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Standard Model plus Gravity from Octonion Creators and Annihilators

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Abstract

Octonion creation and annihilation operators are used to construct the Standard Model plus Gravity. The resulting phenomenological model is the $D_4 - D_5 - E_6$ model described in hep-ph/9501252.

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1 Introduction.

The purpose of this paper is to outline a way to build a model of the Standard Model plus Gravity from the Heisenberg algebra of fermion creators and annihilators.

We want to require that the superposition space of charged fermion creation operators be represented by multiplication on a continuous unit sphere in a division algebra. That limits us to:

the complex numbers \mathbf{C} , with parallelizable S^1 ,

the quaternions \mathbf{Q} , with parallelizable S^3 , and

the octonions \mathbf{O} , with parallelizable S^7 .

We choose the octonions because they are big enough to make a realistic physics model.

Octonions are described in Geoffrey Dixon's book [3] and subsequent papers [4, 5, 6, 7], and in Ian Porteous's book [11]. Essential mathematical tools include the octonion X-product of Martin Cederwall [2] and the octonion XY-product of Geoffrey Dixon [7].

The purpose of this paper is to build a physics model, not to do mathematics, so I ignore mathematical details and subtle points. For them, see the references.

This paper is the result of discussions with Ioannis Raptis and Sarah Flynn, and reading a preprint of Steve Selesnick on fermion creation operators as fundamental to the Quantum Net of David Finkelstein. John Caputlu-Wilson has discussed the role of propagator phase. Igor Kulikov and Tang Zhong have also discussed the paper, and Igor has made it clear that I should not misspell Shilov as Silov.

2 Octonion Creators and Annihilators.

Consider the octonions \mathbf{O} and their unit sphere S^7 .

Our starting point is the creation operator $\alpha_{\mathbf{O}L}$ for the first generation octonion fermion particles. In the octonion case, the L denotes only the helicity of the neutrino, which is a Weyl fermion. The other fermions are Dirac fermions, and can exist in either helicity state L or R .

If a basis for the octonions is $\{1, e_1, e_2, e_3, e_4, e_5, e_6, e_7, \}$, then the first generation fermion particles are represented by:

<i>Octonion basis element</i>	<i>Fermion Particle</i>
1	<i>e – neutrino</i>
e_1	<i>red up quark</i>
e_2	<i>green up quark</i>
e_6	<i>blue up quark</i>
e_4	<i>electron</i>
e_3	<i>red down quark</i>
e_5	<i>green down quark</i>
e_7	<i>blue down quark</i>

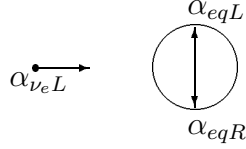
(1)

Graphically, represent the neutral left-handed Weyl e-neutrino creation operator $\alpha_{\nu_e L}$ by

$$\alpha_{\nu_e L} \longrightarrow$$

Now, represent the charged left-handed and right-handed Dirac electron-

quark creation operators α_{eqL} and α_{eqR} by vectors to a point on the sphere S^7 (represented graphically by a circle):



Any superposition of charged fermion particle creation operators α_{eqL} and α_{eqR} can be represented as a point on the sphere S^7 defined by their representative vectors. The sphere S^7 should be thought of as being orthogonal to the vector $\alpha_{\nu_e L}$.

We will represent the superposition of creation of e-neutrinos (represented by a vector on a line) and charged particles (represented by vectors to a sphere S^7) by letting the magnitude of the amplitude $|\alpha_{\nu_e L}|$ of the e-neutrino creator vector run from 0 to 1 and then determining the radius r of the sphere S^7 in octonion space \mathbf{O} by

$$|\alpha_{\nu_e L}|^2 + r^2 = 1 \quad (2)$$

We now have as representation space for the octonion creation operators $S^7 \times \mathbf{R}P^1$, where we have parameterized $\mathbf{R}P^1$ by the interval $[0, 1)$ rather than the conventional $[0, \pi)$.

The octonion first-generation fermion annihilation operator, or antiparticle creation operator, is $\alpha_{\mathbf{O}R}^\dagger$.

Therefore, for the octonions, we have the nilpotent Heisenberg algebra matrix:

$$\begin{pmatrix} 0 & \alpha_{\mathbf{O}L} & \beta \\ 0 & 0 & \alpha_{\mathbf{O}R}^\dagger \\ 0 & 0 & 0 \end{pmatrix} \quad (3)$$

How does this correspond to the $D_4 - D_5 - E_6$ model described in hep-ph/9501252 [13]?

The octonion fermion creators and annihilators,

$$\begin{pmatrix} 0 & \alpha_{\mathbf{O}L} & 0 \\ 0 & 0 & \alpha_{\mathbf{O}R}^\dagger \\ 0 & 0 & 0 \end{pmatrix} \quad (4)$$

are both together represented in the $D_4 - D_5 - E_6$ model by the Shilov boundary of the bounded complex homogeneous domain corresponding to the Hermitian symmetric space $E_6/(D_5 \times U(1))$.

(A good reference on Shilov boundaries is Helgason [10].)

The Shilov boundary is two copies of $S^7 \times \mathbf{R}P^1$. The $\mathbf{R}P^1$ part represents the Weyl neutrino, and the S^7 part represents the Dirac electron and red, green, and blue up and down quarks.

The $\mathbf{R}P^1$ part is represented by $[0, 1)$ in our parameterization (or $[0, \pi)$ on the unit circle in the complex plane in a more conventional one), and the S^7 part can be represented by the unit sphere S^7 in the octonions \mathbf{O} .

Also, mathematically, we can regard

$$S^7 \times \mathbf{R}P^1 = (S^7 \times \mathbf{R}P^1)^\dagger \quad (5)$$

Therefore, the creator-annihilator part of the nilpotent Heisenberg 3×3 matrix can be represented as:

$$\begin{pmatrix} 0 & S^7 \times \mathbf{R}P^1 & 0 \\ 0 & 0 & S^7 \times \mathbf{R}P^1 \\ 0 & 0 & 0 \end{pmatrix} \quad (6)$$

What about the β part?

β is given by the commutator

$$\beta = [S^7 \times \mathbf{R}P^1, S^7 \times \mathbf{R}P^1] \quad (7)$$

Since $\mathbf{R}P^1$ is only the interval $[0, 1)$ in our parameterization (or $[0, \pi)$ on the unit circle in the complex plane in a more conventional one), it is equivalent to a real number and can therefore be absorbed into the real R scalar field of the 3×3 matrices.

It commutes with everything and produces no gauge bosons by its commutators.

From a physical point of view, we can say that $\mathbf{R}P^1$ represents the neutrino, which has no charge and therefore does not interact with or produce any gauge bosons by commutation.

Whichever point of view you prefer, the result is that the full 3×3 nilpotent Heisenberg matrix looks like:

$$\begin{pmatrix} 0 & S^7 & \beta \\ 0 & 0 & S^7 \\ 0 & 0 & 0 \end{pmatrix} \quad (8)$$

Therefore, β is given by

$$\beta = [S^7, S^7] \quad (9)$$

Unlike the parallelizable spheres S^1 and S^3 of the associative algebras \mathbf{C} and \mathbf{Q} , the 7-sphere S^7 of the nonassociative octonions \mathbf{O} does not close under commutator and does not form a Lie algebra.

To deal with the situation, we need to use Martin Cederwall's octonion X-product [2] and Geoffrey Dixon's XY-product [7].

Martin Cederwall and his coworkers [2] have shown that $[S^7, S^7]$ does form an algebra, but not a Lie algebra:

Consider a basis $\{e_{iX}\}$ of the tangent space of S^7 at the point X on S^7 . Following Cederwall and Preitschopf [2], we have

$$[e_{iX}, e_{jX}] = 2T_{ijk}(X)e_{kX} \quad (10)$$

Due to the nonassociativity of the octonions, the "structure constants" $T_{ijk}(X)$ are not constant, but vary with the point X on S^7 , producing torsion.

Effectively, each point of S^7 has its own X-product algebra.

The X-product algebra takes care of the case of $[e_{iX}, e_{jX}]$ where both of the elements are in the tangent space of the same point X of S^7 , but since different points have really different tangent spaces due to nonassociativity of the octonions, it does not take care of the case of $[e_{iX}, e_{jY}]$ where e_{iX} is an element of the tangent space at X and e_{jY} is an element of the tangent space at Y .

To take care of this case, we must use Geoffrey Dixon's XY-product and "expand" $[S^7, S^7]$ from S^7 to at least two copies of S^7 (one for the commutator algebra at each of the points of the other one). That is, if \bowtie denotes a fibration "product":

$$[S^7, S^7] \supset S^7 \bowtie S^7 \quad (11)$$

We are still not quite through, because even though we have used the XY-product to take care of the case of $[e_{iX}, e_{jY}]$ where e_{iX} is an element of the tangent space at X and e_{jY} is an element of the tangent space at Y , we have not taken into account that the octonion basis for the tangent space at X may be significantly different from the octonion basis for the tangent space at Y .

The extra structure that must be "added" to $S^7 \rtimes S^7$ to "transform" the tangent space at X into the tangent space at Y is the automorphism group G_2 of the octonions. Unlike the cases of the associative algebras, the action the automorphism group cannot be absorbed into the products we have already used. So, we see that the Lie algebra of $[S^7, S^7]$ is

$$[S^7, S^7] = S^7 \rtimes S^7 \rtimes G_2 = Spin(8) \quad (12)$$

The fibrations represented by the \rtimes are:

$$Spin(7) \rightarrow Spin(8) \rightarrow S^7 \quad (13)$$

and

$$G_2 \rightarrow Spin(7) \rightarrow S^7 \quad (14)$$

Now, our octonionic version of the nilpotent Heisenberg algebra looks like:

$$\begin{pmatrix} 0 & S^7 & Spin(8) \\ 0 & 0 & S^7 \\ 0 & 0 & 0 \end{pmatrix} \quad (15)$$

Here, $Spin(8)$ is the 28-dimensional adjoint representation of $Spin(8)$. Its 28 infinitesimal generators represent 28 gauge bosons acting on the fermions that we have created, all as in the $D_4 - D_5 - E_6$ model.

The action of the $Spin(8)$ gauge bosons takes place within the arena of the 8-dimensional vector representation of $Spin(8)$, again as in the $D_4 - D_5 - E_6$ model.

We now have the picture of fermion creators and annihilators forming gauge bosons, and all of them interacting in accord with the $D_4 - D_5 - E_6$ model.

However, what about spacetime?

Since by triality (Porteous [11] describes triality) the vector representation of $Spin(8)$ is isomorphic to each of the half-spinor representations that we use for fermion creators and annihilators, we can form a vector representation version of the octonionic nilpotent Heisenberg algebra.

$$\begin{pmatrix} 0 & S^7 & S^7 \\ 0 & 0 & S^7 \\ 0 & 0 & 0 \end{pmatrix} \quad (16)$$

If we put back explicitly the factors of \mathbf{RP}^1 that we had merged into the real scalar field for ease of calculation of the S^7 commutators, we get:

$$\begin{pmatrix} 0 & S^7 \times \mathbf{RP}^1 & S^7 \times \mathbf{RP}^1 \\ 0 & 0 & S^7 \times \mathbf{RP}^1 \\ 0 & 0 & 0 \end{pmatrix} \quad (17)$$

The vector $Spin(8)$ spacetime part is

$$\begin{pmatrix} 0 & 0 & S^7 \times \mathbf{RP}^1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (18)$$

It is represented in the $D_4 - D_5 - E_6$ model by the Shilov boundary of the bounded complex homogeneous domain corresponding to the Hermitian symmetric space $D_5/(D_4 \times U(1))$.

The Shilov boundary is $S^7 \times \mathbf{RP}^1$. The \mathbf{RP}^1 part represents the time axis, and the S^7 part represents a 7-dimensional space.

NOW, we have reproduced the structure of the $D_4 - D_5 - E_6$ model by starting from octonion fermion creators and annihilators.

We can therefore incorporate herein by reference all the phenomenological results of the $D_4 - D_5 - E_6$ model as described in hep-ph/9501252 [13].

3 Complexified Octonions.

Recall that the octonion fermion creators and annihilators are of the form

$$\begin{pmatrix} 0 & S^7 \times \mathbf{R}P^1 & 0 \\ 0 & 0 & S^7 \times \mathbf{R}P^1 \\ 0 & 0 & 0 \end{pmatrix} \quad (19)$$

and that both of the entries $S^7 \times \mathbf{R}P^1$ taken together are represented in the $D_4 - D_5 - E_6$ model by the Shilov boundary of the bounded complex homogeneous domain corresponding to the Hermitian symmetric space $E_6/(D_5 \times U(1))$.

Also recall that the vector $Spin(8)$ spacetime part

$$\begin{pmatrix} 0 & 0 & S^7 \times \mathbf{R}P^1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (20)$$

is also represented in the $D_4 - D_5 - E_6$ model by a Shilov boundary of a bounded complex homogeneous domain. This entry $S^7 \times \mathbf{R}P^1$ corresponds to the Hermitian symmetric space $D_5/(D_4 \times U(1))$.

What if, instead of representing the 3×3 nilpotent Heisenberg matrix structure by Shilov boundaries, we represent them by the linearized tangent spaces of the corresponding Hermitian symmetric spaces?

Then we would have:

$$\begin{pmatrix} 0 & \mathbf{C} \otimes \mathbf{O} & \mathbf{C} \otimes \mathbf{O} \\ 0 & 0 & \mathbf{C} \otimes \mathbf{O} \\ 0 & 0 & 0 \end{pmatrix} \quad (21)$$

Sarah Flynn uses such 3×3 matrix structures in her work [8]. Note that complexified octonions $\mathbf{C} \otimes \mathbf{O}$ are not a division algebra. That is because signature is indistinguishable in complex spaces. Therefore, both the octonions and the split octonions are subspaces of $\mathbf{C} \otimes \mathbf{O}$. Since the split octonions contain nonzero null vectors, the complexified octonions $\mathbf{C} \otimes \mathbf{O}$ may be a normed algebra, but they are not a division algebra. The only complex division algebra is the complex numbers \mathbf{C} themselves.

4 Dimensional Reduction.

Now, going back to the Shilov boundary uncomplexified representations, recall that the vector $Spin(8)$ spacetime is represented by

$$\begin{pmatrix} 0 & 0 & S^7 \times \mathbf{R}P^1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (22)$$

Here, the spacetime of the vector representation of $Spin(8)$ is $S^7 \times \mathbf{R}P^1$, which can be represented by the octonions if $\mathbf{R}P^1$ is the real axis and S^7 is the imaginary octonions.

How do we move a fermion created at one point to another point?

If we move a particle along a lightcone path, how do we tell how "far" it has gone?

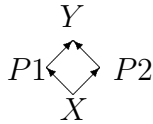
Following the approach of John Caputlu-Wilson [1], we should measure how much its propagator phase has advanced.

Since the phase advance may be greater than 2π , the propagator phase should take values, not on the unit circle, but on the infinite helical multi-valued covering space of the unit circle.

Recognizing that it may be difficult to do an experiment that will distinguish phases θ greater than 2π from phases $\theta - 2\pi$, we will look at very short paths such that the phase advance along the path is much less than 2π .

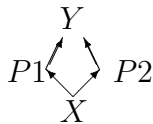
Now that we have a way to tell how "long" is a lightcone path segment, we can look at some paths.

Consider the following two lightcone paths $P1$ and $P2$, each beginning at X and ending at Y and each made up of two "short" lightcone segments:



Since the octonion spacetime $S^7 \times \mathbf{R}P^1$ is nonassociative, it has (as Martin Cederwall and his coworkers have shown [2]) torsion.

Since it has torsion, the end-point Y may not be well-defined, and we may have the diagram:



Since we want paths and lightcones to be consistently defined in the

Minkowski vacuum spacetime
 (before gravity has acted to effectively distort spacetime)
 we must modify our octonionic spacetime so that it is torsion-free at the
 Minkowski vacuum level.

How do we get rid of the torsion?

We must get rid of the nonassociativity.

To do that, reduce the octonionic spacetime $S^7 \times \mathbf{R}P^1$ to its maximal
 associative subspace.

How do we determine the maximal associative subspace of the octonionic
 spacetime $S^7 \times \mathbf{R}P^1$?

Following Reese Harvey [9], define the associative 3-form $\phi(x, y, z)$ for
 $x, y, z \in S^7$ by:

$$\phi(x, y, z) = \langle x, yz \rangle \tag{23}$$

where $\langle x, yz \rangle$ is the octonion inner product $Re(x\bar{y}z)$.

The associative form $\phi(x, y, z)$ is a calibration that defines an associative
 submanifold of S^7 .

When combined with the real axis part $\mathbf{R}P^1$ of octonion spacetime, the
 associative submanifold of S^7 gives us a 4-dimensional quaternionic associa-
 tive spacetime submanifold of the type $S^3 \times \mathbf{R}P^1$.

**4-DIMENSIONAL QUATERNIONIC SPACETIME $S^3 \times \mathbf{R}P^1$
 IS THE ASSOCIATIVE PHYSICAL SPACETIME.**

This structure is the same as that of the $D_4 - D_5 - E_6$ model. A detailed
 description of how dimensional reduction works in the $D_4 - D_5 - E_6$ model,
 including its effects on fermions and gauge bosons, is given in hep-ph/9501252
 [13].

5 Spacetime and Internal Symmetries.

Much of the material in this section is taken from the book of Reese Harvey [9]. To the extent that this section is good, he deserves credit. To the extent that this section is wrong or bad, it is not his fault that I made mistakes using his book.

The 4-dimensional associative physical spacetime is determined by the associative 3-form $\phi(x, y, z)$ on S^7 defined in the previous section.

What happens to the rest of the original 8-dimensional spacetime?

It is the orthogonal 4-dimensional space determined by the coassociative 4-form $\psi(x, y, z, w)$ on S^7 defined for x, y, z, w in S^7 as

$$\psi(x, y, z, w) = (1/2)(x, y(\bar{z}w) - w(\bar{z}y)) \quad (24)$$

That means that the original 8-dimensional spacetime $S^7 \times \mathbf{R}P^1$ is decomposed into an associative physical spacetime $\Phi = S^3 \times \mathbf{R}P^1$ and a coassociative internal space Ψ determined by the coassociative 4-form $\psi(x, y, z, w)$ on S^7 .

If the associative physical spacetime Φ is taken to be the real part and the coassociative internal space Ψ is taken to be the imaginary part of a complex space $\Phi + i\Psi$, then the full spacetime is transformed from a real 8-dimensional space, locally \mathbf{R}^8 , to a complex 4-dimensional space, locally \mathbf{C}^4 .

The gauge group $Spin(8)$ acting locally on \mathbf{R}^8 is then reduced to $U(4)$ acting locally on \mathbf{C}^4 .

5.1 Spacetime, Gravity, and Phase

We now have the gauge group $U(4)$ acting on the associative physical spacetime $\Phi = Re(\mathbf{C}^4)$.

Since $U(4) = Spin(6) \times U(1)$, and $Spin(6)$ is the compact version of the 15-dimensional conformal group, we can now build a model of gravity by gauging the conformal group $Spin(6)$ and use the $U(1)$ for the phase of propagators in the associative physical spacetime.

Note that only the 10-dimensional de Sitter gauge group $Spin(5)$ subgroup of the $Spin(6)$ conformal group is used to build gravity.

The other 5 degrees of freedom are 4 special conformal transformations and 1 scale dilatation. The 4 special conformal transformations are gauge-fixed to pick the $SU(2)$ symmetry-breaking direction of the Higgs mechanism, and the scale dilatation is gauge-fixed to set the Higgs mass scale.

For details, see hep-ph/9501252 [13] and [12].

5.2 Internal Space and Symmetries.

Now we have:

associative physical spacetime $\Phi = S^3 \times \mathbf{R}P^1 = Re(\mathbf{C}^4)$;

gravity from the conformal group $Spin(6)$;

Higgs symmetry breaking and mass scale from conformal $Spin(6)$; and

propagator $U(1)$ phase.

We have not yet built anything from:

the coassociative imaginary space $\Psi = Im(\mathbf{C}^4)$; or

the part of the gauge group $Spin(8)$ that is in the 12-dimensional coset space $Spin(8)/U(4)$.

Let the coassociative imaginary space $\Psi = Im(\mathbf{C}^4)$ be the internal symmetry space on which the internal gauge groups act transitively.

That means that $\Psi = Im(\mathbf{C}^4)$ plays a role similar to the internal sym-

metry spheres of Kaluza-Klein models.

Let the part of the gauge group $Spin(8)$ that is in the 12-dimensional coset space $Spin(8)/U(4)$ be the internal symmetry gauge groups.

A problem is presented here:
The coset space is just a coset space, with no group action.
How does it represent internal symmetry gauge groups?

The 12-dimensional coset space $Spin(8)/U(4)$ is the set of oriented complex structures $Cpx^+(4)$ on \mathbf{R}^8 , and is also the Grassmannian $G_{\mathbf{R}}(2, \mathbf{O})$.

Each element of the Grassmannian $G_{\mathbf{R}}(2, \mathbf{O})$ can be represented by a simple unit vector in $\wedge^2 \mathbf{O}$.

Each simple unit vector in $\wedge^2 \mathbf{O}$ determines a reflection, and all those reflections generate the group $Spin(8)$.

Geometrically, what we have is that the 12-dimensional coset space $Spin(8)/U(4)$ can be represented by 12 "positive" root vectors in the 4-dimensional root vector space of the D_4 Lie algebra of $Spin(8)$,

while the 16-dimensional $U(4)$ subgroup of $Spin(8)$ can be represented by the 12 "negative" root vectors plus the 4-dimensional Cartan subalgebra of the 4-dimensional root vector space of the D_4 Lie algebra of $Spin(8)$.

Using quaternionic coordinates for the root vector space,

$$\{\pm 1, \pm i, \pm j, \pm k, (\pm 1 \pm i \pm j \pm k)\}$$

are the 24 root vectors, and the 12-dimensional coset space $Spin(8)/U(4)$ can be represented by the 12 root vectors

$$\{+1, +i, +j, +k, (+1 \pm i \pm j \pm k)/2\}$$

What internal symmetry gauge groups do the 12 coset space $Spin(8)/U(4)$ generators of $Spin(8)$ form?

Since the 12 coset space $Spin(8)/U(4)$ generators can be represented by the quaternions

$$\{+1, +i, +j, +k, (+1 \pm i \pm j \pm k)/2\}$$

and since they do not together form a simple Lie group, consider what cartesian product of simple Lie groups might be formed.

The 8 quaternions

$$\{(+1 \pm i \pm j \pm k)/2\}$$

should form the Lie group $SU(3)$, with, for example, $(+1 + i + j + k)/2$ and $(+1 - i - j - k)/2$ as its Cartan sualgebra.

The 3 quaternions

$$\{+i, +j, +k\}$$

should form the Lie group $SU(2)$, with, for example, $+j$ as its Cartan sualgebra.

The remaining quaternion

$$\{+1\}$$

should form the Lie group $U(1)$, which is Abelian and equal to its Cartan sualgebra.

Therefore, in this model the 12-dimensional coset space $Spin(8)/U(4)$ represents the internal symmetry group of the Standard Model

$$SU(3) \times SU(2) \times U(1)$$

.

The 4-dimensional internal symmetry space Ψ is the representation space on which each of the internal symmetry groups acts transitively.

The de Sitter $Spin(5)$ of the $U(4) = Spin(6) \times U(1)$ also acts transitively on the imaginary internal symmetry space Ψ .

Each of the 4 groups $Spin(5), SU(3), SU(2), U(1)$ act transitively on the 4-dimensional internal symmetry space Ψ with its own measure.

Effectively, each measure is determined by the way in which the gauge bosons of each of the 4 forces "see" the 4-dimensional internal symmetry space Ψ .

The way each "sees" the space is determined by the geometry of the 4-dimensional symmetric space Ψ_{force} on which each force acts transitively:

<i>Gauge Group</i>	<i>Symmetric Space</i>	Ψ_{force}
$Spin(5)$	$\frac{Spin(5)}{Spin(4)}$	S^4
$SU(3)$	$\frac{SU(3)}{SU(2) \times U(1)}$	CP^2
$SU(2)$	$\frac{SU(2)}{U(1)}$	$S^2 \times S^2$
$U(1)$	$U(1)$	$S^1 \times S^1 \times S^1 \times S^1$

(25)

More about this is in

WWW URL <http://www.gatech.edu/tsmith/See.html> [12].

The ratios of the respective measures are used to calculate the relative force strength constants in this $D_4 - D_5 - E_6$ model. For detailed calculations of force strengths (and also particle masses and K-M parameters), see hep-ph/9501252 [13] and [12].

Not only does the 10-dimensional de Sitter $Spin(5)$ of the $U(4) = Spin(6) \times U(1)$ act on the imaginary internal symmetry space, but the (4+1)-dimensional conformal Higgs mechanism acts on the internal symmetry space to give mass to

the $SU(2)$ weak bosons, and
the $U(1)$ propagator phase acts to give phases to the gauge bosons.

NOW, we have constructed the $D_4 - D_5 - E_6$ model that includes the Standard Model plus Gravity, all from the beginning point of fermion creators and annihilators.

This construction of the model uses a continuous spacetime.
A future paper will deal with a discrete HyperDiamond lattice generalized Feynman checkerboard version of the model.

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