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Higgs and Fermions in $D_4 - D_5 - E_6$ Model based on $Cl(0,8)$ Clifford Algebra

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Abstract

In the $D_4 - D_5 - E_6$ model of a series of papers (hep-ph/9301210, hep-th/9302030, hep-th/9306011, and hep-th/9402003) [8, 9, 10, 11] an 8-dimensional spacetime with Lagrangian action

$$\int_{V_8} F_8 \wedge \star F_8 + \partial_8^2 \overline{\Phi_8} \wedge \star \partial_8^2 \Phi_8 + \overline{S_{8\pm}} \not{\partial}_8 S_{8\pm} + GF + GH$$

is reduced to a 4-dimensional Lagrangian.
In [11], the gauge boson terms were seen to give $SU(3) \times SU(2) \times U(1)$ for the color, weak, and electromagnetic forces and gravity of the MacDowell-Mansouri type [4], which has recently been shown by Nieto, Obregón, and Socorro [6] in gr-qc/9402029 to be equivalent, up to a Pontrjagin topological term, to the Ashtekar formulation.

This paper discusses the Higgs and spinor fermion terms.

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1 Introduction

The $D_4-D_5-E_6$ model of physics starts out with an 8-dimensional spacetime that is reduced to a 4-dimensional spacetime.

The 8-dimensional Lagrangian (up to gauge-fixing and ghost terms) is:

$$\int_{V_8} F_8 \wedge \star F_8 + \partial_8^2 \overline{\Phi_8} \wedge \star \partial_8^2 \Phi_8 + \overline{S_{8\pm}} \; \partial\!\!\!/_8 S_{8\pm} + GF + GH$$

where F_8 is the 28-dimensional $Spin(8)$ curvature, $\star$ is the Hodge dual,
∂_8 is the 8-dimensional covariant derivative,
Φ_8 is the 8-dimensional scalar field,
$\partial\!\!\!/_8$ is the 8-dimensional Dirac operator,
V_8 is 8-dimensional spacetime,
$S_{8\pm}$ are the $+$ and $-$ 8-dimensional half-spinor fermion spaces, and
GF and GH are gauge-fixing and ghost terms.

(hep-th/9402003 [11] had a typo error of S_{8+} or S_{8-} instead of $S_{8\pm}$.)

This paper describes the Higgs mechanism and the spinor fermions of the 4-dimensional Lagrangian. Results of the preceding papers in this series [8, 9, 10, 11] are assumed. They are hep-ph/9301210, hep-th/9302030, hep-th/9306011, and hep-th/9402003).

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Summary of Some Material from Earlier Papers:

$S_{8\pm} = \mathbf{R}P^1 \times S^7$ [9]);

$S_{8\pm} \oplus S_{8\pm} = (\mathbf{R}P^1 \times S^7) \oplus (\mathbf{R}P^1 \times S^7)$ is the full fermion space of first generation particles and antiparticles, and is the Silov boundary of the 32(real)-dimensional bounded complex domain corresponding to the $TypeV$ HJTS $E_6/(Spin(10) \times U(1)$ [9]
(hep-th/9302030 [9] had erroneously used $\times$ instead of $\oplus$.);

after dimensional reduction, the weak force gauge group is $SU(2)$ [11];

with respect to $SU(2)$ of the Higgs and weak force, the 4-dimensional spacetime manifold has global type $M = S^2 \times S^2$ [9];

the Higgs and weak force $SU(2)$ acts effectively on a submanifold of the half-spinor fermion space $S_{8\pm} = \mathbf{R}P^1 \times S^7$, that is, $Q_3 = \mathbf{R}P^1 \times S^2$, which is Silov boundary of the 6(real)-dimensional bounded complex domain corresponding to the $TypeIV_3$ HJTS $\overline{D_3} = Spin(5)/(SU(2) \times U(1)$ [9]
(hep-th/9302030 [9] had erroneously listed $SU(3)/SU(2) \times U(1)$ instead of $Spin(5)/SU(2) \times U(1) = Spin(5)/Spin(3) \times U(1)$.);

after dimensional reduction, the Higgs scalar is the 4th component of the column minimal ideal of $Cle(0,6)$ that contains W_+, W_-, and W_0 of the $SU(2)$ weak force, and so is an $SU(2)$ scalar [11];
the $SU(2)$ gauge group of the vector bosons W_+, W_-, and W_0 and the $SU(2)$ gauge group of the scalar Higgs, insofar as they are independent, can be considered as one $Spin(4) = SU(2) \times SU(2)$ gauge group [9, 11]; and

the electromagnetic $U(1)$ of the $D_4 - D_5 - E_6$ model comes from the $U(3)$ containing the color force $SU(3)$, and so is "unified" with the $SU(3)$ color force rather than with the $SU(2)$ weak force [11].

The last statement is different from most formulations of the standard model, but is similar to the formulation of O'Raifeartaigh (section 9.4 of [7]) of the standard model as $S(U(3) \times U(2))$ rather than $SU(3) \times U(2)$ or $SU(3) \times SU(2) \times U(1)$. O'Raifeartaigh states that the unbroken gauge symmetry is actually $U(3)$ rather than $SU(3) \times U(1)$.

2 Scalar part of the Lagrangian

The scalar part of the 8-dimensional Lagrangian is

$$\int_{V_8} \partial_8^2 \overline{\Phi_8} \wedge \star \partial_8^2 \Phi_8$$

As shown in chapter 4 of Göckeler and Schücker [2], $\partial_8^2 \Phi_8$ can be represented as an 8-dimensional curvature F_{H8}, giving

$$\int_{V_8} F_{H8} \wedge \star F_{H8}$$

When spacetime is reduced to 4 dimensions, denote the surviving 4 dimensions by 4 and the reduced 4 dimensions by $\perp 4$.

Then, $F_{H8} = F_{H44} + F_{H4\perp 4} + F_{H\perp 4\perp 4}$, where
F_{H44} is the part of F_{H8} entirely in the surviving spacetime;
$F_{H4\perp 4}$ is the part of F_{H8} partly in the surviving spacetime and partly in the reduced spacetime; and
$F_{H\perp 4\perp 4}$ is the part of F_{H8} entirely in the reduced spacetime;

The 4-dimensional Higgs Lagrangian is then:
$\int (F_{H44} + F_{H4\perp 4} + F_{H\perp 4\perp 4}) \wedge \star (F_{H44} + F_{H4\perp 4} + F_{H\perp 4\perp 4}) =$
$= \int (F_{H44} \wedge \star F_{H44} + F_{H4\perp 4} \wedge \star F_{H4\perp 4} + F_{H\perp 4\perp 4} \wedge \star F_{H\perp 4\perp 4})$.

As all possible paths should be taken into account in the sum over histories path integral picture of quantum field theory, the terms involving the reduced 4 dimensions, $\perp 4$, should be integrated over the reduced 4 dimensions.

Integrating over the reduced 4 dimensions, $\perp 4$, gives
$\int (F_{H44} \wedge \star F_{H44} + \int_{\perp 4} F_{H4\perp 4} \wedge \star F_{H4\perp 4} + \int_{\perp 4} F_{H\perp 4\perp 4} \wedge \star F_{H\perp 4\perp 4})$.

2.1 First term $F_{H44} \wedge \star F_{H44}$

The first term is just $\int F_{H44} \wedge \star F_{H44}$.

Since they are both $SU(2)$ gauge boson terms, this term, in 4-dimensional spacetime, just merges into the $SU(2)$ weak force term $\int F_w \wedge \star F_w$.

2.2 Third term $\int_{\perp 4} F_{H\perp 4\perp 4} \wedge \star F_{H\perp 4\perp 4}$

The third term, $\int \int_{\perp 4} F_{H\perp 4\perp 4} \wedge \star F_{H\perp 4\perp 4}$, after integration over $\perp 4$, produces terms of the form

$\lambda(\overline{\Phi}\Phi)^2 - \mu^2\overline{\Phi}\Phi$ by a process similar to the Mayer mechanism.

The Mayer mechanism is based on Proposition 11.4 of chapter 11 of volume I of Kobayashi and Nomizu [3], stating that:
$2F_{H\perp 4\perp 4}(X,Y) = [\Lambda(X), \Lambda(Y)] - \Lambda([X,Y])$,
where Λ takes values in the $SU(2)$ Lie algebra.

If the action of the Hodge dual $\star$ on Λ is such that
$\star\Lambda = -\Lambda$ and $\star[\Lambda, \Lambda] = [\Lambda, \Lambda]$,
then
$F_{H\perp 4\perp 4}(X,Y) \wedge \star F_{H\perp 4\perp 4}(X,Y) = (1/4)([\Lambda(X), \Lambda(Y)]^2 - \Lambda([X,Y])^2)$.

If integration of Λ over $\perp 4$ is $\int_{\perp 4} \Lambda \propto \Phi = (\Phi^+, \Phi^0)$, then

$$\int_{\perp 4} F_{H\perp 4\perp 4} \wedge \star F_{H\perp 4\perp 4} = (1/4) \int_{\perp 4} [\Lambda(X), \Lambda(Y)]^2 - \Lambda([X,Y])^2 =$$

$$= (1/4)[\lambda(\overline{\Phi}\Phi)^2 - \mu^2\overline{\Phi}\Phi],$$

where λ is the strength of the scalar field self-interaction, μ^2 is the other constant in the Higgs potential, and where Φ is a 0-form taking values in the $SU(2)$ Lie algebra.

The $SU(2)$ values of Φ are represented by complex $SU(2) = Spin(3)$ doublets $\Phi = (\Phi^+, \Phi^0)$.

In real terms, $\Phi^+ = (\Phi_1 + i\Phi_2)/\sqrt{2}$ and $\Phi^0 = (\Phi_3 + i\Phi_4)/\sqrt{2}$,
so Φ has 4 real degrees of freedom.

In terms of real components, $\overline{\Phi}\Phi = (\Phi_1^2 + \Phi_2^2 + \Phi_3^2 + \Phi_4^2)/2$.

The nonzero vacuum expectation value of the
$\lambda(\overline{\Phi}\Phi)^2 - \mu^2\overline{\Phi}\Phi$ term is $v = \mu/\sqrt{\lambda}$, and
$< \Phi^0 >=< \Phi_3 >= v/\sqrt{2}$.

In the unitary gauge, $\Phi_1 = \Phi_2 = \Phi_4 = 0$,
and
$\Phi = (\Phi^+, \Phi^0) = (1/\sqrt{2})(\Phi_1 + i\Phi_2, \Phi_3 + i\Phi_4) = (1/\sqrt{2})(0, v + H)$,

where $\Phi_3 = (v + H)/\sqrt{2}$,
v is the Higgs potential vacuum expectation value, and
H is the real surviving Higgs scalar field.

Since $\lambda = \mu^2/v^2$ and $\Phi = (v + H)/\sqrt{2}$,

$(1/4)[\lambda(\overline{\Phi}\Phi)^2 - \mu^2\overline{\Phi}\Phi] =$

$= (1/16)(\mu^2/v^2)(v + H)^4 - (1/8)\mu^2(v + H)^2 =$

$= (1/16)[\mu^2v^2 + 4\mu^2vH + 6\mu^2H^2 + 4\mu^2H^3/v + \mu^2H^4/v^2 - 2\mu^2v^2-$
$-4\mu^2vH - 2\mu^2H^2] =$

$= (1/4)\mu^2H^2 - (1/16)\mu^2v^2[1 - 4H^3/v^3 - H^4/v^4]$.

2.3 Second term $F_{H4\perp 4} \wedge \star F_{H4\perp 4}$

The second term,
$\int_{\perp 4} F_{H4\perp 4} \wedge \star F_{H4\perp 4}$,
gives $\int \partial\overline{\Phi}\partial\Phi$, by a process similar to the Mayer mechanism.

From Proposition 11.4 of chapter 11 of volume I of Kobayashi and Nomizu [3]:
$2F_{H4\perp 4}(X, Y) = [\Lambda(X), \Lambda(Y)] - \Lambda([X, Y])$,
where Λ takes values in the $SU(2)$ Lie algebra.

For example, if the X component of $F_{H4\perp 4}(X, Y)$ is in the surviving 4 spacetime and the Y component of $F_{H4\perp 4}(X, Y)$ is in $\perp 4$, then

the Lie bracket product $[X, Y] = 0$ so that $\Lambda([X, Y]) = 0$ and therefore
$F_{H4\perp 4}(X, Y) = (1/2)[\Lambda(X), \Lambda(Y)] = (1/2)\partial_X\Lambda(Y)$.

The total value of $F_{H4\perp 4}(X, Y)$ is then $F_{H4\perp 4}(X, Y) = \partial_X\Lambda(Y)$.

Integration of Λ over $\perp 4$ gives
$\int_{Y \epsilon \perp 4} \partial_X \Lambda(Y) = \partial_X \Phi$,
where, as above, Φ is a 0-form taking values in the $SU(2)$ Lie algebra.

As above, the $SU(2)$ values of Φ are represented by complex $SU(2) = Spin(3)$ doublets $\Phi = (\Phi^+, \Phi^0)$.

In real terms, $\Phi^+ = (\Phi_1 + i\Phi_2)/\sqrt{2}$ and $\Phi^0 = (\Phi_3 + i\Phi_4)/\sqrt{2}$,
so Φ has 4 real degrees of freedom.

As discussed above, in the unitary gauge, $\Phi_1 = \Phi_2 = \Phi_4 = 0$, and
$\Phi = (\Phi^+, \Phi^0) = (1/\sqrt{2})(\Phi_1 + i\Phi_2, \Phi_3 + i\Phi_4) = (1/\sqrt{2})(0, v + H)$,
where $\Phi_3 = (v + H)/\sqrt{2}$,
v is the Higgs potential vacuum expectation value, and
H is the real surviving Higgs scalar field.

The second term is then:
$\int(\int_{\perp 4} -F_{H4\perp 4} \wedge \star F_{H4\perp 4}) =$
$= \int(\int_{\perp 4}(-1/2)[\Lambda(X), \Lambda(Y)] \wedge \star[\Lambda(X), \Lambda(Y)]) = \int \partial\overline{\Phi} \wedge \star\partial\Phi$

where the $SU(2)$ covariant derivative ∂ is
$\partial = \partial + \sqrt{\alpha_w}(W_+ + W_-) + \sqrt{\alpha_w} \cos\theta_w{}^2 W_0$, and θ_w is the Weinberg angle.

Then $\partial\Phi = \partial(v + H)/\sqrt{2} =$
$= [\partial H + \sqrt{\alpha_w} W_+(v + H) + \sqrt{\alpha_w} W_-(v + H) + \sqrt{\alpha_w} W_0(v + H)]/\sqrt{2}$.

In the $D_4 - D_5 - E_6$ model the W_+, W_-, W_0, and H terms are considered to be linearly independent.

$v = v_+ + v_- + v_0$ has linearly independent components v_+, v_-, and v_0 for W_+, W_-, and W_0.

H is the Higgs component.

$\partial\overline{\Phi} \wedge \star\partial\Phi$ is the sum of the squares of the individual terms.

Integration over $\perp 4$ involving two derivatives $\partial_X \partial_X$ is taken to change the sign by $i^2 = -1$.

Then:
$\partial\overline{\Phi}\wedge\star\partial\Phi=(1/2)(\partial H)^2+$
$+(1/2)[\alpha_w v_+^2\overline{W_+}W_+ + \alpha_w v_-^2\overline{W_-}W_- + \alpha_w v_0^2\overline{W_0}W_0]+$
$+(1/2)[\alpha_w\overline{W_+}W_+ + \alpha_w\overline{W_-}W_- + \alpha_w\overline{W_0}W_0][H^2+2vH].$

Then the full curvature term of the weak-Higgs Lagrangian,
$\int F_w\wedge\star F_w+\partial\overline{\Phi}\wedge\star\partial\Phi+\lambda(\overline{\Phi}\Phi)^2-\mu^2\overline{\Phi}\Phi,$

is, by the Higgs mechanism:

$\int[F_w\wedge\star F_w+$
$+(1/2)[\alpha_w v_+^2\overline{W_+}W_+ + \alpha_w v_-^2\overline{W_-}W_- + \alpha_w v_0^2\overline{W_0}W_0]+$
$+(1/2)[\alpha_w\overline{W_+}W_+ + \alpha_w\overline{W_-}W_- + \alpha_w\overline{W_0}W_0][H^2+2vH]+$
$+(1/2)(\partial H)^2+(1/4)\mu^2H^2-$
$-(1/16)\mu^2v^2[1-4H^3/v^3-H^4/v^4]].$

The weak boson Higgs mechanism masses, in terms of $v=v_++v_-+v_0$, are:

$(\alpha_w/2)v_+^2=m_{W_+}^2$;

$(\alpha_w/2)v_-^2=m_{W_-}^2$; and

$(\alpha_w/2)v_0^2=m_{W_{+0}}^2,$

with $(v=v_++v_-+v_0)=((\sqrt{2})/\sqrt{\alpha_w})(m_{W_+}+m_{W_-}+m_{W_0}).$

Then:

$\int[F_w\wedge\star F_w+$
$+m_{W_+}^2W_+W_+ + m_{W_-}^2W_-W_- + m_{W_0}^2W_0W_0+$
$+(1/2)[\alpha_w\overline{W_+}W_+ + \alpha_w\overline{W_-}W_- + \alpha_w\overline{W_0}W_0][H^2+2vH]+$
$+(1/2)(\partial H)^2+(1/2)(\mu^2/2)H^2-$
$-(1/16\mu^2v^2[1-4H^3/v^3-H^4/v^4].$

2.4 Higgs Mass

The Higgs vacuum expectation value $v = (v_+ + v_- + v_0)$ is the only particle mass free parameter.

In the $D_4 - D_5 - E_6$ model, v is set so that the electron mass $m_e = 0.5110 MeV$.

Therefore, $(\sqrt{\alpha_w})/\sqrt{2})v = m_{W_+} + m_{W_-} + m_{W_0} = 260.774 GeV$,

the value chosen so that the electron mass (which is to be determined from it) will be 0.5110 MeV.

In the $D_4 - D_5 - E_6$ model, α_w is calculated to be $\alpha_w = 0.2534577$,
so $\sqrt{\alpha_w} = 0.5034458$ and $v = 732.53$ GeV.

The Higgs mass m_H is given by the term
$(1/2)(\partial H)^2 - (1/2)(\mu^2/2)H^2 = (1/2)[(\partial H)^2 - (\mu^2/2)H^2]$
to be $m_H^2 = \mu^2/2 = \lambda v^2/2$, so that $m_H = \sqrt{(\mu^2/2)} = \sqrt{\lambda} v^2/2)$.

λ is the scalar self-interaction strength. It should be the product of the "weak charges" of two scalars coming from the reduced 4 dimensions in $Spin(4)$, which should be the same as the weak charge of the surviving weak force $SU(2)$ and therefore just the square of the $SU(2)$ weak charge, $\sqrt{(\alpha_w^2)} = \alpha_w$, where α_w is the $SU(2)$ geometric force strength.

Therefore $\lambda = \alpha_w = 0.2534576$, $\sqrt{\lambda} = 0.5034458$, and $v = 732.53$ GeV,
so that the mass of the Higgs scalar is $m_H = v\sqrt{(\lambda/2)} = 260.774$ GeV.

3 Spinor Fermion part of the Lagrangian

Consider the spinor fermion term $\int \overline{S_{8\pm}}\ \partial\!\!\!/_8 S_{8\pm}$

For each of the surviving 4-dimensional 4 and reduced 4-dimensional $\perp 4$ of 8-dimensional spacetime, the part of $S_{8\pm}$ on which the Higgs $SU(2)$ acts locally is $Q_3 = \mathbf{R}P^1 \times S^2$.

It is the Silov boundary of the bounded domain D_3 that is isomorphic to the symmetric space $\overline{D_3} = Spin(5)/SU(2) \times U(1)$.

The Dirac operator $\partial\!\!\!/_8$ decomposes as $\partial\!\!\!/ = \partial\!\!\!/_4 + \partial\!\!\!/_{\perp 4}$, where
$\partial\!\!\!/_4$ is the Dirac operator corresponding to the surviving spacetime 4 and
$\partial\!\!\!/_{\perp 4}$ is the Dirac operator corresponding to the reduced $4 \perp 4$.

Then the spinor term is $\int \overline{S_{8\pm}}\ \partial\!\!\!/_4 S_{8\pm} + \overline{S_{8\pm}}\ \partial\!\!\!/_{\perp 4} S_{8\pm}$
The Dirac operator term $\partial\!\!\!/_{\perp 4}$ in the reduced $\perp 4$ has dimension of mass.

After integration $\int \overline{S_{8\pm}}\ \partial\!\!\!/_{\perp 4} S_{8\pm}$ over the reduced $\perp 4$,
$\partial\!\!\!/_{\perp 4}$ becomes the real scalar Higgs scalar field $Y = (v + H)$ that comes from the complex $SU(2)$ doublet Φ after action of the Higgs mechanism.

If integration over the reduced $\perp 4$ involving two fermion terms $\overline{S_{8\pm}}$ and $S_{8\pm}$ is taken to change the sign by $i^2 = -1$, then, by the Higgs mechanism,
$\int \overline{S_{8\pm}}\ \partial\!\!\!/_{\perp 4} S_{8\pm} \rightarrow \int (\int_{\perp 4} \overline{S_{8\pm}}\ \partial\!\!\!/_{\perp 4} S_{8\pm}) \rightarrow$
$\rightarrow - \int \overline{S_{8\pm}} Y Y S_{8\pm} = - \int \overline{S_{8\pm}} Y (v + H) S_{8\pm}$,

where:
H is the real physical Higgs scalar, $m_H = v\sqrt{(\lambda/2)} = 261$ GeV, and v is the vacuum expectation value of the scalar field Y, the free parameter in the theory that sets the mass scale.

Denote the sum of the three weak boson masses by Σ_{m_W}.

$v = \Sigma_{m_W}((\sqrt{2})/\sqrt{\alpha_w}) = 260.774 \times \sqrt{2}/0.5034458 = 732.53 GeV$,
a value chosen so that the electron mass will be 0.5110 MeV.

3.1 Yukawa Coupling and Fermion Masses

Y is the Yukawa coupling between fermions and the Higgs field.

Y acts on all 28 elements
(2 helicity states for each of the 7 Dirac particles and 7 Dirac antiparticles) of the Dirac fermions in a given generation, because all of them are in the same Spin(8) spinor representation.

Denote the sum of the first generation Dirac fermion masses by Σ_{f_1}.

Then $Y = (\sqrt{2})\Sigma_{f_1}/v$, just as $\sqrt{(\alpha_w)} = (\sqrt{2})\Sigma_{m_W}/v$.

Y should be the product of two factors:

e^2, the square of the electromagnetic charge $e = \sqrt{\alpha_E}$, because in the term $\int(\int_{\perp 4}\overline{S_{8\pm}}\ \partial\!\!\!/_{\perp 4} S_{8\pm}) \rightarrow -\int \overline{S_{8\pm}} Y(v+H)S_{8\pm}$ each of the Dirac fermions $S_{8\pm}$ carries electromagnetic charge proportional to e ; and

$1/g_w$, the reciprocal of the weak charge $g_w = \sqrt{\alpha_w}$, because an $SU(2)$ force, the Higgs $SU(2)$, couples the scalar field to the fermions.

Therefore $\Sigma_{f_1} = Yv/\sqrt{2} = (e^2/g_w)v/\sqrt{2} = 7.508$ GeV and

$\Sigma_{f_1}/\Sigma_{m_W} = (e^2/g_w)v/g_w v = e^2/g_w^2 = \alpha_E/\alpha_w$.

The Higgs term $-\int \overline{S_{8\pm}} Y(v+H)\ S_{8\pm} = -\int \overline{S_{8\pm}}\ YvS_{8\pm} - \int \overline{S_{8\pm}} YHS_{8\pm} = = -int\overline{S_{8\pm}}(\sqrt{2}\Sigma_{f_1})S_{8\pm} - \int \overline{S_{8\pm}}(\sqrt{2}\Sigma_{f_1}/v)S_{8\pm}$.

The resulting spinor term is of the form $\int[\overline{S_{8\pm}}(\partial\!\!\!/ - Yv)S_{8\pm} - \overline{S_{8\pm}}YHS_{8\pm}]$
,

where $(\partial\!\!\!/ - Yv)$ is a massive Dirac operator.

How much of the total mass $\Sigma_{f_1} = Yv/\sqrt{2} = 7.5 GeV$ is allocated to each of the first generation Dirac fermions is determined by calculating the individual fermion masses in the $D_4 - D_5 - E_6$ model, and

those calculations also give the values of

$\Sigma_{f_2} = 32.9 GeV$, $\Sigma_{f_3} = 1,629 GeV$, and
individual second and third generation fermion masses.

The individual tree-level lepton masses and quark constituent masses [8] are:
$m_e = 0.5110$ MeV (assumed);
$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$;
$m_d = m_u = 312.8$ MeV (constituent quark mass);
$m_\mu = 104.8$ MeV;
$m_s = 625$ MeV (constituent quark mass);
$m_c = 2.09$ GeV (constituent quark mass);
$m_\tau = 1.88$ GeV;
$m_b = 5.63$ GeV (constituent quark mass);
$m_t = 130$ GeV (constituent quark mass).

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:
phase angle $\epsilon = \pi/2$
$\sin\alpha = [m_e + 3m_d + 3m_u]/\sqrt{[m_e^2 + 3m_d^2 + 3m_u^2] + [m_\mu^2 + 3m_s^2 + 3m_c^2]}$
$\sin\beta = [m_e + 3m_d + 3m_u]/\sqrt{[m_e^2 + 3m_d^2 + 3m_u^2] + [m_\tau^2 + 3m_b^2 + 3m_t^2]}$
$\sin\tilde{\gamma} = [m_\mu + 3m_s + 3m_c]/\sqrt{[m_\tau^2 + 3m_b^2 + 3m_t^2] + [m_\mu^2 + 3m_s^2 + 3m_c^2]}$
$\sin\gamma = \sin\tilde{\gamma}\sqrt{\Sigma_{f_2}/\Sigma_{f_1}}$

The resulting Kobayashi-Maskawa parameters are:

	d	s	b
u	0.975	0.222	$-0.00461i$
c	$-0.222 - 0.000191i$	$0.974 - 0.0000434i$	0.0423
t	$0.00941 - 0.00449i$	$-0.0413 - 0.00102i$	0.999

4 Parity Violation, W-Boson Masses, and θ_w

In the $D_4 - D_5 - E_6$ model prior to dimensional reduction, the fermion particles are all massless at tree level.

The neutrinos obey the Weyl equation and must remain massless and left-handed at tree level after dimensional reduction.

The electrons and quarks obey the Dirac equation and acquire mass after dimensional reduction.

After dimensional reduction, the charged $W_{\pm}$ of the $SU(2)$ weak force can interchange Weyl fermion neutrinos with Dirac fermion electrons.

4.1 Massless Neutrinos and Parity Violation

It is required (as an ansatz or part of the $D_4 - D_5 - E_6$ model)

that the charged $W_{\pm}$ neutrino-electron interchange must be symmetric with the electron-neutrino interchange, so that the absence of right-handed neutrino particles requires that the charged $W_{\pm}$ $SU(2)$ weak bosons act only on left-handed electrons.

It is also required (as an ansatz or part of the $D_4 - D_5 - E_6$ model) that each gauge boson must act consistently on the entire Dirac fermion particle sector, so that the charged $W_{\pm}$ $SU(2)$ weak bosons act only on left-handed fermions of all types.

Therefore, for the charged $W_{\pm}$ $SU(2)$ weak bosons, the 4-dimensional spinor fields $S_{8\pm}$ contain only left-handed particles and right-handed antiparticles.

So, for the charged $W_{\pm}$ $SU(2)$ weak bosons, $S_{8\pm}$ can be denoted $S_{8\pm L}$.

4.2 W_0, Z, and θ_w

The neutral W_0 weak bosons do not interchange Weyl neutrinos with Dirac fermions, and so may not entirely be restricted to left-handed spinor particle fields $S_{8\pm L}$, but may have a component that acts on the full right-handed and left-handed spinor particle fields $S_{8\pm} = S_{8\pm L} + S_{8\pm R}$.

However, the neutral W_0 weak bosons are related to the charged $W_\pm$ weak bosons by custodial $SU(2)$ symmetry, so that the left-handed component of the neutral W_0 must be equal to the left-handed (entire) component of the charged $W_\pm$.

Since the mass of the W_0 is greater than the mass of the $W_\pm$, there remains for the W_0 a component acting on the full $S_{8\pm} = S_{8\pm L} + S_{8\pm R}$ spinor particle fields.

Therefore the full W_0 neutral weak boson interaction is proportional to $(m^2_{W_\pm}/m^2_{W_0})$ acting on $S_{8\pm L}$ and $(1 - (m^2_{W_\pm}/m^2_{W_0}))$ acting
on $S_{8\pm} = S_{8\pm L} + S_{8\pm R}$.

If $(1 - (m_{W_\pm} 2/m^2_{W_0}))$ is defined to be $\sin\theta_w{}^2$ and denoted by ξ, and

if the strength of the $W_\pm$ charged weak force (and of the custodial $SU(2)$ symmetry) is denoted by T,

then the W_0 neutral weak interaction can be written as:
$W_{0L} \sim T + \xi$ and $W_{0R} \sim \xi$.

The $D_4 - D_5 - E_6$ model allows calculation of the Weinberg angle θ_w, by $m_{W_+} = m_{W_-} = m_{W_0} \cos\theta_w$.

The Hopf fibration of S^3 as
$S^1 \rightarrow S^3 \rightarrow S^2$
gives a decomposition of the W bosons into the neutral W_0 corresponding to S^1 and the charged pair W_+ and W_- corresponding to S^2.

The mass ratio of the sum of the masses of W_+ and W_- to the mass of W_0 should be the volume ratio of the S^2 in S^3 to the S^1 in $S3$.

The unit sphere $S^3 \subset R^4$ is normalized by $1/2$.

The unit sphere $S^2 \subset R^3$ is normalized by $1/\sqrt{3}$.

The unit sphere $S^1 \subset R^2$ is normalized by $1/\sqrt{2}$.

The ratio of the sum of the W_+ and W_- masses to the W_0 mass should then be $(2/\sqrt{3})V(S^2)/(2/\sqrt{2})V(S^1) = 1.632993$.

The sum $\Sigma_{m_W} = m_{W_+} + m_{W_-} + m_{W_0}$ has been calculated to be $v\sqrt{\alpha_w} = 517.798\sqrt{0.2534577} = 260.774GeV$.

Therefore, $\cos\theta_w{}^2 = m^2_{W_\pm}/m^2_{W_0} = (1.632993/2)^2 = 0.667$, and $\sin\theta_w{}^2 = 0.333$, so $m_{W_+} = m_{W_-} = 80.9GeV$, and $m_{W_0} = 98.9GeV$.

4.3 Corrections for m_Z and θ_w

The above values must be corrected for the fact that only part of the w_0 acts through the parity violating $SU(2)$ weak force and the rest acts through a parity conserving $U(1)$ electromagnetic type force.

In the $D_4 - D_5 - E_6$ model, the weak parity conserving $U(1)$ electromagnetic type force acts through the $U(1)$ subgroup of $SU(2)$, which is not exactly like the $D_4 - D_5 - E_6$ electromagnetic $U(1)$ with force strength $\alpha_E = 1/137.03608 = e^2$.

The W_0 mass m_{W_0} has two parts:
the parity violating $SU(2)$ part $m_{W_{0\pm}}$ that is equal to $m_{W_\pm}$; and
the parity conserving part $m_{W_{00}}$ that acts like a heavy photon.

As $m_{W_0} = 98.9$ GeV $= m_{W_{0\pm}} + m_{W_{00}}$, and as $m_{W_{0\pm}} = m_{W_\pm} = 80.9GeV$, we have $m_{W_{00}} = 18GeV$.

Denote by $\tilde{\alpha_E} = \tilde{e}^2$ the force strength of the weak parity conserving $U(1)$ electromagnetic type force that acts through the $U(1)$ subgroup of $SU(2)$.

The $D_4 - D_5 - E_6$ electromagnetic force strength $\alpha_E = e^2 = 1/137.03608$ was calculated using the volume $V(S^1)$ of an $S^1 \subset R^2$, normalized by $1/\sqrt{2}$.

The $\tilde{\alpha_E}$ force is part of the $SU(2)$ weak force whose strength $\alpha_w = w^2$

was calculated using the volume $V(S^2) of an S^2 \subset R^3$, normalized by $1/\sqrt{3}$.

Also, the $D_4 - D_5 - E_6$ electromagnetic force strength $\alpha_E = e^2$ was calculated using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,

while the $SU(2)$ weak force strength $\alpha_w = w^2$ was calculated using two 2-spheres $S^2 \times S^2$, each of which contains one 1-sphere of the $\tilde{\alpha_E}$ force.

Therefore $\tilde{\alpha_E} = \alpha_E(\sqrt{2}/\sqrt{3})(2/4) = \alpha_E/\sqrt{6}$, $\tilde{E} = e/4\sqrt{6} = e/1.565$, and the mass $m_{W_{00}}$ must be reduced to an effective value
$m_{W_{00}eff} = m_{W_{00}}/1.565 = 18/1.565 = 11.5$ GeV
for the $\tilde{\alpha_E}$ force to act like an electromagnetic force in the 4-dimensional spacetime of the $D_4 - D_5 - E_6$ model:
$\tilde{E}m_{W_{00}} = e(1/5.65)m_{W_{00}} = em_{Z_0}$,

where the physical effective neutral weak boson is denoted by Z rather than W_0.

Therefore, the correct $D_4 - D_5 - E_6$ values for weak boson masses and the Weinberg angle are:

$m_{W_+} = m_{W_-} = 80.9 GeV$;

$m_Z = 80.9 + 11.5 = 92.4 GeV$; and

$\sin\theta_w{}^2 = 1 - (m_{W_\pm}/m_Z)^2 = 1 - 6544.81/8537.76 = 0.233$.

Radiative corrections are not taken into account here, and may change the $D_4 - D_5 - E_6$ value somewhat.

5 Some Errata for Previous Papers

hep-th/9402003 [11] had a typographical error of only S_{8+} or S_{8-} instead of $S_{8\pm}$. The correct 8-dim Lagrangian is:

$$\int_{V_8} F_8 \wedge \star F_8 + \partial_8^2 \Phi_8 \star \partial_8^2 \Phi_8 + \overline{S_{8\pm}} \not\partial_8 S_{8\pm} + GF + GH$$

hep-th/9302030 [9] had erroneously used $\times$ instead of $\oplus$ for the fermion spinor space. The correct full fermion space of first generation particles and antiparticles is

$S_{8+} \oplus S_{8\pm} = (\mathbf{R}P^1 \times S^7) \oplus (\mathbf{R}P^1 \times S^7)$.

It is the Silov boundary of the 32(real)-dimensional bounded complex domain corresponding to the $TypeV$ HJTS $E_6/(Spin(10) \times U(1)$ [9]

hep-th/9302030 [9] had erroneously listed $SU(3)/SU(2) \times U(1)$, instead of $Spin(5)/SU(2) \times U(1) = Spin(5)/Spin(3) \times U(1)$, as the $TypeIV_3$ HJTS corresponding to the 6(real)-dimensional bounded complex domain on whose Silov boundary the gauge group $SU(2)$ naturally acts.

The corrected table is:

The Q and D manifolds for the gauge groups of the four forces are:

Gauge Group	*Hermitian Symmetric Space*	*Type of D*	m	Q
$Spin(5)$	$\frac{Spin(7)}{Spin(5)\times U(1)}$	IV_5	4	$\mathbf{R}P^1 \times S^4$
$SU(3)$	$\frac{SU(4)}{SU(3)\times U(1)}$	B^6 (*ball*)	4	S^5
$SU(2)$	$\frac{Spin(5)}{SU(2)\times U(1)}$	IV_3	2	$\mathbf{R}P^1 \times S^2$
$U(1)$	$-$	$-$	1	$-$

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