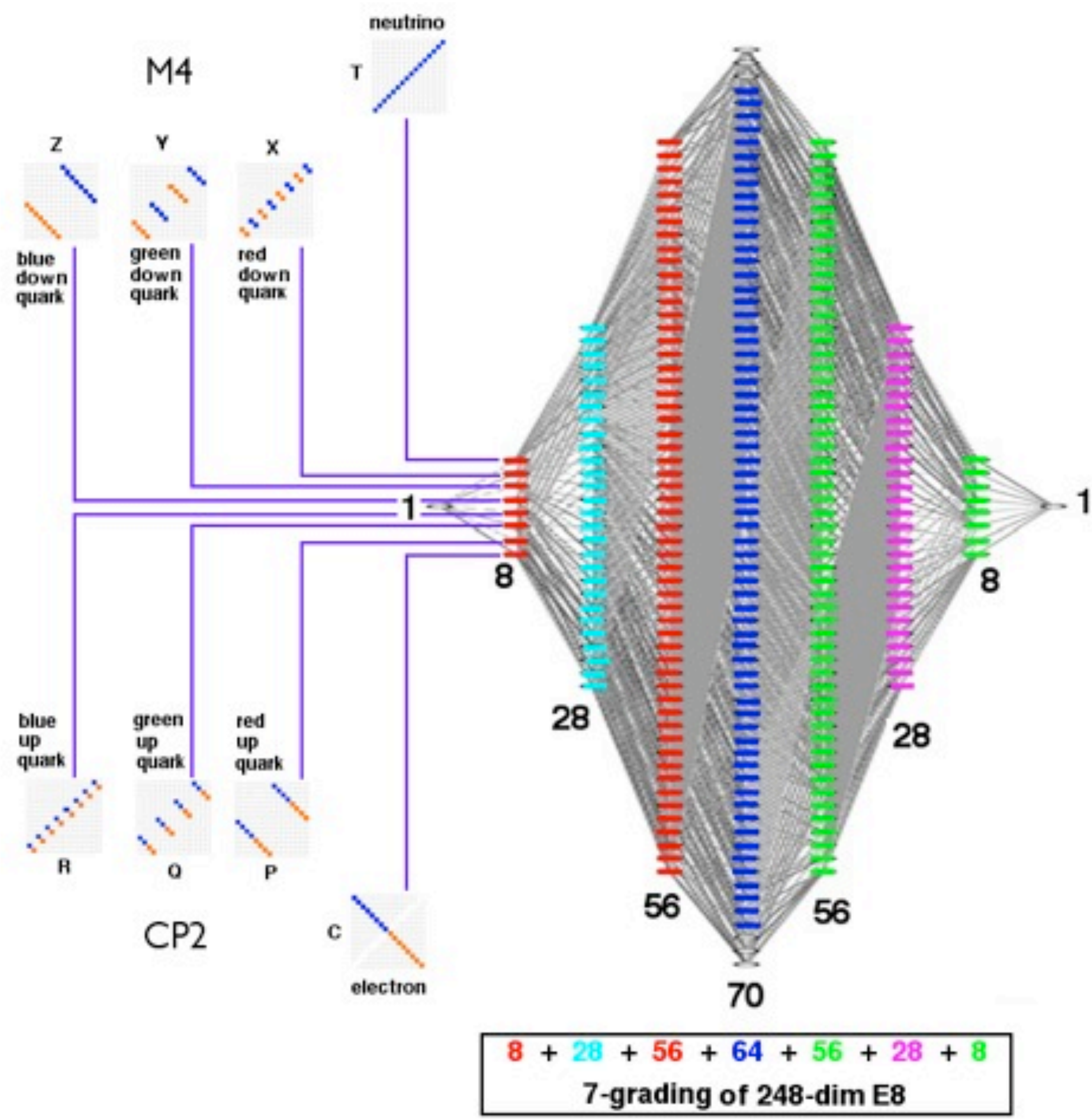
http://demonstrations.wolfram.com/DiracMatricesInHigherDimensions/ Contributed by: Enrique Zeleny

Here is how the 8 grade-1 elements fit among all 16x16 = 256 elements of Cl(8):



The correspondence between M4xCP2 spacetime and fermions is due to Triality.

In addition to the 16x16 = 256 element array shown above arranged in binary number order (similar to the Fu Xi ordering of the I Ching),
the 256 elements can also be arranged according to their Clifford Algebra graded strucure, and here is how the 8 grade-1 elements fit that way:



The colored elements are the 248 of the 256 that correspond to E8.

Triality makes the correspondence of Cl(8) with E8 consistent with physics.
Here is my current (somewhat conjectural) view of how that works:

The **8** of E8 represents the primary covariant component of 8 fermion particles in 8-dim spacetime, and
the grade-1 8 of Cl(8) represents the 8-dim spacetime vector space,
which is OK by Triality between 8v and 8s;

The **8** of E8 represents the primary covariant component of 8 fermion anti-particles in 8-dim spacetime, and
the grade-7 8 of Cl(8) represents the 8-dim pseudovectors,
which is OK by Triality between 8v and 8c;

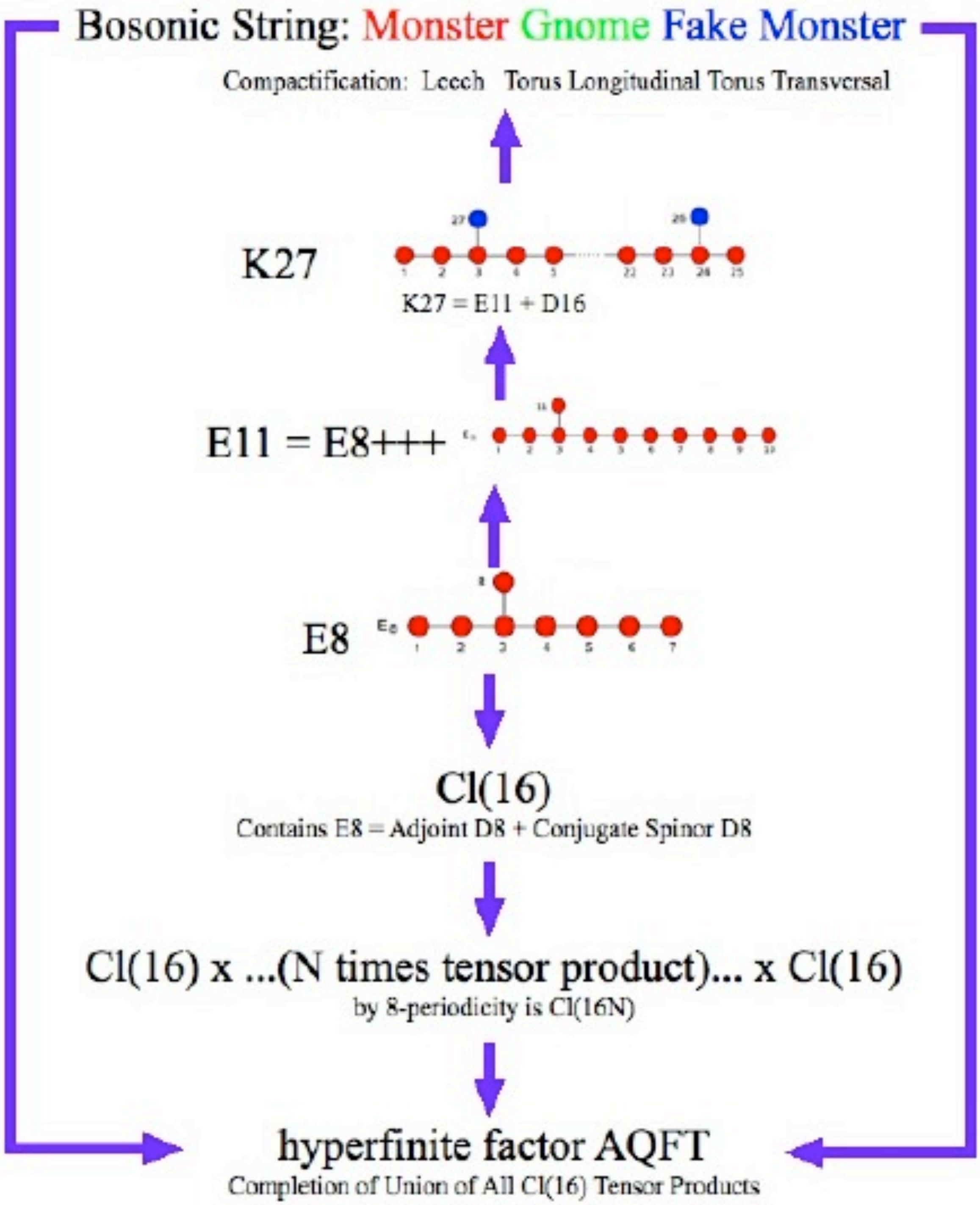
The **56** of E8 represents 7 of the 8 covariant components of 8 fermion particles in 8-dim spacetime, and
the grade-3 56 of Cl(8) represents the 8-dim spacetime vector space acted on by the bivector spacetime transformations from the primary component to the other 7 covariant components of fermion particles in 8-dim spacetime,
which is OK by Triality between 8v and 8s;

The **56** of E8 represents 7 of the 8 covariant components of 8 fermion anti-particles in 8-dim spacetime, and
the grade-5 56 of Cl(8) represents the 8-dim spacetime pseudovector space acted on by dual of the transformations described with respect to **56**,
which is OK by Triality between 8v and 8c;

The **64** of E8 represents 8v x 8v of 8-dim spacetime, and
the grade-4 70 of Cl(8) contains 64 elements 8s x 8c,
which is OK by Triality between 8v and 8s and 8c;

The remaining 6 = 70 - **64** of grade-4 and 1 grade-0 and 1 grade-8 of Cl(8) are, as indicated in the above chart, not included in E8.

The direct inclusion of E8 in Cl(8)xCl(8) = Cl16)
is useful in the following outline of how the E8 of this Physics model lives
inside a larger Cl(16)-based hyperfinite factor Algebraic Quantum Field Theory
whose structure is related to a Bosonic String Theory in which
Strings are physically interpreted as representing World-Lines:


Bosonic String: Monster Gnome Fake Monster
Compactification: Leech Torus Longitudinal Torus Transversal
K27
K27 = E11 + D16
E11 = E8+++
E8
Cl(16)
Contains E8 = Adjoint D8 + Conjugate Spinor D8
Cl(16) x ...(N times tensor product)... x Cl(16)
by 8-periodicity is Cl(16N)
hyperfinite factor AQFT
Completion of Union of All Cl(16) Tensor Products