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Calculation of 130 GeV Mass for T-Quark

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Abstract

A 130 GeV mass is calculated for the t-quark, also called the truth quark or the top quark.

The 130 GeV T-quark mass is consistent with the analysis of Dalitz and Goldstein [1] (131 GeV (-11, +22)) of a CDF T-quark candidate event. It is not consistent with the theoretical calculations of Dimopoulos, Hall, and Raby [2] that the T-quark mass range should be 176 to 190 GeV.

The calculation uses a model in which the 28-dimensional adjoint representation of Spin(8) forms the gauge group of the model with an 8-dimensional spacetime whose tangent vector space is represented by the vector representation of Spin(8); the first generation fermion particles and antiparticles are represented by the two mirror image 8-dimensional half-spinor representations of Spin(8). The 8-dimensional spacetime of the model is reduced to 4 dimensions. Dimensional reduction gives the fermions a 3-generation structure.

1 Introduction

The model used in the theoretical calculations is based on the Lie group Spin(8).

The 28-dimensional adjoint representation of Spin(8) forms the gauge group of the model with an 8-dimensional spacetime whose tangent vector space is represented by the vector representation of Spin(8).

The first generation fermion particles and antiparticles are represented by the two mirror image 8-dimensional half-spinor representations of Spin(8), denoted by S8+ and S8-.

The 8-dimensional spacetime of the model is reduced to 4 dimensions.

After dimensional reduction:
MacDowell-Mansouri gravity becomes an effective nonrenormalizable theory;

a natural Higgs scalar field appears that gives mass to the weak bosons and Dirac fermions; and there are three generations of fermions, being effectively represented by:
octonions,
pairs of octonions, and
triples of octonions.

2 3-Generation Structure of Fermions.

The model puts the 8 first generation fermion particles in one 8-dimensional half-spinor representation S8+ of Spin(8), with the 8 antiparticles being in the mirror image 8-dimensional half-spinor representation S8- of Spin(8). Given a basis (1, i, j, k, ke, je, ke) for the octonions O , where 1 is the basis element for the real axis and, as to the seven imaginaries, i, j, and k are just the three imaginary quaternions, and e, ie, je, and ke are constructed from the four quaternionic basis elements 1, i, j, and k by introducing an octonionic imaginary e.

The octonionic basis for S8+ corresponds to fermion particles as follows: 1 is the electron neutrino; i, j, k are the red, blue, and green up quarks; e is the electron; and ie, je, ke are the red, blue, and green down quarks. The antiparticle correpsondence for S8- is similar.

Consider the model from a lattice gauge theory point of view. It looks like an 8-dim lattice of vertices connected by links. The spinor fermions are assigned to the vertices of the lattice spacetime. The fermions go from place to place by moving from an origin vertex to a destination vertex
(origin) *—–* (destination) along a link that connects them.

The gauge bosons are assigned to the links, and are represented as Lie group elements of the gauge group, which then acts as a transport. Effectively, dimensional reduction does not give a generation structure to gauge bosons because the transport
—–(g1)—–—–(g2)—–* is the same as
—–(g1 g2)—–
where g1 and g2 are gauge Lie group elements and g1 g2 is the Lie group product.

Consider fermion particles (similar arguments apply to antiparticles) represented by the 8 basis octonions (1,i,j,k,e,ie,je,ke) of S8+. For this discussion of how the three generations are formed, ignore helicity and the distinction between the Weyl neutrino (1) and the Dirac electron and quarks (i,j,k,e,ie,je,ke).

In 8-dim lattice F4 model, a fermion particle * going from (origin) to (destination) can be represented by an octonion
(origin)*—–(destination), or in short
*—– , where * is an octonion.

This notation uses *, o, and o' as notations for octonions representing fermion particles at the vertices also denoted by *, o, and o'. Since the fermions live on vertices, the abuse of notation (which is useful) should not be misleading.

What happens when the spacetime is reduced to 4 dimensions?

The 8-dim E8 lattice, with octonionic (1,i,j,k,e,ie,je,ke) vertices, goes to a 4-dim lattice with quaternionic (1,i,j,k) vertices.

The dimensional reduction is like a projection

(1,i,j,k,e,ie,je,ke) —– (1,i,j,k)

The subspace (1,i,j,k) is invariant and the subspace (e,ie,je,ke) is projected into (1,i,j,k) .

Consider a given fermion particle going in 8-dim (origin)*—–(destination)

THERE ARE 4 CASES:

CASE 1. (origin) and (destination) are both in the (1,i,j,k) subspace of (1,i,j,k,e,ie,je,ke). Then the fermion:

(origin)*—–(destination) IS NOT CHANGED by the dimensional reduction and is still represented by the single octonion *.

THE CASE 1. FERMIONS ARE FIRST GENERATION FERMIONS.

CASE 2. (origin) is in the (1,i,j,k) subspace of (1,i,j,k,e,ie,je,ke) BUT (destination) has components in the (e,ie,je,ke) subspace of (1,i,j,k,e,ie,je,ke). Then the fermion:

(origin)*—–(destination) IS CHANGED because dimensional reduction takes the (destination)o vertex o in (1,i,j,k,e,ie,je,ke) into its image in (1,i,j,k) under the dimensional reduction map, denoted as (reduced destination). The result is fermion:

(origin)*—–(destination)o—–(reduced destination).

After dimensional reduction, there is a new intermediate vertex o on which another octonion fermion can live.

THEREFORE, IT TAKES TWO (2) OCTONIONS TO REPRESENT SUCH A CASE 2. FERMION AFTER DIMENSIONAL REDUCTION.

IT IS A SECOND GENERATION FERMION, REPRESENTED BY A PAIR (*,o) OF OCTONIONS.

CASE 3. (origin) has components in the (e,ie,je,ke) subspace of (1,i,j,k,e,ie,je,ke) BUT (destination) is in the (1,i,j,k) subspace of (1,i,j,k,e,ie,je,ke). Then the fermion:

(origin)*—–(destination) IS CHANGED because dimensional reduction takes the (origin)* vertex * in (1,i,j,k,e,ie,je,ke) into its image in (1,i,j,k) under the dimensional reduction map, denoted as (reduced origin)o, before the fermion goes to its (destination) in (1,i,j,k). The result is fermion:

(origin)*—–(reduced origin)o—–(destination).

After dimensional reduction, there is a new intermediate vertex o on which another octonion fermion can live.

THEREFORE, IT TAKES TWO (2) OCTONIONS TO REPRESENT SUCH A CASE 3. FERMION AFTER DIMENSIONAL REDUCTION.

IT IS ALSO A SECOND GENERATION FERMION, REPRESENTED BY A PAIR (*,o) OF OCTONIONS.

The only remaining possibility is

CASE 4. (origin) has components in the (e,ie,je,ke) subspace of (1,i,j,k,e,ie,je,ke) AND (destination) has components in the (e,ie,je,ke) subspace of (1,i,j,k,e,ie,je,ke). Then the fermion:

(origin)*—–(destination) IS CHANGED because dimensional reduction takes the (origin)* vertex * in (1,i,j,k,e,ie,je,ke) into its image in (1,i,j,k) under the dimensional reduction map, denoted as (reduced origin)o, before the fermion goes to its (destination) in (1,i,j,k,e,ie,je,ke), which is denoted by (destination)o'.

THEN THE DIMENSIONAL REDUCTION MAP takes the (destination)o' vertex o' in (1,i,j,k,e,ie,je,ke) into its image in (1,i,j,k) under the dimensional reduction map, denoted as (reduced destination). The result is fermion:

(origin)*—-(reduced origin)o—-(destination)o'—-(reduced destination).

After dimensional reduction, there is are two (2) new intermediate vertices o and o' on which two more octonion fermions can live.

THEREFORE, IT TAKES THREE (3) OCTONIONS TO REPRESENT SUCH A CASE 4. FERMION AFTER DIMENSIONAL REDUCTION.

IT IS A THIRD GENERATION FERMION, REPRESENTED BY A TRIPLE (*,o,o') OF OCTONIONS.

SINCE THERE ARE NO MORE CASES, THERE ARE ONLY 3 GENERATIONS.

Gauge bosons do not get a 3-generation structure because the corresponding pair or triple of links (g1 , g2) or (g1 , g2 , g3) can be reduced to a single gauge boson by the Lie group product g1 g2 or g1 g2 g3. (On a finite lattice, gauge Lie group elements, not infinitesimal Lie algebra elements, live on the links.)

The important difference here between adjoint rep gauge bosons and spinor rep fermions is that the adjoint rep gauge bosons inherit the gauge Lie group product, and the spinor rep fermions have no such product.

3 First-Generation Quark Consitituent Masses.

In the model, the Weyl fermion neutrino has at tree level only the left-handed state, whereas the Dirac fermion electron and quarks can have both left-handed and right-handed states, so that the total number of states corresponding to each of the half-spinor Spin(8) representations Spin(8) is 15.

Neutrinos are massless at tree level in all generations.

In the model, the first generation fermions correspond to octonions O, while second generation fermions correspond to pairs of octonions OO and third generation fermions correspond to triples of octonions OOO.

To calculate the fermion masses in the model, the volume of a compact manifold representing the spinor fermions S8+is used. It is the parallelizable manifold $S^7 \times RP^1$.

Also, since gravitation is coupled to mass, the infinitesimal generators of the MacDowell-Mansouri gravitation group, Spin(5), are used in the fermion mass calculations.

The calculated quark masses are constituent masses, not current masses.

In the model, fermion masses are calculated as a product of four factors:

$V(Q) \times N(Graviton) \times N(octonion) \times Sym$

V(Q) is the volume of the part of the half-spinor fermion particle manifold $S^7 \times RP^1$ that is related to the fermion particle by photon, weak boson, and gluon interactions.

N(Graviton) is the number of types of Spin(5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of Spin(5) = Sp(2). 2 of them are in the Cartan subalgebra. 6 of them carry color charge, and may therefore be considered as corresponding to quarks. The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons.

One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron.

The graviton number ratio of the down quark to the first-generation electron is therefore $6/1 = 6$.

N(octonion) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

The ratio of the down quark constituent mass to the electron mass is then calculated as follows: Consider the electron, e. By photon, weak boson, and gluon interactions, e can only be taken into 1, the massless neutrino. The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks. The neutrino, being massless, does not add anything to the mass formula for the electron. Since the electron cannot be related to any other massive Dirac fermion, its volume V(Q) is taken to be 1.

Next consider a red down quark ie. By gluon interactions, ie can be taken into je and ke, the blue and green down quarks. By weak boson interactions, it can be taken into i, j, and k, the red, blue, and green up quarks. Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos. Therefore the red down quark (similarly, any down quark) is related to any part of $S^7 \times RP^1$, the compact manifold corresponding to (1, i, j, k, e, ie, je, ke), and therefore a down quark should have a spinor manifold volume factor of the volume of $S^7 \times RP^1$. The ratio of the down quark spinor manifold volume factor to the electron spinor manifold volume factor is just $V(S^7 \times RP^1)/1 = \pi^5/3$.

Since the first generation graviton factor is 6,

$md/me = 6V(S^7 \times RP^1) = 2\pi^5 = 612.03937$.

As the up quarks correspond to i, j, and k, which are isomorphic to ie, je, and ke of the down quarks, the up quarks and down quarks have the same constituent mass mu = md.

Antiparticles have the same mass as the corresponding particles.

Since the model only gives ratios of massses, the mass scale is fixed by assuming that the electron mass me = 0.5110 MeV. Then, the constituent mass of the down quark md = 312.75 MeV, and the constituent mass for the up quark mu = 312.75 MeV.

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks, m(proton) = mu + mu + md = 938.25 MeV, the model calculation is close to the experimental value of 938.27 MeV.

4 T-Quark Mass Calculation.

The third generation fermion particles correspond to triples of octonions. There are $8^3 = 512$ such triples. The triple (1,1,1) corresponds to the tau-neutrino. The other 7 triples involving only 1 and e correspond to the tauon: (e,e,e), (e,e,1), (e,1,e), (1,e,e), (1,1,e), (1,e,1), and (e,1,1).

The symmetry of the 7 tauon triples is the same as the symmetry of the 3 down quarks, the 3 up quarks, and the electron, so the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass 1.87704 GeV.

Note that all triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples. They are triples of the form (1,1,ie), (1,ie,1), (ie,1,1), (ie,ie,1), (ie,1,ie), (1,ie,ie), and (ie,ie,ie), and the similar triples for 1 and je and for 1 and ke.

Note particularly that triples of the type (1,ie,je), (ie,je,ke), etc., do not correspond to the beauty quark, but to the truth quark.

The red beauty quark is defined as the seven triples (1,1,ie), (1,ie,1), (ie,1,1), (ie,ie,1), (ie,1,ie), (1,ie,ie), and (ie,ie,ie), because ie is the red down quark. The seven triples of the red beauty quark correspond to the seven triples of the tauon, except that the beauty quark interacts with 6 Spin(5) gravitons while the tauon interacts with only two. The beauty quark constituent mass should be the tauon mass times the third generation graviton factor $6/2 = 3$, so the B-quark mass is 5.63111 GeV.

The blue beauty quarks correspond to the seven triples involving je, and the green beauty quarks correspond to the seven triples involving ke.

The truth quark corresponds to the remaining 483 triples, so the constituent mass of the red truth quark is $161/7 = 23$ times the red beauty quark mass, and the red T-quark mass is 129.5155 GeV.

The blue and green truth quarks are defined similarly.

The tree level T-quark constituent mass rounds off to 130 GeV.

5 Gauge Bosons and Further Results

Dimensional reduction also acts on the gauge bosons, giving an effective SU(3) x SU(2)L x U(1) standard model gauge group plus an effective Spin(5) MacDowell-Mansouri gravity.

The physically realistic way to decompose the 28 infinitesimal generators of Spin(8) after dimensional reduction is to group them according to Weyl group symmetry into groups of 10, 6, 8, and 4 members.

Then the group of 10 becomes Spin(5) with base manifold S^4, the group of 6 becomes Spin(4) with base manifold $S^2 \times S^2$, the group of 8 becomes SU(3) with base manifold CP^2, and the group of 4 becomes $U(1)^4$ with base manifold T^4. The Weyl group of Spin(8) is the semidirect product of the Weyl groups of the groups into which Spin(8) is decomposed. Each group is then considered to be independent, with the effect that the Spin(5) gives MacDowell-Mansouri gravity,

the SU(3) is the color force SU(3),

the Spin(4) has two copies of SU(2), one of which becomes the effective weak force SU(2)L and the other of which is integrated over the 4 "lost" dimensions to give an effective Higgs scalar field, and

the 4 copies of U(1) become the 4 covariant components of the electromagnetic photon.

Second-generation fermion masses (constituent masses for quarks), force strengths, the Weinberg angle, and Kobayashi-Maskawa parameters can also be calculated using the model, with the following results [3]:

$m_\mu = 104.8$ MeV
$m_{\mu-neutrino} = 0$
$m_s = ...625$ MeV
$m_c = 2.09$ GeV

U(1) electromagnetism: $\alpha_E = \frac{1}{137.03608}$

SU(2) weak force: $G_F = G_W m^2_{proton} = 1.02 \times 10^{-5}$

SU(3) color force:
$\alpha_C = 0.629$ at 0.24 GeV
$\alpha_C = 0.168$ at 5.3 GeV
$\alpha_C = 0.122$ at 34 GeV
$\alpha_C = 0.106$ at 91 GeV

Higgs scalar mass = 260.8 GeV

$m_{W+} = m_{W-} = 80.9 GeV$
$m_Z = 92.4 GeV$

Weinberg angle: $Sin^2\theta_W = 0.233$

The Kobayashi-Maskawa Parameters are:
phase angle: $e = \frac{\pi}{2}$
$V_{ud} = 0.975$
$V_{us} = 0.222$
$V_{ub} =$ -0.00461 i
$V_{cd} =$ -0.222 -0.000190 i
$V_{cs} =$ 0.974 -0.0000434 i
$V_{cb} = 0.0423$
$V_{td} =$ 0.00941 -0.00449 i

V_{ts} = -0.0413 -0.00102 i

$V_{tb} = 0.999$

The same K-M mixing angles apply to both leptons and quarks, but are only effective for leptons if neutrinos have nonzero mass.

Beyond tree level, neutrinos can get mass by radiative processes related to the Planck mass [3]:

$m_{e-neutrino} = 2.2 \times 10^{-6} eV$

$m_{\mu-neutrino} = 4.5 \times 10^{-4} eV$

$m_{\tau-neutrino} = 8.1 \times 10^{-3} eV$

Therefore, the model has a natural MSW mechanism that may solve the solar neutrino problem.

6 Chronology of T-quark Mass Calculations

At the request of others who have done theoretical calculations, I am adding this section in January 1993:

I do not represent that this section is a complete history of calculations of the T-quark mass. It is about the chronology of some theoretical T-quark mass calculations of which I am now aware. It includes only purely theoretical calculations giving a result of about 130 GeV, and does not include calculations of bounds on the T-quark mass resulting from applying the standard model (or other models) to experimental results such as B-mixing, Z-width, etc.

1982:

Harvey, Reiss, and Ramond write Mass Relations and Neutrino Oscillations in an SO(10) Model (Nuc. Phys. **B199** (1982) 223-268) (revised 3 Feb 82).

Eq. 3.33, tan $\alpha = \sqrt{\frac{m_c}{m_t}}$, where tan $\alpha = V_{cb}$, relates the T-quark mass m_t to the K-M parameter V_{cb} and the C-quark mass m_c.

As the paper states (p. 237):

"Unfortunately, it does not predict m_t except through τ_B ... ."

Inoue, Kakuto, Komatsu, and Takeshita write Aspects of Grand Unified Models with Softly Broken Suypersymmetry (Prog. Theor. Phys. **68** (1982) 927) (received 10 May 82)

They relate supersymmetry to electro-weak symmetry breaking by radiative corrections and renormalization group equations, and find that the renormalization group equations have a fixed point.

The fixed point is related to a T-quark mass of about 125 GeV, as was explicitly discussed in 1983 by Alvarez-Gaume, Polchinski, and Wise.

1983:

Alvaerez-Gaume, Polchinski, and Wise write Minimal Low-Energy Supergravity (Nuc. Phys. **B221** (1983) 495-523) (received 8 Feb 83).

Their calculations show that, for electro-weak symmetry breaking to occur, the T-quark mass must be from 100 GeV to 195 GeV.

Moreover, they also note (p. 511) that the renormalization group equation

"... tends to attract the top quark mass towards a fixed point of about 125 GeV."

Work similar to that of Alvarez-Gaume, Polchinski, and Wise was done by Ibanez and Lopez in N=1 Supergravity, the Weak Scale and the Low-Energy Spectrum
(Nuc. Phys. **B233** (1984) 511-544) (received 8 Aug 83).

As far as I know, the works of Inoue, Kakuto, Komatsu, and Takeshita; of Alvarez-Gaume, Polchinski, and Wise; and of Ibanez and Lopez are the first purely theoretical calculations of the T-quark mass to be about 130 GeV.

1984:
I wrote Particle Masses, Force Constants, and Spin(8)
(Int. J. Theor. Phys. **24** (1985) 155-174) (received 27 Feb 84).
Using a model similar, but not identical, to the model I am now using, I calculated the T-quark mass to be 129.5 GeV. (The models are similar with respect to the T-quark mass.)

Nature (**310** (12 July 84) 97) article about CERN discovering the T-quark at 40 GeV.

At the 31 Oct- 3 Nov 84 APS DPF Santa Fe meeting I gave a 10-minute talk about my theoretical work on the T-quark mass, and interpreted the CERN experimental results as being consistent with T-quark mass of 120 to 160 GeV, rather than 40 GeV.

1986:
Mohapatra, in his book *Unification and Supersymmetry, The Frontiers of Quark-Lepton Physics* (Springer-Verlag 1986), in Section 15.3 on Electro-Weak Symmetry Breaking and Supergravity, discussed the work of Alvarez-Gaume, Polchinski, and Wise, and stated (pp. 287-288 (323-324 in 1992 second edition)):
"It is interesting that m_t lies in the range 100 GeV $\leq m_t \leq$ 190 GeV. The recent discovery of the t-quark in the mass range of 40-60 GeV therefore rules out the simple-minded analysis carried out here."

1991:
Arason, Castano, Kesthelyi, Mikaelian, Piard, Ramond, and Wright write
Top-Quark and Higgs-Boson Mass Bounds from a Numerical Study of Sypersymetric Grand Unified Theories (Phys. Rev. Lett. **67** (1991) 2933-2936) (received 7 Aug 91).
They calculate T-quark and Higgs masses for SUSY scale of 1 TeV to be:
if $m_b = 4.6$ GeV then $162 \leq m_t \leq 176$ GeV and $106 \leq m_H \leq 111$ GeV; and
if $m_b = 5.0$ GeV then $116 \leq m_t \leq 147$ GeV and $93 \leq m_H \leq 101$ GeV.

1992:
I submitted a preprint to SLAC (10 July 92)
T PRINT-92-0226 [GEORGIA-TECH] and
ET PRINT-92-0227 [GEORGIA-TECH] in SLAC index
and to CERN (week beginning 22 Sep 92)
PRE 33611 in CERN index.
This preprint is an extensive presentation of my current work,
including T-quark mass = 130 GeV.

Arason, Castano, Piard, and Ramond give a renormalization group analysis of mixing angles and masses including the T-quark mass in Phys. Rev. **D 47** (1993) 232-240.

Supersymmetric theories have been constrained by CDF to have
squarks $\geq$ 126 GeV and gluinos $\geq$ 141 GeV
(Search for Squarks and Gluinos from p bar-p Collisions at $\sqrt{s} = 1.8$ TeV, Phys. Rev. Lett. **69** (1992) 3439-3443) (received 17 Aug 92).

1993: I submitted the replaced version of this paper to hep-ph@xxx.lanl.gov as hep-ph/9301210 and to clf-alg@stars.sfsu.edu bulletin boards. (Jan 93)

Further details of my work [3] are available as a paper preprint or
as a Mathematica 2.1 notebook on a Macintosh HD disk.

References

[1] Dalitz, R. H.; and Goldstein, Gary R. 1992. Analysis of top-antitop production and dilepton events and the top quark mass. *Phys. Lett.* **B 287**, 225-230.

[2] Dimopoulos, S.; Hall, L. J.; and Raby, S. 1992. Predictive Ansatz for fermion mass matrices in supersymmetricgrand unified theories. *Phys. Rev.* **D 45**, 4192-4200.

[3] Smith, F. D. T. 1992. An 8-Dimensional F_4 Model that reduces in 4 Dimensions to the Standard Model and Gravity. PREPRINT: PRINT-92-0227 [GEORGIA-TECH] in SLAC index and PRE 33611 in CERN index - approx.: 186 pages + 52 pages of comments..