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From Sets to Quarks:

Deriving the Standard Model plus Gravitation from Simple Operations on Finite Sets

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Abstract

From sets and simple operations on sets,
a Feynman Checkerboard physics model is constructed
that allows computation of force strength constants and
constituent mass ratios of elementary particles,
with a Lagrangian structure that gives a Higgs scalar
particle mass of about 146 GeV and a Higgs scalar
field vacuum expectation value of about 252 GeV,
giving a tree level constituent Truth Quark (top quark) mass
of roughly 130 GeV, which is (in my opinion) supported
by dileptonic events and some semileptonic events.
See http://galaxy.cau.edu/tsmith/HDFCmodel.html
and http://www.innerx.net/personal/tsmith/HDFCmodel.html
Chapter 1 - Introduction.
Chapter 2 - From Sets to Clifford Algebras.
Chapter 3 - Octonions and E8 lattices.
Chapter 4 - E8 spacetime and particles.
Chapter 5 - HyperDiamond Lattices.
Chapter 6 - Internal Symmetry Space.
Chapter 7 - Feynman Checkerboards.
Chapter 8 - Charge = Amplitude to Emit Gauge Boson.
Chapter 9 - Mass = Amplitude to Change Direction.
Chapter 10 - Protons, Pions, and Physical Gravitons.

Contents

1 Introduction.

From finite sets and some simple operations on sets, a Feynman Checkerboard physics model, whose continuous version is the Lie algebra $D_4 - D_5 - E_6$ model, is constructed.

1.1 Particle Masses and Force Constants.

Force strength constants and constituent mass ratios of elementary particles can be calculated. If the electron mass is taken as the base mass, and assumed to be about 0.511 MeV, then the model gives roughly the following particle masses (the massless gluons are confined, the massless Spin(5) gravitons are confined, but the massless physical gravitons propagate):

$10^{19} GeV$			$PlanckMass$	
10^{15}			$HiggsMax$	
500			SSB	
$250-260$	$T-T$		$HiggsVEV$	$W^+ + W^- + Z_0$
$130-146$	$TruthQ$		$HiggsBoson$	$\sqrt{W_+^2 + W_-^2 + Z_0^2}$
$80-92$				W^+, W^-, Z_0
$5-6$		$BeautyQ$		
2	$CharmQ$		$Tauon$	
1		$Proton$		
0.6		$StrangeQ$		
0.3	UpQ	$DownQ$		
0.24		QCD		
0.14		$Pion$		
0.10			$Muon$	
0.0005			$Electron$	
0			$Neutrinos$ (e, μ, τ)	$photon$ $gluon$ $graviton$

(1)

The fermion masses give the following Kobayashi-Maskawa parameters:

	d	s	b
u	0.975	0.222	$-0.00461i$
c	$-0.222 - 0.000191i$	$0.974 - 0.0000434i$	0.0423
t	$0.00941 - 0.00449i$	$-0.0413 - 0.00102i$	0.999

(2)

If the geometric gravitational force volume is set at unity, then the model gives the following force strengths, at the characteristic energy level of each force:

Gauge Group	*Force*	*Characteristic Energy*	*Geometric Force Strength*	*Total Force Strength*
$Spin(5)$	*gravity*	$\approx 10^{19} GeV$	1	$G_G m_{proton}^2 \approx 5 \times 10^{-39}$
$SU(3)$	*color*	$\approx 245 MeV$	0.6286	0.6286
$SU(2)$	*weak*	$\approx 100 GeV$	0.2535	$G_W m_{proton}^2 \approx \approx 1.05 \times 10^{-5}$
$U(1)$	$e - mag$	$\approx 4KV$	1/137.03608	1/137.03608

(3)

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.

The effect is particularly pronounced with the color force.

The color force strength was calculated at various energies according to renormalization group equations, with the following results:

Energy Level	*Color Force Strength*
$245 MeV$	0.6286
$5.3 GeV$	0.166
$34 GeV$	0.121
$91 GeV$	0.106

(4)

1.1.1 Confined Quark Hadrons.

Since particle masses can only be observed experimentally for particles that can exist in a free state ("free" means "not strongly bound to other particles, except for virtual particles of the active vacuum of spacetime"),

and

since quarks do not exist in free states, interpret the calculated quark masses as constituent masses (not current masses).

Constituent particles are Pre-Quantum particles in the sense that their properties are calculated without using sum-over-histories Many-Worlds quantum theory.

("Classical" is a commonly-used synonym for "Pre-Quantum".)

Since experiments are quantum sum-over-histories processes, experimentally observed particles are Quantum particles.

Consider the experimentally observed proton.

A proton is a Quantum particle containing 3 constituent quarks:

two up quarks and one down quark; one Red, one Green, and one Blue.

The 3 Pre-Quantum constituent quarks are called "valence" quarks. They are bound to each other by SU(3) QCD. The constituent quarks "feel" the effects of QCD by "sharing" virtual gluons and virtual quark-antiquark pairs that come from the vacuum in sum-over-histories quantum theory.

Since the 3 valence constituent quarks within the proton are constantly surrounded by the shared virtual gluons and virtual quark-antiquark pairs, the 3 valence constituent quarks can be said to "swim" in a "sea" of virtual gluons and quark-antiquark pairs, which are called "sea" gluons, quarks, and antiquarks.

In the model, the proton is the most stable bound state of 3 quarks, so that the virtual sea within the proton is at the lowest energy level that is experimentally observable.

The virtual sea gluons are massless SU(3) gauge bosons. Since the lightest quarks are up and down quarks, the virtual sea quark-antiquark pairs that most often appear from the vacuum are up or down pairs, each of which have the same constituent mass, 312.75 MeV. If you stay below the threshold energy of the strange quark, whose constituent mass is about 625 MeV, the low energy sea within the proton contains only the lightest (up and down) sea quarks and antiquarks, so that the Quantum proton lowest-energy background sea has a density of 312.75 MeV. (In the model, "density" is mass/energy per unit volume, where the unit volume is Planck-length in size.)

Experiments that observe the proton as a whole do not "see" the proton's internal virtual sea, because the paths of the virtual sea gluon, quarks, and antiquarks begin and end within the proton itself. Therefore, the experimentally observed mass of the proton is the sum of the 3 valence quarks, 3 x 32.75 MeV, or 938.25 MeV which is very close to the experimental value of about 938.27 MeV.

To study the internal structure of hadrons, mesons, etc., you should use sum-over-histories quantum theory of the SU(3) color force SU(3). Since that is computationally very difficult,

you can use approximate theories that correspond to your experimental energy range.

For instance, the internal structure of a proton looks like a nonperturbative QCD soliton. See WWW URLs

http://galaxy.cau.edu/tsmith/SolProton.html

http://www.innerx.net/personal/tsmith/SolProton.html

For high energy experiments, such as Deep Inelastic Scattering, you can use Perturbative QCD. For low energies, you can use Chiral Perturbation Theory.

To do calculations in theories such as Perturbative QCD and Chiral Perturbation Theory, you need to use effective quark masses that are called current masses. Current quark masses are different from the Pre-Quantum constituent quark masses of the model.

The current mass of a quark is defined in the model as the difference between the constituent mass of the quark and the density of the lowest-energy sea of virtual gluons, quarks, and antiquarks, or 312.75 MeV.

Since the virtual sea is a quantum phenomenon, the current quarks of Perturbative QCD and Chiral Perturbation Theory are, in my view, Quantum particles.

Therefore, the model is unconventional in that:

the input current quarks of Perturbative QCD and Chiral Perturbation Theory are Quantum, and not Pre-Quantum, so that Perturbative QCD and Chiral Perturbation Theory are effectively "second-order" Quantum theories (rather than fundamental theories) that are most useful in describing phenomena at high and low energy levels, respectively; and

a current quark is a composite combination of a fundamental constituent quark and Quantum virtual sea gluon, quarks, and antiquarks (compare the conventional picture of, for example, hep-ph/9708262, in which current quarks are Pre-Quantum and constituent quarks are Quantum composites).

Assuming the accepted values of the gravitational force strength constant (Newton's constant) and the electron mass (0.511 MeV), then the calculationed ratios give values of all the other force strength constants and particle masses. These calculated force strengths and particle masses agree with conventionally accepted experimental results within at most about 10 percent in all but one case:

the mass of the Truth quark (sometimes called the Top quark).

The tree level constituent mass of the Truth quark is computed to be roughly 130 GeV, as opposed to the roughly 175 GeV figure advocated by FermiLab.

In my opinion, the FermiLab figure is incorrect.

The Fermilab figure is based on analysis of semileptonic events. I think that the Fermilab semileptonic analysis does not handle background correctly, and ignores signals in the data that are in rough agreement with the tree level constituent mass of about 130 GeV.

Further, I think that dileptonic events are more reliable for Truth quark mass determination, even though there are fewer of them than semileptonic events.

I think that analysis of dileptonic events gives a Truth quark mass that is in rough agreement with the tree level constituent mass of about 130 GeV.

More details about these issues, including gif images of Fermilab data histograms and other relevant experimental results, can be found on the World Wide Web at URLs

http://galaxy.cau.edu/tsmith/TCZ.html

http://www.innerx.net/personal/tsmith/TCZ.html

I consider the mass of the Truth quark to be a good test of the model, as the model can be falsified if my interpretation of experimental results turns out to be wrong.

1.2 D4-D5-E6 Lagrangian and 146 GeV Higgs.

The D4-D5-E6 Lagrangian in 8-dimensional SpaceTime, prior to dimensional reduction, is the Integral over 8-dim SpaceTime of

$$\int_{V_8} \partial_8^2 \overline{\Phi_8} \wedge \star \partial_8^2 \Phi_8 + F_8 \wedge \star F_8 + \overline{S_{8\pm}} \not{\partial}_8 S_{8\pm} + GF + GH$$

where F_8 is the 28-dimensional $Spin(8)$ curvature, $\star$ is the Hodge dual,
∂_8 is the 8-dimensional covariant derivative,
Φ_8 is the 8-dimensional scalar field,
$\not{\partial}_8$ is the 8-dimensional Dirac operator,
V_8 is 8-dimensional spacetime,
$S_{8\pm}$ are the + and − 8-dimensional half-spinor fermion spaces, and
GF and GH are gauge-fixing and ghost terms.

1.2.1 Ni-Lou-Lu-Yang R-R Method.

Before I read quant-ph/9806009,

To Enjoy the Morning Flower in the Evening - What does the Appearance of Infinity in Physics Imply?

by Guang-jiong Ni of the Department of Physics, Fudan University, Shanghai 200433, P. R. China,

and the related paper hep-ph/9801264 by Guang-jiong Ni, Sen-yue Lou, Wen-fa Lu, and Ji-feng Yang,

I did not correctly understand the Higgs mechanism.

Therefore, in my earlier papers I had wrongly stated that the D4-D5-E6 model gives a Higgs scalar mass of about 260 GeV and a Higgs scalar field vacuum expectation value of about 732 GeV.

I now see that my earlier values were wrong, and that the correct values under the D4-D5-E6 model are a Higgs scalar mass of about 146 GeV and a Higgs scalar field vacuum expectation value of about 252 GeV. The fault was not with the D4-D5-E6 model itself, but with my incorrect understanding of it with respect to the Higgs mechanism.

Guang-jiong Ni, Sen-yue Lou, Wen-fa Lu, and Ji-feng Yang, in hep-ph/9801264 [23], used a new Regularization-Renormalization (R-R) method to calculate the Higgs mass in the Standard Model to be about 140 GeV. Guang-jiong Ni has further described the R-R method in quant-ph/9806009 [22].

When the R-R method of Ni is applied to the D4-D5-E6 physics model, it is seen that, at tree level, the mass of the Higgs scalar is about 146 GeV.

Here is a description of the R-R method of Ni:

For example , in the calculation of "self-energy" of election in Quantum ElectroDynamics (QED) , there are Feynman Diagram Itegrals (FDI) of the form:

$$I = \int \frac{d^4K}{(2\pi)^4} \frac{1}{(K^2 - M^2)^2} \tag{5}$$

$$M^2 = p^2x^2 + \left(m^2 - p^2\right) x \tag{6}$$

where x is the Feynman parameter , $K = k - xp$.

Consider the integration range of K to be $(-\infty \to \infty)$. Since the denominator $(K^2 - M^2)^2 \sim K^4$, so the integral diverges as

$$I \sim \int \frac{dK^4}{K^4} \sim \int_0^\Lambda \frac{dK}{K} \tag{7}$$

where a radius Λ of large sphere in four dimensional space is introduced and is called as the cutoff of momentum in integration . When $\Lambda \to \infty$, there is a logarithmic divergence. In other FDIs, one may encounter the linear divergence ($\sim \Lambda$) and the quadratic divergence ($\sim \Lambda^2$) etc.

Taking the partial derivative of divergent integral I with respect to the parameter $M^2$with dimension of mass square,

$$\frac{\partial I}{\partial M^2} = 2 \int \frac{d^4K}{(2\pi)^4} \frac{1}{(K^2 - M^2)^3} \tag{8}$$

As the denominator behaves as K^6, the integral becomes convergent:

$$\frac{\partial I}{\partial M^2} = \frac{-i}{(4\pi)^2}\frac{1}{M^2} \tag{9}$$

To get back to I, integrate with respect to M^2:

$$I = \frac{-i}{(4\pi)^2}\left(\ln M^2 + C_1\right) = \frac{-i}{(4\pi)^2}\ln\frac{M^2}{\mu_1^2} \tag{10}$$

Here an arbitrary constant of integration C_1 appears.

Rewrite $C_1 = -\ln\mu_1^2$ with μ_1 carrying a mass dimension so that the argument of logarithmic function is dimensionless.

By the chain approximation, derive a renormalized mass $m_R = m + \delta m$. When the freely moving particle is on the mass-shell , i.e. , $p^2 = m^2$, we have ($\alpha \equiv e^2/4\pi$):

$$\delta m = \frac{\alpha m}{4\pi}\left(5 - 3\ln\frac{m^2}{\mu_1^2}\right) \tag{11}$$

The arbitrary constant μ_1 is fixed as follows. We want the parameter m in the Lagrangian of original theory to be understood as the observed mass m_R . Since the mass cannot be calculated by perturbative QFT, the mass can only be fixed by experiment . Hence , the condition $\delta m = 0$ gives

$$\ln\frac{m^2}{\mu_1^2} = 5/3, \mu_1 = e^{-5/6}m \tag{12}$$

The condition $m_R = m$ does not mean that the calculation of self - energy FDI is useless. When the motion of particle deviates from the mass - shell , i.e. ,$p^2 \neq m$, the combination of the self - energy formula with other FDI$_s$ in QED gives useful results. For instance, we are able to calculate quickly the (qualitative) energy shift of $2S_{1/2}$ state upward with respect to $2P_{1/2}$ state in Hydrogen atom being 997 MHz , so called Lamb shift (with experimental result 1057.8 MHz). The latter should be viewed as a mass modification for an electron in a bound state. By using perturbative QFT , mass modification can be evaluted, even though mass generation cannot.

Therefore, we replace the divergence by an arbitrary constant μ_1 , and the constant μ_1 is fixed by the mass m measured in the experiment.

With respect to the trick of taking derivative for reducing the degree of divergence, the crucial point that we should act from the beginning , act before the counterterm is introduced, act until the bottom is reached .

That is, take the derivative of integral I with respect to M^2
(or to a parameter σ added by hand, say $M^2 \rightarrow M^2 + \sigma$)
enough times until it becomes convergent,
then perform the corresponding integrations with respect to M^2
(or σ then setting $\sigma \rightarrow 0$ again)
to get back to I .

Now instead of divergences,there are arbitrary constants C_i .
Note that each divergence is now resolved into one of the constants C_i to be fixed, so that each C_i has its own unique meaning and role.

The difference between the D4-D5-E6 model Higgs mass of 146 GeV and the Ni, Lou, Lu, and Yang calculated value of 138 GeV is due to these facts:
the D4-D5-E6 model value of 145.789 GeV (about 146 GeV) is a tree level value, while Ni, Lou, Lu, and Yang calculate the Higgs mass beyond tree level using a nonperturbative Gaussian Effective Potential (GEP) method; Ni, Lou, Lu, and Yang use as input parameters: the particle masses $m_{W+} = m_{W-} = 80.359GeV$ and $m_{Z0} = 91.1884GeV$, the Weinberg angle $sin^2\theta_W = s2w = 0.2317$, and the Electromagnetic Fine Structure Constant at the Z_0-mass energy level $\alpha_{E(mZ)} = 1/128.89$; the D4-D5-E6 model uses as input parameter the vacuum expectation value of the Higgs scalar field $v = 252.514GeV$, which is based on the D4-D5-E6 model identity $v = m_{W+} + m_{W-} + m_{Z0}$, and which leads to the tree-level particle masses $m_{W+} = m_{W-} = 80.326GeV$ and $m_{Z0} = 91.862GeV$ as well as the other calculated particle masses and force strength constants of the D4-D5-E6 model; Ni, Lou, Lu, and Yang [23] use a T-quark mass of 175 GeV, giving them a larger 1-loop level fermion contribution than would be the case for the D4-D5-E6 model which has a T-quark mass of 130 GeV.

1.2.2 Lagrangian Scalar Term-1.

As shown in chapter 4 of Gockeler and Schucker [13],
the scalar part of the Lagrangian

$$\int_{V_8} \partial_8^2 \overline{\Phi_8} \wedge \star \partial_8^2 \Phi_8$$

becomes

$$F_{h8} \wedge \star F_{h8}$$

where F_8 is an 8-dimensional Higgs curvature term.

After dimensional reduction to 4-dim SpaceTime, the scalar $F_8 \wedge \star F_8$ term becomes

$$\int_{V_4} \int_I (F_{h44} + F_{h4I} + F_{hII}) \wedge \star (F_{h44} + F_{h4I} + F_{hII}) =$$

$$= \int_{V_4} \int_I F_{h44} \wedge \star F_{h44} + F_{h4I} \wedge \star F_{h4I} + F_{hII} \wedge \star F_{hII}$$

where cross-terms are eliminated by antisymmetry of the wedge $\wedge$ product and 4 denotes 4-dim SpaceTime and I denotes 4-dim Internal Symmetry Space.

The Internal Symmetry Space terms I should be integrated over the 4-dimensional Internal Symmetry Space I, to get 3 terms.

The first term is

$$\int_{V_4} F_{h44} \wedge \star F_{h44}$$

Since it, like the weak force curvature term, is an SU(2) gauge group terms, this term merges into the SU(2) weak force term

$$\int_{V_4} F_w \wedge \star F_w$$

(where w denotes Weak Force).

1.2.3 Lagrangian Scalar Term-3.

The third term is

$$\int_{V_4} \int_I F_{hII} \wedge \star F_{hII}$$

The third term after integration over the 4-dim Internal Symmetry Space I, produces, by a process similar to the Mayer Mechanism [21], terms of the form

$$\int_{V_4} -2\mu^2 \Phi^2 + \lambda \Phi^4 \tag{13}$$

where the wrong-sign $\Lambda\Phi^4$ theory potential term describes the Higgs Mechanism.

The form and notation above is used by Kane and Barger and Phillips.

Ni, and Ni, Lou, Lu, and Yang, use a different notation

$$V(\Phi) = -\frac{1}{2}\sigma\Phi^2 + \frac{1}{4!}\lambda_N \Phi^4 \tag{14}$$

Proposition 11.4 of chapter II of volume 1 of Kobayashi and Nomizu [18] states that

$2F_{hII}(X, Y) = [\Lambda(X), \Lambda(Y)] - \Lambda([X, Y])$,

where Λ takes values in the $SU(2)$ Lie algebra.

If the action of the Hodge dual $\star$ on Λ is such that

$\star\Lambda = -\Lambda$ and $\star[\Lambda, \Lambda] = [\Lambda, \Lambda]$,

then $F_{hII}(X, Y) \wedge \star F_{hII}(X, Y) = (1/4)([\Lambda(X), \Lambda(Y)]^2 - \Lambda([X, Y])^2)$.

If integration of Λ over I is $\int_I \Lambda \propto \Phi = (\Phi^+, \Phi^0)$, where (Φ^+, Φ^0) is the complex doublet HIggs scalar field, then

$$\int_I F_{hII} \wedge \star F_{hII} = (1/4) \int_I [\Lambda(X), \Lambda(Y)]^2 - \Lambda([X, Y])^2 =$$

$$= (1/4)[\lambda(\overline{\Phi}\Phi)^2 - \mu^2 \overline{\Phi}\Phi]$$

which is the Higgs Mechanism potential term.

1.2.4 Physical Interpretation and Mass Scale.

In my notation (and that of Kane [17] and Barger and Phillips [4]), $2\mu^2$ is the square m_H^2 of the tree-level Higgs scalar particle mass, and Φ is the Higgs scalar field, and its tree-level vacuum expectation value V is given by

$$v^2/2 = P^2 = M^2/2L$$

or

$$M^2 = Lv^2$$

The tree-level value of the fundamental mass scale vacuum expectation value v of the Higgs scalar field is set in the D4-D5-E6 model as the sum of the tree-level physical masses of the weak bosons

$$v = m_{W+} + m_{W-} + m_{Z0} = 80.326 + 80.326 + 91.862 = 252.514GeV$$

so that in the D4-D5-E6 model the electron mass will be 0.5110 MeV.

The resulting equations, in my notation (and that of Kane [17] and Barger and Phillips [4]), are:

$$m_H^2 = 2\mu^2$$

$$\mu^2 = \lambda v^2$$

$$m_H^2/v^2 = 2\lambda$$

In their notation, with $m_\sigma = m_H$ and with ϕ_1 as the fundamental Higgs scalar field mass scale, Ni, Lou, Lu, and Yang [23] have

$$m_H^2 = 2\sigma$$

$$\phi_1^2 = 6\sigma/\lambda_N$$

so that for the tree-level value of the Higgs scalar particle mass they have

$$m_H^2/\phi_1^2 = \lambda_N/3$$

By combining the non-perturbative Gaussian Effective Potential (GEP) approach with their Regularization-Renormalization (R-R) method, Ni, Lou, Lu, and Yang [23] find that:

m_H and ϕ_1 are the two fundamental mass scales of the Higgs mechanism, and

the fundamental Higgs scalar field mass scale ϕ_1 of Ni, Lou, Lu, and Yang [23] is equivalent to the vacuum expectation value v of the Higgs scalar field in my notation (and that of Kane [17] and Barger and Phillips [4]), and

λ_N (and the corresponding λ) can not only be interpreted as the Higgs scalar field self-coupling constant, but also can be interpreted as determining the invariant ratio between the mass squares of the Higgs mechanism fundamental mass scales, m_H^2 and $\phi_1^2 = v^2$.

Since the tree-level value of λ_N is $\lambda_N = 1$,
and since

$$\lambda_N/3 = m_H^2/\phi_1^2 = m_H^2/v^2 = 2\lambda$$

then, at tree-level,

$$\lambda = \lambda_N/6 = 1/6$$

so that, at tree-level

$$m_H^2/\phi_1^2 = m_H^2/v^2 = 2/6 = 1/3$$

In the D4-D5-E6 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model, and v is set to be

$$v = 252.514GeV$$

which is the sum of the tree-level physical masses of the weak bosons

$$v = m_{W+} + m_{W-} + m_{Z0} = 80.326 + 80.326 + 91.862 = 252.514GeV$$

so that in the D4-D5-E6 model the electron mass will be 0.5110 MeV.

Then, the tree-level mass m_H of the Higgs scalar particle is given by

$$m_H = v/\sqrt{3} = 145.789 GeV$$

Ni, Lou, Lu, and Yang use their Quantum Field Theory model to calculate two more important mass scales:

The Critical Mass (or Energy, or Temperature) M_{SSB} for restoration of the Spontaneous Symmetry Breaking (SSB) symmetry, which is $M_{SSB} = m_H \sqrt{12/Ln}$, so that, for the tree-level value $\lambda_N = 1$,

$$M_{SSB} = m_H \sqrt{12} = 505 GeV$$

The High-Energy Singularity of the Higgs Mechanism model, M_{SING}, beyond which the Higgs field vanishes, and

the Maximum Energy Scale M_{MAX} that can be calculated in the Higgs Mechanism model.

The fact that the Higgs Mechanism model is not calculable and the Higgs field vanishes above Msing and Mmax may justify regarding the Higgs Mechanism model as a low energy effective theory, just as the D4-D5-E6 model is fundamentally a low (with respect to the Planck energy) energy effective theory.

The values calculated by Ni, Lou, Lu, and Yang are

$$M_{SING} = 0.55 \times 10^{15} GeV$$

$$M_{MAX} = 0.87 \times 10^{15} GeV$$

The Planck energy is

$$M_{PLANCK} = 1.22 \times 10^{19} GeV$$

1.2.5 Complex Doublet.

The Higgs scalar field Φ is a Complex Doublet that can be expressed in terms of a vacuum expectation value v and a real Higgs field H.

The Complex Doublet

$$\Phi = (\Phi^+, \Phi^0) = (1/\sqrt{2})(\Phi_1 + i\Phi_2, \Phi_3 + i\Phi_4) = (1/\sqrt{2})(0, v + H)$$

so that

$$\Phi_3 = (1/\sqrt{2})(v + H)$$

where v is the vacuum expectation value and H is the real surviving Higgs field.

The value of the fundamental mass scale vacuum expectation value v of the Higgs scalar field is in the D4-D5-E6 physics model set to be 252.514 GeV, so that the electron mass will turn out to be 0.5110 MeV.

Now, to interpret the term

$$\int_I F_{hII} \wedge \star F_{hII} = (1/4) \int_I [\Lambda(X), \Lambda(Y)]^2 - \Lambda([X,Y])^2 =$$

$$= (1/4)[\lambda(\overline{\Phi}\Phi)^2 - \mu^2\overline{\Phi}\Phi]$$

in terms of v and H,

note that $\lambda = \mu^2/v^2$
and that $\Phi = \Phi_3 = (1/\sqrt{2})(v + H)$,
so that

$$F_{hII}(X,Y) \wedge \star F_{hII}(X,Y) == (1/4)[\lambda(\overline{\Phi}\Phi)^2 - \mu^2\overline{\Phi}\Phi] =$$

$$= (1/16)((\mu^2/v^2)(v + H)^4 - (1/8)M^2(v + H)^2 =$$

$$= (1/4)\mu^2 H^2 - (1/16)\mu^2 v^2(1 - 4H^3/v^3 - H^4/v^4)$$

Disregarding some terms in v and H,

$$F_{hII}(X,Y) \wedge \star F_{hII}(X,Y) = (1/4)\mu^2 H^2 - (1/16)\mu^2 v^2$$

1.2.6 Lagrangian Scalar Term-2.

The second term is

$$\int_{V4}\int_{I} F_{h4I} \wedge *F_{h4I}$$

The second term, after integration over the 4-dim Internal Symmetry Space I, produces, by a process similar to the Mayer Mechanism [21], terms of the form

$$\int_{V4} \partial\overline{\Phi}\partial\Phi$$

Proposition 11.4 of chapter II of volume 1 of Kobayashi and Nomizu [18] states that

$$2F_{h4I}(X,Y) = [\Lambda(X), \Lambda(Y)] - \Lambda([X,Y])$$

where $\Lambda(X)$ takes values in the SU(2) Lie algebra.

If the X component of $F_{h4I}(X,Y)$ is in the surviving 4-dim SpaceTime and the Y component of $F_{h4I}(X,Y)$ is in the 4-dim Internal Symmetry Space I, then the Lie bracket product $[X,Y] = 0$ so that $\Lambda([X,Y]) = 0$ and therefore

$$F_{h4I}(X,Y) = (1/2)[\Lambda(X), \Lambda(Y)] = (1/2)\partial_X \Lambda(Y)$$

Integration over Internal Symmetry Space I of

$$(1/2)\partial_X \Lambda(Y)$$

gives $(1/2)\partial_X \Phi$.

Taking into account the Complex Doublet structure of Φ, the second term is the Integral over 4-dim SpaceTime of

$$F_{h4I}(X,Y) \wedge \star F_{h4I} = (1/2)\partial\Phi \wedge *(1/2)\partial\Phi = (1/4)\partial\Phi \wedge *\partial\Phi =$$

$$= (1/4)(1/2)\partial(v+H)/\ *\partial(v+H) = (1/8)\partial H \partial H + (sometermsinv, H)$$

Disregarding some terms in v and H,

$$F_{h4I}(X,Y) \wedge \star F_{h4I} = (1/8)\partial H \partial H$$

1.2.7 Total Higgs Lagrangian.

Combining the second and third terms, since the first term is merged into the weak force part of the Lagrangian:

$$F_{h4I}(X,Y) \wedge \star F_{h4I} + F_{hII}(X,Y) \wedge \star F_{hII}(X,Y) =$$

$$= (1/8)\partial H \partial H + (1/4)\mu^2 H^2 - (1/16)\mu^2 v^2 =$$

$$= (1/8)(dHdH + 2\mu^2 H^2 - (1/2)\mu^2 v^2)$$

This is the form of the Higgs Lagrangian in Barger and Phillips [4] for a Higgs scalar particle of mass

$$m_H = \mu\sqrt{2} = v/\sqrt{3} = 145.789 GeV$$

1.2.8 Weak Force.

What about the Weak Force Strength and Weak Boson Masses?

In the D4-D5-E6 model, the geometric part of the weak force strength, and the geometric weak charge that is its square root, are:

$$\alpha_W = 0.2535$$

$$\sqrt{\alpha_W} = 0.5035$$

In more customary particle physics notation, such as that found in Kane [17], there are two weak charges, g_1 and g_2, such that their squares are weak force strengths. In the D4-D5-E6 model, the geometric weak charge is the average of the customary two weak charges g_1 and g_2:

$$\sqrt{\alpha_W} = (1/2)(g_1 + g_2)$$

so that the numerical values are

$$g_1 = 0.33566$$
$$g_1^2 = 0.11267$$
$$g_2 = 0.66434$$
$$g_2^2 = 0.44135$$

Combining some aspects of the D4-D5-E6 model and some aspects of the customary picture gives tree-level estimate results that are off by a few percent. Estimated Weak Boson masses are approximately

$$m_W = g_2 v/2 = 83.88 GeV$$

$$m_Z = \sqrt{g_1^2 + g_2^2} v/2 = 93.98 GeV$$

Some other relations given by Kane [17], and the results of using in them some D4-D5-E6 model values, and the estimates immediately above, are

$$e = \sqrt{4\pi\alpha_E} = 0.30286$$

$$g_2 = e/\sin\theta_W = 0.6247$$

$$g_1 = e/\cos\theta_W = 0.3463$$

$$G_F = \sqrt{2} g_2^2/8m_W^2 = 1.11 \times 10^{-5} GeV^{-2}$$

For comparison, the D4-D5-E6 model value of the Fermi constant is

$$G_F = \alpha_W/M_W^2 = 1.188 \times 10^{-5} GeV^{-2}$$

where

$$M_W = \sqrt{(m_{W+}^2 + m_{W-}^2 + m_{Z0}^2)} =$$

$$= \sqrt{(80.326^2 + 80.326^2 + 92.862^2)} = 146.09298 GeV$$

Note that M_W is very close to the Higgs scalar particle mass $m_H = 145.789 GeV$.

1.2.9 Higgs, Truth Quark, and Weak Bosons.

The tree level mass m_H of a Higgs scalar, about 146 GeV, is somewhat higher than, but roughly similar to, the tree level Truth quark mass m_T of about 130 GeV.

In the D4-D5-E6 physics model, the sum of the tree level masses $m^2_{W+} + m^2_{W-} + m^2_{Z0}$ of the 3 weak bosons W_+, W_-, and Z_0, that is, the physical weak bosons below the Higgs mass scale, is the fundamental energy level vacuum expectation value v of the Higgs scalar field.

To give the tree-level particle masses $m_{W+} = m_{W-} = 80.326 GeV$ and $m_{Z0} = 91.862 GeV$ (as well as the other calculated particle masses and force strength constants of the D4-D5-E6 model), v is set equal to 252.514 GeV.

The D4-D5-E6 model assumed value of v is about 80+80+92 = 252 GeV which is close to the tree level mass of the truth quark T-T(bar) meson of about 260 GeV.

$M_W = \sqrt{(m^2_{W+} + m^2_{W-} + m^2_{Z0})}$, the square root of the sum of the squares of the tree level masses of the 3 weak bosons W_+, W_-, and Z_0, that is, the physical weak bosons below the Higgs mass scale, is 146.09298 GeV, which is very close to the mass of the tree level Higgs scalar mass of 145.789 GeV.

The tree level mass of a pair of Higgs scalars, about 292 GeV, is somewhat higher than, but roughly similar to, the fundamental energy level vacuum expectation value v of the Higgs scalar field, about 252 GeV, and the truth quark T-T(bar) meson mass of about 260 GeV.

1.3 Mathematical Structures.

The D4-D5-E6 physics model emerges from bits just as it does from the points of Simplex Physics above the Planck Energy which is similar to its emergence from the arrows of quantum set theory and from the structure of Metaclifford algebras.

The mathematical structures used in the D4-D5-E6 model are much like the structures of:

IFA, whose $256 = 2^8 = 2^4 \times 2^4 = 16 \times 16$ elements correspond to the Cl(8) Clifford Algebra;

Wei Qi, whose board grid and stones correspond to the Spin(8) vector HyperDiamond lattice;

I Ching, whose 28 antisymmetrized off-diagonal hexagrams, out of the total $64 = 2^6 = 2^3 \times 2^3 = 8 \times 8$ hexagrams, correspond to the 28Spin(8) bivector gauge bosons;

Tai Hsuan Ching, whose $81 = 3^4$ ternary power-of-3 structure corresponds to 16 first-generation Spin(8) spinor fermions, and whose ternary structure corresponds to the 3-generation structure of fermions in the D4-D5-E6 model; and

Tarot, whose 78 cards correspond to 78-dimensional E6.

1.4 Outline of Chapters.

The purpose of this paper is to describe the model in some detail.

After this introductory overview,

Chapter 2 describes the construction of discrete Clifford algebras from set theory and some simple natural operations.

Begin with set theory to get the Natural Numbers N.

Then use reflection to get the integers Z.

Then use the set of subsets and the XOR operator to get the Discrete Clifford Group ($DClG(n)$).

$DClG(n)$ is extended to its Z-Group Algebra, thus producing a discrete real Clifford Algebra (DCl(0,n)) over the Z_n hypercubic lattice.

For some n, DCl(0,n) is naturally extended from the Z_n hypercubic lattice to larger lattices, such as the D_4 quaternionic integer lattices for $n = 4$ and an E_8 octonionic integer lattices for $n = 8$.

The real Clifford Algebras have periodicity 8, so the fundamental real Clifford Algebra produced by this process is $DCl(0, 8)$.

The vector, +half-spinor and -half-spinor representations $DCl(0, 8)$ are all isomorphic by triality to the discrete integral octonions.

Chapter 3 describes the octonionic structure of E_8 lattices.

Chapter 4 describes the scalar representation of $DCl(0, 8)$ as physically representing the Higgs scalar particle;

the vector representation of $DCl(0, 8)$ as an E_8 lattice physically representing an 8-dimensional spacetime;

the bivector representation of $DCl(0, 8)$ as having 28 basis bivectors that represent the 28 gauge boson infinitesimal generators of a $Spin(0, 8)$ gauge group;

and the two half-spinor representations of $DCl(0,8)$ as two E_8 lattices, in which the 8 octonion basis vectors of each physically represent 8 fundamental first-generation fermion particles (neutrino; red, blue, green up quarks; red, blue green down quarks; electron) and 8 fermion antiparticles.

Chapter 5 describes a 4-dimensional HyperDiamond lattice spacetime that comes from a 4-dimensional sub-lattice of the E_8 lattice spacetime.

Chapter 6 describes a 4-dimensional internal symmetry space that comes from the rest of the original E_8 lattice spacetime. A separate copy of the internal symmetry space lives on each vertex of the spacetime 4-dim HyperDiamond lattice. Each copy of the internal symmetry space looks like a 4-dim HyperDiamond lattice.

Chapter 7 describes how a Feynman Checkerboard construction on the HyperDiamond structures gives the physics of the Standard Model plus Gravity.

Chapter 8 describes the numerical calculation of charge as the amplitude for a particle to emit a gauge boson, with the force strength constant being the square of the charge.

Chapter 9 describes the numerical calculation of particle mass as the amplitude for a particle to change direction.

Chapter 10 describes HyperDiamond Feynman Checkerboard configurations that represent protons as triples of confined quarks, pions as confined quark-antiquark pairs, and physical gravitons as quadruples of confined Spin(0,5) gravitons.

Appendix A describes some earlier papers, including some errata for them.

The $D_4 - D_5 - E_6$ model that is described in some detail on the World Wide Web at URL http://xxx.lanl.gov/abs/hep-ph/9501252

is a continuum version of the HyperDiamond Feynman Checkerboard model at URL

http://xxx.lanl.gov/abs/hep-ph/9501252

Both of those papers, and all my papers written prior to June 1998, contain an incorrect value (about 260 GeV) of the Higgs scalar mass, which should be about 146 GeV.

The $D_4 - D_5 - E_6$ model is also described on the World Wide Web at URLs

http://galaxy.cau.edu/tsmith/d4d5e6hist.html

http://www.innerx.net/personal/tsmith/d4d5e6hist.html

Briefly, roughly, and non-rigorously, the $D_4 - D_5 - E_6$ model is constructed from E_6, $D_5 = Spin(10$, and $D_4 = Spin(8)$:

The first generation of fermions are constructed from $E_6/Spin(10) \times U(1)$, whose dimension is 78-45-1=32, the real dimensionality of a bounded complex domain whose Shilov boundary has real dimension 16=8+8 for 8 fermion particles and 8 for antiparticles;

An 8-dimensional spacetime is constructed from $Spin(10)/Spin(8) \times U(1)$, whose dimension is 45-28-1=16, the real dimensionality of a bounded complex domain whose Shilov boundary has real dimension 8 for 8-diemnsional spacetime;

28 gauge bosons are constructed directly from 28-dimensional $Spin(8)$.

Then reduce spacetime from 8 dimensions to 4 dimensions, by choosing a quaternionic subspace of octonionic 8-dimensional spacetime.

The result of that spacetime symmetry breaking is:

The fermions get 3 generations, corresponding to E_6, E_7, and E_8;

The 28 = 16+12 gauge bosons split into two parts:

16 of them form $U(4)$, which is $U(1) \times SU(4)$, the $U(1)$ to give a complex phase to propagator amplitudes, the $SU(4) = Spin(6)$ to give the conformal group, the $Spin(6)$ (compact version of Spin(4,2) conformal group has 5 dimensions of conformal and scale transformations that give a mass scale and a Higgs scalar

and has 10 dimensions that give the $Spin(5)$ deSitter group, which is gauged to give Gravity;

12 = 8+3+1 of them form $SU(3) \times SU(2) \times U(1)$ of the Standard Model.

The D4-D5-E6 model coset spaces $E6/(D5 \times U(1))$ and $D5/(D4 \times U(1))$ are Conformal Spaces.

You can continue the chain to $D4/(D3 \times U(1))$ where D3 is the 15-dimensional Conformal Group whose compact version is Spin(6),

and to $D3/(D2 \times U(1))$ where D2 is the 6-dimensional Lorentz Group whose compact version is Spin(4).

Electromagnetism, Gravity, and the Zero Point Fluctuation of the vacuum all have in common the symmetry of the 15-dimensional D3 Conformal Group whose compact version is Spin(6), as can be seen by the following structures with D3 Conformal Group symmetry:

Maxwell's equations of Electromagnetism;

Gravity derived from the Conformal Group using the MacDowell-Mansouri mechanism;

the Quantum Theoretical Hydrogen atom; and

the Lie Sphere geometry of SpaceTime Correlations in the Many-Worlds picture.

Further, the 12-dimensional Standard Model Lie Algebra $U(1) \times SU(2) \times SU(3)$ may be related to the D3 Conformal Group Lie Algebra in the same way that the 12-dimensional Schrodinger Lie Algebra is related to the D3 Conformal Group Lie Algebra.

2 From Sets to Clifford Algebras.

2.1 Sets, Reflections, Subsets, and XOR.

Start with von Neumann's Set Theoretical definition of the Natural Numbers N:

$$0 = \emptyset, 1 = \{\emptyset\}, 2 = \{\emptyset, \{\emptyset\}\}, ..., n+1 = n \cup \{n\} \ .$$

in which each Natural Number n is a set of n elements.

By reflection through zero, extend the Natural Numbers N to include the negative numbers, thus getting an Integral Domain, the Ring Z.

Now, following the approach of Barry Simon [27], consider a set $S_n = \{e_1, e_2, ..., e_n\}$ of n elements.

Consider the set 2^{S_n} of all of its 2^n subsets, with a product on 2^{S_n} defined as the symmetric set difference XOR.

Denote the elements of 2^{S_n} by m_A where A!is in ;^{S_n}.

2.2 Discrete Clifford Algebras.

To go beyond set theory to Discrete Clifford Algebras, enlarge 2^{S_n} to $DClG(n)$ by:

order the basis elements of S_n,

and then give each element of 2^{S_n} a sign, either +1 or -1, so that $DClG(n)$ has 2^{n+1} elements.

This amounts to orientation of the signed unit basis of S_n.

Then define a product on $DClG(n)$ by

$(x_1 e_A)(x_2 e_B) = x_3 e_{A\,XOR\,B})$

where A and B are in 2^{S_n} with ordered elements, and x_1, x_2, and x_3 determine the signs.

For given x_1 and x_2, $x_3 = x_1 x_2 x(A,B)$ where $x(A,B)$ is a function that determines sign by using the rules

$e_i e_i = +1$ for i in S_n

and $e_i e_j = -e_j e_i$ for $i \neq j$ in S_n ,

then writing (A,B) as an ordered set of elements of S_n,

then using $e_i e_j = -e_j e_i$ to move each of the B-elements to the left until it:

either meets a similar element and then cancelling it with the similar element by using $e_i e_i = +1$

or it is in between two A-elements in the proper order.

$DClG(n)$ is a finite group of order 2^{n+1}.

It is the Discrete Clifford Group of n signed ordered basis elements of S_n.

Now we can construct a discrete Group Algebra of $DClG(n)$ by extending $DClG(n)$ by the integral domain ring Z

and using the relations

$e_i e_j + e_j e_i = 2\delta(i,j)1$

where $\delta(i,j)$ is the Kronecker delta.

Since $DClG(n)$ is of order 2^{n+1}, and since two of its elements are -1 and $+1$, which act as scalars, the discrete Group Algebra of $DClG(n)$ is 2^n-dimensional. The vector space on which it acts is the n-dimensional hypercubic lattice Z^n.

The discrete Group Algebra of discrete Clifford Group $DClG(n)$ is the discrete Clifford Algebra $DCl(n)$.

Here is an explicit example showing how to assign the elements of the Clifford Group to the basis elements of the Clifford Algebra Cl(3):

First, order the $2^{3+1} = 16$ group strings into rows lexicographically:

$$\boxed{\begin{array}{c} 0000 \\ 0001 \\ 0010 \\ 0011 \\ 0100 \\ 0101 \\ 0110 \\ 0111 \\ 1000 \\ 1001 \\ 1010 \\ 1011 \\ 1100 \\ 1101 \\ 1110 \\ 1111 \end{array}} \tag{15}$$

Then discard the first bit of each string, because it corresponds to sign which is redundant in defining the Algebra basis. This reduces the number of different strings to $2^3 = 8$:

$$\boxed{\begin{array}{c} 000 \\ 001 \\ 010 \\ 011 \\ 100 \\ 101 \\ 110 \\ 111 \end{array}} \tag{16}$$

Then separate them into columns by how many 1's they have:

000				
	001			
	010			
		011		(17)
	100			
		101		
		110		
			111	

Now they are broken down into the 1 3 3 1 graded pattern of the Clifford Algebra Cl(3).

The associative Cl(3) product can be deformed into the 1 x 1 Octonion nonassociative product by changing EE from 1 to -1, and IJ from K to -k, JK from I to -i, KI from J to -j, and changing the cross-terms accordingly.

If you want to make an Octonion basis without the graded structure, and with the 7 imaginary octonions all on equal footing, all you have to do is to assign them, one-to-one in the order starting from the left column and from the top of each row, to the 8 Octonion Algebra strings:

1	000	0000000
I	001	1000000
J	010	0100000
K	100	0010000
i	110	0001000
j	101	0000100
k	011	0000010
E	111	0000001

(18)

To give an example of how to write an octonion product in terms of XOR operations, look at the 7 associative triangles:

$$\boxed{\begin{array}{c} ijk \\ JiK \\ jIK \\ JIk \\ IEi \\ JEj \\ KEk \end{array}} \tag{19}$$

which, in string terms, are each represented by 3 element strings and 1 Asssociative Triangle string:

$Octonion$	$3Elements$	$Associative$ $Triangle$	$Coassociative$ $Square$
I	$i - 0001000$ $J - 0100000$ $K - 0010000$	0111000	1000111
J	$I - 1000000$ $j - 0000100$ $K - 0010000$	1010100	0101011
K	$I - 1000000$ $J - 0100000$ $k - 0000010$	1100010	0011101
i	$E - 0000001$ $I - 1000000$ $i - 0001000$	1001001	0110110
j	$E - 0000001$ $J - 0100000$ $j - 0000100$	0100101	1011010
k	$E - 0000001$ $K - 0010000$ $k - 0000010$	0010011	1101100
E	$i - 0001000$ $j - 0000100$ $k - 0000010$	0001110	1110001

(20)

Here is Onar Aam's method for calculating the octonion product ab of octonion basis elements a and b in terms of these strings:

To multiply using triangles, note that there are 7 octonion imaginary elements and that XOR of two triangles give a square and that the Hodge dual (within imaginary octonions) of that square is the triangle that represents the product of the two triangles, so that:

$$ab = \star(aXORb)$$

For instance, here is an example of multiplying by triangles (up to +/- sign determined by ordering):

$$EI = \star(0001110XOR0111000) = \star(0110110) = 1001001 = i$$

On the other hand, the XOR of two squares is a square, so that multiplying by squares simply becomes an XOR. and we have (up to +/- sign determined by ordering):

$$ij = 0110110XOR1011010 = 1101100 = k$$

The Clifford Algebra product $\cdot$ combines the vector space exterior $\wedge$ product and the vector space interior product $\vdash$ so that, if a is a 1-vector and B is a k-vector,

$$a \cdot B = a \wedge B - a \vdash B$$

The Clifford Algebra has the same graded structure as the exterior $\wedge$ algebra of the vector space, and the underlying exterior product antisymmetry rule that, for p-vector A and q-vector B

$$A \wedge B = (-1)^{pq} B \wedge A$$

The associative Cl(3) product can be deformed into the 1 x 1 Octonion nonassociative product by changing EE from 1 to -1, and IJ from K to $-k$, JK from I to $-i$, KI from J to $-j$, and changing the cross-terms accordingly.

A fundamental reason for the deformation is that the graded structure of the Clifford algebra gives it the underlying exterior product antisymmetry rule that while, for Octonions, you want for all unequal imaginary octonions to have the antisymmetry rule $AB = -BA$ and for equal ones to have $AA = -1$.

The discrete Clifford Algebra $DCl(n)$ acts on the n-dimensional hypercubic lattice Z^n.

2.3 Real Clifford and Division Algebras.

Compare the Discrete Clifford Algebra, with the Real Group Algebra of $DClG(n)$ made by extending it by the real numbers R, which is the usual real Euclidean Clifford Algebra $Cl(n)$ of dimension 2^n, acting on the real n-dimensional vector space R^n.)

The empty set $\emptyset$ corresponds to a vector space of dimension -1, so that $Cl(-1)$ corresponds to the VOID.

$Cl(0)$ has dimension $2^0 = 1$ and corresponds to the real numbers and to time. Its even subalgebra is EMPTY. $Cl(0)$ is the 1×1 real matrix algebra.

$Cl(1)$ has dimension $2^1 = 2 = 1 + 1 = 1 + i$ and corresponds to the complex numbers and to 2-dim space-time. Its even subalgebra is $Cl(0)$. $Cl(1)$ is the 1×1 complex matrix algebra.

$Cl(2)$ has dimension $2^2 = 4 = 1 + 2 + 1 = 1 + \{j, k\} + i$ and corresponds to the quaternions and to 4-dim space-time. Its even subalgebra is $Cl(1)$. $Cl(2)$ is the 1×1 quaternion matrix algebra.

$Cl(3)$ has dimension $2^3 = 8 = 1{+}3{+}3{+}1 = 1{+}\{I, J, K\}{+}\{i, j, k\}{+}E$ and corresponds to the octonions and to 8-dim space-time. Its even subalgebra is $Cl(2)$. $Cl(3)$ is the direct sum of two 1×1 quaternion matrix algebras.

The associative Cl(3) product can be deformed into the 1 x 1 Octonion nonassociative product by changing EE from 1 to -1, and IJ from K to $-k$, JK from I to $-i$, KI from J to $-j$, and changing the cross-terms accordingly.

A fundamental reason for the deformation is that the graded structure of the Clifford algebra gives it the underlying exterior product antisymmetry rule that while, for Octonions, you want for all unequal imaginary octonions to have the antisymmetry rule $AB = -BA$ and for equal ones to have $AA = -1$.

There are a number of choices you can make in writing the octonion multiplication table:

Given 1, i, and j, there are 2 inequivalent quaternion multiplication tables, one with $ij = k$ and the reverse with $ji = k$, or $ij = -k$.

To get an octonion multiplication table, start with an orthonormal basis of 8 octonions $\{1, i, j, k, E, I, J, K\}$, and

pick a scalar real axis 1

and pick (2 sign choices) a pseudoscalar axis E or $-E$.

Then you have 6 basis elements to designate as i or $-i$, which is 6 element choices and 2 sign choices.

Then you have 5 basis elements to designate as j or $-j$, which is 5 element choices and 2 sign choices.

Then the underlying quaternionic product fixes ij as k or $-k$, which is 2 sign choices.

How many inequivalent octonion multiplication tables are there?

You had 6 i-element choices, 5 j-element choices, and 4 sign choices,
for a total of $6 \times 5 \times 2^4 = 30 \times 16 = 480$ octonion products.

$Cl(4)$ has dimension $2^4 = 16 = 1 + 4 + 6 + 4 + 1 =$

$= 1 + \{S, T, U, V\} + \{i, j, k, I, J, K\} + \{W, X, Y, Z\} + E$

$Cl(4)$ corresponds to the sedenions. Its even subalgebra is $Cl(3)$. $Cl(4)$ is the 2×2 quaternion matrix algebra.

$Cl(4)$ is the first Clifford algebra that is NOT made of 1×1 matrices or the direct sum of 1×1 matrices, and the $Cl(4)$ sedenions do NOT form a division algebra.

For more details see these WWW URLs, web pages, and references therein: Clifford Algebras:

http://galaxy.cau.edu/tsmith/clfpq.html

http://www.innerx.net/personal/tsmith/clfpq.html

McKay Correspondence:

http://galaxy.cau.edu/tsmith/DCLG-McKay.html

http://www.innerx.net/personal/tsmith/DCLG-McKay.html

Octonions:

http://galaxy.cau.edu/tsmith/3x3OctCnf.html

http://www.innerx.net/personal/tsmith/3x3OctCnf.html

Sedenions:

http://galaxy.cau.edu/tsmith/sedenion.html

http://www.innerx.net/personal/tsmith/sedenion.html

Cross-Products:

http://galaxy.cau.edu/tsmith/clcroct.html

http://www.innerx.net/personal/tsmith/clcroct.html

NonDistributive Algebras:

http://galaxy.cau.edu/tsmith/NDalg.html

http://www.innerx.net/personal/tsmith/NDalg.html

2.4 Discrete Division Algebra Lattices.

The discrete Clifford Algebras $DCl(n)$ can be extended from hypercubic lattices to lattices based on the Discrete Division Algebras.

For $n = 2$, the 2-dimensional hypercubic lattice Z^2 can be thought of as the Gaussian lattice of the complex numbers.

For n greater than 2, the action of the discrete Clifford Algebra $DCl(n)$ on the n-dimensional hypercubic lattice Z^n can be extended to action on the D_n lattice with $2n(n-1)$ nearest neighbors to the origin, corresponding to the second-layer, or norm-square 2, vertices of the hypercubic lautice Z_n. The D_n lattice is called the Checkerboard lattice, because it can be represented as one half of the vertices of Z^n.

For $n = 4$, the extension of the 4-dimensional hypercubic lattice Z^4 to the D_4 lattice produces the lattice of integral quaternions, with 24 vertices nearest the origin, forming a 24-cell.

For $n = 8$, the 8-dimensional lattice D_8 can be extended to the E_8 lattice of integral octonions, with 240 vertices nearest the origin, forming a Witting polytope, by fitting together two copies of the D_8 lattice, each of whose vertices are at the center of the holes of the other.

For $n = 16$, the 16-dimensional lattice D_{16} can be extended to the Λ_{16} Barnes-Wall lattice. with 4,320 vertices nearest the origin.

For $n = 24$, the 24-dimensional lattice D_{24} can be extended to the Λ_{24} Leech lattice. with 196,560 vertices nearest the origin.

2.5 Spinors.

The discrete Clifford Group $DClG(n)$ is a subgroup of the discrete Clifford Algebra $DCl(n)$.

There is a $1-1$ correspondence between the representations of $DCl(n)$ and those representations of $DClG(n)$ such that $U(-1) = -1$.

$DClG(n)$ has 2^n 1-dimensional representations, each with $U(-1) = +1$.

The irreducible representations of $DClG(n)$ with dimension greater than 1 have $U(-1) = -1$, and are representations of $DCl(n)$.

If n is even, there is one such representation, of degree $2^{n/2}$, the full spinor representation of $DCl(n)$. It is reducible into two half-spinor representations, each of degree $2^{(n-1)/2}$.

If n is odd, there are two such representations, each of degree $2^{(n-1)/2}$, One of them is the spinor representation of $DCl(n)$.

2.6 Signature.

So far, we have been discussing only Euclidean space with positive definite signature, $DCl(n) = DCl(0, n)$. Symmetries for the general signature cases include:

$DCl(p-1, q) = DCl(q-1, p)$;

The even subalgebras of $DCl(p, q)$ and $DCl(q, p)$ are isomorphic;

$DCl(p, q)$ is isomoprphic to both the even subalgebra of $DCl(p+1, q)$ and the even subalgebra of $DCl(p, q+1)$.

Signature is not meaningful for complex vector spaces. The complex Clifford algebra $DCl(2p)_{\mathbf{C}}$ is the complexification $DCl(p, p) \otimes_{\mathbf{R}} \mathbf{C}$

2.7 Periodicity 8.

$DCl(p,q)$ has the periodicity properties:

$DCl(n,n+8) = DCl(n,n) \otimes M(\mathbf{R},16) = DCl(n,n) \otimes DCl(0,8) = DCl(n+8,n)$

$DCl(n-4,n+4) = DCl(n,n)$ $DCl(n,n+8) = DCl(n,n) \otimes M(\mathbf{R},16) = DCl(n,n) \otimes DCl(0,8) = DCl(n+8,n)$

Therefore any discrete Clifford algebra $DCl(p,q)$ of any size can first be embedded in a larger one with p and q multiplex of 8, and then the larger one can be "factored" into

$DCl(0,p) \otimes DCl(0,8) \otimes ... \otimes DCl(0,8)$

so that the fundamental building block of the real discrete Clifford algebras is $DCl(0,8)$.

The vector, +half-spinor, and -half-spinor representations

of $DCl(0,8)$ are each 8-dimensional

and can be represented by an octonionic E8 lattice.

2.8 Many-Worlds Quantum Theory.

To see how Many-Worlds Quantum Theory arises naturally in the D4-D5-E6 HyperDiamond Feynman Checkerboard physics model, note that the model is basically built by using the discrete Clifford Algebra $DCL(0,8)$ as its basic building block, due to the Periodicity 8 property, so that the model looks like a tensor product of Many Copies of $DCl(0,8)$:

$$DCl(0,p) \otimes DCl(0,8) \otimes ... \otimes DCl(0,8)$$

How do the Many Copies of $DCl(0,8)$ fit together?

Consider that each Copy of $DCl(0,8)$ has graded structure:

$$1 \quad 8 \quad 28 \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1 \tag{21}$$

The vector 8 space corresponds to an 8-dimensional spacetime that is a discrete E8 lattice.

Take any two Copies of $DCl(0,8)$ and consider the origin of the E8 lattice of each Copy.

From each origin, there are 240 links to nearest-neighbor vertices.

The two Copies naturally fit together if the origin of the E8 lattice of the vector 8 space of one Copy and the origin of the E8 lattice of the vector 8 space of the other Copy are nearest neighbors, one at each end of a single link in the E8 lattice.

If you start with a Seed Copy of $DCl(0,8)$, and repeat the fitting-together process with other copies, the result is one large E8 lattice spacetime, with one Copy of $DCl(0,8)$ at each vertex.

Since there are 7 TYPES OF E8 LATTICE, 7 different types of E8 lattice spacetime neighborhoods can be constructed.

WHAT HAPPENS at boundaries of different E8 neighborhoods?

ALL the E8 lattices have in common links of the form

$$\pm V$$

(where $V = 1, i, j, k, E, I, J, K$)

but they DO NOT AGREE for all links of the form

$$(+ \pm W \pm X \pm Y \pm Z)/2$$

(where $W = 1, E; X = i, I; Y = j, J; Z = k, K$)

ALL the E8 lattices BECOME CONSISTENT if they are DECOMPOSED into two 4-dimensional HyperDiamond lattices so that

$$E8 = 8HD = 4HDa + 4HDca$$

where $4HDa$ is the 4-dimensional associative Physical Spacetime and $4HDca$ is the 4-dimensional coassociative Internal Symmetry space.

Therefore, the D4-D5-E6 HyperDiamond Feynman Checkerboard model is physically represented on a $4HD$ lattice Physical Spacetime, with an Internal Symmetry space that is also another $4HD$.

BIVECTOR GAUGE BOSON STATES ON LINKS:

The bivector 28 space corresponds to the 28-dimensional D4 Lie algebra Spin(0,8), which, after Dimensional Reduction of Physical Spacetime, corresponds to 28 gauge bosons:

12 for the Standard Model,

15 for Conformal Gravity and the Higgs Mechanism, and

1 for propagator phase.

Define a Bivector State of a given Copy of $DCl(0,8)$ to be a configuration of the 28 gauge bosons at its vertex.

Now look at any link in the E8 lattice, and at the two Copies of $DCl(0,8)$ at each end.

The gauge boson state on that link is given by the Lie algebra bracket product of the Bivector States of the two Copies of $DCL(0,8)$ at each end.

Now define the Total Superposition Bivector State of a given Copy of $DCl(0,8)$ to be the superposition of all configurations of the 28 gauge bosons at its vertex

and the Total Superposition Gauge Boson State on a link to be the superposition of all gauge boson states on that link.

SPINOR FERMION STATES AT VERTICES:

The 8+8 = 16 fermions corresponding to spinors do not correspond to any single grade of $DCl(0,8)$ but correspond to the entire Clifford algebra as a whole.

Its total dimension is $2^8 = 256 = 16 \times 16$

and there are, in the first generation,

8 half-spinor fermion particles and

8 half-spinor fermion antiparticles,

for a total of 16 fermions.

Dimensional Reduction of Physical Spacetime produces 3 Generations of spinor fermion particles and antiparticles.

Define a Spinor Fermion State at a vertex occupied by a given Copy of $DCl(0,8)$ to be a configuration of the spinor fermion particles and antiparticles of all 3 generations at its vertex.

Now define the Total Superposition Spinor Fermion State at a vertex to be the superposition of all Spinor fermion states at that vertex.

Now we have:

$4HD$ HyperDiamond lattice Physical Spacetime

and for each link, a Total Superposition Gauge Boson State

and for each vertex, a Total Superposition Spinor Fermion State.

Now:

Define Many-Worlds Quantum Theory by specifying each of its many Worlds, as follows:

Each World of the Many-Worlds is determined by:

for each link, picking one Gauge Boson State from the Total Superposition

and

for each vertex, picking one Spinor Fermion State from the Total Superposition.

To get an idea of how to think about the D4-D5-E6 HyperDiamond Feynman Checkerboard lattice model, here is a rough outline of how the Uncertainty Principle works:

Do NOT (as is conventional) say that a particle is sort of "spread out" around a given location in a given space-time due to "quantum uncertainty".

Instead, say that the particle is really at a point in space-time

BUT that the "uncertainty spread" is not a property of the particle, but is due to dynamics of the space-time, in which particle-antiparticle pairs x-o are being created sort of at random. For example, in one of the Many-Worlds, the spacetime might not be just an empty vertex but would have created a particle-antiparticle pair

If the original particle is where we put it to start with, then in this World we would have the original particle plus a new particle plus a new antiparticle.

Now, if the new antiparticle annihilates the original particle, we would see a particle at the position of the new particle,

and, since the particles are indistinguishable from each other, it would APPEAR that the original particle was at the different location of the new particle, and the probabilities of such appearances would look like the conventional uncertainty in position.

In the D4-D5-E6 model, correlated states, such as a particle-antiparticle pair coming from the non-trivial vacuum, or an amplitude for two entangled particles, extend over a part of the lattice that includes both particles. The stay in the same World of the Many-Worlds until they become uncorrelated.

3 Octonions and E_8 lattices.

The E_8 lattice is made up of one hypercubic Checkerboard D_8 lattice plus another D_8 shifted by a glue vector (1/2, 1/2, 1/2, 1/2, 1/2, 1/2, 1/2, 1/2).

If octonionic coordinate are chosen so that a given minimal vector in E_8 is +1, the vectors in E_8 that are perpendicular to +1 make up a spacelike E_7 lattice.

The E_8 lattice nearest neighbor vertices have only 4 non-zero coordinates, like 4-dimensional spacetime with speed of light $c = \sqrt{3}$,

rather than 8 non-zero coordinates, like 8-dimensional spacetime with speed of light $c = \sqrt{7}$, so the E_8 lattice light-cone structure appears to be 4-dimensional rather than 8-dimensional.

To build the E_8 Lattice:

Begin with an 8-dimensional octonionic spacetime R^8, where a basis for the octonions is $\{1, i, j, k, E, I, J, K\}$.

The vertices of the E_8 lattice are of the form

$(a_0 1 + a_1 E + a_2 i + a_3 j + a_4 I + a_5 K + a_6 k + a_7 J)/2$,

where the a_i may be either all even integers, all odd integers,

or four of each (even and odd),

with residues mod 2 in the four-integer cases being

$(1; 0, 0, 0, 1, 1, 0, 1)$

or $(0; 1, 1, 1, 0, 0, 1, 0)$

or the same with the last seven cyclically permuted. E_8 forms an integral domain of integral octonions.

The E_8 lattice integral domain has 240 units:

$\pm 1, \pm i, \pm j, \pm k \pm E, \pm I, \pm J \pm K,$

$(\pm 1 \pm I \pm J \pm K)/2, (\pm E \pm i \pm j \pm k)/2,$

and the last two with cyclical permutations of

$\{i, j, k, E, I, J, K\}$ in the order (E, i, j, I, K, k, J).

The cyclical permutation (E, i, j, I, K, k, J) preserves the integral domain E_8, but is not an automorphism of the octonions since it takes the associative triad $\{i, j, k\}$ into the anti-associative triad $\{j, ie, je\}$.

The cyclical permutation (E, I, J, i, k, K, j) is an automorphism of the octonions but takes the E_8 integral domain defined above into another of seven integral domains.

Denote the integral domain described above as $7E_8$,

and the other six by iE_8 , $i = 1, ..., 6$.

The 240 units of the $7E_8$ lattice corresponding to the integral domain $7E_8$ represent the 240 lattice points in the shell at unit distance (also commonly normalized as 2):

$$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K,$$
$$(\pm 1 \pm I \pm J \pm K)/2$$
$$(\pm e \pm i \pm j \pm k)/2$$
$$(\pm 1 \pm K \pm E \pm k)/2$$
$$(\pm i \pm j \pm I \pm J)/2$$
$$(\pm 1 \pm k \pm i \pm J)/2$$
$$(\pm j \pm I \pm K \pm E)/2$$
$$(\pm 1 \pm J \pm j \pm E)/2$$
$$(\pm I \pm K \pm k \pm i)/2$$
$$(\pm 1 \pm E \pm I \pm i)/2$$
$$(\pm K \pm k \pm J \pm j)/2$$
$$(\pm 1 \pm i \pm K \pm j)/2$$
$$(\pm k \pm J \pm E \pm I)/2$$
$$(\pm 1 \pm j \pm k \pm I)/2$$
$$(\pm J \pm E \pm i \pm K)/2$$

The other six integral domains iE_8 are:

$1E_8$:

$$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K,$$
$$(\pm 1 \pm J \pm i \pm j)/2$$
$$(\pm k \pm E \pm I \pm K)/2$$
$$(\pm 1 \pm j \pm I \pm K)/2$$
$$(\pm i \pm k \pm E \pm J)/2$$
$$(\pm 1 \pm K \pm k \pm i)/2$$
$$(\pm j \pm E \pm I \pm J)/2$$
$$(\pm 1 \pm i \pm E \pm I)/2$$
$$(\pm j \pm k \pm J \pm K)/2$$
$$(\pm 1 \pm I \pm J \pm k)/2$$
$$(\pm i \pm j \pm E \pm K)/2$$
$$(\pm 1 \pm k \pm j \pm E)/2$$
$$(\pm i \pm I \pm J \pm K)/2$$
$$(\pm 1 \pm E \pm K \pm J)/2$$
$$(\pm i \pm j \pm k \pm I)/2$$

$2E_8$:

$$\begin{gathered}
\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K, \\
(\pm 1 \pm i \pm k \pm E)/2 \\
(\pm j \pm I \pm J \pm K)/2 \\
(\pm 1 \pm E \pm J \pm j)/2 \\
(\pm i \pm k \pm I \pm K)/2 \\
(\pm 1 \pm j \pm K \pm k)/2 \\
(\pm i \pm E \pm I \pm J)/2 \\
(\pm 1 \pm k \pm I \pm J)/2 \\
(\pm i \pm j \pm E \pm I)/2 \\
(\pm 1 \pm J \pm i \pm K)/2 \\
(\pm j \pm k \pm E \pm I)/2 \\
(\pm 1 \pm K \pm E \pm I)/2 \\
(\pm i \pm j \pm k \pm J)/2 \\
(\pm 1 \pm I \pm j \pm i)/2 \\
(\pm k \pm E \pm J \pm K)/2
\end{gathered}$$

$3E_8$:

$$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K,$$
$$(\pm 1 \pm k \pm K \pm I)/2$$
$$(\pm i \pm j \pm E \pm J)/2$$
$$(\pm 1 \pm I \pm i \pm E)/2$$
$$(\pm j \pm k \pm J \pm K)/2$$
$$(\pm 1 \pm E \pm j \pm K)/2$$
$$(\pm i \pm k \pm I \pm J)/2$$
$$(\pm 1 \pm K \pm J \pm i)/2$$
$$(\pm j \pm k \pm E \pm I)/2$$
$$(\pm 1 \pm i \pm k \pm j)/2$$
$$(\pm e \pm I \pm J \pm K)/2$$
$$(\pm 1 \pm j \pm I \pm J)/2$$
$$(\pm i \pm k \pm E \pm K)/2$$
$$(\pm 1 \pm J \pm E \pm k)/2$$
$$(\pm i \pm j \pm I \pm K)/2$$

$4E_8$:

$$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K,$$
$$(\pm 1 \pm K \pm j \pm J)/2$$
$$(\pm i \pm k \pm E \pm I)/2$$
$$(\pm 1 \pm J \pm k \pm I)/2$$
$$(\pm i \pm j \pm E \pm K)/2$$
$$(\pm 1 \pm I \pm E \pm j)/2$$
$$(\pm i \pm k \pm J \pm K)/2$$
$$(\pm 1 \pm j \pm i \pm k)/2$$
$$(\pm e \pm I \pm J \pm K)/2$$
$$(\pm 1 \pm k \pm K \pm E)/2$$
$$(\pm i \pm j \pm I \pm J)/2$$
$$(\pm 1 \pm E \pm J \pm i)/2$$
$$(\pm j \pm k \pm I \pm K)/2$$
$$(\pm 1 \pm i \pm I \pm K)/2$$
$$(\pm j \pm k \pm E \pm J)/2$$

$5E_8$:

$$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K,$$
$$(\pm 1 \pm j \pm E \pm i)/2$$
$$(\pm k \pm I \pm J \pm K)/2$$
$$(\pm 1 \pm i \pm K \pm J)/2$$
$$(\pm j \pm k \pm E \pm I)/2$$
$$(\pm 1 \pm J \pm I \pm E)/2$$
$$(\pm i \pm j \pm k \pm K)/2$$
$$(\pm 1 \pm E \pm k \pm K)/2$$
$$(\pm i \pm j \pm I \pm J)/2$$
$$(\pm 1 \pm K \pm j \pm I)/2$$
$$(\pm i \pm k \pm E \pm J)/2$$
$$(\pm 1 \pm I \pm i \pm k)/2$$
$$(\pm j \pm E \pm J \pm K)/2$$
$$(\pm 1 \pm k \pm J \pm j)/2$$
$$(\pm i \pm E \pm I \pm K)/2$$

$6E_8$:

$$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K,$$
$$(\pm 1 \pm E \pm I \pm k)/2$$
$$(\pm i \pm j \pm J \pm K)/2$$
$$(\pm 1 \pm k \pm j \pm i)/2$$
$$(\pm e \pm I \pm J \pm K)/2$$
$$(\pm 1 \pm i \pm J \pm I)/2$$
$$(\pm j \pm k \pm E \pm K)/2$$
$$(\pm 1 \pm I \pm K \pm j)/2$$
$$(\pm i \pm k \pm E \pm J)/2$$
$$(\pm 1 \pm j \pm E \pm J)/2$$
$$(\pm i \pm k \pm I \pm K)/2$$
$$(\pm 1 \pm J \pm k \pm K)/2$$
$$(\pm i \pm j \pm E \pm I)/2$$
$$(\pm 1 \pm K \pm i \pm E)/2$$
$$(\pm j \pm k \pm I \pm J)/2$$

The vertices that appear in more than one lattice are:

$\pm 1, \pm i, \pm j, \pm k, \pm E, \pm I, \pm J, \pm K$ in all of them;

$(\pm 1 \pm i \pm j \pm k)/2$ and $(\pm e \pm I \pm J \pm K)/2$ in $3E_8$, $4E_8$, and $6E_8$;

$(\pm 1 \pm i \pm E \pm I)/2$ and $(\pm j \pm k \pm J \pm K)/2$ in $7E_8$, $1E_8$, and $3E_8$;

$(\pm 1 \pm j \pm E \pm J)/2$ and $(\pm i \pm k \pm I \pm K)/2$ in $7E_8$, $2E_8$, and $6E_8$;

$(\pm 1 \pm k \pm E \pm K)/2$ and $(\pm i \pm j \pm I \pm J)/2$ in $7E_8$, $4E_8$, and $5E_8$;

$(\pm 1 \pm i \pm J \pm K)/2$ and $(\pm j \pm k \pm E \pm I)/2$ in $2E_8$, $3E_8$, and $5E_8$;

$(\pm 1 \pm j \pm I \pm K)/2$ and $(\pm i \pm k \pm e \pm J)/2$ in $1E_8$, $5E_8$, and $6E_8$;

$(\pm 1 \pm k \pm I \pm J)/2$ and $(\pm i \pm j \pm E \pm K)/2$ in $1E_8$, $2E_8$, and $4E_8$;

The 240 unit vertices in the E_8 lattices do not include any of the 256 E_8 light cone vertices, of the form $(\pm 1 \pm i \pm j \pm k \pm E \pm I \pm J \pm K)/2$.

They appear in the next layer out from the origin, at radius sqrt 2, which layer contains in all 2160 vertices.

The E8 lattice is, in a sense, fundamentally 4-dimensional.

For instance:

The E8 lattice nearest neighbor vertices have only 4 non-zero coordinates, like 4-dimensional spacetime with speed of light $c = \sqrt{3}$, rather than 8 non-zero coordinates, like 8-dimensional spacetime with speed of light $c = \sqrt{7}$, so the E8 lattice light-cone structure appears to be 4-dimensional rather than 8-dimensional.

The E8 lattice can be represented by quaternionic icosians, as described by Conway and Sloane [5].

The E8 lattice can be constructed, using the Golden ratio, from the D4 lattice, which has a 24-cell nearest neighbor polytope. The construction starts with the 24 vertices of a 24-cell, then adds Golden ratio points on each of the 96 edges of the 24-cell, then extends the space to 8 dimensions by considering the algebraicaly independent $\sqrt{5}$ part of the coordinates to be geometrically independent, and finally doubling the resulting 120 vertices in 8-dimensional space by considering both the D4 lattice and its dual D4* to get the 240 vertices of the E8 lattice nearest neighbor polytope (the Witting polytope).

The 240-vertex Witting polytope, the E8 lattice nearest neighbor polytope, most naturally lives in 4 complex dimensions, where it is self-dual, rather than in 8 real dimensions.

4 E_8 Spacetime and Particles.

The 256-dimensional Clifford algebra $DCl(0,8)$ has graded structure

$$\mathbf{1} \quad \mathbf{8} \quad \mathbf{28} \quad 56 \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$$

The grade-0 scalar of $DCl(0,8)$ is 1-dimensional, representing the Higgs scalar field.

The grade-1 vectors of $DCl(0,8)$ are 8-dimensional, representing spacetime prior to dimensional reduction.

The grade-2 bivectors of $DCl(0,8)$ are 28-dimensional, representing a $Spin(0,8)$ gauge group prior to dimensional reduction.

The entire 256-dimensional $DCl(0,8)$ can be represented by $16x16$ matrices.

Each row or column of $DCl(0,8)$ is a 16-dimensional minimal left or right ideal of $DCl(0,8)$, and can be represented by two integral octonions.

Each of the two integral octonions in a row minimal left ideal is a half-spinor, one +half-spinor and the other a mirror image -half-spinor.

The basis elements for the +half-spinor integral octonions correspond to first-generation fermion particles.

Octonion basis element	*Fermion Particle*
1	$e-neutrino$
i	$red\ up\ quark$
j	$green\ up\ quark$
k	$blue\ up\ quark$
E	$electron$
I	$red\ down\ quark$
J	$green\ down\ quark$
K	$blue\ down\ quark$

(22)

The basis elements for the -half-spinor integral octonions correspond to the first-generation fermion antiparticles.

The column minimal right ideal half-spinors correspond to the Clifford algebra gammas of spacetime transformations.

In calculations, it is sometimes convenient to use the volumes of compact manifolds that represent spacetime, internal symmetry space, and fermion representation space.

The compact manifold that represents 8-dim spacetime is $\mathbf{R}P^1 \times S^7$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(10)/(Spin(8) \times U(1))$.

The compact manifold that represents 4-dim spacetime is $\mathbf{R}P^1 \times S^3$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(6)/(Spin(4) \times U(1))$.

The compact manifold that represents 4-dim internal symmetry space is $\mathbf{R}P^1 \times S^3$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(6)/(Spin(4) \times U(1))$.

The compact manifold that represents the 8-dim fermion representation space is $\mathbf{R}P^1 \times S^7$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(10)/(Spin(8) \times U(1))$.

The manifolds $\mathbf{R}P^1 \times S^3$ and $\mathbf{R}P^1 \times S^7$ are homeomorphic to $S^1 \times S^3$ and $S^1 \times S^7$, which are untwisted trivial sphere bundles over S^1. The corresponding twisted sphere bundles are the generalized Klein bottles $Klein(1,3)Bottle$ and $Klein(1,7)Bottle$.

4.1 Row and Column Spinors.

The entire 256-dimensional DCl(0,8) can be represented by 16x16 real matrices.

Each column or row of DCl(0,8) is a 16-dimensional minimal left or right ideal of DCl(0,8), and can be represented by two integral octonions.

The column minimal ideal spinors, as described above, correspond to fermion particles and antiparticles,

while the row minimal ideals correspond to SpaceTime Dirac gammas.

Each of the two integral octonions in a row minimal right ideal is a half-spinor, one +half-spinor and the other a mirror image -half-spinor.

The row minimal right ideal half-spinors correspond to the Clifford algebra gammas of spacetime transformations. Each of the two integral octonions in a row minimal right ideal is a half-spinor, one +half-spinor and the other a mirror image -half-spinor.

The 8 basis elements for the +half-spinor integral octonions and for the -half-spinor integral octonions each correspond, by triality, to the 8 basis elements of the Cl(8) vector space, and therefore to the 8-dimensional spacetime gamma matrices:

After Dimensional Reduction to 4-dimensional spacetime, the 8 Gammas of the 8-dimensional Octonionic vector space of Cl(8) are replaced by 4 Gammas of the 4-dimensional SpaceTime, which can be seen to have Quaternionic structure.

Physical 4-dimensional spinors live in the even subalgebra of the Cl(4) Clifford algebra, and so are either bivectors, or bivectors plus the scalar or the pseudoscalar.

Since bivectors generate rotations (or boosts), physical spinors transform under transformations of spacetime like spinning (rotating) spheres.

Since gravity can be represented in 4-dimensional spacetime by geometric transformations of spacetime, the interaction between spinors and gravity can be represented by curvature in a Clifford Manifold, as done by William Pezzaglia in gr-qc/9710027 in his derivation of the Papapetrou Equations. Pezzaglia's picture may be consistent with the picture of G. Sardanashvily and of the D4-D5-E6 model with respect to Gravity and the Higgs Mechanism.

For a more conventional derivation of the Papapetrou Equations, see, for example, Geometric Quantization, by N. Woodhouse, second edition Clarendon Press 1992, pages 127-129.

To me, the Clifford method of William Pezzaglia semms clearer than the more conventional derivation, particularly with respect to fundamental fermions and massless (at tree level) neutrinos.

The Papapetrou Equations are important because they show that gravity acts on spinor particles, even if they are massless.

Torsion comes from spin, and both are related to physics models using Conformal Weyl curvature.

4.2 Gauge Bosons in 4-dim SpaceTime.

After Dimensional Reduction to 4-dimensional spacetime, Bivector Gauge Bosons and the Higgs Scalar take the form of the Standard Model plus Gravity.

You can use Clifford Algebras to see how Dimensional Reduction affects the bivector Gauge Bosons and the Higgs scalar,

First go from DCl(0,8) to its even subalgebra DCle(0,8) which is the Clifford algebra DCl(0,7).

Then go to its even subalgebra DCle(0,7) which is the Clifford algebra DCl(0,6).

Then go to its even subalgebra DCle(0,6), which is the Clifford algebra DCl(0,5):

The two components of DCl(0,5) can be regarded as Real and Imaginary parts.

The 4x4 = 16 components of the Real part of DCl(0,5) are:

1-dimensional U(1) propagator phase (light blue); and

15-dimensional Conformal group that gauges to produce Gravity from

the MacDowell-Mansouri mechanism and 10 Poincare group generators (red), with gravitons that see their Symmetry Space of Spacetime according to their group symmetry, and

5 Conformal degrees of freedom (brown) that, when gauge-fixed, combine with the scalar to produce the Higgs mechanism;

The 1 + 12 + 3 components of the Imaginary part of DCl(0,5) are:

1-dimensional Higgs scalar, that combines with the gauge-fixed Conformal degrees of freedom to produce the Higgs mechanism;

12 Standard Model gauge bosons, 3 for weak SU(2) (red), 9 for 8 SU(3) gluons, and 1 U(1) photon (yellow), that see Internal Symmetry Space according to their group symmetry; and

3 components that are 4-vectors (gray), not bivectors, and do not contribute to the physics of gauge bosons and the scalar.

The Gauge Boson dimensional reduction process can also be described by Division Algebras:

Start with the 28 generators of the Spin(8) Lie Algebra acting on its 8-dimensional Vector space.

Reduce the 8-dimensional Vector space to 4-dimensional Spacetime.

Reduce Spin(8) to U(4) = U(1)xSU(4) = U(1)xSpin(6), where the U(1) represents the phase of propagators and the Spin(6) = Spin(4,2) acts as the Conformal Group (or its Euclidean version) on 4-dimensional Spacetime that produce Gravity and the Higgs Mechanism.

Since U(4) has 16 generators that act on Spacetime as phases or as Conformal Group elements, that leaves 28 - 16 = 12 Spin(8) generators to be accounted for.

The 12 remaining generators should not act on Spacetime, but on the 4-dimensional Internal Symmetry Space.

The 4-dimensional Spacetime is a subspace of the 6-dimensional Spin(6) Vector space.

The Internal Symmetry Space on which the 12 remaining generators act should correspond to the Spinor representation space of Spin(6).

Since the Spin(6) Clifford Algebra Cl(6) is the 8x8 real matrix algebra M(8,R), the full Spinor space of Spin(6) is 8-dimensional.

If the full 8-dimensional Spinor space of Cl(6) is given Octonionic structure, then it should transform under both G2, the automorphism group of the Octonions, and Spin(6)=SU(4).

Octonionic Cl(6) Spinor space is 1 copy of the Quaternions, and corresponds to a full 8-dimensional real generalized Spinor space of Cl(6).

14-dimensional G2 and 15-dimensional Spin(6)=SU(4) both have 8-dimensional SU(3) as a subgroup, and SU(3) is the intersection of G2 and Spin(6)=SU(4).

SU(3) acts globally on the full CP2 Internal Symmetry Space through the fibration SU(3)/S(U(1)xU(2)) = CP2.

SU(3) is made up of 8 of the remaining 12 Spin(8) generators, and physically is the local gauge symmetry of the Color Force.

If the full 8-dimensional Spinor space of Cl(6) is given Quaternionic structure, then it should transform under both Sp(1)=SU(2)=Spin(3)=S3, the automorphism group of the Quaternions, and Spin(6)=SU(4).

Quaternionic Cl(6) Spinor space is 2 copies of the Quaternions, and corresponds to 2 4-dimenisional real generalized half-Spinor spaces of Cl(6).

3-dimensional SU(2) is a subgroup of Spin(6)=SU(4).

SU(2) acts globally on a CP1 = S2 subspace of CP2 Internal Symmetry Space through the fibration SU(2)/U(1) = S2. 2 copies of SU(2) correspond to the 2 copies of the Quaternions in Cl(6) Spinor space and to the 2 copies of S2 needed to make a 4-dimensional Internal Symmetry Space.

If the 2 copies of SU(2) are synchronized so as to be consistent throughout the full Cl(6) Spinor Space and the full 4-dimensional Internal Symmetry Space, then SU(2) is made up of 3 of the remaining 12 - 8 = 4 Spin(8) generators, and physically is the local gauge symmetry of the Weak force.

If the full 8-dimensional Spinor space of Cl(6) is given Complex structure, then it should transform under both U(1)=S1, the automorphism group of the Complex numbers, and Spin(6)=SU(4).

Complex Cl(6) Spinor space is 4 copies of the Complex numbers, and corresponds to the 4 Complex dimensions of the full (spinor pus conjugate spinor) Cl(6) Spinor space derived from the even subalgebra Cl(5) = M(4,C) of the Cl(6) Clifford algebra.

1-dimensional U(1) is a subgroup of Spin(6)=SU(4). U(1)=S1 acts globally on a S1 subspace of CP2 Internal Symmetry Space. 4 copies of U(1) correspond to the 4 copies of the Complex numbers in Cl(6) Spinor space and to the 4 copies of S1 needed to make a 4-dimensional Internal Symmetry Space.

If the 4 copies of U(1) are synchronized so as to be consistent throughout the full Cl(6) Spinor Space and the full 4-dimensional Internal Symmetry Space, then:

U(1) is made up of the 1 remaining 12 - 8 - 3 = 1 Spin(8) generator, and physically is the local gauge symmetry of Electromagnetism.

The Standard Model U(1)xSU(2)xSU(3) Lie algebra structure acts on the 8-dimensional generalized half-Spinor space of Cl(6) through:

1 copy of Octonionic SU(3), acting on the Octonions O.

2 synchronized copies of Quaternionic SU(2), acting as SU(2) on the Quaternions Q.

4 synchronized copies of Complex U(1), acting as U(1) on the Complex numbers C.

If each of U(1), SU(2), and SU(3) are considered to act respectively on C, Q, and O, then the Standard Model Lie algebra U(1)xSU(2)xSU(3) can be considered as acting on the 2x4x8=64-dimensional tensor product space T = C x Q x O (which is a non-alternative algebra).

Using T = C x Q x O to describe the Standard Model is the idea of Geoffrey Dixon.

To see how his approach works, let the basis for the Complex numbers be 1,i, the basis for the Quaternions be 1,i,j,k, and the basis for the Octonions be 1,i,j,k,E,I,J,K, and denote by q an imaginary unit quaternion, and then decompose the Identity of T by using

L0 = (1 + i i) / 2

L1 = (1 - i i) / 2

L2 = (1 + i q) / 2

L3 = (1 - i q) / 2

R+ = (1+ i E) / 2

R- = (1 - i E) / 2

to form 4 orthogonal associative primitive idempotent projection operators

D0 = L0 R+ = (1 + i i) (1+ i E) / 4 = (1 + i i + i E - i E) / 4

D1 = L1 R+ = (1 - i i) (1+ i E) / 4 = (1 - i i + i E + i E) / 4

D2 = L2 R- = (1 + i q) (1- i E) / 4 = (1 + i q - i E + q E) / 4

D3 = L3 R- = (1 - i q) (1- i E) / 4 = (1 - i q - i E - q E) / 4

whose symmetries are

D0 - U(1)xSU(2)xSU(3)

D1 - U(1)xSU(2)xSU(3) x Ui(1)

D2 - U(1)xSU(2)xSU(3) x Uq(1) x SUq(2)

D3 - U(1)xSU(2)xSU(3) x UE(1) x SUq(2)

SUq(2) is an SU(2) symmetry due to the variability of the imaginary unit Quaternion q over the entire S3 = SU(2) of imaginary unit Quaternions. If q were fixed (with respect to i) then the 2 SU(2) symmetries would be synchronized, and the resulting symmetries would be

D0 - U(1)xSU(2)xSU(3)

D1 - U(1)xSU(2)xSU(3) x Ui(1)

D2 - U(1)xSU(2)xSU(3) x Uq(1)

D3 - U(1)xSU(2)xSU(3) x UE(1)

Ui(1), Uq(1), and UE(1) are U(1) symmetries due to the variability of each imaginary unit Quaternions over the S1 spanned by it in the parallelizable S3, and to the variability of the imaginary unit Octonion over the S1 spanned by it in the parallelizable S7. If the imaginary unit Quaternions and Octonion were fixed, then the 4 U(1) symmetries would be synchronized, and the resulting symmetries would be

D0 - U(1)xSU(2)xSU(3)

D1 - U(1)xSU(2)xSU(3)

D2 - U(1)xSU(2)xSU(3)

D3 - U(1)xSU(2)xSU(3)

which is the Standard Model Lie algebra U(1)xSU(2)xSU(3).

In his book, Geoffrey Dixon noted that his approach, like mine, produced the Lie algebra

U(1)xU(1)xU(1)xU(1) x SU(2)xSU(2) x SU(3)

prior to synchronization of the U(1)'s and the SU(2)'s.

Note - My terms, generalized Spinor and generalized half-Spinor space of Cl(6), are not standard mathematical terms but are used here because they seem to me to be useful.

From a more geometric Weyl Group - Root Vector Space point to view, the Gauge Boson dimensional reduction process can also be described this way:

Start with the 24-cell whose vertices are the 24-vertex root vectors of the Spin(8) Lie algebra in 4-dimensional root vector space with 4 origin root vectors.

The 12 vertices of the central cuboctahedron of the 24-cell plus the 4 origin root vectors correspond to the U(4) = U(1)xSU(4) = U(1)xSpin(6) Lie subalgebra, where the U(1) represents the phase of propagators and the Spin(6) = Spin(4,2) acts as the Conformal Group (or its Euclidean version) on 4-dimensional Spacetime that produce Gravity and the Higgs Mechanism. Since the 4 origin root vectors of Spin(8) are included here, the U(1)xSpin(6) Lie subalgebra acts on 4-dimensional Spacetime.

The remaining 12 vertices are in the two octahedra of the 24-cell. Since they do not include any of the 4 origin root vectors of Spin(8), they do not represent Lie algebras that act on 4-dimensional Spacetime, but rather represent Lie algebras that act on CP2 Internal Symmetry Space.

The 6 vertices of one octahedron can be projected into interpenetrating triangles in a plane corresponding to the 6 outer vertices of the SU(3) Lie algebra root vectors in 2-dimensional space:

2 opposite vertices of the other octahedron correspond to the 2 center vertices of the SU(3) Lie algebra root vector space in 2-dimensional space, giving us the entire root vector diagram of 8-dimensional SU(3).

The remaining 4 vectors of the (blue) octahedron correspond to the 2 outer vertices and 2 center vertices of the U(2) Lie Algebra.

Now we have decomposed the 28 generators of Spin(8) into:

16 generators of U(4) = U(1)xSU(4) = U(1)xSpin(6) for propagator phase, Gravity, and the Higgs Mechanism, acting on 4-dimensional Spacetime.

8 generators of SU(3) for the Color Force, acting on 4-dimensional Internal Symmetry Space.

4 generators of U(2) = U(1)xSU(2) for Electromagnetism and the Weak Force, acting on 4-dimensional Internal Symmetry Space.

4.3 Cl(8) Multivectors of grades 3, 4, 5, 6, 7, 8.

As to the remaining Multivectors of DCl(0,8):

35 of the 70 4-vectors in the 4-grade subspace are the symmetric parts of the +1 8x8 component of even subalgebraof DCl(8) that remain after taking out the 1-dimensional scalar 0-vector Higgs scalar.

The 1-dimensional scalar 0-vector representing the Higgs scalar can be thought of as the trace of the full symmetric 1+35=36-dimensional space of symmetric 8x8 real matrices.

The 35 4-vectors are the traceless symmetric 8x8 matrices. They are related to the coassociative 4-form that is fixed in the dimensional reduction process to determine the internal symmetry space. At our low energy levels, below the Planck-scale at which dimensional reduction occurs in the D4-D5-E6 model, the 35 4-vectors do not play a dynamical role that we can test experimentally here and now.

The 56-dimensional trivector 3-grade subspace is related to the structure of 3+1 = 4-dimensional subspaces of 1+7 = 8-dimensional spacetime that are connected with the E8 HyperDiamond lattice links that are (normalized) sums of 4 of the basis octonions.

To reduce the dimension of spacetime to 1+3=4 dimensions, an associative 3-form is used. This effectively fixes a particular trivector, so the 56 trivectors do not play a dynamical role in the 4-dimensional phase of the D4-D5-E6 model.

The other 35 56 28 8 1 are (by Hodge * duality) located symmetrically on the opposite side of the 16x16 DCl(8)matrix.

By Hodge duality, they can be viewed as the structures of the D4-D5-E6 model that act on the antiparticle half-spinor fermions, while the 1 8 28 56 35 can be viewed as acting on the particle half-spinor fermions.

4.4 Subtle Triality Supersymmetry.

The Dynkin diagram for Spin(8) has 1 central vertex with 3 lines extending from it to 3 external vertices.

Each vertex represents a representation of Spin(8), with the center vertex (Spin(8)) corresponding to the 28-dimensional adjoint representation that I identified with gauge bosons.

The three representations for spacetime, fermion particles, and fermion antiparticles are EACH 8-dimensional with Octonionic structure.

They are ALL isomorphic by the Spin(8) Triality Automorphism, which can be represented by rotating or interchanging the 3 arms of the Dynkin diagram of Spin(8).

The Triality isomorphism between spacetime and fermion particles and fermion antiparticles constitutes a SUBTLE SUPERSYMMETRY between fermions and spacetime.

Ultraviolet finiteness of the D4-D5-E6 physics model may be seen by considering, prior to dimensional reduction, the generalized supersymmetry relationship between the 28 gauge bosons

and the 8 (first-generation) fermion particles and antiparticles.

In the 8-dimensional spacetime, the dimension of each of the 28 gauge bosons in the Lagrangian is 1,

and the dimension of each of the 8 fermion particles is 7/2,

so that the total dimension 28x1 = 28 of the gauge bosons is equal to the total dimension 8x(7/2) = 28 of the fermion particles.

After dimensional reduction of spacetime to 4 dimensions, the 8 fermions get a 3-generation structure

and the 28 gauge bosons are decomposed to produce the Standard Model of U(1) electromagnetism, SU(2) weak force, and SU(3) color force,

plus a Spin(5) = Sp(2) gauge field that can produce gravity by the MacDowell-Mansouri mechanism.

Note that both Gravity and the Standard Model forces are required for the cancellations that produce the ultraviolet finiteness that is useful in the Sakharov Zero Point Fluctuation model of gravity.

5 HyperDiamond Lattices.

n-dimensional HyperDiamond structures nHD are constructed from D_n lattices.

An n-dimensional HyperDiamond structures nHD is a lattice if and only if n is even.

If n is odd, the nHD structure is only a "packing", not a "lattice", because a nearest neighbor link from an origin vertex to a destination vertex cannot be extended in the same direction to get another nearest neighbor link.

n-dimensional HyperDiamond structures nHD are constructed from D_n lattices.

The lattices of type D_n are n-dimensional Checkerboard lattices, that is, the alternate vertices of a $\mathbf{Z}^n$ hypercubic lattice.

A general reference on lattices is Conway and Sloane [5].

For the n-dimensional HyperDiamond lattice construction from D_n, Conway and Sloane use an n-dimensional glue vector with n coordinates of length 0.5:

$$[1] = (0.5, ..., 0.5)$$

Consider the 3-dimensional structure $3HD$.

Start with D_3, the fcc close packing in 3-space.

Make a second D_3 shifted by the glue vector $(0.5, 0.5, 0.5)$.

Then form the union $D_3 \cup ([1] + D_3)$.

That is the 3-dimensional crystal structure that is made up of tetrahedral bonds.

The crystal structure of H2O Water Ice has such tetrahedral bonds. The bonds from Oxygen to Oxygen are each half covalent and half Hydrogen.

Carbon Diamonds have tetrahedral bonds from Carbon to Carbon that are purely covalent Carbon-Carbon.

HyperDiamond lattices were so named by David Finkelstein because of the 3-dimensional Diamond crystal structure.

5.1 From E8 to 4HD.

When you construct an 8-dimensional HyperDiamond 8HD lattice, you get

$$D8 \cup ([1] + D8) = E8$$

The E8 lattice is in a sense fundamentally 4-dimensional, and the E8 HyperDiamond lattice naturally reduces to the 4-dimensional HyperDiamond lattice.

To get from E8 to 4HD, reduce each of the D8 lattices in the $E8 = 8HD = D8 \cup ([1] + D8)$ lattice to D4 lattices.

We then get a 4-dimensional HyperDiamond $4HD = D4 \cup ([1] + D4)$ lattice.

To see this, start with $E8 = 8HD = D8 \cup ([1] + D8)$.

We can write:

$$D8 = ((D4, 0, 0, 0, 0) \cup (0, 0, 0, 0, D4)) \cup (1, 0, 0, 0, 1, 0, 0, 0) + (D4, D4)$$

The third term is the diagonal term of an orthogonal decomposition of D8, and the first two terms are orthogonal to each other:

associative 4-dimensional Physical Spacetime

and coassociative 4-dimensional Internal Symmetry space.

Now, we see that the orthogonal decomposition of 8-dimensional spacetime into 4-dimensional associative Physical Spacetime plus 4-dimensional Internal Symmetry space gives a decomposition of D8 into D4 + D4.

Since $E8 = D8 \cup ([1] + D8)$, and since

$$[1] = (0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5)$$

can be decomposed by

$$[1] = (0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0) + (0, 0, 0, 0, 0.5, 0.5, 0.5, 0.5)$$

we have

$$E8 = D8 \cup ([1] + D8) =$$

$$= (D4, 0, 0, 0, 0) + (0, 0, 0, 0, D4))$$
$$\cup$$
$$(((0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0) + (0, 0, 0, 0, 0.5, 0.5, 0.5, 0.5))+$$
$$((D4, 0, 0, 0, 0) + (0, 0, 0, 0, D4))) =$$
$$= ((D4, 0, 0, 0, 0) \cup ((0.5, 0.5, 0.5, 0.5, 0, 0, 0, 0) + (D4, 0, 0, 0, 0)))+$$
$$((0, 0, 0, 0, D4) \cup ((0, 0, 0, 0, 0.5, 0.5, 0.5, 0.5) + (0, 0, 0, 0, D4)))$$

Since 4HD is $D4 \cup ([1] + D4)$,

$E8 = 8HD = 4HDa + 4HDca$

where $4HDa$ is the 4-dimensional associative Physical Spacetime

and $4HDca$ is the 4-dimensional coassociative Internal Symmetry space.

5.2 4-dimensional HyperDiamond Lattice.

The 4-dimensional HyperDiamond lattice HyperDiamond is
HyperDiamond $= D_4 \cup ([1] + D_4)$.

The 4-dimensional HyperDiamond HyperDiamond $= D_4 \cup ([1] + D_4)$ is the $\mathbf{Z}^4$ hypercubic lattice with null edges.

It is the lattice that Michael Gibbs [12] used in his 1994 Georgia Tech Ph.D. thesis advised by David Finkelstein.

The 8 nearest neighbors to the origin in the 4-dimensional HyperDiamond HyperDiamond lattice can be written in octonion coordinates as:

$$\begin{array}{c} (1+i+j+k)/2 \\ (1+i-j-k)/2 \\ (1-i+j-k)/2 \\ (1-i-j+k)/2 \\ (-1-i+j+k)/2 \\ (-1+i-j+k)/2 \\ (-1+i+j-k)/2 \\ (-1-i-j-k)/2 \end{array} \tag{23}$$

Here is an explicit construction of the 4-dimensional HyperDiamond HyperDiamond lattice nearest neighbors to the origin.

Start with 24 vertices of a 24-CELL D_4 with squared norm 2:

$$
\begin{array}{cccc}
+1 & +1 & 0 & 0 \\
+1 & 0 & +1 & 0 \\
+1 & 0 & 0 & +1 \\
+1 & -1 & 0 & 0 \\
+1 & 0 & -1 & 0 \\
+1 & 0 & 0 & -1 \\
-1 & +1 & 0 & 0 \\
-1 & 0 & +1 & 0 \\
-1 & 0 & 0 & +1 \\
-1 & -1 & 0 & 0 \\
-1 & 0 & -1 & 0 \\
-1 & 0 & 0 & -1 \\
0 & +1 & +1 & 0 \\
0 & +1 & 0 & +1 \\
0 & +1 & -1 & 0 \\
0 & +1 & 0 & -1 \\
0 & -1 & +1 & 0 \\
0 & -1 & 0 & +1 \\
0 & -1 & -1 & 0 \\
0 & -1 & 0 & -1 \\
0 & 0 & +1 & +1 \\
0 & 0 & +1 & -1 \\
0 & 0 & -1 & +1 \\
0 & 0 & -1 & -1
\end{array}
\tag{24}
$$

Shift the 24 vertices by the glue vector to get 24 more vertices $[1] + D_4$:

$$
\begin{array}{cccc}
+1.5 & +1.5 & 0.5 & 0.5 \\
+1.5 & 0.5 & +1.5 & 0.5 \\
+1.5 & 0.5 & 0.5 & +1.5 \\
+1.5 & -0.5 & 0.5 & 0.5 \\
+1.5 & 0.5 & -0.5 & 0.5 \\
+1.5 & 0.5 & 0.5 & -0.5 \\
-0.5 & +1.5 & 0.5 & 0.5 \\
-0.5 & 0.5 & +1.5 & 0.5 \\
-0.5 & 0.5 & 0.5 & +1.5 \\
-0.5 & -0.5 & 0.5 & 0.5 \\
-0.5 & 0.5 & -0.5 & 0.5 \\
-0.5 & 0.5 & 0.5 & -0.5 \\
0.5 & +1.5 & +1.5 & 0.5 \\
0.5 & +1.5 & 0.5 & +1.5 \\
0.5 & +1.5 & -0.5 & 0.5 \\
0.5 & +1.5 & 0.5 & -0.5 \\
0.5 & -0.5 & +1.5 & 0.5 \\
0.5 & -0.5 & 0.5 & +1.5 \\
0.5 & -0.5 & -0.5 & 0.5 \\
0.5 & -0.5 & 0.5 & -0.5 \\
0.5 & 0.5 & +1.5 & +1.5 \\
0.5 & 0.5 & +1.5 & -0.5 \\
0.5 & 0.5 & -0.5 & +1.5 \\
0.5 & 0.5 & -0.5 & -0.5
\end{array}
\tag{25}
$$

Of the 48 vertices of $D_4 \cup ([1] + D_4)$, these 6 are nearest neighbors to the origin:

$$\begin{array}{llll} -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 0.5 & -0.5 & 0.5 \\ -0.5 & 0.5 & 0.5 & -0.5 \\ 0.5 & -0.5 & -0.5 & 0.5 \\ 0.5 & -0.5 & 0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \end{array} \tag{26}$$

Two more nearest neighbors, also of squared norm 1, of the origin

$$\begin{array}{llll} 0.5 & 0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & -0.5 & -0.5 \end{array} \tag{27}$$

come from adding the glue vector to the origin

$$0 \quad 0 \quad 0 \quad 0 \tag{28}$$

and to the squared norm 4 point

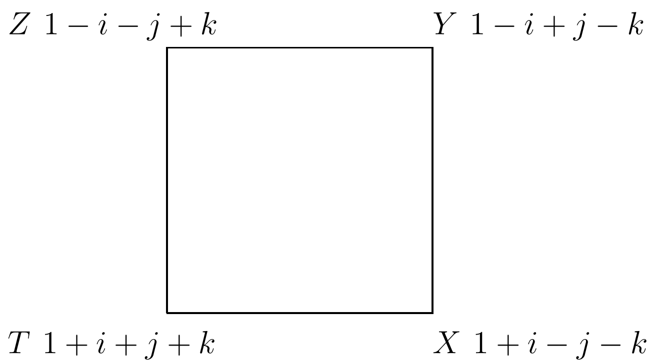
$$-1 \quad -1 \quad -1 \quad -1 \tag{29}$$

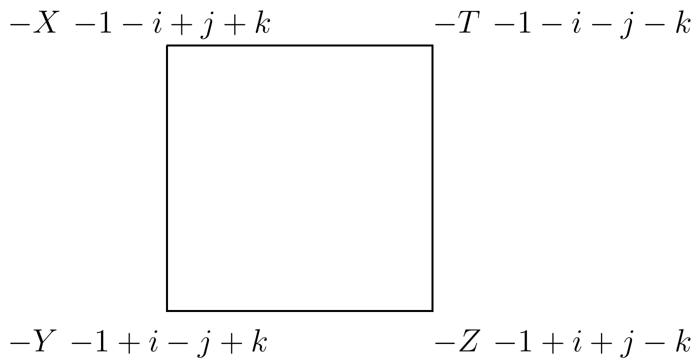
6 Spacetime and Internal Symmetry Space.

The 4-dimensional spacetime of this HyperDiamond Feynman Checkerboard physics model is a HyperDiamond lattice that comes from 4 dimensions of the 8-dimensional E_8 lattice spacetime. If the basis of the E_8 lattice is $\{1, i, j, k, E, I, J, K\}$, then the basis of the 4-dimensional spacetime is the associative part with basis $\{1, i, j, k\}$.

Therefore, the 4-dimensional spacetime lattice is called the associative spacetime and denoted by $4HD_a$.

The 1-time and 3-space dimensions of the $4HD_a$ spacetime can be represented by the 4 future lightcone links and the 4 past lightcone links as in the following pair of "Square Diagrams" of the 4 lines connecting the future ends of the 4 future lightcone links and of the 4 lines connecting the past ends of the 4 past lightcone links:





The 8 links $\{\mathbf{T}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}, -\mathbf{T}, -\mathbf{X}, -\mathbf{Y}, -\mathbf{Z}\}$ correspond to the 8 root vectors of the $Spin(5)$ de Sitter gravitation gauge group, which has an 8-element Weyl group $S_2^2 \times S_2$.
The symmetry group of the 4 links of the future lightcone is S_4, the Weyl group of the 15-dimensional Conformal group $SU(4) = Spin(6)$.
10 of the 15 dimensions make up the de Sitter $Spin(5)$ subgroup, and the other 5 fix the "symmetry-breaking direction" and scale of the Higgs mechanism.

For more on this, see [33] and WWW URLs

http://galaxy.cau.edu/tsmith/cnfGrHg.html

http://www.innerx.net/personal/tsmith/cnfGrHg.html

The Internal Symmetry Space of this HyperDiamond Feynman Checkerboard physics model is a HyperDiamond lattice that comes from 4 dimensions of the 8-dimensional E_8 lattice spacetime. If the basis of the E_8 lattice is $\{1, i, j, k, E, I, J, K\}$, then the basis of the 4-dimensional Internal Symmetry Space is the coassociative part with basis $\{E, I, J, K\}$.

Therefore, the 4-dimensional spacetime lattice is called the associative spacetime and denoted by $4HD_{ca}$.

Physically, the $4HD_{ca}$ Internal Symmetry Space should be thought of as a space ”inside” each vertex of the $4HD_a$ HyperDiamond Feynman Checkerboard spacetime, sort of like a Kaluza-Klein structure.

The 4 dimensions of the $4HD_{ca}$ Internal Symmetry Space are:
electric charge;
red color charge;
green color charge; and
blue color charge.

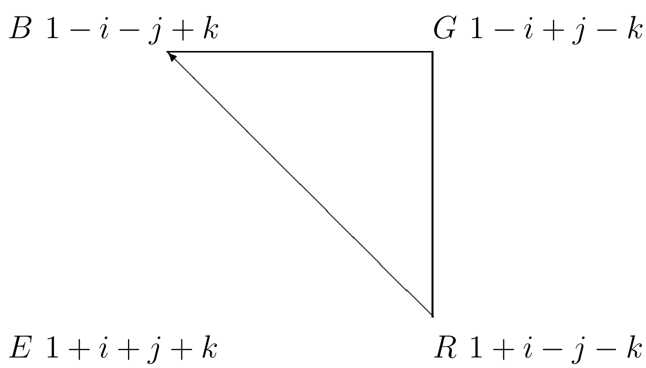
Each vertex of the $4HD_{ca}$ lattice has 8 nearest neighbors, connected by lightcone links. They have the algebraic structure of the 8-element quaternion group $< 2, 2, 2 >$. [6]

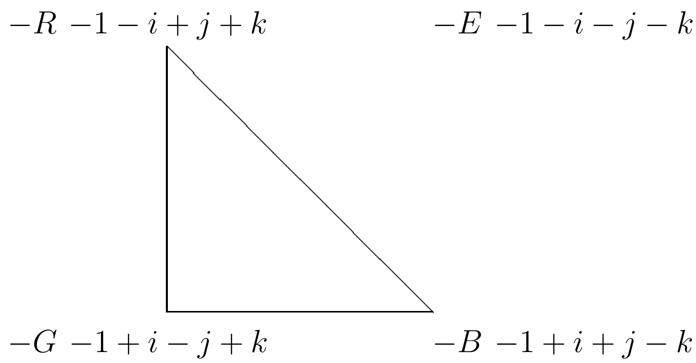
Each vertex of the $4HD_{ca}$ lattice has 24 next-to-nearest neighbors, connected by two lightcone links. They have the algebraic structure of the 24-element binary tetrahedral group $< 3, 3, 2 >$ that is associated with the 24-cell and the D_4 lattice. [6]

As with the time as space dimensions of the $4HD_a$ spacetime, the E-electric and RGB-color dimensions of the $4HD_{ca}$ Internal Symmetry Space can be represented by the 4 future lightcone links and the 4 past lightcone links.

However, in the $4HD_{ca}$ Internal Symmetry Space the E Electric Charge should be treated as independent of the RGB Color Charges.

As a result the following pair of "Square Diagrams" look more like "Triangle plus Point Diagrams".





The 2+6 links $\{\mathbf{E}, -\mathbf{E}; \mathbf{R}, \mathbf{G}, \mathbf{B}, -\mathbf{R}, -\mathbf{G}, -\mathbf{B}\}$ correspond to:

the 2 root vectors of the weak force $SU(2)$, which has a 2-element Weyl group S_2; and

the 6 root vectors of the color force $SU(3)$, which has a 6-element Weyl group S_3.

In calculations, it is sometimes convenient to use the volumes of compact manifolds that represent spacetime, internal symmetry space, and fermion representation space.

The compact manifold that represents 8-dim spacetime is $\mathbf{R}P^1 \times S^7$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(10)/(Spin(8) \times U(1))$.

The compact manifold that represents 4-dim spacetime is $\mathbf{R}P^1 \times S^3$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(6)/(Spin(4) \times U(1))$.

The compact manifold that represents 4-dim internal symmetry space is

$\mathbf{R}P^1 \times S^3$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(6)/(Spin(4) \times U(1))$.

The compact manifold that represents the 8-dim fermion representation space is $\mathbf{R}P^1 \times S^7$, the Shilov boundary of the bounded complex homogeneous domain that corresponds to $Spin(10)/(Spin(8) \times U(1))$.

The manifolds $\mathbf{R}P^1 \times S^3$ and $\mathbf{R}P^1 \times S^7$ are homeomorphic to $S^1 \times S^3$ and $S^1 \times S^7$, which are untwisted trivial sphere bundles over S^1. The corresponding twisted sphere bundles are the generalized Klein bottles $Klein(1,3)Bottle$ and $Klein(1,7)Bottle$.

6.1 Global Internal Symmetry Space and SpaceTime.

Matti Pitkanen has suggested that the global structure of 4-dimensional Spacetime and Internal Symmetry Space should be given by 8-dimensional SU(3), which decomposes into CP2 base and U(2) fibre, both of which are 4-dimensional, by SU(3) / U(2) = CP2.

Associative 4-dimensional Spacetime, with Minkowski signature, is topologically U(2) = SU(2)xU(1) = S3 x S1

which is consistent with the D4-D5-E6 model Minkowski Spacetime of RP1 x S3.

If you do a Wick rotation to Euclidean signature, you get a Spacetime that is naturally globally S4.

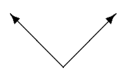
Coassociative 4-dimensional Internal Symmetry Space is CP2.

7 Feynman Checkerboards.

The 2-dimensional Feynman Checkerboard [9, 10] is a notably successful and useful representation of the Dirac equation in 2-dimensional spacetime.

To build a Feynman 2-dimensional Checkerboard,
start with a 2-dimensional Diamond Checkerboard with
two future lightcone links and two past lightcone links at each vertex.

The future lightcone then looks like



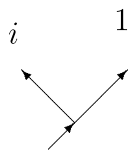
If the 2-dimensional Feynman Checkerboard is
coordinatized by the complex plane $\mathbf{C}$:
the real axis 1 is identified with the time axis t;
the imaginary axis i is identified with the space axis x; and
the two future lightcone links are $(1/\sqrt{2})(1+i)$ and $(1/\sqrt{2})(1-i)$.

In cylindrical coordinates t, r with $r^2 = x^2$,
the Euclidian metric is $t^2 + r^2 = t^2 + x^2$ and
the Wick-Rotated Minkowski metric with speed of light c is
$(ct)^2 - r^2 = (ct)^2 - x^2$.

For the future lightcone links to lI on
the 2-dimensional Minkowski lightcone, $c = 1$.

Either link is taken into the other link by complex multiplication by $\pm i$.

Now, consider a path in the Feynman Checkerboard.
At a given vertex in the path, denote the future lightcone link in
the same direction as the past path link by 1, and
the future lightcone link in the (only possible) changed direction by i.



The Feynman Checkerboard rule is that
if the future step at a vertex point of a given path is in a different direction from the immediately preceding step from the past,
then the path at the point of change gets a weight of $-im\epsilon$,
where m is the mass (only massive particles can change directions),and
ϵ is the length of a path segment.

Here I have used the Gersch [11] convention
of weighting each turn by $-im\epsilon$
rather than the Feynman [9, 10] convention
of weighting by $+im\epsilon$, because Gersch's convention gives a better nonrelativistic limit in the isomorphic 2-dimensional Ising model [11].

HOW SHOULD THIS BE GENERALIZED
TO HIGHER DIMENSIONS?

The 2-dim future light-cone is the 0-sphere $S^{2-2} = S^0 = \{i, 1\}$,

with 1 representing a path step to the future in
the same direction as the path step from the past, and

i representing a path step to the future in a
(only 1 in the 2-dimensional Feynman Checkerboard lattice)
different direction from the path step from the past.

The 2-dimensional Feynman Checkerboard lattice spacetime can be represented by the complex numbers $\mathbf{C}$,
with $1, i$ representing the two future lightcone directions and
$-1, -i$ representing the two past lightcone directions.

Consider a given path in
the Feynman Checkerboard lattice 2-dimensional spacetime.

At any given vertex on the path in the lattice 2-dimensional spacetime, the future lightcone direction representing the continuation of the path in the same direction can be represented by 1, and the future lightcone direction representing the (only 1 possible) change of direction can be represented by i since either of the 2 future lightcone directions can be taken into the other by multiplication by $\pm i$, $+$ for a left turn and $-$ for a right turn.

If the path does change direction at the vertex, then the path at the point of change gets a weight of $-im\epsilon$, where i is the complex imaginary, m is the mass (only massive particles can change directions),and ϵ is the timelike length of a path segment, where the 2-dimensional speed of light is taken to be 1.

Here I have used the Gersch [11] convention of weighting each turn by $-im\epsilon$ rather than the Feynman [9, 10] convention of weighting by $+im\epsilon$, because Gersch's convention gives a better nonrelativistic limit in the isomorphic 2-dimensional Ising model [11].

For a given path, let C be the total number of direction changes, and c be the cth change of direction, and i be the complex imaginary representing the cth change of direction.

C can be no greater than the timelike Checkerboard distance D between the initial and final points.

The total weight for the given path is then

$$\prod_{0\leq c\leq C} -im\epsilon = (m\epsilon)^C(\prod_{0\leq c\leq C} -i) = (-im\epsilon)^C \tag{30}$$

The product is a vector in the direction ± 1 or $\pm i$.

Let $N(C)$ be the number of paths with C changes in direction.

The propagator amplitude for the particle to go from the initial vertex to the final vertex is the sum over all paths of the weights, that is the path integral sum over all weighted paths:

$$\sum_{0 \leq C \leq D} N(C)(-im\epsilon)^C \tag{31}$$

The propagator phase is the angle between
the amplitude vector in the complex plane and the complex real axis.

Conventional attempts to generalize the Feynman Checkerboard from 2-dimensional spacetime to k-dimensional spacetime are based on the fact that the 2-dimensional future light-cone directions are the 0-sphere $S^{2-2} = S^0 = \{i, 1\}$.

The k-dimensional continuous spacetime lightcone directions are the $(k-2)$-sphere S^{k-2}.

In 4-dimensional continuous spacetime, the lightcone directions are S^2.

Instead of looking for a 4-dimensional lattice spacetime, Feynman and other generalizers went from discrete S^0 to continuous S^2 for lightcone directions, and then tried to construct a weighting using changes of directions as rotations in the continuous S^2, and never (as far as I know) got any generalization that worked.

The HyperDiamond HyperDiamond generalization has discrete lightcone directions.

If the 4-dimensional Feynman Checkerboard is coordinatized by the quaternions **Q**:
the real axis 1 is identified with the time axis t;
the imaginary axes i, j, k are identified with the space axes x, y, z; and
the four future lightcone links are
$(1/2)(1 + i + j + k)$,
$(1/2)(1 + i - j - k)$,
$(1/2)(1 - i + j - k)$, and
$(1/2)(1 - i - j + k)$.

In cylindrical coordinates t, r
with $r^2 = x^2 + y^2 + z^2$,
the Euclidian metric is $t^2 + r^2 = t^2 + x^2 + y^2 + z^2$ and
the Wick-Rotated Minkowski metric with speed of light c is
$(ct)^2 - r^2 = (ct)^2 - x^2 - y^2 - z^2$.

For the future lightcone links to lI on
the 4-dimensional Minkowski lightcone, $c = \sqrt{3}$.

Any future lightcone link is taken into any other future lightcone link by quaternion multiplication by $\pm i$, $\pm j$, or $\pm k$.

For a given vertex on a given path,
continuation in the same direction can be represented by the link 1, and
changing direction can be represented by the
imaginary quaternion $\pm i, \pm j, \pm k$ corresponding to
the link transformation that makes the change of direction.

Therefore, at a vertex where a path changes direction,
a path can be weighted by quaternion imaginaries
just as it is weighted by the complex imaginary in the 2-dimensional case.

If the path does change direction at a vertex, then
the path at the point of change gets a weight of $-im\epsilon$, $-jm\epsilon$, or $-km\epsilon$
where i, j, k is the quaternion imaginary representing the change of direction,

m is the mass (only massive particles can change directions), and
$\sqrt{3}\epsilon$ is the timelike length of a path segment,
where the 4-dimensional speed of light is taken to be $\sqrt{3}$.

For a given path,
let C be the total number of direction changes,
c be the cth change of direction, and
e_c be the quaternion imaginary i, j, k representing the cth change of direction.

C can be no greater than the timelike Checkerboard distance D between the initial and final points.

The total weight for the given path is then

$$\prod_{0 \leq c \leq C} -e_c m\sqrt{3}\epsilon = (m\sqrt{3}\epsilon)^C (\prod_{0 \leq c \leq C} -e_c) \tag{32}$$

Note that since the quaternions are not commutative,
the product must be taken in the correct order.

The product is a vector in the direction ± 1, $\pm i$, $\pm j$, or $\pm k$. and

Let $N(C)$ be the number of paths with C changes in direction.

The propagator amplitude for the particle to go
from the initial vertex to the final vertex is
the sum over all paths of the weights,
that is the path integral sum over all weighted paths:

$$\sum_{0 \leq C \leq D} N(C)(m\sqrt{3}\epsilon)^C (\prod_{0 \leq c \leq C} -e_c) \tag{33}$$

The propagator phase is the angle between
the amplitude vector in quaternionic 4-space and
the quaternionic real axis.

The plane in quaternionic 4-space defined by
the amplitude vector and the quaternionic real axis
can be regarded as the complex plane of the propagator phase.

8 Charge = Amplitude to Emit Gauge Boson.

8.1 What Factors Determine Charge?

In the HyperDiamond Feynman Checkerboard model the charge of a particle is the amplitude for a particle to emit a gauge boson of the relevant force. Neutral particles do not emit gauge bosons.

Force strengths are probalilities, or squares of amplitudes for emission of gauge bosons, or squares of charges, so that calculation of charges is equivalent to calculation of force strengths.

Three factors determine the probability for emission of a gauge boson from an origin spacetime vertex to a target vertex:

the part of the Internal Symmetry Space of the target spacetime vertex that is available for the gauge boson to go to from the origin vertex;

the volume of the spacetime link that is available for the gauge boson to go through from the origin vertex to the target vertex; and

an effective mass factor for forces (such as the Weak force and Gravity) that, in the low-energy ranges of our experiments, are carried effectively by gauge bosons that are not massless high-energy.

In the $D_4 - D_5 - E_6$ Lagrangian continuum version of this physics model, force strength probabilities are calculated in terms of relative volumes of bounded complex homogeneous domains and their Shilov boundaries.

The relationship between the $D_4 - D_5 - E_6$ Lagrangian continuum approach and the HyperDiamond Feynman Checkerboard discrete approach is that:

the bounded complex homogeneous domains correspond to

harmonic functions of generalized Laplacians that determine heat equations, or diffusion equations;

while the amplitude to emit gauge bosons in the HyperDiamond Feynman Checkerboard is a process that is similar to diffusion, and therefore also corresponds to a generalized Laplacian.

Details of the $D_4 - D_5 - E_6$ Lagrangian continuum approach can be found on the World Wide Web at URLs

http://xxx.lanl.gov/abs/hep-ph/9501252

http://galaxy.cau.edu/tsmith/d4d5e6hist.html

For the discrete HyperDiamond Feynman Checkerboard approach of this paper, the only free charge parameter is the charge of the $Spin(5)$ gravitons in the MacDowell-Mansouri formalism of Gravity. Note that these $Spin(5)$ gravitons are NOT the ordinary spin-2 gravitons of the low-energy region in which we live. The charge of the $Spin(5)$ gravitons is taken to be unity, 1, so that its force strength is also unity, 1. All other force strengths are determined as ratios with respect to the $Spin(5)$ gravitons and each other.

The four forces of the HyperDiamond Feynman Checkerboard model are Gravity, the Color force, the Weak force, and Electromagnetism.

The charge of each force is the amplitude for one of its gauge bosons to be emitted from a given origin vertex of the spacetime HyperDiamond lattice and go to a neighboring target vertex.

The force strength of each force is the square of the charge amplitude, or the probablility for one of its gauge bosons to be emitted from a given origin vertex of the spacetime HyperDiamond lattice and go to a neighboring target vertex.

The HyperDiamond Feynman Checkerboard model calculations are actually done for force strengths, or probablilities, because it is easier to calculate probablilities.

The force strength probability for a gauge boson to be emitted from an origin spacetime HyperDiamond vertex and go to a target vertex is the product of three things:

the volume $Vol(M_{IS_{force}})$ of the target Internal Symmetry Space, that is, the part of the Internal Symmetry Space of the target spacetime vertex that is available for the gauge boson to go to from the origin vertex;

the volume $\frac{Vol(Q_{force})}{Vol(D_{force})^{\left(\frac{1}{m_{force}}\right)}}$ of the spacetime link to the target spacetime vertex from the origin vertex; and

an effective mass factor $\frac{1}{\mu^2_{force}}$ for forces (such as the Weak force and Gravity) that, in the low-energy ranges of our experiments, are carried effectively by gauge bosons that are not massless high-energy $SU(2)$ or $Spin(5)$ gauge bosons, but are either massive Weak bosons due to the Higgs mechanism or effective spin-2 gravitons. For other forces, the effective mass factor is taken to be unity, 1.

Therefore, the force strength of a given force is

$$\alpha_{force} = \left(\frac{1}{\mu^2_{force}}\right)\left(Vol(M_{IS_{force}})\right)\left(\frac{Vol(Q_{force})}{Vol(D_{force})^{\left(\frac{1}{m_{force}}\right)}}\right) \tag{34}$$

The symbols have the following meanings:

α_{force} represents the force strength;

μ_{force} represents the effective mass;

$M_{IS_{force}}$ represents the part of the target Internal Symmetry Space that is available for the gauge boson to go to;

$Vol(M_{IS_{force}})$ stands for volume of $M_{IS_{force}}$;

Q_{force} represents the link from the origin to the target that is available for the gauge boson to go through;

$Vol(Q_{force})$ stands for volume of Q_{force};

D_{force} represents the complex bounded homogeneous domain of which Q_{force} is the Shilov boundary;

m_{force} is the dimensionality of Q_{force}, which is 4 for Gravity and the Color force, 2 for the Weak force (which therefore is considered to have two copies of Q_W for each spacetime HyperDiamond link), and 1 for Electromagnetism (which therefore is considered to have four copies of Q_E for each spacetime HyperDiamond link)

$Vol(D_{force})^{\left(\frac{1}{m_{force}}\right)}$ stands for a dimensional normalization factor (to reconcile the dimensionality of the Internal Symmetry Space of the target vertex with the dimensionality of the link from the origin to the target vertex).

The force strength formula is stated in terms of continuum structures, such as volumes of manifolds, Shilov Boundaries, etc., rather than in discrete terms. We recognize that a discrete version of the calculations would be the fundamentally correct way to calculate in the discrete HyperDiamond Feynman Checkerboard model, but it is easier for us to look up relevant manifold volumes than to write and execute computer code to do the discrete calculations.

However, since the HyperDiamond Feynman Checkerboard lattice spacing is Planck length and therefore much smaller than the relevant distances for any experiments that we want to describe, we think that the continuum calculations are good approximations of the fundamental discrete HyperDiamond Feynman Checkerboard calculations.

The geometric volumes needed for the calculations, mostly taken from Hua [15], are

$Force$	M	$Vol(M)$	Q	$Vol(Q)$	D	$Vol(D)$
$gravity$	S^4	$8\pi^2/3$	$\mathbf{R}P^1 \times S^4$	$8\pi^3/3$	IV_5	$\pi^5/2^4 5!$
$color$	$\mathbf{C}P^2$	$8\pi^2/3$	S^5	$4\pi^3$	$B^6\ (ball)$	$\pi^3/6$
$weak$	$S^2 \times S^2$	$2 \times 4\pi$	$\mathbf{R}P^1 \times S^2$	$4\pi^2$	IV_3	$\pi^3/24$
$e - mag$	T^4	$4 \times 2\pi$	$-$	$-$	$-$	$-$

(35)

Using these numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

$Gauge\ Group$	$Force$	$Characteristic\ Energy$	$Geometric\ Force\ Strength$	$Total\ Force\ Strength$
$Spin(5)$	$gravity$	$\approx 10^{19} GeV$	1	$G_G m_{proton}^2 \approx 5 \times 10^{-39}$
$SU(3)$	$color$	$\approx 245 MeV$	0.6286	0.6286
$SU(2)$	$weak$	$\approx 100 GeV$	0.2535	$G_W m_{proton}^2 \approx \approx 1.05 \times 10^{-5}$
$U(1)$	$e - mag$	$\approx 4 KV$	1/137.03608	1/137.03608

(36)

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.

The effect is particularly pronounced with the color force.

The color force strength was calculated at various energies according to renormalization group equations, with the following results:

$Energy\ Level$	$Color\ Force\ Strength$
$245 MeV$	0.6286
$5.3 GeV$	0.166
$34 GeV$	0.121
$91 GeV$	0.106

(37)

Shifman [26] in a paper at

http://xxx.lan.gov/abs/hep-ph/9501222

has noted that Standard Model global fits at the Z peak, about 91 GeV, give a color force strength of about 0.125 with $\Lambda_{QCD} \approx 500\ MeV$, whereas low energy results and lattice calculations give a color force strength at the Z peak of about 0.11 with $\Lambda_{QCD} \approx 200\ MeV$.

In the remainder of this Chapter 8, we discuss further the concepts of Target Internal Symmetry Space, Link to Target, and Effective Mass Factors, and then we discuss in more detail each of the four forces.

In this HyperDiamond Feynman Checkerboard model, all force strengths are represented as ratios with respect to the geometric force strength of Gravity (that is, the force strength of Gravity without using the Effective Mass factor).

8.1.1 Volume of Target Internal Symmetry Space.

$M_{IS_{force}}$ represents the part of the target Internal Symmetry Space that is available for the gauge boson to go to; and

$Vol(M_{IS_{force}})$ stands for volume of $M_{IS_{force}}$.

What part of the target Internal Symmetry Space is available for the gauge boson to go to?

Each vertex of the spacetime HyperDiamond lattice is not just a point, but also contains its own Internal Symmetry Space (also a HyperDiamond lattice), so the amplitude for a gauge boson to go from one vertex to another depends

not only on the spacetime link between the vertices

but also on the degree of connection the gauge boson has with the Internal Symmetry Spaces of the vertices.

If the gauge boson can connect any vertex in the origin Internal Symmetry Space with any vertex in the destination Internal Symmetry Space, then the gauge boson has full connectivity between the Internal Symmetry Spaces.

However, if the gauge boson can connect a vertex in the origin Internal Symmetry Space only with some, but not any, of the vertices in the destination Internal Symmetry Space, then the gauge boson has only partial connectivity between the Internal Symmetry Spaces, and has a lower amplitude to be emitted from the origin spacetime vertex to the destination spacetime vertex.

The amount of connectivity between the Internal Symmetry Spaces is the geometric measure of the charge of a force, and therefore of its force strength.

To represent the gauge boson connectivity between Internal Symmetry Spaces, it is useful to label the basis of the Internal Symmetry Space by the degrees of freedom of the forces:

electric charges $\{+1, -1\}$; and

color (red, green, blue) charges $\{+r, -r; +g, -g; +b, -b\}$.

so that the total basis of the Internal Symmetry Space is

$\{+1, -1, +r, -r; +g, -g; +b, -b\}$

This connectivity can be measured by comparing

the full target Internal Symmetry Space HyperDiamond lattice

with the subspace of the target Internal Symmetry Space HyperDiamond lattice that is the image of a given point of the origin Internal Symmetry Space under all the transformations of the Internal Symmetry Space of the gauge group of the force.

The $M_{IS_{force}}$ target Internal Symmetry Space manifolds, each irreducible component of which has dimension m_{force}, for the four forces are:

Gauge Group	*Symmetric Space*	m_{force}	$M_{IS_{force}}$
$Spin(5)$	$\frac{Spin(5)}{Spin(4)}$	4	S^4
$SU(3)$	$\frac{SU(3)}{SU(2) \times U(1)}$	4	$\mathbf{C}P^2$
$SU(2)$	$\frac{SU(2)}{U(1)}$	2	$S^2 \times S^2$
$U(1)$	$U(1)$	1	$S^1 \times S^1 \times S^1 \times S^1$

(38)

8.1.2 Volume of Link to Target.

If, as in the case of the Electromagnetic $U(1)$ photon, there is only one gauge boson that can go through the link from an origin spacetime HyperDiamond vertex to a target vertex, then the link to the target vertex is like a one-lane road.

For the Weak $SU(2)$ force, the Color $SU(3)$ force, and $Spin(5)$ Gravity, there are, respectively, 3, 8, and 10 gauge bosons that can go through the link from an origin spacetime HyperDiamond vertex to a target vertex, so that the link to the target vertex is like a multi-lane highway.

The volume of the link to the target vertex is not measured just by the number of gauge bosons, but by the volume of the minimal manifold Q_{force} that can carry two things:

the gauge bosons, with gauge group G_{force}; and

the $U(1)$ phase of the propagator from origin to target.

The first step in constructing Q_{force} is to find a manifold whose local isotropy symmetry group is $G_{force} \times U(1)$ so that it can carry both G_{force} and $U(1)$.

To do that, look for the smallest Hermitian symmetric space of the form $K/(G_{force} \times U(1))$.

Having found that Hermitian symmetric space, then go to its corrresponding complex bounded homogeneous domain, D_{force}.

Then take Q_{force} to be the Shilov boundary of D_{force}.

Q_{force} is then the minimal manifold that can carry both G_{force} and $U(1)$, and so is the manifold that should represent the link from origin to target vertex.

The Q_{force}, Hermitian symmetric space, and D_{force} manifolds for the four forces are:

Gauge Group	*Hermitian Symmetric Space*	*Type of* D_{force}	m_{force}	Q_{force}
$Spin(5)$	$\frac{Spin(7)}{Spin(5)\times U(1)}$	IV_5	4	$\mathbf{R}P^1 \times S^4$
$SU(3)$	$\frac{SU(4)}{SU(3)\times U(1)}$	$B^6\ (ball)$	4	S^5
$SU(2)$	$\frac{Spin(5)}{SU(2)\times U(1)}$	IV_3	2	$\mathbf{R}P^1 \times S^2$
$U(1)$	$-$	$-$	1	$-$

(39)

The geometric volumes of the target Internal Symmetry Space $M_{IS_{force}}$, the link volume Q_{force}, and the bounded complex domains D_{force} of which the link volume is the Shilov boundary, mostly taken from Hua [15], are:

$Force$	M	$Vol(M)$	Q	$Vol(Q)$	D	$Vol(D)$
$gravity$	S^4	$8\pi^2/3$	$\mathbf{R}P^1 \times S^4$	$8\pi^3/3$	IV_5	$\pi^5/2^4 5!$
$color$	$\mathbf{C}P^2$	$8\pi^2/3$	S^5	$4\pi^3$	$B^6\ (ball)$	$\pi^3/6$
$weak$	$S^2 \times S^2$	$2 \times 4\pi$	$\mathbf{R}P^1 \times S^2$	$4\pi^2$	IV_3	$\pi^3/24$
$e - mag$	T^4	$4 \times 2\pi$	$-$	$-$	$-$	$-$

(40)

The geometric part of the force strength is formed from the product of the volumes of the target Internal Symmetry Space $M_{IS_{force}}$ and the link volume Q_{force}.

To take the product properly, we must take into account how the target Internal Symmetry Space $M_{IS_{force}}$, which lives at the target vertex, and the link volume Q_{force}, which lives on the link connecting the origin vertex to the target vertex and so can be regarded as containing both the origin vertex and the target vertex, fit together.

The dimension m_{force} of each irreducible component of the target Internal Symmetry Space $M_{IS_{force}}$ is less than the dimension of the link volume Q_{force},

which in turn is less than the dimension of the bounded complex domain D_{force} of which the link volume Q_{force} is the Shilov boundary.

Since the link volume Q_{force} is a Shilov boundary, it can be regarded as a shell whose Shilov interior is the bounded complex domain D_{force}.

If we were to merely take the product of the volume of the target Internal Symmetry Space $M_{IS_{force}}$ with the volume of the link volume Q_{force}, we would be overcounting the contribution of the link volume Q_{force} because of its dimension is higher than the dimension of each irreducible component of the target Internal Symmetry Space $M_{IS_{force}}$.

To get rid of the overcounting, we should make the link volume Q_{force} compatible with each irreducible component of the target Internal Symmetry Space $M_{IS_{force}}$.

Since the target Internal Symmetry Space $M_{IS_{force}}$ has fundamentally a 4-dimensional HyperDiamond structure, each irreducible component is fundamentally made up of m_{force}-dimensional hypercubic cells,

where the irreducible component hypercubic cells are

4-dimensional for Gravity and the Color force,

2-dimensional for the Weak force,

and 1-dimensional for Electromagnetism.

Since D_{force} is the Shilov interior of Q_{force}, Q_{force} would be compatible with $M_{IS_{force}}$ if D_{force} were mapped 1-1 onto an m_{force}-dimensional hypercubic cell in an irreducible component of the target Internal Symmetry Space $M_{IS_{force}}$.

To do this, first construct a hypercube of the same dimension m_{force} as an irreducible component of the target Internal Symmetry Space $M_{IS_{force}}$ and the same volume as D_{force}.

The edge length of such a hypercube is the m_{force}-th root of the volume of the bounded complex domain D_{force}.

Since the hypercubes in the fundamental HyperDiamond structures of the target Internal Symmetry Space $M_{IS_{force}}$ and its irreducible components are unit hypercubes with edge length 1,

we must, to make the link volume Q_{force} compatible with each irreducible component of the target Internal Symmetry Space $M_{IS_{force}}$,

divide the volume of the link volume Q_{force} by the m_{force}-th root of the volume of the bounded complex domain D_{force}

to make D_{force} the right size to fit the hypercubes in the fundamental HyperDiamond structures of the target Internal Symmetry Space $M_{IS_{force}}$ and its irreducible components.

In other words, to reconcile the dimensionality of the link volume Q_{force} to the dimensionality of the target Internal Symmetry Space, divide Q_{force} by

$$Vol(D_{force})^{\left(\frac{1}{m_{force}}\right)}.$$

The resulting volume

$$\frac{Vol(Q_{force})}{Vol(D_{force})^{\left(\frac{1}{m_{force}}\right)}}$$

is then the correctly normalized volume of the spacetime link that is available for the gauge boson to go through to get to the target spacetime vertex from the origin vertex, which correctly normalized volume should be used in multiplying by the volume of he target Internal Symmetry Space $M_{IS_{force}}$ to get the geometric part of the force strength.

8.1.3 Effective Mass Factors.

In the low-energy ranges of our experiments, the Weak force and Gravity are carried effectively by gauge bosons that are not massless high-energy $SU(2)$ or $Spin(5)$ gauge bosons, but are either massive Weak bosons due to the Higgs mechanism or effective spin-2 gravitons.

μ_{force} represents the effective mass for the Weak force and Gravity in the low-energy range.

In the low-energy range, the forece strengths of the Weak force and Gravity have an effective mass factor $\frac{1}{\mu^2_{force}}$.

For other forces, the Color force and Electromagnetism, the effective mass factor is taken to be unity, 1.

8.2 Gravity.

For Gravity, $Spin(5)$ gravitons can carry all of the Internal Symmetry Space charges

$\{+1, -1; +r, -r; +g, -g; +b, -b\}$

$Spin(5)$ gravitons act transitively on the 4-dimensional manifold $S^4 = Spin(5)/Spin(4)$, so that they can take any given point in the origin Internal Symmetry Space into any point in the target Internal Symmetry Space.

The full connectivity of the Gravity of $Spin(5)$ gravitons is geometrically represented by M_{IS_G} as the 4-dimensional sphere $S^4 = Spin(5)/Spin(4)$ whose volume is $8\pi^2/3$.

The link manifold to the target vertex is

$Q_G = \mathbf{R}P^1 \times S^4 = ShilovBdy(D_G)$ with volume $8\pi^3/3$

The bounded complex homogeneous domain D_G

is of type IV_5 with volume $\pi^5/2^4 5!$.

It corresponds to the Hermitian Symmetric Space

$$Spin(7)/(Spin(5) \times U(1)$$

For $Spin(5)$ Gravity, m_G is 4.

For $Spin(5)$ Gravity, $\mu_G = m_{Planck}$, so that

$\frac{1}{\mu_G^2} = \frac{1}{m_{Planck}^2}$.

Therefore, the force strength of Gravity is:

$\alpha_G = \left(\frac{1}{\mu_G^2}\right)(Vol(M_{IS_G}))\left(\frac{Vol(Q_G)}{Vol(D_G)^{\left(\frac{1}{m_G}\right)}}\right)$

which is $G_G m_{proton}^2 \approx 5 \times 10^{-39}$

at the characteristic energy level of $\approx 10^{19} GeV$, the Planck energy.

The only factor different from 1 in the force strength of Gravity is the Effective Mass factor, because force strengths in the HyperDiamond Feynman Checkerboard model are represented as ratios with respect to the strongest geometric force, Gravity, so that the geometric force strength factors for Gravity are cancelled to unity by taking the ratio with themselves.

For an estimate of the Planck mass calculated in the spirit of the HyperDiamond Feynman Checkerboard model, see

http://galaxy.cau.edu/tsmith/Planck.html

http://www.innerx.net/personal/tsmith/Planck.html

The action of Gravity on Spinors is given by the Papapetrou Equations.

8.3 Color Force.

For the Color force, $SU(3)$ gluons carry the Internal Symmetry Space color charges

$\{+r, -r; +g, -g; +b, -b\}$

$SU(3)$ gluons act transitively on the 4-dimensional manifold $\mathbf{C}P^2 = SU(3)/(SU(2) \times U(1))$, so that they can take any given point in the origin Internal Symmetry Space into any point in the target Internal Symmetry Space.

The full connectivity of the Color force of $SU(3)$ gluons is geometrically represented by M_{IS_C} as $\mathbf{C}P^2 = SU(3)/(SU(2) \times U(1))$ whose volume is $8\pi^2/3$.

The link manifold to the target vertex is

$Q_C = S^5 = ShilovBdy(B^6)$ with volume $4\pi^3$

The bounded complex homogeneous domain D_C is of type $B^6(ball)$ with volume $\pi^3/6$

For the $SU(3)$ Color force, m_C is 4.

For the $SU(3)$ Color force, $\mu_C = 1$, so that

for the $SU(3)$ Color force, $\frac{1}{\mu_C^2} = 1$

Therefore, the force strength of the Color force, which is sometimes conventionally denoted by α_S (for strong) as well as by α_C, is:

$$\alpha_C = \alpha_S = \left(\frac{1}{\mu_C^2}\right)(Vol(M_{IS_C}))\left(\frac{Vol(Q_C)}{Vol(D_C)^{\left(\frac{1}{m_C}\right)}}\right)$$

which, when divided by the geometric force strength of Gravity, is 0.6286

at the characteristic energy level of $\approx 245 MeV$.

Force strength constants and particle masses are not really "constant" when you measure them, as the result of your measurement will depend on the energy at which you measure them. Measurements at one energy level can be related to measurements at another by renormalization equations. The lightest experimentally observable particle based on the color force is the pion, which is a quark-antiquark pair made up of the lightest quarks, the up and down quarks. A quark-antiquark pair is the carrier of the strong force, and mathematically resembles a bivector gluon, which is the carrier of the color force. The charactereistic energy level of pions is the square root of the sum of the squares of the masses of the two charged and one neutral pion. It is about 245 MeV (to more accuracy 241.4 MeV). The gluon-carried color force strength is renormalized to higher energies from about 245 MeV in the conventional way.

The Color force strength was calculated at various energies according to renormalization group equations, as shown at

http://galaxy.cau.edu/tsmith/cweRen.html

http://www.innerx.net/personal/tsmith/cweRen.html

with the following results:

Energy Level	*Color Force Strength*
$245 MeV$	0.6286
$5.3 GeV$	0.166
$34 GeV$	0.121
$91 GeV$	0.106

(41)

Shifman [26], in a paper at

http://xxx.lan.gov/abs/hep-ph/9501222

has noted that Standard Model global fits at the Z_0 peak, about 91 GeV, give a color force strength of about 0.125 with $\Lambda_{QCD} \approx 500\ MeV$,

whereas low energy results and lattice calculations give a color force strength at the Z_0 peak of about 0.11 with $\Lambda_{QCD} \approx 200\ MeV$.

The low energy results and lattice calculations are closer to the tree level HyperDiamond Feynman Checkerboard model value at 91 GeV of 0.106.

Also, the HyperDiamond Feynman Checkerboard model has $\Lambda_{QCD} \approx 245\ MeV$

For the pion mass, upon which the Λ_{QCD} calculation depends, see

http://galaxy.cau.edu/tsmith/SnGdnPion.html

http://www.innerx.net/personal/tsmith/SnGdnPion.html

8.4 Weak Force.

For the Weak force, $SU(2)$ Weak bosons carry the Internal Symmetry Space electric charges

$\{+1, -1\}$

$SU(2)$ Weak bosons act transitively on the 2-dimensional manifold $S^2 = SU(2)/U(1)$, so that two copies of S^2, in the form of $S^2 \times S^2$, are required so that they can take any given point in the origin Internal Symmetry Space into any point in the target Internal Symmetry Space.

Each of the two connectivity components of the Weak force of $SU(2)$ Weak bosons is geometrically represented by M_{IS_W} as $S^2 = SU(2)/U(1)$ whose volume is 4π.

The total manifold M_{IS_W} is $S^2 \times S^2$, with volume $2 \times 4\pi$

The link manifold to the target vertex is

$Q_W = \mathbf{R}P^1 \times S^2 = ShilovBdy(D_W)$ with volume $4\pi^3$.

The bounded complex homogeneous domain D_W is of type IV_3 with volume $4\pi^2$.

It corresponds to the Hermitian Symmetric Space

$Spin(5)/(SU(2) \times U(1)$.

For the $SU(2)$ Weak force, m_W is 2.

For the $SU(2)$ Weak force, due to the Higgs mechanism,

$\mu_W = \sqrt{m_{W+}^2 + m_{W-}^2 + m_{W_0}^2}$,

so that

$\frac{1}{\mu_W^2} = \frac{1}{m_{W+}^2 + m_{W-}^2 + m_{W_0}^2}$.

Therefore, the force strength of the Weak force is:

$\alpha_W = \left(\frac{1}{\mu_W^2}\right)(Vol(M_{IS_W}))\left(\frac{Vol(Q_W)}{Vol(D_W)^{\left(\frac{1}{m_W}\right)}}\right)$

which, when divided by the geometric force strength of Gravity, is $G_W m_{proton}^2 \approx 1.05 \times 10^{-5}$

at the characteristic energy level of $\approx 100GeV$.

The geometric component of the Weak force strength, that is, everything but the effective mass factor, has the value 0.2535 when divided by the geometric force strength of Gravity,

8.5 Electromagnetism.

For Electromagnetixm, $U(1)$ photons carry no Internal Symmetry Space charges.

$U(1)$ Weak bosons act transitively on the 1-dimensional manifold $S^1 = U(1)$, so that four copies of S^1, in the form of the 4-torus T^4, are required so that they can take any given point in the origin Internal Symmetry Space into any point in the target Internal Symmetry Space.

Each of the four connectivity components of the Electromagnetism of $U(1)$ photons is geometrically represented by M_{IS_E} as $S^1 = U(1)$ whose volume is 2π.

The total manifold M_{IS_E} is $T^4 = S^1 \times S^1 \times S^1 \times S^1$, with volume $4 \times 2\pi$

The link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take Q_E and D_E to be equal to unity.

For $U(1)$ Electromagnmetism, m_E is 1.

For $U(1)$ Electromagnmetism, $\mu_W = 1$, so that

$\frac{1}{\mu_E^2} = 1$.

Therefore, the force strength of Electromagnetism is:

$$\alpha_E = \left(\frac{1}{\mu_E^2}\right)(Vol(M_{IS_E}))\left(\frac{Vol(Q_E)}{Vol(D_E)^{\left(\frac{1}{m_E}\right)}}\right)$$

which, when divided by the geometric force strength of Gravity, is $\alpha_E = 1/137.03608$

at the characteristic energy level of $\approx 4KeV$.

9 Mass = Amplitude to Change Direction.

In the HyperDiamond Feynman Checkerboard model the mass parameter m is the amplitude for a particle to change its spacetime direction. Massless particles do not change direction, but continue on the same lightcone path.

In the $D_4 - D_5 - E_6$ Lagrangian continuum version of this physics model, particle masses are calculated in terms of relative volumes of bounded complex homogeneous domains and their Shilov boundaries.

The relationship between the $D_4 - D_5 - E_6$ Lagrangian continuum approach and the HyperDiamond Feynman Checkerboard discrete approach is that:

the bounded complex homogeneous domains correspond

to harmonic functions of generalized Laplacians

that determine heat equations, or diffusion equations;

while the amplitude to change directions in the HyperDiamond Feynman Checkerboard is a diffusion process in the HyperDiamond spacetime, also corresponding to a generalized Laplacian.

Details of the $D_4 - D_5 - E_6$ Lagrangian continuum approach can be found on the World Wide Web at URLs

http://xxx.lanl.gov/abs/hep-ph/9501252

http://galaxy.cau.edu/tsmith/d4d5e6hist.html

For the discrete HyperDiamond Feynman Checkerboard approach of this paper, the only free mass parameter is the mass of the Higgs scalar. All other particle masses are determined as ratios with respect to the Higgs scalar and each other.

The Higgs mass is set equal to 259.031 GeV in the HyperDiamond Feynman Checkerboard model, a figure chosen so that the ratios will give an electron mass of 0.5110 MeV.

Effectively, in the HyperDiamond Feynman Checkerboard model, the electron mass is fixed at 0.5110 MeV

and all other masses are determined from it by the ratios calculated in the model.

9.1 Higgs Scalar, Gauge Bosons, and Fermions.

Recall from Chapter 4 that 16x16 = 256-dimensional $DCl(0,8)$ has structure

1 **8** **28** 56 70 56 28 8 1

that gives three types of particles for which mass ratios can be calculated in the HyperDiamond Feynman Checkerboard model:

the Higgs Scalar;

the 28 bivector gauge bosons;

and the 8 + 8 = 16 half-spinor fermions.

9.2 Higgs Scalar Mass.

There is only one Higgs scalar, and we have chosen its mass to be 259.031 GeV.

9.3 Gauge Boson Masses.

After dimensional reduction to 4-dimensional spacetime, the 28 $Spin(0,8)$ gauge bosons split into two groups:

12 Standard Model gauge bosons, which are

8 $SU(3)$ gluons,

3 $SU(2)$ weak bosons, and

1 $U(1)$ photon;

and

16 $U(4)$ gauge bosons,

which reduce to 15 $SU(4) = Spin(6)$ gauge bosons plus one $U(1)$ phase for particle propagator amplitudes (this phase is what makes the sum-over-histories quantum theory interferences work).

The 15 $SU(4) = Spin(6)$ gauge bosons further reduce to 10 $Spin(5)$ deSitter gravitons that give physical gravity by the MacDowell-Mansouri mechanism as described in

http://xxx.lanl.gov/abs/hep-ph/9501252

and 4 conformal generators and 1 scale generator.

The 4 conformal generators couple to the Higgs scalar so that it becomes the mass-giver by the Higgs mechanism as described in

http://xxx.lanl.gov/abs/hep-ph/9501252

and the 1 scale generator represents the scale that we chose by setting the Higgs scalar mass at 259.031 GeV.

All these gauge bosons are massless at very high energies, but at energies comparable to the Higgs mass and below, the Higgs scalar couples to the $SU(2)$ weak bosons to give them mass.

9.3.1 Weak Boson Masses.

Denote the 3 $SU(2)$ massless high-energy weak bosons by W_+, W_-, and W_0.

The triplet $\{W_+, W_-, W_0\}$ couples directly with the Higgs scalar, so that the total mass of the triplet $\{W_+, W_-, W_0\}$ is equal to the mass of the Higgs scalar, 259.031 GeV.

What are individual masses of members of the triplet $\{W_+, W_-, W_0\}$?

The entire triplet $\{W_+, W_-, W_0\}$ can be represented by the 3-sphere S^3.

The Hopf fibration of S^3 as $S^1 \rightarrow S^3 \rightarrow S^2$ gives a decomposition of the W bosons into the neutral W_0 corresponding to S^1 and the charged pair W_+ and W_- corresponding to S^2.

The mass ratio of the sum of the masses of W_+ and W_- to the mass of W_0 should be the volume ratio of the S^2 in S^3 to the S^1 in $S3$.

The unit sphere $S^3 \subset R^4$ is normalized by $1/2$.

The unit sphere $S^2 \subset R^3$ is normalized by $1/\sqrt{3}$.

The unit sphere $S^1 \subset R^2$ is normalized by $1/\sqrt{2}$.

The ratio of the sum of the W_+ and W_- masses to the W_0 mass should then be $(2/\sqrt{3})V(S^2)/(2/\sqrt{2})V(S^1) = 1.632993$.

Since the total mass of the triplet $\{W_+, W_-, W_0\}$ is 259.031 GeV, and the charged wek bosons have equal mass, we have

$m_{W_+} = m_{W_-} = 80.326\ GeV$, and $m_{W_0} = 98.379\ GeV$.

9.3.2 Parity Violation, Effective Masses, and Weinberg Angle.

The charged $W_\pm$ neutrino-electron interchange
must be symmetric with the electron-neutrino interchange,
so that the absence of right-handed neutrino particles requires
that the charged $W_\pm$ $SU(2)$ weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector, so that the charged $W_\pm$ $SU(2)$ weak bosons act only on left-handed fermions of all types.

The neutral W_0 weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral W_0 weak bosons are related to the charged $W_\pm$ weak bosons by custodial $SU(2)$ symmetry, so that the left-handed component of the neutral W_0 must be equal to the left-handed (entire) component of the charged $W_\pm$.

Since the mass of the W_0 is greater than the mass of the $W_\pm$, there remains for the W_0 a component acting on both types of fermions.

Therefore the full W_0 neutral weak boson interaction is proportional to $(m_{W_\pm}^2/m_{W_0}^2)$ acting on left-handed fermions and

$(1-(m_{W_\pm}^2/m_{W_0}^2))$ acting on both types of fermions.

If $(1-(m_{W_\pm}2/m_{W_0}^2))$ is defined to be $\sin\theta_w{}^2$ and denoted by ξ, and

if the strength of the $W_\pm$ charged weak force (and of the custodial $SU(2)$ symmetry) is denoted by T,

then the W_0 neutral weak interaction can be written as:

$W_{0L} \sim T+\xi$ and $W_{0LR} \sim \xi$.

Since the W_0 acts as W_{0L} with respect to the parity violating $SU(2)$ weak force and

as W_{0LR} with respect to the parity conserving $U(1)$ electromagnetic force of the $U(1)$ subgroup of $SU(2)$,

the W_0 mass m_{W_0} has two components:

the parity violating $SU(2)$ part $m_{W_{0L}}$ that is equal to $m_{W_\pm}$; and the parity conserving part $m_{W_{0LR}}$ that acts like a heavy photon.

As $m_{W_0} = 98.379$ GeV $= m_{W_{0L}} + m_{W_{0LR}}$, and

as $m_{W_{0L}} = m_{W_\pm} = 80.326\ GeV$,

we have $m_{W_{0LR}} = 18.053\ GeV$.

Denote by $\tilde{\alpha_E} = \tilde{e}^2$ the force strength of the weak parity conserving $U(1)$ electromagnetic type force that acts through the $U(1)$ subgroup of $SU(2)$.

The electromagnetic force strength $\alpha_E = e^2 = 1/137.03608$ was calculated in Chapter 8 using the volume $V(S^1)$ of an $S^1 \subset R^2$, normalized by $1/\sqrt{2}$.

The $\tilde{\alpha_E}$ force is part of the $SU(2)$ weak force whose strength $\alpha_w = w^2$ was calculated in Chapter 8 using the volume $V(S^2)$ of an $S^2 \subset R^3$, normalized by $1/\sqrt{3}$.

Also, the electromagnetic force strength $\alpha_E = e^2$ was calculated in Chapter 8 using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,

while the $SU(2)$ weak force strength $\alpha_w = w^2$ was calculated in Chapter 8 using two 2-spheres $S^2 \times S^2$, each of which contains one 1-sphere of the $\tilde{\alpha_E}$ force.

Therefore $\tilde{\alpha_E} = \alpha_E(\sqrt{2}/\sqrt{3})(2/4) = \alpha_E/\sqrt{6}$,

$\tilde{e} = e/\sqrt[4]{6} = e/1.565$, and

the mass $m_{W_{0LR}}$ must be reduced to an effective value

$m_{W_{0LReff}} = m_{W_{0LR}}/1.565 = 18.053/1.565 = 11.536$ GeV

for the $\tilde{\alpha_E}$ force to act like an electromagnetic force in the 4-dimensional spacetime HyperDiamond Feynman Checkerboard model:

$\tilde{e} m_{W_{0LR}} = e(1/5.65) m_{W_{0LR}} = e m_{Z_0}$, where the physical effective neutral weak boson is denoted by Z_0.

Therefore, the correct HyperDiamond Feynman Checkerboard values for weak boson masses and the Weinberg angle θ_w are:

$m_{W_+} = m_{W_-} = 80.326\ GeV$;

$m_{Z_0} = 80.326 + 11.536 = 91.862\ GeV$; and

$\sin\theta_w{}^2 = 1 - (m_{W_\pm}/m_{Z_0})^2 = 1 - (6452.2663/8438.6270) = 0.235.$

Radiative corrections are not taken into account here, and may change these tree-level HyperDiamond Feynman Checkerboard values somewhat.

9.4 Fermion Masses.

First generation fermion particles are represented by octonions as follows:

Octonion basis element	*First Generaton Fermion Particle*
1	*e − neutrino*
i	*red up quark*
j	*green up quark*
k	*blue up quark*
E	*electron*
I	*red down quark*
J	*green down quark*
K	*blue down quark*

(42)

First generation fermion antiparticles are represented by octonions in a similiar way.

Second generation fermion particles and antiparticles are represented by pairs of octonions.

Third generation fermion particles and antiparticles are represented by triples of octonions.

In the HyperDiamond Feynman Checkerboard model, there are no higher generations of fermions than the Third.

This can be seen algebraically as a consequence of the fact that the Lie algebra series E_6, E_7, and E_8, has only 3 algebras, which in turn is a consequence of non-associativity of octonions, as described in

http://galaxy.cau.edu/tsmith/E678.html

http://www.innerx.net/personal/tsmith/E678.html

or geometrically as a consequence of the fact that,

if you reduce the original 8-dimensional spacetime

into associative 4-dimensional physical spacetime

and coassociative 4-dimensional Internal Symmetry Space,

then, if you look in the original 8-dimensional spacetime at a fermion (First-generation represented by a single octonion) propagating from one vertex to another, there are only 4 possibilities for the same propagation after dimensional reduction:

1 - the origin and target vertices are both in the associative 4-dimensional physical spacetime, in which case the propagation is unchanged, and the fermion remains a FIRST generation fermion represented by a single octonion;

2 - the origin vertex is in the associative spacetime, and the target vertex in in the Internal Symmetry Space, in which case there must be a new link from the original target vertex in the Internal Symmetry Space to a new target vertex in the associative spacetime, and a second octonion can be introduced at the original target vertex in connection with the new link, so that the fermion can be regarded after dimensional reduction as a pair of

octonions, and therefore as a SECOND generation fermion;

3 - the target vertex is in the associative spacetime, and the origin vertex in in the Internal Symmetry Space, in which case there must be a new link to the original origin vertex in the Internal Symmetry Space from a new origin vertex in the associative spacetime, so that a second octonion can be introduced at the original origin vertex in connection with the new link, so that the fermion can be regarded after dimensional reduction as a pair of octonions, and therefore as a SECOND generation fermion; and

4 - both the origin vertex and the target vertex are in the Internal Symmetry Space, in which case there must be a new link to the original origin vertex in the Internal Symmetry Space from a new origin vertex in the associative spacetime, and a second new link from the original target vertex in the Internal Symmetry Space to a new target vertex in the associative spacetime, so that a second octonion can be introduced at the original origin vertex in connection with the first new link, and a third octonion can be introduced at the original target vertex in connection with the second new link, so that the fermion can be regarded after dimensional reduction as a triple of octonions, and therefore as a THIRD generation fermion.

As there are no more possibilities, there are no more generations.

9.4.1 Renormalization.

Particle masses and force strength constants are not really "constant" when you measure them, as the result of your measurement will depend on the energy at which you measure them. Measurements at one energy level can be related to measurements at another by renormalization equations.

The particle masses calculated in the $D_4 - D_5 - E_6$ model are, with respect to renormalization, each defined at the energy level of the calculated particle mass.

For leptons, such as the electron, muon, and tauon, which carry no color charge, you can renormalize conventionally from that energy level to "translate" the result to another energy level, because those particles are not "confined" and so can be experimentally observed as "free particles" ("free" means "not strongly bound to other particles, except for virtual particles of the active vacuum of spacetime").

For quarks, which are confined and cannot be experimentally observed as free particles, the situation is more complicated. In the $D_4 - D_5 - E_6$ model, the calculated quark masses are considered to be constituent masses.

In hep-ph/9802425, Di Qing, Xiang-Song Chen, and Fan Wang, of Nanjing University, present a qualitative QCD analysis and a quantitative model calculation to show that the constituent quark model [after mixing a small amount (15remains a good approximation even taking into account the nucleon spin structure revealed in polarized deep inelastic scattering.

The effectiveness of the NonRelativistic model of light-quark hadrons is explained by, and affords experimental Support for, the Quantum Theory of David Bohm (see quant-ph/9806023).

Consitituent particles are Pre-Quantum particles in the

sense that their properties are calculated without

using sum-over-histories Many-Worlds quantum theory.

(”Classical” is a commonly-used synonym for ”Pre-Quantum”.)

Since experiments are quantum sum-over-histories processes, experimentally observed particles are Quantum particles.

The lightest experimentally observable particle containing quarks is the pion, which is a quark-antiquark pair made up of the lightest quarks, the up and down quarks.

A quark-antiquark pair is the carrier of the strong force, and mathematically resembles a bivector gluon, which is the carrier of the color force.

The charactereistic energy level of pions is the square root of the sum of the squares of the masses of the two charged and one neutral pion. It is about 245 MeV (to more accuracy 241.4 MeV).

The gluon-carried color force strength is renormalized to higher energies from about 245 MeV in the conventional way.

What about quarks, as opposed to gluons?

Gluons are represented by quark-antiquark pairs, but a quark is a single quark.

The lightest particle containing a quark that is not coupled to an antiquark is the proton, which is a stable (except with respect to quantum gravity) 3-quark color neutral particle.

The characteristic energy level of the proton is about 1 GeV (to more accuracy 938.27 MeV).

Quark masses are renormalized to higher energies from about 1 GeV (or from their calculated mass, below which they do not exist except virtually) in the conventional way.

What about the 3 quarks (up, down, and strange) that have constituent masses less than 1 GeV?

Below 1 GeV, they can only exist (if not bound to an antiquark) within a proton, so their masses are ”flat”, or do not ”run”, in the energy range below 1 GeV.

Since the 3 quarks, up, down, and strange, are the only ones lighter than a proton, they can be used as the basis for a useful low-energy theory, Chiral Perturbation Theory, that uses the group $SU(3) \times SU(3)$, or, if based only on the lighter up and down quarks that uses the group $SU(2) \times SU(2)$.

A useful theory at high energies, much above 1 GeV, is Perturbative QCD, that treats the quarks and gluons as free, which they are asymptotically as energies become very high.

9.4.2 Perturbative QCD, Chiral PT, Lattice GT.

To do calculations in theories such as Perturbative QCD and Chiral Perturbation Theory, you need to use effective quark masses that are called current masses. Current quark masses are different from the Pre-Quantum constituent quark masses of our model.

The current mass of a quark is defined in our model as the difference between the constituent mass of the quark and the density of the lowest-energy sea of virtual gluons, quarks, and antiquarks, or 312.75 MeV.

Since the virtual sea is a quantum phenomenon, the current quarks of Perturbative QCD and Chiral Perturbation Theory are, in my view, Quantum particles.

The relation between current masses and constituent masses may be explained, at least in part, by the Quantum Theory of David Bohm.

Therefore, our model is unconventional in that:

a current quark is viewed as a composite combination of a fundamental constituent quark and Quantum virtual sea gluon, quarks, and antiquarks (compare the conventional picture of, for example, hep-ph/9708262, in which current quarks are Pre-Quantum and constituent quarks are Quantum composites); and the input current quarks of Perturbative QCD and Chiral Perturbation Theory are Quantum, and not Pre-Quantum, so that we view Perturbative QCD and Chiral Perturbation Theory as effectively "second-order" Quantum theories (rather than fundamental theories) that are most useful in describing phenomena at high and low energy levels, respectively.

Therefore, in Perturbative QCD and Chiral Perturbation Theory, the up and down quarks roughly massless. One result is that the current masses can then be used as input for the $SU(3) \times SU(3)$ Chiral Perturbation Theory that, although it is only approximate because the constituent mass of the strange quark is about 312 MeV, rather than nearly zero, can be useful in calculating meson properties.

WHAT ARE THE REGIONS OF VALIDITY OF PERTURBATIVE QCD and CHIRAL PERTURBATION THEORY?

Perturbative QCD is useful at high energies. If Perturbative QCD is valid at energies above 4.5 GeV, then Yndurian in hep-ph/9708300 has shown that lower bounds for current quark masses are:

m_s at least 150 MeV (compare $D_4 - D_5 - E_6$ 312 MeV)

$m_u + m_d$ at least 10 MeV (compare $D_4 - D_5 - E_6$ 6.5 MeV)

$m_u - m_d$ at least 2 MeV (compare $D_4 - D_5 - E_6$ 2.1 MeV)

Yndurian bases his estimates on positivity, and uses $\bar{M}S$ masses defined at $1GeV^2$.

Chiral Perturbation Theory is useful at low energies. Lellouch, Rafael, and Taron in hep-ph/9707523 have shown roughly similar lower bounds using Chiral Perturbation Theory.

Lattice Gauge Theory calculations of Gough et al in hep-ph/9610223 give light quark current $\bar{M}S$ masses at $1GeV^2$ as:

m_s 59 to 101 MeV

$m_u + m_d$ 4.6 to 7.8 MeV

Clearly, the strange quark current masses of Lattice Gauge Theory are a lot lighter than those calculated by Perturbative QCD and Chiral Perturbation Theory, as well as the $D_4 - D_5 - E_6$l.

Further, recent observation of the decay of K+ to pion+ and a muon-antimuon pair gives a branching ratio with respect to the decay of K+ to pion+ and e+ e- that is 0.167, which is about 2 sigma below the Chiral Perturbation Theory prediction of 0.236.

These facts indicate that Perturbative QCD, Chiral Perturbation Theory, and Lattice Gauge Theory are approximations to fundamental theory, each useful in some energy regions, but not fully understood in all energy regions.

Conventional Lattice Gauge Theory for fermions, such as quarks, has some fundamental problems:

The conventional lattice Dirac operator is afflicted with the Fermion Doubling Problem, in which nearest-neighbor lattice sites are occupied with (for 4-dim spacetime) $2^4 = 16$ times too many fermions;.

The conventional solutions to the Fermion Doubling Problem are to add non-local terms that violate Chiral Symmetry on the lattice. If you are trying to do Chiral Perturbation Theory on the lattice, that seems to be a bad idea.

To solve the Fermion Doubling Problem without violating Chiral Symmetry, Bo Feng, Jianming Li, and Xingchang Song have proposed to modify the conventional lattice Dirac operator by adding a non-local term that (like an earlier approach of Drell et. al., Phys. Rev. D 14 (1976) 1627) couples all lattice sites along a given direction instead of coupling only nearest-neighbor sites. Their modified lattice Dirac operator not only preserved Chiral Symmetry, it also gives the conventional D'Alembertian operator, and they are able to construct the Weinberg-Salam Electro-Weak model on a lattice.

Conventional Lattice Gauge Theory is formulated somewhat differently from another approach to formulating physics models on lattices: Feynman Checkerboards,

9.4.3 Truth Quark Mass.

In the HyperDiamond Feynman Checkerboard model, the Higgs scalar couples directly with the particle-antiparticle pair made of femions with the most charge in the highest generation.

That means Third Generation fermions made of triples of octonions,

carrying both color and electric charge, and therefore quarks rather than leptons,

and carrying electric charge of magnitude 2/3 rather than 1/3, and therefore:

the Higgs scalar couples most strongly with the Truth quark, whose tree-level constituent mass of 129.5155 GeV is somewhat lower than, but close to, the Higgs scalar mass of about 146 GeV.

The HyperDiamond Feynman Checkerboard model value of about 130 GeV is substantially different from the roughly 175 GeV figure advocated by FermiLab.

I think that the FermiLab figure is incorrect.

The Fermilab figure is based on analysis of semileptonic events. We think that the Fermilab semileptonic analysis does not handle background correctly, and ignores signals in the data that are in rough agreement with our tree level constituent mass of about 130 GeV.

Further, I think that dileptonic events are more reliable for Truth quark mass determination, even though there are fewer of them than semileptonic events.

I disagree with the Fermilab D0 analysis of dileptonic events, which Fermilab says are in the range of 168.3 GeV.

My analysis of those dileptonic events gives a Truth quark mass of about 136.7 GeV, in rough agreement with the D4-D5-E6 model tree level Truth quark constituent mass of about 130 GeV.

More details about these issues, including gif images of Fermilab data histograms and other relevant experimental results, can be found on the World Wide Web at URLs

http://galaxy.cau.edu/tsmith/TCZ.html

http://www.innerx.net/personal/tsmith/TCZ.html

I consider the mass of the Truth quark to be a good test of our theory, as our theory can be falsified if we turn out to be wrong in our interpretation of experimental results.

9.4.4 Other Fermion Masses.

In the HyperDiamond Feynman Checkerboard model, the masses of the other fermions are calculated from the mass of the Truth quark, with the following results for individual tree-level lepton masses and quark constituent masses:
$m_e = 0.5110$ MeV;
$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0$;
$m_d = m_u = 312.8$ MeV (constituent quark mass);
$m_\mu = 104.8$ MeV;
$m_s = 625$ MeV (constituent quark mass);
$m_c = 2.09$ GeV (constituent quark mass);
$m_\tau = 1.88$ GeV;
$m_b = 5.63$ GeV (constituent quark mass);
These results when added up give a total mass of first generation fermions:

$$\Sigma_{f_3} = 7.508\ GeV \tag{43}$$

Here is how the individual fermion mass calculations are done:

The Weyl fermion neutrino has at tree level only the left-handed state, whereas the Dirac fermion electron and quarks can have both left-handed and right-handed states, so that the total number of states corresponding to each of the half-spinor $Spin(0,8)$ representations is 15.

In all generations, neutrinos are massless at tree level. However, even though massless at tree level, neutrinos are spinors and therefore are acted upon by Gravity as shown by the Papapetrou Equations.

Further, in Quantum Field Theory at Finite Temperature, the gravitational equivalence principle may be violated, causing mixing among neutrinos of different generations.

In the HyperDiamond Feynman Checkerboard model, the first generation fermions correspond to octonions $\mathbf{O}$, while second generation fermions correspond to pairs of octonions $\mathbf{O} \times \mathbf{O}$ and third generation fermions correspond to triples of octonions $\mathbf{O} \times \mathbf{O} \times \mathbf{O}$.

To calculate the fermion masses in the model, the volume of a compact manifold representing the spinor fermions S_{8+} is used. It is the parallelizable manifold $S^7 \times RP^1$.

Also, since gravitation is coupled to mass, the infinitesimal generators of the MacDowell-Mansouri gravitation group, $Spin(0,5)$, are relevant.

The calculated quark masses are constituent masses, not current masses.

In the HyperDiamond Feynman Checkerboard model, fermion masses are calculated as a product of four factors:

$$V(Q_{fermion}) \times N(Graviton) \times N(octonion) \times Sym \tag{44}$$

$V(Q_{fermion})$ is the volume of the part of the half-spinor
fermion particle manifold $S^7 \times RP^1$ that is
related to the fermion particle by photon, weak boson, and gluon interactions.

$N(Graviton)$ is the number of types of $Spin(0,5)$ graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of $Spin(0,5) = Sp(2)$.
2 of them are in the Cartan subalgebra.
6 of them carry color charge, and may therefore be considered as corresponding to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons.

One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino.
Therefore only one graviton should correspond to the mass of the first-generation electron.
The graviton number ratio of the down quark to the first-generation electron is therefore $6/1 = 6$.

$N(octonion)$ is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions.
It is not used in first-generation calculations.

The ratio of the down quark constituent mass to the electron mass is then calculated as follows:
Consider the electron, e.
By photon, weak boson, and gluon interactions, e can only be taken into 1, the massless neutrino.
The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.
The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.
Since the electron cannot be related to any other massive Dirac fermion, its volume $V(Q_{electron})$ is taken to be 1.

Next consider a red down quark I.

By gluon interactions, I can be taken into J and K, the blue and green down quarks.

By also using weak boson interactions, it can be taken into i, j, and k, the red, blue, and green up quarks.

Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos.

Therefore the red down quark (similarly, any down quark) is related to any part of $S^7 \times \mathbf{R}P^1$, the compact manifold corresponding to

$$\{1, i, j, k, I, J, K, E\}$$

and therefore a down quark should have a spinor manifold volume factor $V(Q_{downquark}$ of the volume of $S^7 \times \mathbf{R}P^1$.

The ratio of the down quark spinor manifold volume factor to the electron spinor manifold volume factor is just

$$V(Q_{downquark})/V(Q_{electron}) = V(S^7 \times \mathbf{R}P^1)/1 = \pi^5/3. \quad (45)$$

Since the first generation graviton factor is 6,

$$md/me = 6V(S^7 \times \mathbf{R}P^1) = 2\pi^5 = 612.03937 \quad (46)$$

As the up quarks correspond to i, j, and k, which are the octonion transforms under E of I, J, and K of the down quarks, the up quarks and down quarks have the same constituent mass $m_u = m_d$.

Antiparticles have the same mass as the corresponding particles.

Since the model only gives ratios of massses, the mass scale is fixed by assuming that the electron mass $m_e = 0.5110$ MeV.

Then, the constituent mass of the down quark is $m_d = 312.75$ MeV, and the constituent mass for the up quark is $m_u = 312.75$ MeV.

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

$$m_{proton} = m_u + m_u + m_d = 938.25\ MeV \quad (47)$$

The $D_4 - D_5 - E_6$ model calculation is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions. There are $8^3 = 512$ such triples.

The triple $\{1, 1, 1\}$ corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

$$\{E, E, E\}, \{E, E, 1\}, \{E, 1, E\}, \{1, E, E\}, \{1, 1, E\}, \{1, E, 1\}, \{E, 1, 1\}$$

,

The symmetry of the 7 tauon triples is the same as the symmetry of the 3 down quarks, the 3 up quarks, and the electron, so the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass 1.87704 GeV.

The calculated Tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the Tauon mass is about 2 GeV, the effective Tauon mass should be renormalized from the energy level of 1 GeV (where the mass is 1.88 GeV) to the energy level of 2 GeV.

Such a renormalization should reduce the mass. If the renormalization reduction were about 5 percent, the effective Tauon mass at 2 GeV would be about 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a Tauon mass of 1.777 GeV.

Note that all triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples, but for 1 and I, 1 and J, and 1 and$K$, which correspond to the red, green, and blue beauty quarks, respectively.

The seven triples of the red beauty quark correspond to the seven triples of the tauon, except that the beauty quark interacts with 6 $Spin(0,5)$ gravitons while the tauon interacts with only two.

The beauty quark constituent mass should be the tauon mass times the third generation graviton factor $6/2 = 3$, so the B-quark mass is
$m_b = 5.63111$ GeV.

The calculated Beauty Quark mass of 5.63 GeV is a consitituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Beauty Quark mass of 5.63 GeV corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a lattice gauge theory Beauty Quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant $alpha_s$ is known. The conventional value of $alpha_s$ at about 5 GeV is about 0.22. Using $alpha_{s(5GeV)} = 0.22$, a pole mass of 5.0 GeV gives an MSbar 1-loop mass of 4.6 GeV, and an MSbar 1,2-loop mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV, the MSbar mass decreases by about 1.3 GeV, giving an expected MSbar mass of about 3.0 GeV at 90 GeV. DELPHI at LEP has observed the Beauty Quark and found a 90 GeV MSbar mass of about 2.67 GeV, with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

Note that the D4-D5-E6 model calculated mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV.

However, the D4-D5-E6 model calculated value of the color force strength constant $alpha_s$ at about 5 GeV is about 0.166,

while the conventional value of the color force strength constant $alpha_s$ at about 5 GeV is about 0.216, and the D4-D5-E6 model calculated value of the color force strength constant $alpha_s$ at about 90 GeV is about 0.106,

while the conventional value of the color force strength constant $alpha_s$ at about 90 GeV is about 0.118.

The D4-D5-E6 model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV),

and a color force strength $alpha_s$ at 5 GeV (0.166) such that 1 + $alpha_s$ = 1.166 is about 4 percent lower than the conventional value of 1 + $alpha_s$ = 1.216 at 5 GeV.

Note particularly that triples of the type $\{1, I, J\}$, $\{I, J, K\}$, etc., do not correspond to the beauty quark, but to the truth quark.

The truth quark corresponds to the remaining 483 triples, so the constituent mass of the red truth quark is 161/7 = 23 times the red beauty quark mass, and the red T-quark mass is

$$m_t = 129.5155\ GeV \tag{48}$$

The blue and green truth quarks are defined similarly.

The tree level T-quark constituent mass gives a Truth quark-antiquark mass of 259.031 GeV.

The tree level T-quark constituent mass rounds off to 130 GeV.

These results when added up give a total mass of third generation fermions:

$$\Sigma_{f_3} = 1,629\ GeV \tag{49}$$

The second generation fermion calculations are:

The second generation fermion particles correspond to pairs of octonions. There are 82 = 64 such pairs.
The pair $\{1, 1\}$ corresponds to the μ-neutrino.
the pairs $\{1, E\}$, $\{E, 1\}$, and $\{E, E\}$ correspond to the muon.
Compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles.
The pair $\{E, E\}$ should correspond to the E electron.
The other two muon pairs have a symmetry group S2, which is 1/3 the size of the color symmetry group S3 which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of the $\{E, E\}$ electron mass and the $\{1, E\}$, $\{E, 1\}$ symmetry mass, which is 1/3 of the up or down quark mass.

Therefore, $m_\mu = 104.76$ MeV.

Note that all pairs corresponding to the muon and the μ-neutrino are colorless.

The red, blue and green strange quark each corresponds to the 3 pairs involving 1 and I, J, or K.

The red strange quark is defined as the three pairs 1 and I, because I is the red down quark.
Its mass should be the sum of two parts: the $\{I, I\}$ red down quark mass, 312.75 MeV, and the product of the symmetry part of the muon mass, 104.25 MeV, times the graviton factor.

Unlike the first generation situation, massive second and third generation leptons can be taken, by both of the colorless gravitons that may carry electric charge, into massive particles.

Therefore the graviton factor for the second and third generations is $6/2 = 3$.

Therefore the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and the red strange quark constituent mass is

$$m_s = 312.75\ MeV + 312.75\ MeV = 625.5\ MeV$$

The blue strange quarks correspond to the three pairs involving J, the green strange quarks correspond to the three pairs involving K, and their masses are determined similarly.

The charm quark corresponds to the other 51 pairs. Therefore, the mass of the red charm quark should be the sum of two parts:

the $\{i, i\}$, red up quark mass, 312.75 MeV; and

the product of the symmetry part of the strange quark mass, 312.75 MeV, and

the charm to strange octonion number factor 51/9, which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is

$$m_c = 312.75\ MeV + 1,772.25\ MeV = 2.085\ GeV$$

The blue and green charm quarks are defined similarly, and their masses are calculated similarly.

The calculated Charm Quark mass of 2.09 GeV is a constituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant $alpha_s$ is known.

The conventional value of $alpha_s$ at about 2 GeV is about 0.39, which is somewhat lower than the D4-D5-E6 model value.

Using $alpha_{s(2GeV)} = 0.39$, a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

These results when added up give a total mass of second generation fermions:

$$\Sigma_{f_2} = 32.9 \; GeV \tag{50}$$

9.5 K-M Parameters.

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

$$phase\ angle\ \epsilon = \pi/2 \tag{51}$$

$$\sin\alpha = [m_e + 3m_d + 3m_u]/\sqrt{[m_e^2 + 3m_d^2 + 3m_u^2] + [m_\mu^2 + 3m_s^2 + 3m_c^2]} \tag{52}$$

$$\sin\beta = [m_e + 3m_d + 3m_u]/\sqrt{[m_e^2 + 3m_d^2 + 3m_u^2] + [m_\tau^2 + 3m_b^2 + 3m_t^2]} \tag{53}$$

$$\sin\tilde{\gamma} = [m_\mu + 3m_s + 3m_c]/\sqrt{[m_\tau^2 + 3m_b^2 + 3m_t^2] + [m_\mu^2 + 3m_s^2 + 3m_c^2]} \tag{54}$$

$$\sin\gamma = \sin\tilde{\gamma}\sqrt{\Sigma_{f_2}/\Sigma_{f_1}} \tag{55}$$

The resulting Kobayashi-Maskawa parameters are:

	d	s	b
u	0.975	0.222	$-0.00461i$
c	$-0.222 - 0.000191i$	$0.974 - 0.0000434i$	0.0423
t	$0.00941 - 0.00449i$	$-0.0413 - 0.00102i$	0.999

(56)

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix can be labelled either by

(u c t) input and (u'c't') output,

or by

(d s b) input and (d's'b') output:

Since neutrinos of all three generations are massless, the lepton sector has no K-M mixing.

10 Protons, Pions, and Physical Gravitons.

In his 1994 Georgia Tech Ph. D. thesis under David Finkelstein, *Spacetime as a Quantum Graph*, Michael Gibbs [12] describes some 4-dimensional Hyper-Diamond lattice structures, that he considers likely candidates to represent physical pcrticles.

The terminology used by Michael Gibbs in his thesis [12] is useful with respect to the model he constructs. Since his model is substantially different from my HyperDiamond Feynman Checkerboard in some respects, I use a different terminology here. However, I want to make it clear that I have borrowed these particular structures from his thesis.

Three useful HyperDiamond structures are:

3-link Rotating Propagator, useful for building a proton out of 3 quarks;

2-link Exchange Propagator, useful for building a pion out of a quark and an antiquark; and

4-link Propagator, useful for building a physical spin-2 physical graviton out of $Spin(5)$ Gauge bosons..

In the 2-dimensional Feynman Checkerboard, there is only one massive particle, the electron.

What about the $D_4-D_5-E_6$ model, or any other model that has different particles with different masses?

In the context of Feynman Checkerboards, mass is just the amplitude for a particle to have a change of direction in its path.

More massive particles will change direction more often.

In the $D_4 - D_5 - E_6$ model, the HyperDiamond Feynman Checkerboard fundamental path segment length ϵ of any particle the Planck length L_{PL}.

However, in the sum over paths for a particle of mass m,
it is a useful approximation to consider the path segment length to be the

Compton wavelength L_m of the mass m,

$$L_m = h/mc$$

That is because the distances between direction changes in the vast bulk of the paths will be at least L_m, and
those distances will be approximately integral multiples of L_m,
so that L_m can be used as the effective path segment length.

This is an important approximation because the Planck length L_{PL} is about 10^{-33} cm,
while the effective length L_{100GeV} for a particle of mass 100 GeV is about 10^{-16} cm.

In this section, the HyperDiamond lattice is given quaternionic coordinates.
The orgin 0 designates the beginning of the path.
The 4 future lightcone links from the origin are given the coordinates
$1+i+j+k$, $1+i-j-k$, $1-i+j-k$, $1-i-j+k$

The path of a "Particle" at rest in space, moving 7 steps in time, is denoted by

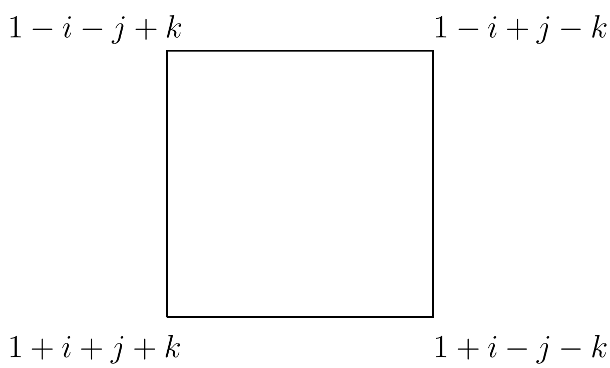
$Particle$
0
1
2
3
4
5
6
7

Note that since the HyperDiamond speed of light is $\sqrt{3}$, the path length is $7\sqrt{3}$.

The path of a "Particle" moving along a lightcone path in the $1+i+j+k$ direction for 7 steps with no change of direction is

$Particle$
0
$1+i+j+k$
$2+2i+2j+2k$
$3+3i+3j+3k$
$4+4i+4j+4k$
$5+5i+5j+5k$
$6+6i+6j+6k$
$7+7i+7j+7k$

At each step in either path, the future lightcone can be represented by a "Square Diagram" of lines connecting the future ends of the 4 future lightcone links leading from the vertex at which the step begins.



In the following subsections, protons, pions, and physical gravitons will be represented by multiparticle paths.
The multiple particles representing protons, pions, and physical gravitons will be shown on sequences of such Square Diagrams, as well as by a sequence of coordinates.

The coordinate sequences will be given only for a representative sequence of timelike steps, with no space movement, because
the notation for a timelike sequence is clearer and
it is easy to transform a sequence of timelike steps into a sequence of lightcone link steps, as shown above.

Only in the case of gravitons will it be useful to explicitly discuss a path that moves in space as well as time.

10.1 3-Quark Protons.

Since particle masses can only be observed experimentally for particles that can exist in a free state (”free” means ”not strongly bound to other particles, except for virtual particles of the active vacuum of spacetime”), and since quarks do not exist in free states, the quark masses that we calculate are interpreted as constituent masses (not current masses).

The relation between current masses and constituent masses may be explained, at least in part, by the Quantum Theory of David Bohm.

In hep-ph/9802425, Di Qing, Xiang-Song Chen, and Fan Wang, of Nanjing University, present a qualitative QCD analysis and a quantitative model calculation to show that the constituent quark model [after mixing a small amount (15remains a good approximation even taking into account the nucleon spin structure revealed in polarized deep inelastic scattering.

The effectiveness of the NonRelativistic model of light-quark hadrons is explained by, and affords experimental Support for, the Quantum Theory of David Bohm (see quant-ph/9806023).

Consitituent particles are Pre-Quantum particles

in the sense that their properties are calculated without
using sum-over-histories Many-Worlds quantum theory.

(”Classical” is a commonly-used synonym for ”Pre-Quantum”.)

Since experiments are quantum sum-over-histories processes, experimentally observed particles are Quantum particles.

Consider the experimentally observed proton. A proton is a Quantum particle containing 3 constituent quarks: two up quarks and one down quark; one Red, one Green, and one Blue. The 3 Pre-Quantum constituent quarks are called ”valence” quarks. They are bound to each other by $SU(3)$ QCD. The constituent quarks ”feel” the effects of QCD by ”sharing” virtual gluons and virtual quark-antiquark pairs that come from the vacuum in sum-over-histories quantum theory.

Since the 3 valence constituent quarks within the proton are constantly surrounded by the shared virtual gluons and virtual quark-antiquark pairs, the 3 valence constituent quarks can be said to "swim" in a "sea" of virtual gluons and quark-antiquark pairs, which are called "sea" gluons, quarks, and antiquarks.

In the model, the proton is the most stable bound state of 3 quarks, so that the virtual sea within the proton is at the lowest energy level that is experimentally observable.

The virtual sea gluons are massless $SU(3)$ gauge bosons. Since the lightest quarks are up and down quarks, the virtual sea quark-antiquark pairs that most often appear from the vacuum are up or down pairs, each of which have the same constituent mass, 312.75 MeV. If you stay below the threshold energy of the strange quark, whose constituent mass is about 625 MeV, the low energy sea within the proton contains only the lightest (up and down) sea quarks and antiquarks, so that the Quantum proton lowest-energy background sea has a density of 312.75 MeV. (In the model, "density" is mass/energy per unit volume, where the unit volume is Planck-length in size.)

Experiments that observe the proton as a whole do not "see" the proton's internal virtual sea, because the paths of the virtual sea gluon, quarks, and antiquarks begin and end within the proton itself. Therefore, the experimentally observed mass of the proton is the sum of the 3 valence quarks, 3 x 32.75 MeV, or 938.25 MeV which is very close to the experimental value of about 938.27 MeV.

To study the internal structure of hadrons, mesons, etc., you should use sum-over-histories quantum theory of the $SU(3)$ color force $SU(3)$. Since that is computationally very difficult

(For instance, in my view, the internal structure of a proton looks like a nonperturbative QCD soliton.

See WWW URLs http://galaxy.cau.edu/tsmith/SolProton.html

http://www.innerx.net/personal/tsmith/SolProton.html) you can use approximate theories that correspond to your experimental energy range.

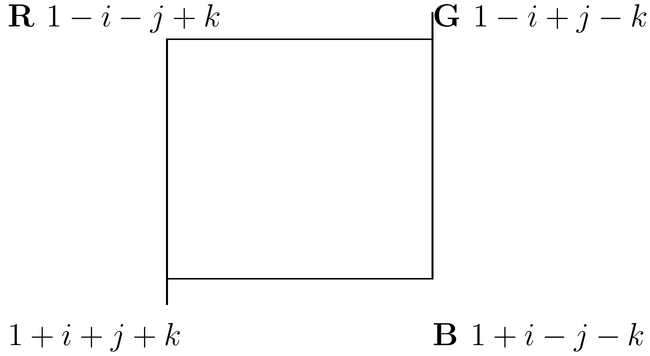
For high energy experiments, such as Deep Inelastic Scattering, you can use Perturbative QCD. For low energies, you can use Chiral Perturbation Theory. Renormalization equations are conventionally used to relate experimental observations that are made at different energy levels.

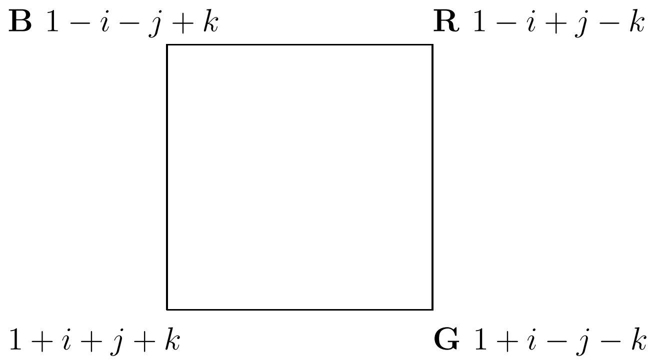
The HyperDiamond structure used to approximate the proton is the 3-link Rotating Propagator, in which 3 quarks orbit their center somewhat like the 3 balls of an Argentine bola.

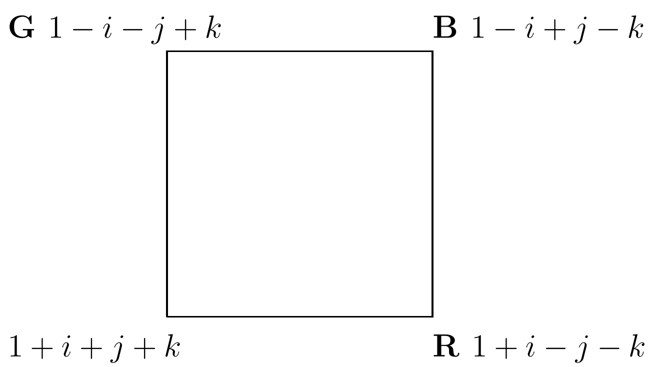
Here is a coordinate sequence representation of the approximate HyperDiamond Feynman Checkerboard path of a proton:

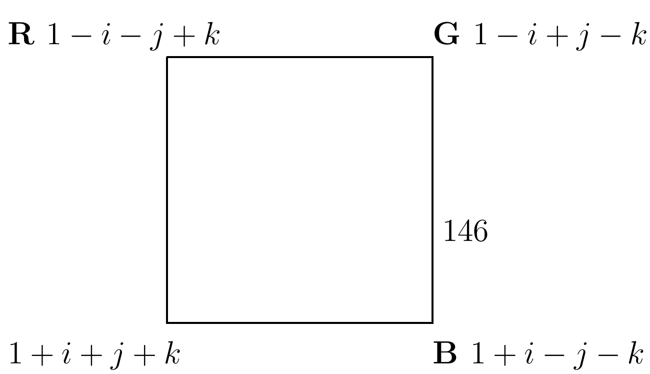
$R-Quark$	$G-Quark$	$B-Quark$
$1+i-j-k$	$1-i+j-k$	$1-i-j+k$
$2-i+j-k$	$2-i-j+k$	$2+i-j-k$
$3-i-j+k$	$3+i-j-k$	$2-i+j-k$
$4+i-j-k$	$4-i+j-k$	$4-i-j+k$

The following page contains a Square Diagram representation of the approximate HyperDiamond Feynman Checkerboard path of a proton at times 1,2, 3, and 4:


R $1-i-j+k$
G $1-i+j-k$
$1+i+j+k$
B $1+i-j-k$


B $1-i-j+k$
R $1-i+j-k$
$1+i+j+k$
G $1+i-j-k$


G $1-i-j+k$
B $1-i+j-k$
$1+i+j+k$
R $1+i-j-k$


R $1-i-j+k$
G $1-i+j-k$
$1+i+j+k$
B $1+i-j-k$

In the HyperDiamond Feynman Checkerboard model, where the proton is represented by two up quarks and one down quark, and quark masses are constituent masses:

the spins of the quarks should be in the lowest energy state, with one spin anti-parallel to the other two, so that the spin of the proton is

$$+1/2 + 1/2 - 1/2 = +1/2$$

the color charge of the proton is

$$+red + blue + green = 0$$

so that the pion is color-neutral;

the electric charge of the proton is

$$+2/3 + 2/3 - (-1/3) = +1$$

the theoretical tree-level mass is

$$3x312.75MeV = 938.25MeV$$

while the experimental mass is 938.27 MeV;

the proton is stable with respect to decay by the color, weak, and electromagnetic forces, while decay by the gravitational force is so slow that it cannot be observed with present technology.

10.1.1 Neutron Mass.

WHAT ABOUT THE NEUTRON MASS?

According to the 1986 CODATA Bulletin No. 63, the experimental value of the neutron mass is 939.56563(28) Mev, and the experimental value of the proton is 938.27231(28) Mev.

The neutron-proton mass difference 1.3 Mev is due to the fact that the proton consists of two up quarks and one down quark, while the neutron consists of one up quark and two down quarks.

The magnitude of the electromagnetic energy difference $m_N - m_P$ is about 1 Mev,

but the sign is wrong:

$$m_N - m_P = -1 Mev$$

and the proton's electromagnetic mass is greater than the neutron's.

The difference in energy between the bound states, neutron and proton, is not due to a difference between the Pre-Quantum constituent masses of the up quark and the down quark, calculated in the theory to be equal.

It is due to the difference between the Quantum color force interactions of the up and down constituent valence quarks with the gluons and virtual sea quarks in the neutron and the proton.

An up valence quark, constituent mass 313 Mev, does not often swap places with a 2.09 Gev charm sea quark, but a 313 Mev down valence quark can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force constituent mass of the down valence quark is heavier by about

$$(m_s - m_d)(m_d/m_s)^2 a(w) V_{12} = 312 \times 0.25 \times 0.253 \times 0.22 Mev = 4.3 Mev$$

(where $a(w) = 0.253$ is the geometric part of the weak force strength and $V_{12} = 0.22$ is the K-M parameter mixing generations 1 and 2)

so that the Quantum color force constituent mass Qm_d of the down quark is

$$Qm_d = 312.75 + 4.3 = 317.05 MeV$$

Similarly, the up quark Quantum color force mass increase is about

$$(m_c - m_u)(m_u/m_c)^2 a(w) V_{12} = 1777 \times 0.022 \times 0.253 \times 0.22 Mev = 2.2 Mev$$

so that the Quantum color force constituent mass Qm_u of the up quark is

$$Qm_u = 312.75 + 2.2 = 314.95 MeV$$

The Quantum color force Neutron-Proton mass difference is

$$m_N - m_P = Qm_d - Qm_u = 317.05 Mev - 314.95 Mev = 2.1 Mev$$

Since the electromagnetic Neutron-Proton mass difference is roughly $m_N - m_P = -1 MeV$

the total theoretical Neutron-Proton mass difference is

$$m_N - m_P = 2.1 Mev - 1 Mev = 1.1 Mev$$

an estimate that is fairly close to the experimental value of 1.3 Mev.

Note that in the equation

$(ms - md)(md/ms)^2 a(w)|Vds| = 4.3 Mev$

Vds is a mixing of down and strange by a neutral Z0, compared to the more conventional Vus mixing by charged W.

Although real neutral Z0 processes are suppressed by the GIM mechanism, which is a cancellation of virtual processes, the process of the equation is strictly a virtual process.

Note also that the K-M mixing parameter $|Vds|$ is linear.

Mixing (such as between a down quark and a strange quark) is a two-step process, that goes approximately as the square of $|Vds|$:

First the down quark changes to a virtual strange quark, producing one factor of $|Vds|$.

Then, second, the virtual strange quark changes back to a down quark, producing a second factor of $|Vsd|$, which is approximately equal to $|Vds|$.

Only the first step (one factor of $|Vds|$) appears in the Quantum mass formula used to determine the neutron mass.

If you measure the mass of a neutron, that measurement includes a sum over a lot of histories of the valence quarks inside the neutron.

In some of those histories, in my view, you will "see" some of the two valence down quarks in a virtual transition state that is at a time after the first action, or change from down to strange,

and before the second action, or change back.

Therefore, you should take into account those histories in the sum in which you see a strange valence quark,

and you get the linear factor $|Vds|$ in the above equation.

Note that if there were no second generation fermions, or if the second generation quarks had equal masses,

then the proton would be heavier than the neutron (due to the electromagnetic difference)

and the hydrogen atom would decay into a neutron, and there would be no stable atoms in our world.

10.2 Quark-AntiQuark Pions.

In this HyperDiamond Feynman Checkerboard version of the $D_4 - D_5 - E_6$ model, pions are made up of first generation valence Quark-AntiQuark pairs. The pion bound state of valence Quark-AntiQuark pairs has a soliton structure that would, projected onto a 2-dimensional spacetime, be a Sine-Gordon breather. See WWW URLs

http://galaxy.cau.edu/tsmith/SnGdnPion.html

http://www.innerx.net/personal/tsmith/SnGdnPion.html

Marin, Eilbeck, and Russell, in their paper

Localized Moving Breathers in a 2-D Hexagonal Lattice,

show "...that highly localized in-plane breathers can propagate in specific directions with minimal lateral spreading ... This one-dimensional behavior in a two-dimensional lattice was called quasi-one-dimensional (QOD) ..."

(In their paper, dimensionality refers to spatial dimensionality.)

They use QOD behavior to describe phenomena in muscovite mica crytstals.

The D4-D5-E6 model uses similar QOD behavior to describe the pion.

Take the pion as the fundamental quark-antiquark structure and assume that the quark masses are constituent masses that include the effects of the complicated structure of sea quarks and binding gluons within the pion at the energy level of about 313 MeV that corresponds to the Compton wavelength of the first generation quark constituent mass.

The sea quarks and binding gluons within the pion constitute the dressing of the proton valence quarks with the quark and gluon sea in which the valence quarks swim within the pion.

The quark and gluon sea has characteristic energy of roughly the constituent quark masses of 312.8 MeV so that the valence quarks float freely within the pion sea.

To express the assumption of free-floating quarks more mathematically, define the relationship between the calculated constituent quark masses, denoted by m_q, and QCD Lagrangian current quark masses, denoted by M_q, by

$$M_q = m_q - m_u = m_q - m_d = m_q - 312.8MeV$$

This makes, for the QCD Lagrangian, the up and down quarks roughly massless.

One result is that the current masses can then be used as input for the $SU(3) \times SU(3)$ chiral theory that, although it is only approximate because the constituent mass of the strange quark is about 312 MeV, rather than nearly zero, can be useful in calculating meson properties.

In hep-ph/9802425, Di Qing, Xiang-Song Chen, and Fan Wang, of Nanjing University, present a qualitative QCD analysis and a quantitative model calculation to show that the constituent quark model [after mixing a small amount (15remains a good approximation even taking into account the nucleon spin structure revealed in polarized deep inelastic scattering.

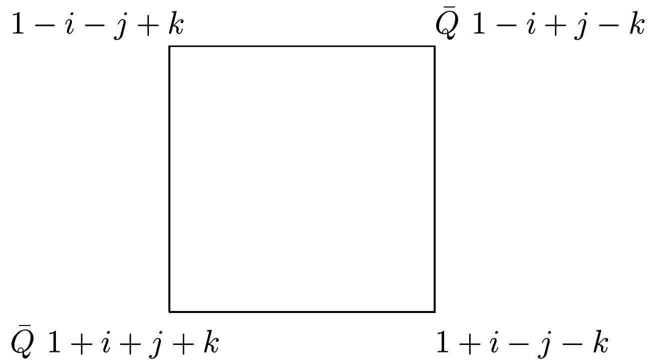
The effectiveness of the NonRelativistic model of light-quark hadrons is explained by, and affords experimental Support for, the Quantum Theory of David Bohm (see quant-ph/9806023).

The HyperDiamond structure used to approximate the pion is the 2-link Exchange Propagator.

Here is a coordinate sequence representation of the approximate HyperDiamond Feynman Checkerboard path of a pion:

$Quark$	$AntiQuark$
0	0
$1+i+j+k$	$1-i+j-k$
2	2
$3-i+j-k$	$3+i+j+k$
4	4
$5+i+j+k$	$5-i+j-k$
6	6

The following page contains a Square Diagram representation of the approximate HyperDiamond Feynman Checkerboard path of a pion at times 1, 3, and 5 (at 0, 2, 4, and 6, both the quark and the antiquark are at the origin):



For more details about pions and other mesons, see WWW URLs

http://galaxy.cau.edu/tsmith/Sets2Quarks10.html

http://www.innerx.net/personal/tsmith/Sets2Quarks10.html

10.3 Spin-2 Physical Gravitons.

In this HyperDiamond Feynman Checkerboard version of the $D_4 - D_5 - E_6$ model, spin-2 physical gravitons are made up of the 4 translation spin-1 gauge bosons of the 10-dimensional $Spin(5)$ de Sitter subgroup of the 15-dimensional $Spin(6)$ Conformal group used to construct Einstein-Hilbert gravity in the $D_4 - D_5 - E_6$ model described in URLs

http://xxx.lanl.gov/abs/hep-ph/9501252

http://xxx.lanl.gov/abs/quant-ph/9503009.

The action of Gravity on Spinors is given by the Papapetrou Equations.

The spin-2 physical gravitons are massless, but they can have energy up to and including the Planck mass.

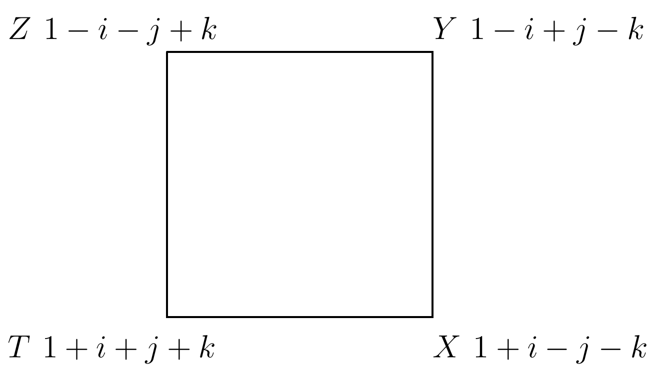
Unlike the pions and protons, which are made up of fermion quarks that live on vertices of the HyperDiamond Lattice, the gravitons are gauge bosons that live on the links of the HyperDiamond Lattice.

The Planck energy spin-2 physical gravitons are really fundamental structures with HyperDiamond Feynman Checkerboard path length L_{Planck}.

Here is a coordinate sequence representation of
the HyperDiamond Feynman Checkerboard path of a fundamental Planck-mass spin-2 physical graviton,
where $\mathbf{T}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}$ represent infinitesimal generators
of the $Spin(5)$ de Sitter group:

$\mathbf{T}$	$\mathbf{X}$	$\mathbf{Y}$	$\mathbf{Z}$
$1+i+j+k$	$1+i-j-k$	$1-i+j-k$	$1-i-j+k$
$2+i+j+k$	$2+i-j-k$	$2-i+j-k$	$2-i-j+k$
$3+i+j+k$	$3+i-j-k$	$3-i+j-k$	$3-i-j+k$
$4+i+j+k$	$4+i-j-k$	$4-i+j-k$	$4-i-j+k$

The following is a Square Diagram representation of the HyperDiamond Feynman Checkerboard path of a fundamental Planck-mass spin-2 physical graviton at times 1, 2, 3, and 4:

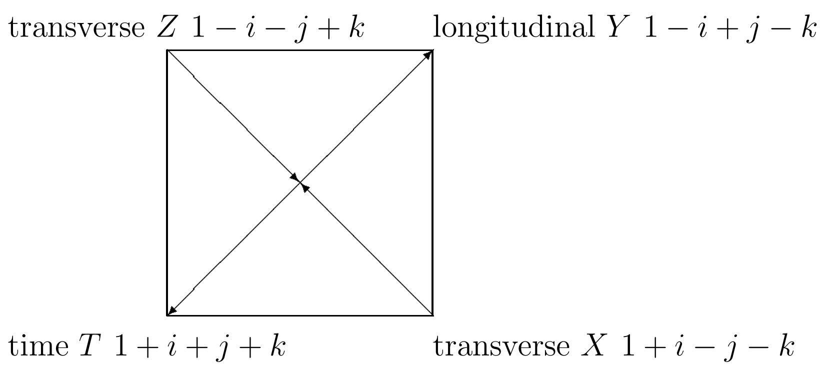


The representation above is for a timelike path at rest in space.

With respect to gravitons, we can see something new and different by letting the path move in space as well.

Let **T** and **Y** represent time and longitudinal space, and
X and **Z** represent transverse space.

Then, as discussed in Feynman's *Lectures on Gravitation*, pp. 41-42 [8], the Square Diagram representation shows that our spin-2 physical graviton is indeed a spin-2 particle.



Spin-2 physical gravitons of energy less than the Planck mass are more complicated composite gauge boson structures with approximate HyperDiamond Feynman Checkerboard path length $L_{gravitonenergy}$.

They can be deformed from a square shape, but retain their spin-2 nature as described by Feynman [8].

10.3.1 Planck Mass.

An estimated calculation of the Planck mass is at WWW URLs

http://galaxy.cau.edu/tsmith/Planck.html

http://www.innerx.net/personal/tsmith/Planck.html

Here is a summary of a combinatorial calculation:

Consider an isolated single point, or vertex in the lattice picture of spacetime. In the HyperDiamond Feynman Checkerboard model, fermions live on vertices, and only first-generation fermions can live on a single vertex.

(The second-generation fermions live on two vertices that act at our energy levels very much like one, and the third-generation fermions live on three vertices that act at our energy levels very much like one.)

At a single spacetime vertex, a Planck-mass black hole

is the Many-Worlds quantum sum of all possible virtual

first-generation particle-antiparticle fermion pairs

permitted by the Pauli exclusion principle to live on that vertex.

The Planck mass in 4-dimensional spacetime is the sum of masses of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle.

There are 8 fermion particles and 8 fermion antiparticles for a total of 64 particle-antiparticle pairs. A typical combination should have several quarks, several antiquarks, a few colorless quark-antiquark pairs that would be equivalent to pions, and some leptons and antileptons.

Due to the Pauli exclusion principle, no fermion lepton or quark could be present at the vertex more than twice unless they are in the form of boson pions, colorless first-generation quark-antiquark pairs that are not subject to the Pauli exclusion principle.

Of the 64 particle-antiparticle pairs, 12 are pions.

A typical combination should have about 6 pions.

If all the pions are independent, the typical combination should have a mass of $0.14 \times 6GeV = 0.84GeV$.

However, just as the pion mass of 0.14 GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as 0.1 GeV,

and if the typical combination has one such pair and 4 other pions, then the typical combination should have a mass in the range of 0.66 GeV.

Summing over all 2^{64} combinations, the total mass of a one-vertex universe should give:

$$m_{Planck} = 1.217 - 1.550 \times 10^{19} GeV$$

There is also a quaternionic calculation of about $1.3 \times 10^{19} GeV$ at WWW URLs

http://galaxy.cau.edu/tsmith/Planck.html

http://www.innerx.net/personal/tsmith/Planck.html

A Errata for Earlier Papers.

Errata for Earlier Papers can be found at WWW URLs

http://galaxy.cau.edu/tsmith/Errata.html

http://www.innerx.net/personal/tsmith/Errata.html

References

[1] J. Frank Adams, *The Selected Works of J. Frank Adams*, Canbridge (1994).

[2] J. Frank Adams, *Lectures on Exceptional Lie Groups*, Chicago (1996).

[3] A. Besse, *Einstein Manifolds*, Springer-Verlag (1987).

[4] V. Barger and R. Phillips, *Collider Physics, updated edition*, Addison Wesley (1997).

[5] J. Conway and N. Sloane, *Sphere Packings, Lattices, and Groups, 2nd ed*, Springer-Verlag (1993).

[6] H. Coxeter, *Regular Complex Polytopes, 2nd ed*, Cambridge (1991).

[7] G. Dixon, *Division Algebras*, Kluwer (1994).

[8] R. Feynman, *Lectures on Gravitation, 1962-63*, Caltech (1971).

[9] R. Feynman, *Notes on the Dirac Equation*, Caltech archives, box 13, folder 6 (1946).

[10] R. Feynman and Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill (1965).

[11] H. Gersch, Int. J. Theor. Phys. 20 (1981) 491.

[12] M. Gibbs, *Spacetime as a Quantum Graph*, Ph. D. thesis of J. Michael Gibbs, with advisor David Finkelstein, Georgia Tech (1994).

[13] M. Gockeler and T. Schucker, *Differential Geometry, Gauge Theories, and Gravity*, Cambridge (1987).

[14] F. Harvey, *Spinors and Calibrations*, Academic (1990).

[15] L.-K. Hua, *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains*, AMS (1963).

[16] J. Humphreys, *Reflection Groups and Coxeter Groups*, Cambridge (1990).

[17] G. Kane, *Modern Elementary Particle Physics, updated edition*, Addison Wesley (1997).

[18] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, vol. 1*, John Wiley (1962).

[19] S. Kobayashi and K. Nomizu, *Foundations of Differential Geometry, vol. 2*, John Wiley (1969).

[20] J. Lohmus, E. Paal, L. Sorgsepp, *Nonassociative Algebras in Physics*, Hadronic Press (1994).

[21] M. Mayer, Hadronic Journal 4 (1981) 108-152, and also articles in New Developments in Mathematical Physics, 20th Universitatswochen fur Kernphysik in Schladming in February 1981 (ed. by Mitter and Pittner), Springer-Verlag 1981, which articles are *A Brief Introduction to the Geometry of Gauge Fields (written with Trautman); The Geometry of Symmetry Breaking in Gauge Theories; Geometric Aspects of Quantized Gauge Theories.*.

[22] Guang-jiong Ni, *To Enjoy the Morning Flower in the Evening - What does the Appearance of Infinity in Physics Imply?*; http://xxx.lanl.gov/absquant-ph/9806009

[23] Guang-jiong Ni, Sen-yue Lou, Wen-fa Lu, and Ji-feng Yang, *Lambda Phi4 model and Higgs mass in standard model calculated by Gaussian effective potential approach with a new regularization-renormalization method*; http://xxx.lanl.gov/abs/hep-ph/9801264

[24] I. Porteous, *Clifford Algebras and the Classical Groups*, Cambridge (1995).

[25] B. Rosenfeld, *Geometry of Lie Groups*, Kluwer (1997).

[26] M. Shifman, *Determining α_s from Measurements at Z: How Nature Prompts us about New Physics*, http://xxx.lanl.gov/abs/hep-ph/9501222

[27] B. Simon, *Representations of Finite and Compact Groups*, American Mathematical Society (1996).

[28] F. Smith, *Calculation of 130 GeV Mass for T-Quark*, preprint: THEP-93-2; clf-alg/smit-93-01(1993); http://xxx.lanl.gov/abs/hep-ph/9301210

[29] F. Smith, *Hermitian Jordan Triple Systems, the Standard Model plus Gravity, and* $\alpha_E = 1/137.03608$, preprint: THEP-93-3; http://xxx.lanl.gov/abs/hep-th/9302030

[30] F. Smith, *Sets and* $\mathbf{C^n}$*; Quivers and* $A - D - E$*; Triality; Generalized Supersymmetry; and* $D_4 - D_5 - E_6$, preprint: THEP-93-5; http://xxx.lanl.gov/abs/hep-th/9306011

[31] F. Smith, $SU(3) \times SU(2) \times U(1)$*, Higgs, and Gravity from* $Spin(0,8)$ *Clifford Algebra* $Cl(0,8)$, preprint: THEP-94-2; http://xxx.lanl.gov/abs/hep-th/9402003

[32] F. Smith, *Higgs and Fermions in* $D_4 - D_5 - E_6$ *Model based on* $Cl(0,8)$ *Clifford Algebra*, preprint: THEP-94-3; http://xxx.lanl.gov/abs/hep-th/9403007

[33] F. Smith, *Gravity and the Standard Model with 130 GeV Truth Quark from* $D_4 - D_5 - E_6$ *Model using* 3×3 *Octonion Matrices*, preprint: THEP-95-1; http://xxx.lanl.gov/abs/hep-ph/9501252

[34] F. Smith, *Standard Model plus Gravity from Octonion Creators and Annihilators*, preprint: THEP-95-2; http://xxx.lanl.gov/abs/quant-ph/9503009

[35] F. Smith, *HyperDiamond Feynman Checkerboard in 4-dimensional Spacetime*, preprint: THEP-95-3; http://xxx.lanl.gov/abs/quant-ph/9503015

[36] Additional references can be found in the bibliographies of these references, and in my web pages, such as URLs
http://galaxy.cau.edu/tsmith/HDFCmodel.html
http://www.innerx.net/personal/tsmith/HDFCmodel.html