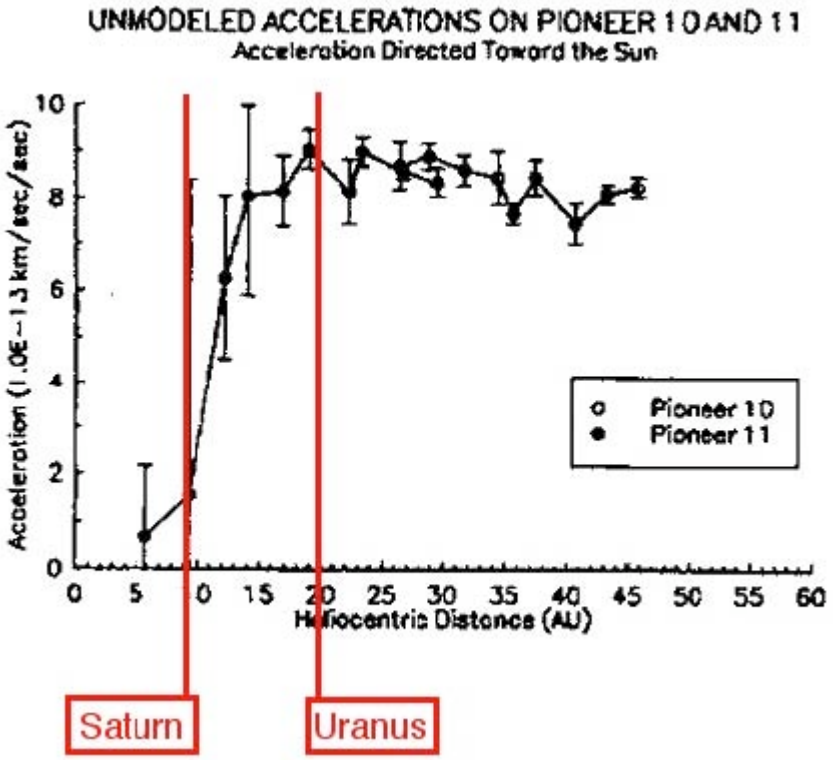
Our Conformal Keplerian Solar System

by Frank Dodd (Tony) Smith, Jr. – May-June 2009

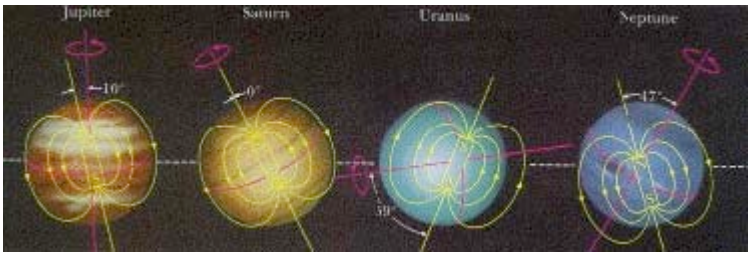
Jack Sarfatti provided a nice graph showing the unmodeled accelerations on Pioneer 10 and 11 vs. distance from the sun in AU.
Here is some stuff relating the Conformal Degrees of Freedom to the onset of the Pioneer anomaly by extending Kepler's ideas to the cuboctahedron, which (it is Buckminster Fuller's Vector Equilibrium) represents Conformal structure, the onset of which occurs at a phase transition (between the two phases of gravity, Minkowski and Conformal, that I. E. Segal described in his work on cosmology) at the onset of the Pioneer Anomaly.

This is a modification of that graph showing the orbits
of Saturn and Uranus.



AS you can see, the increase takes place between Saturn and Uranus,
two unusual planets (Saturn has a lot of rings, and the axis of Uranus has an unusual orientation (see next attachment (from Kaufmann's texbook "Universe")

with Jupiter, Saturn, Uranus, and Neptune spin axes shown in red):



Now, for something completely interesting:

Kepler showed that nesting the 5 regular polytopes gave a roughly accurate picture of the orbit radii of the 6 then-known planets:

Mercury
Octahedron
Venus
Icosahedron
Earth and Moon
Dodecahedron
Mars
Tetrahedron (the only one that is self-dual – it contains asteroid belt)
Jupiter
Cube
Saturn

What seems to be not so well known is that if you extend from the 5 by adding the cuboctahedron and the rhombic dodecahedron (and if you set the radius of the sphere inside the cuboctahedron to be inscribed within the projection of each square face) then you can extend Kepler's results to:

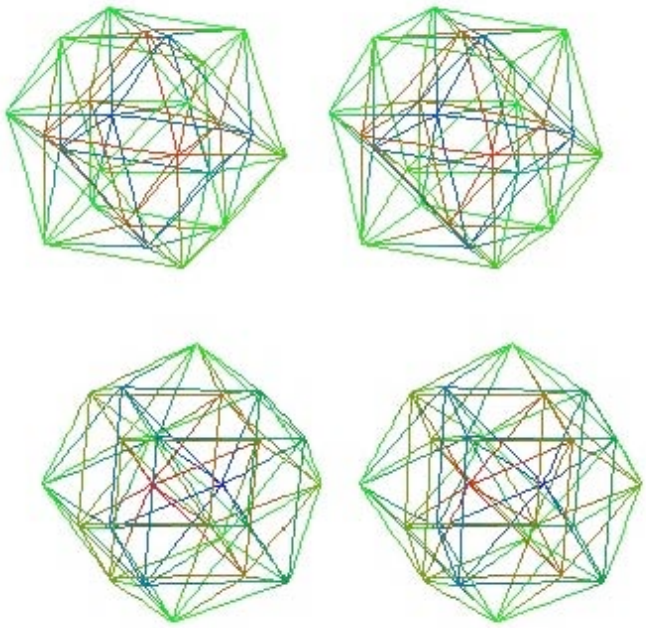
Saturn
cuboctahedron
Uranus
rhombic dodecahedron

Neptune

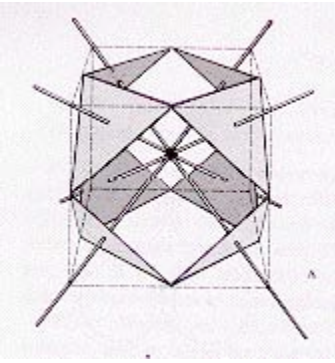
which completes the list of "real" planets in our solar system.

The cuboctahedron and rhombic dodecahedron can each be considered
as a central figure in a 24-cell,
the only type of polytope (if you consider ALL dimensions)
that was omitted from Kepler's original list.

The following two attachments show the cuboctahedron in the 24-cell
and another orientation of the 24-cell showing a rhombic dodecahedron
They are stereo pairs, with colors representing the 4th dimension,
blue = near ; green = middle ; red = far), so that the cuboctahedron
and the rhombic dodecahedron are shown in green:



As this attachment (from Fuller's book Synergistics) shows



the cuboctahedron has 4 axes that (although not orthonormal in 3 dimensions)
are projections of 4 orthonormal axes
in 4-dim space (where the 24-cell lives),
so that the cuboctahedron can be said to represent
the 3-dim sphere S3 in 4-dim space.

Since the Hopf fibration of the 3-sphere describes twistors
(see next attachment – from Penrose and Rindler's book Spinors and Spacetime)

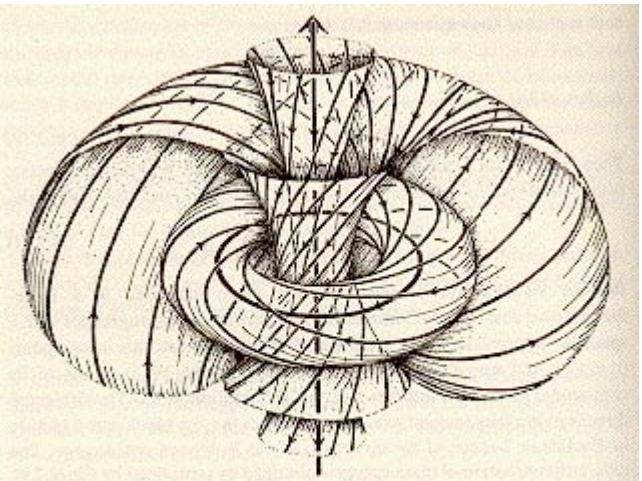
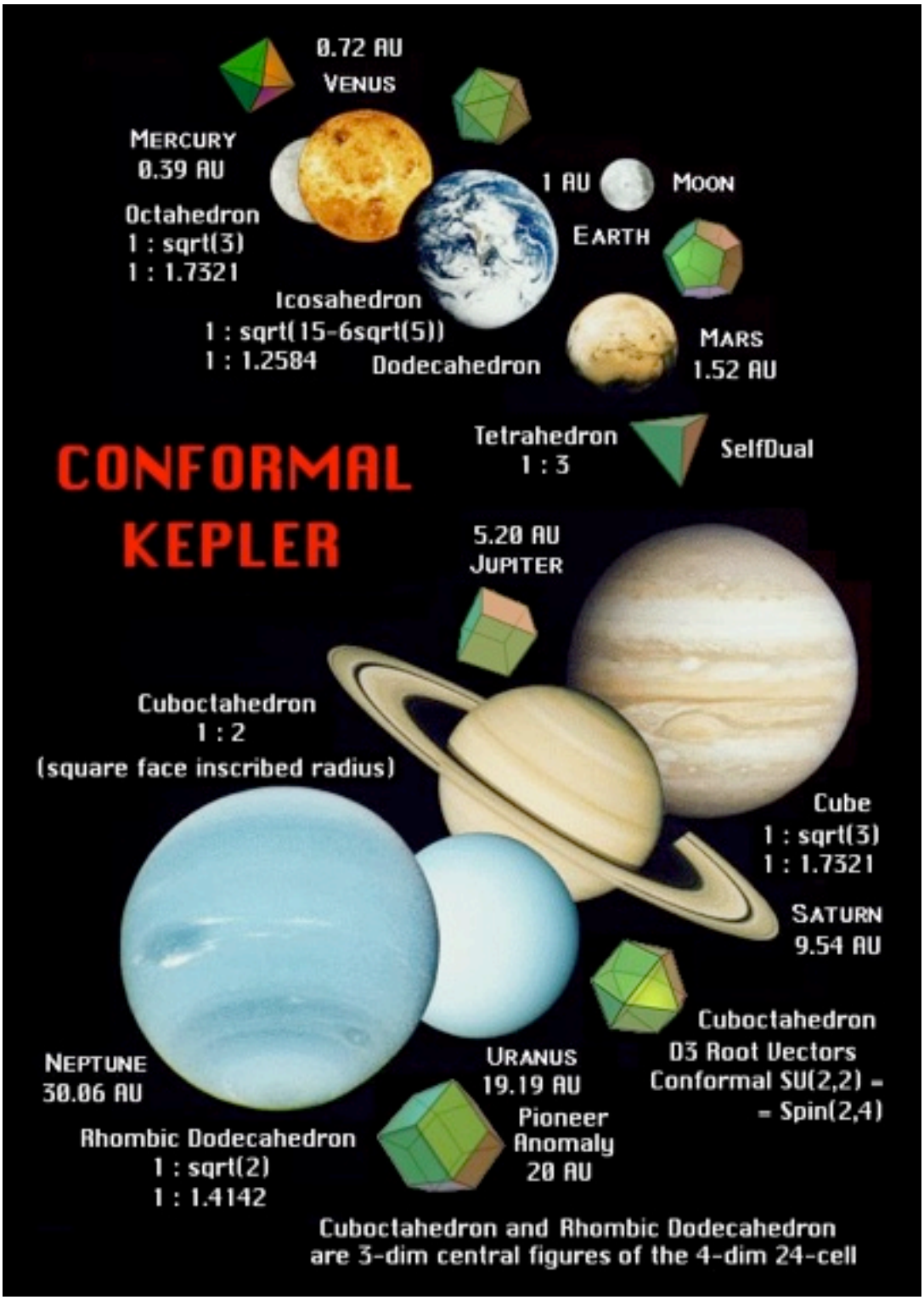


Fig. 6-3. For a general twistor Z^α, the flagpole directions of ω^A point, at any one time, in directions whose spatial projections are tangent to a family of circles, and one straight line, constituting stereographically projected Clifford parallels on S^3. As time progresses, the whole configuration moves with the speed of light in the negative direction of the straight line (i.e. in the projected direction of $\bar{\pi}^A \pi^A$).

and since twistors are well known to be represented in terms
of the Conformal Group Spin(2,4) = SU(2,2)
it is clear that the cuboctahedron represents the Conformal Degrees
of Freedom that come into play at the phase transition
which occurs at the distance from the sun represented
by the cuboctahedron in the generalized Kepler picture,
which is shown in this attachment
(planet images from aerospaceweb.org
and polyhedra images from wikipedia):


0.72 AU
VENUS
MERCURY
0.39 AU
Octahedron
1 : sqrt(3)
1 : 1.7321
1 AU
MOON
EARTH
Icosahedron
1 : sqrt(15-6sqrt(5))
1 : 1.2584
Dodecahedron
MARS
1.52 AU
Tetrahedron
1 : 3
SelfDual
CONFORMAL
KEPLER
5.20 AU
JUPITER
Cuboctahedron
1 : 2
(square face inscribed radius)
Cube
1 : sqrt(3)
1 : 1.7321
SATURN
9.54 AU
Cuboctahedron
D3 Root Vectors
Conformal SU(2,2) =
= Spin(2,4)
NEPTUNE
30.06 AU
URANUS
19.19 AU
Pioneer
Anomaly
20 AU
Rhombic Dodecahedron
1 : sqrt(2)
1 : 1.4142
Cuboctahedron and Rhombic Dodecahedron
are 3-dim central figures of the 4-dim 24-cell

As to Jack's questionabout "the physics of geometrodynamical (Einstein —> Weyl) fields",the two relevant domains are:

Inner – Minkowski – symmetry group Spin(1,3)
(which describes what Jack is calling the Einstein field)
gives a cosmological metric that I. E. Segal calls "Minkowski"
and described in his book
"Mathematical Cosmology and Extragalactic Astronomy" (Academic 1976) as:
Temporal evolution in Minkowski space is

H -> H + s I

Spin(1,3) has 3 spatial dimensions in which you have 5 types of polytope:
octahedron
icosahedron
dodecahdron
tetrahedron
cube

Outer – Conformal symmetry group Spin(2,4) = SU(2,2) = Conformal Group
(which describes what Jack calls the Weyl field – note that I do not
use the Mannheim-type Weyl conformal stuff, but instead use the
related 15-dimensional Conformal group that Segal uses and that
Penrose and Rindler discusse with respect to twistor theory in
their book "Spinors and Spacetime")
gives a cosmological metric that I. E. Segal calls "unispace" and
described in his book as:
unispace temporal evolution is

H -> (H + 2 tan(a/2)) / (1 – (1/2) H tan(a/2)) =
= H + a I + (1/4) a H^2 + O(s^2)

which is similar to the Quadratic Time Augmentation model that
according to Anderson et al in gr-qc/0104064 "... fits ...fairly well ..."
and is given by (in their notation):

ET -> ET + (1/2) a_ET ET^2

Spin(2,4) has 4 spatial dimensions in which you have another
type of polytope – the self-dual 24-cell
which has two (dual) central 3-dim figures:
cuboctahedron
rhombic dodecahedron

So,
the phase boundary in question is between
the Inner Minkowski phase
and
the Outer Conformal Segal Unispace phase
of
the metric of spacetime.

Observationally, the boundary is between Saturn and Uranus
which coincides
with the cuboctahedron in the expanded Kepler polytope picture,
so
it seems more than mere quaint numerological coincidence
that the cuboctahedron's parent polytope, the 24-cell, lives
in the 4-dim space of the Spin(2,4) Conformal group.

From: Tony Smith <f75m17h@bellsouth.net> Date: May-June 2009

To: JACK SARFATTI <sarfatti@pacbell.net>

Jack you say that I need to "explain my reasoning ...
and show exactly how to compute the numbers ...".

Universe Expansion Acceleration = Ua = 8 x 10^(-8) cm/sec^2
(known from cosmology)

Pioneer Anomaly Acceleration = Pa = Ua
(observed and explicitly stated by Anderson et al in gr-qc/0104064)

Wesson and Sirag have studied (and Jack has discussed)
the astrophysics angular momentum problem
which is
that total angular momentum is related to mass
by a clearly defined relationship (see attached image
from a stardrive.org web page)

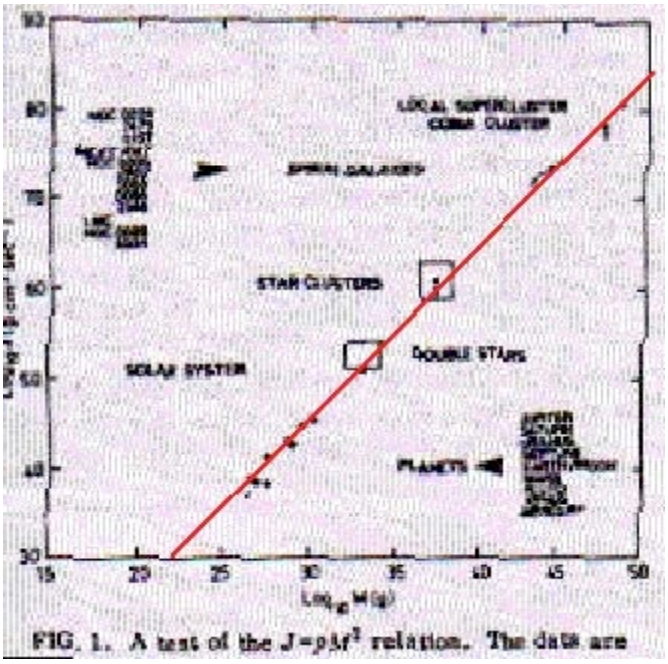


FIG. 1. A test of the $J=pM^2$ relation. The data are

In addition to the angular momentum - mass relationship,

Blackett and Sirag have studied (and Jack has discussed)
a log-log linear relationship for astrophysical bodies
between angular momentum and magnetic dipole moment
(see attached image from Sirag's 1979 Nature paper)

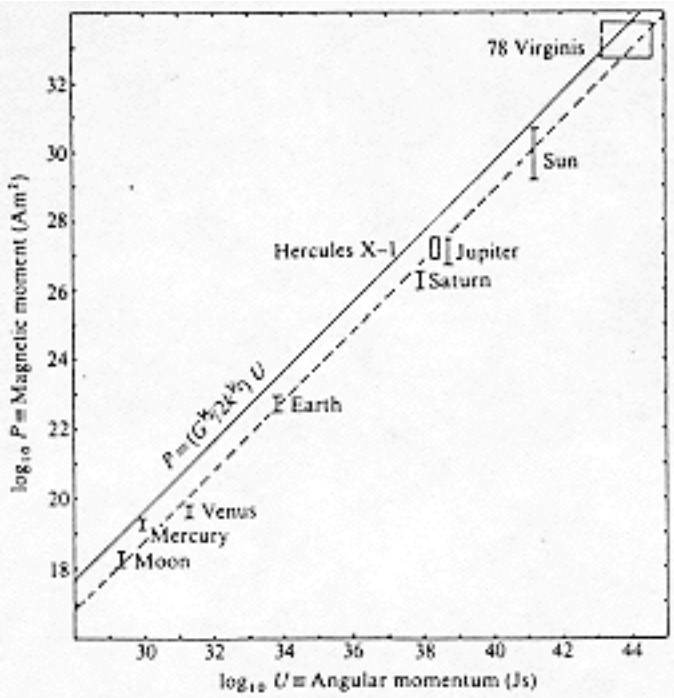


Fig. 1 Magnetic moment/angular momentum diagram. Comparison of gravi-magnetic hypothesis line, $P = (G^{1/2}/2k^{1/2})\,U$, with P and U for massive rotating bodies. The dotted line is the regression line for the data. In $\log_{10}$ form, and in SI units: $G^{1/2}/2k^{1/2} = -10.37$; the mean P/U is -11.13 ± 0.42.

So,
there is a (mysterious to conventional astrophysicists) connection
among static mass, angular momentum, and magnetic dipole moment,
which
suggests the existence of a gravitational/magnetic force
that might be carried by a particle – call it the Wesson particle.

Since it is related to static mass gravity,
and since the strength of gravity is given by (1 / Mass_Planck)^2
the Wesson particle that should define the Wesson force
should have a mass related to that of the pure-static-mass Planck mass.

Since the Wesson particle has an electromagnetic component
(to account for the magnetic dipole relationships)
its mass should be the Planck mass reduced by a factor alpha = 1/137.

Therefore,
the Wesson force should have strength given by
(1 / Mass_Wesson)^2 = (1/alpha)^2 x (1 / Mass_Planck)^2 =
= (1/alpha)^2 x Ordinary Static-Mass Gravity

Further,
the Generalized Gravity Effect of
something with a high ratio of angular momentum to mass
(such as the Pioneer spacecraft)
should be enhanced by the Wesson/Planck factor of (1/alpha)^2
so
that the effective Generalized Gravity strength of the Pioneer spacecraft
should be (1/alpha)^2 x the Ordinary Gravity acceleration due to
the Static-Mass Sun.

Since the Static-Mass Sun Ordinary Gravity acceleration is
G x Msun / R^2 where R is the distance to the Sun,
the Phase Transition for Pioneer should take place at the
distance from the Sun such that

Pioneer Acceleration x (1/alpha)^2 = G x Msun / R^2

which is given by

8 x 10^(-8) x 137 x 137 = 6 x 10^(-8) x 2 x 10^33 / R^2

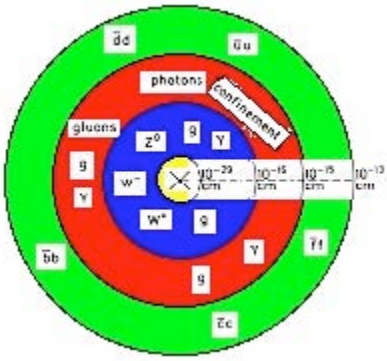
so that

R^2 = 15 x 10^32 / 137^2 cm^2 and R = 0.028 x 10^16 cm

Since 1 AU = 1.5 x 10^13 cm
R = 2.8 x 10^14 / 1.5 x 10^13 = 18.7 AU or roughly 20 AU
so that
THE ORBIT OF URANUS IS ROUGHLY THE PIONEER PHASE TRANSITION.
Note that this ties together a lot of loose ends in
conventional astrophysics
the Pioneer Anomaly
the Angular Momentum – Mass Relation
the Angular Momentum – Magnetic Dipole Relation

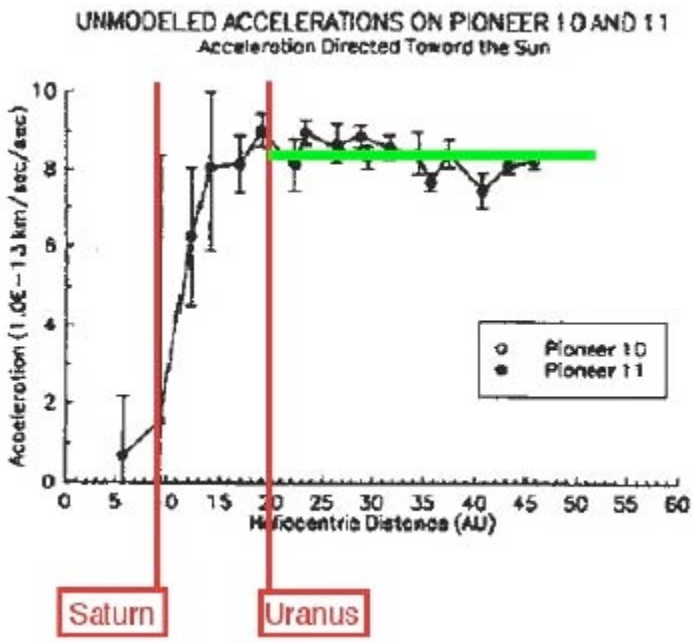
by using Conformal gravity-magnetism stuff,
which conformal stuff also explains
the Dark Energy : Dark Matter : Ordinary Matter ratio.

This also suggests using spinning and/or electromagnetic
techniques to exploit Dark Energy in Earth Laboratory techniques,
as in the work of Chiao and of Tajmar, de Matos et al.



with mass Mplanck / 137 = 1.2 x 10^19 / 137 = 8.8 x 10^16 GeV
and an onion-layer structure described by Borner in his book
The Early Universe (Springer-Verlag 1988): "...
Near the center (about 10^(-29) cm) there is a GUT symmetric vacuum.
... At about 10^(-16) cm, out to the Yukawa tail ... exp(– Mw r) the
field is the electroweak colour field of the (3,2,1) standard model
... at ...[10^(-15) cm]... it is made up of photons and gluons, while
... at the edge [10^(-13) cm] there are fermion-antifermion pairs
... Far beyond nuclear distances it behaves as
a magnetically-charged pole of the Dirac type. ...".

Note that beyond the Pioneer Phase Transition Boundary
the Pioneer Anomalous Acceleration is constant at 8 x 10^(-8) cm/sec^2
(graph adapted from one similar to Fig. 2 of gr-qc/0411077 by Nieto et al)



as is consistent with its fundamental Conformal Cosmological nature related to the Dark Energy Expansion of our Universe.

In light of Carlos Castro's June 2009 paper
“The Clifford Space Geometry behind the Pioneer and Flyby Anomalies”
it seems to me that there are two fundamental boundaries in our Solar System related to gravity and gravitomagnetism:

1 - a Gravity-MASS coupling with boundary at Uranus,
related to the Pioneer anomaly;
and
2 - a Gravity-SPIN coupling with boundary at Earth,
related to Fly-By anomalies.

1 -
The Orbit-of-Uranus boundary (around 20 AU) is the location of a phase transition of gravitational coupling to MASS
such that the Pioneer anomalous acceleration appears outside that phase transition boundary,
and
that the gravitational acceleration of mass at that distance from the sun is
about 1.5 x 10^(-3) cm / sec^2 which is about 137^2 x 8 x 10^(-8) cm/sec^2
where
8 x 10^(-8) cm/sec^2 is the Pioneer anomalous acceleration
and
8 x 10^(-8) cm/sec^2 / c^2 = 1 / (10^28 cm)

2 -
There is also a phase transition of gravitational coupling to SPIN
that is located at the Orbit-of-Earth boundary (around 1 AU)
such that
the gravitational coupling to spin at that distance from the sun is

$(2 G M_{sun} / c^2 r_{es}) (w_{espin}) (w_{espin} r_{er}) (1/c^2) =$

$= 1 / (1.32 \times 10^{28} \text{ cm})$

where
Schwarzchild Radius of Sun = $2 G M_{sun} / c^2$ with units cm
distance earth to sun = r_{es} with units cm
earth axis spin = $w_{espin} = 2 \pi / 24 \times 3600$ with units $s^{(-1)}$
earth radius = r_{er} with units cm
G has units $cm^3 / g s^2$

$(2 G M_{sun} / c^2 r_{es})$ is the dominant gravitational curvature tensor component that couples by the Papapetrou equations to the spin tensor at Orbit-of-Earth distance from the sun

(w_{espin}) is the spin for Earth rotation, a bivector in terms of the Clifford algebra of 4-dimensional physical spacetime

$(w_{espin} r_{er})$ is the angular part of Earth velocity with r_{er} physically representing something like a radius of gyration

$1/c^2$ is a conversion factor between $1 / (10^{28} \text{ cm})$ and
the Pioneer anomalous acceleration $8 \times 10^{(-8)}$ cm/sec^2

The SPIN gravitational coupling can be understood in terms of the Papapetrou equations, which Carlos Castro describes in terms of Bill Pezzaglia's Clifford Polydimensional Lagrangian construction in gr-qc/9912025,
but which can also be described in a more conventional Lagrangian construction in gr-qc/0505021 by M. Leclerc who said:

"… We present a simple method to derive the semiclassical equations of motion for a spinning particle in a gravitational field … starting with a simple Lagrangian …

in general relativity, one usually substitutes the Schwarzchild metric into the Lagrangian
$L = m\, g_{ik}\, u^i\, u^k$
The aim … is … to find suitable generalizations of $L = m\, g_{ik}\, u^i\, u^k$ that allow for the description of particles with spin …
classical spin … is … distinguish[ed] … from the intrinsic spin which is of quantum mechanical origin
…
In the … Papapetrou .. equations, there is no torsion involved. …
Torsion effects will only arise if we consider particles with intrinsic spin .
…
for the coordinate of a mass element of the body we can write … $x^i = X^i + r^i$
and for the velocity of the same mass element $u^i = V^i + w^i{}_k\, r^k$
with antisymmetric w^{ik} and $V^i = dX^i / dt$
Just as in the original work of Papapetrou we suppose $r^0 = 0$ in a certain reference frame.
If we set in the same frame $w^{i0} = 0$, the quantity w^{ik} is clearly the angular velocity in four dimensional form … Consequently, w^{ik} is related to the angular momentum (the classical spin) through $S^{ik} = I\, w^{ik}$
where I is the moment of intertia of the body … $(1/2)\, I\, g^{mn} = -$ INTEGRAL $r^m\, r^n\, dm$.
… leave the frome where $r^0 = w^{i0} = 0$ and consider S^{im} as a four dimensional antisymmetric tensor … we can … write …
$L = (m/2)\, g_{ik}\, u^i\, u^k - (1/2)\, G^l{}_{km}\, S_l{}^k\, u^m$
…
the Euler-Lagrange equations are …
$m\, Du_m = -(1/2)\, (G^l{}_{ki,m} - G^l{}_{km,i})\, S_l{}^k\, u^i + (1/2)\, G^l{}_{km}\, S_l{}^k$
… in a coordinate system where $G^l{}_{ik} = 0$ …[the preceding equation].. can be written as

$m\, D\, u_m = -(1/2)\, R^l{}_{kmi}\, S_l{}^k\, u^i$

… to be covariant, we have to require in addition

$D\, S^{ik} = 0$

…[the last two equations]… are in agreement with the Mathisson-Papapetrou equations in the required order of precision, i.e. if one identifies momentum with $m\, u^i$ … “.

In Leclerc’s formulation:
the curvature part is $-(1/2)\, R^l{}_{kmi}$
the spin part is $S_l{}^k$ since $S^{ik} = I\, w^{ik}$
the angular velocity part is in u^i since $u^i = V^i + w^i{}_k\, r^k$

Sirag (see his image on page 10 hereof)
and Vasiliev in astro-ph/0002048 whose Fig. 2 is

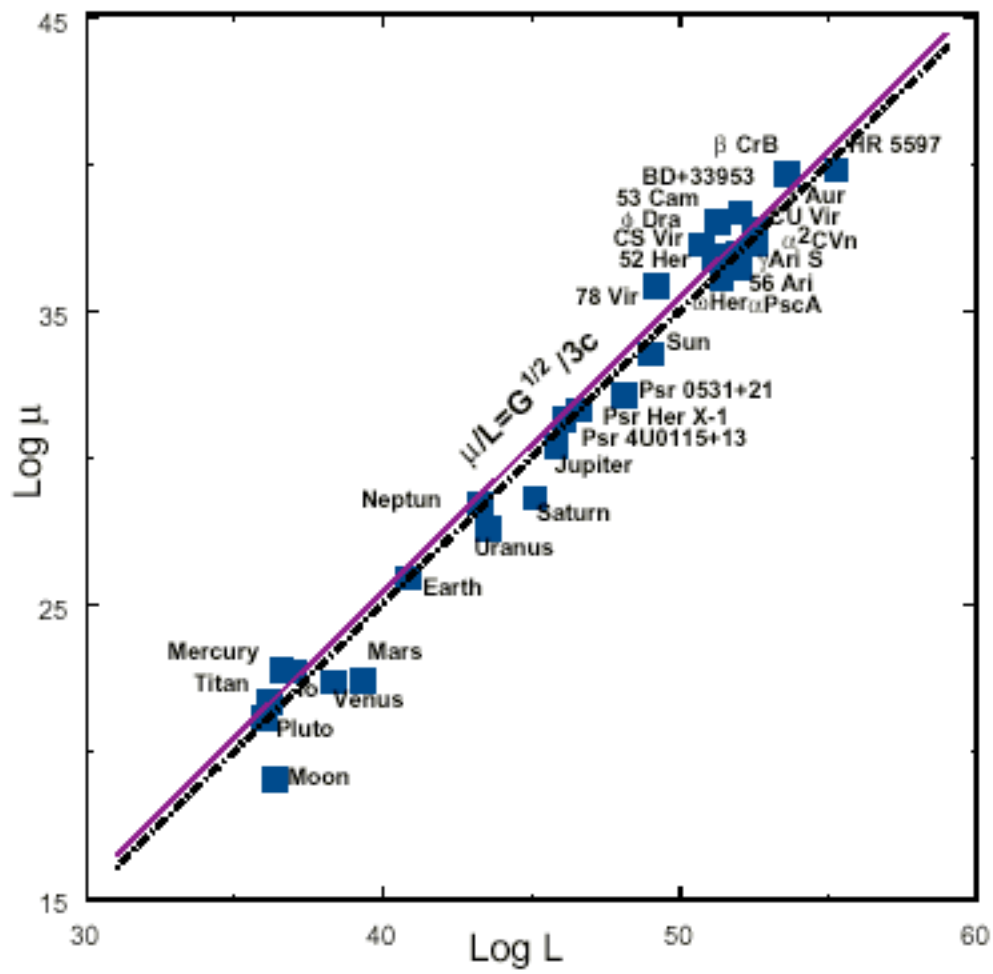
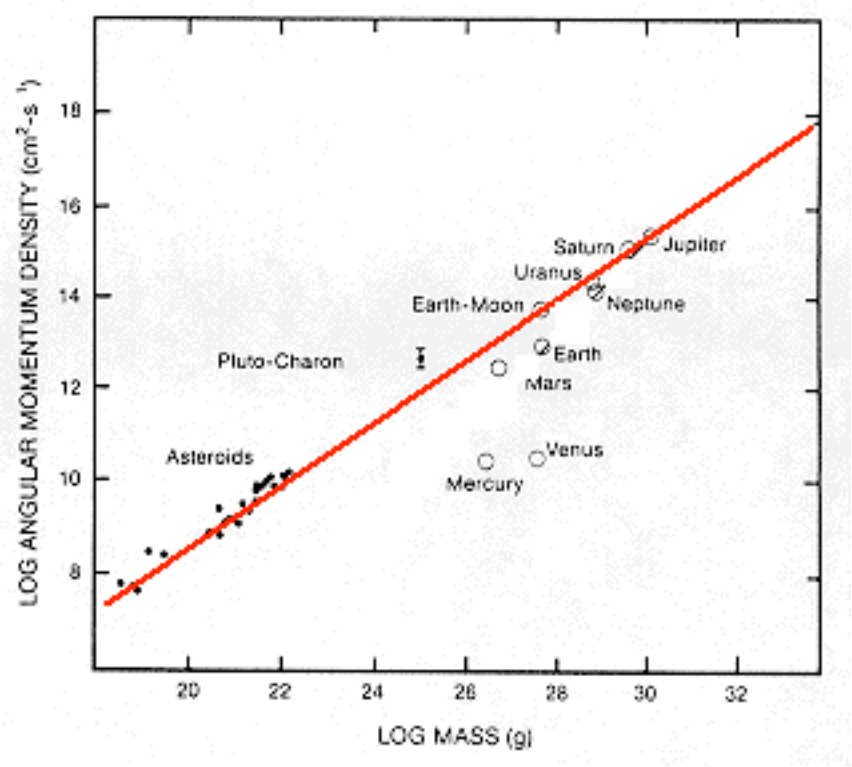


Figure 2: The observed values of the magnetic moments of celestial bodies vs. their angular momenta. On the ordinate, the logarithm of the magnetic moment over $Gs \cdot cm^3$ is plotted; on the abscissa the logarithm of the angular momentum over $erg \cdot s$ is shown. The solid line illustrates Eq.(34). The dash-dotted line is the fitting of the observed values.

have observed that
spin angular momentum is proportional to magnetic moment for planets, stars, and pulsars
so it is possible that electromagnetic processes at the time of formation of our Solar System and of Earth itself dictated that our Earth have the particular size and spin giving the spin tensor described above, which in turn gives the relationship to 1 / (10^28 cm) .

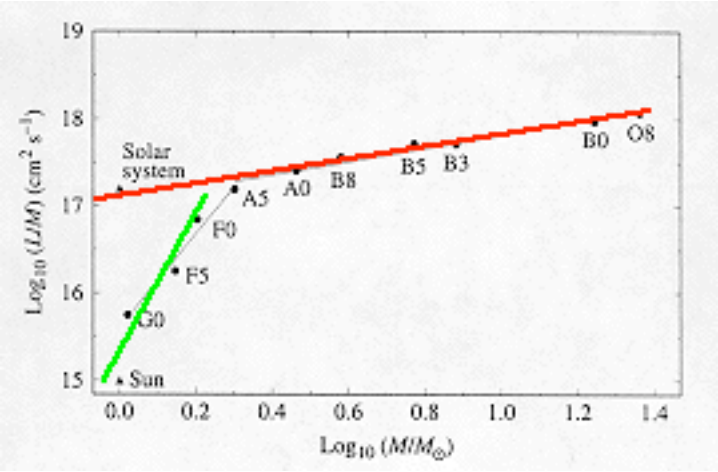
It may be that when the Early Earth formed somewhat over 4 x 10^9 years ago,
it carried more angular momentum than its magnetic moment could support,
in which case the electromagnetic formation processes would have spun off excess mass to form the Moon, which carries no significant magnetic moment but does carry a lot of angular momentum, so that the Earth-Moon system as a whole would be in line with the Mass – Angular Momentum Density relationship that generally holds from Asteroids through the Gas Giant Planets, as shown by



this chart (red line added by me) from the book "Solar System Evolution" by Stuart Taylor Ross (Cambridge 1992).

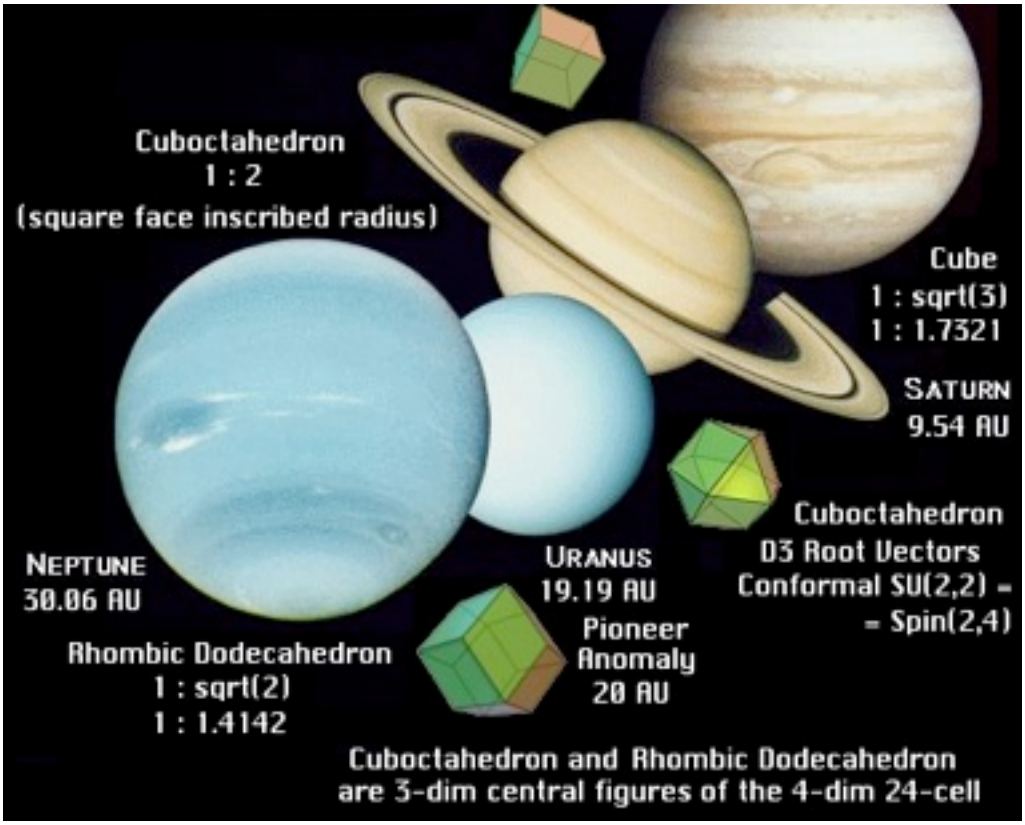
Note that the Planets inside the Earth Orbit Gravity-SPIN coupling phase boundary, that is, Venus and Mercury, are far off the red line and seem to be, for their mass, very deficient in Angular Momentum.

The same Mass – Specific Angular Momentum relationship holds for our Solar System as a whole and for Stars at least as massive as A5, as shown by the red line (added by me) with the same slope (the apparently different angle is due to different scale ratio of x and y axes) on



this chart from the book "An Introduction to Modern Astrophysics" by Carroll and Osterlie (Addison-Wesley 1996). As the green line (added by me) shows, the Sun alone and Stars less massive than A5 have a lesser Specific Angular Momentum, which may indicate that they have Planets that carry some Specific Angular Momentum.

With respect to our Solar System, outside the Uranus Orbit Gravity-MASS coupling phase boundary



we have full Conformal Gravity of the Keplerian Cuboctahedron – Rhombic Dodecahedron Region and the Outer Planets Uranus and Neptune,

while at the Earth Orbit Gravity-SPIN coupling phase boundary at the Keplerian Icosahedron – Dodecahedron Border

we have the Earth-Moon System
and
inside that we have the Inner Planets Venus and Mercury that seem to be very deficient in Angular Momentum.

In their paper "Stabilization of the Earth's obliquity by the Moon" (Nature 361 (18 Feb 1993) 615-617) J. Laskar, F. Joutel and P. Robutel said:
"… the Earth … obliquity is essentially stable, exhibiting only small variations of about 1.3 degrees around the mean value of 23.3 degrees. But if the Moon were not present … the chaotic zone would then extend from nearly 0 degrees up to about 85 degrees …[and]… large variations in obliquity resulting from its chaotic behaviour might have driven dramatic changes in climate …".

In their paper "The Earth-Moon System and the Dynamical Stability of the Inner Solar System" (The Astrophysical Journal, 116:2055-2057, 1998 October) Kimmo Innanen, Seppo Mikkola and Paul Wiegert said:
"… the Earth-Moon system (EM) … appears to play a kind of "gravitational keystone" role in the terrestrial precinct, for without it, the orbits of Venus and Mercury become immediately destabilized. …
EM is … suppressing or "damping out" a secular resonance driven by the giant planets near the Venusian heliocentric distance … That Venus should exist very close to the exact heliocentric

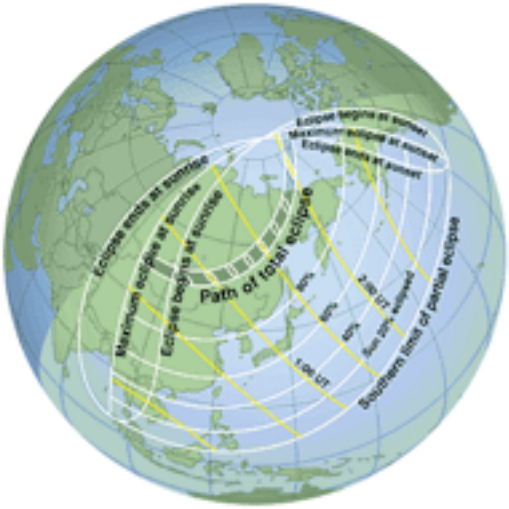
distance of this resonance may … not … be just a coincidence. …
Mercury's orbit is coupled to that of Venus …".

In their paper "Existence of collisional trajectories of Mercury, Mars and Venus with the Earth" (Nature 459 (11 June 2009) 817-819) J. Laskar and M. Gastineau said:
"… we report numerical simulations of the evolution of the Solar System over 5 Gyr
…
without the Moon or relativistic conditions … In 60% of the solutions, we observed large increases in Mercury's eccentricity, beyond 0.9
…
In … simulations … including contributions from the Moon and general relativity … one per cent of the solutions lead to a large increase in Mercury's eccentricity … large enough to allow collisions with Venus or the Sun … in one of these high-eccentricity solutions, a subsequent decrease in Mercury's eccentricity induces a transfer of angular momentum from the giant planets that destabilizes all the terrestrial planets about 3.34 Gyr from now, with possible collisions of Mercury, Mars or Venus with the Earth …".

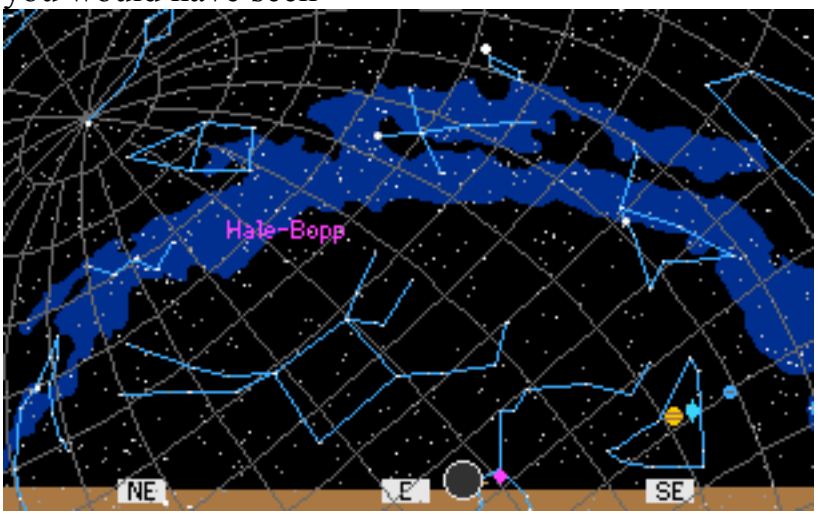
The Extra-Solar Planetary System of 55 Cancri is also consistent with
the Conformal Kepler picture (whch itself is consistent with the Titius -Bode Law),
according to Arcadio Poveda and Patricia Lara, who in arXiv 0803.2240 [astro-ph] said:
"... The recent discovery of a fifth planet bound to 55Cancri ... motivated us to investigate if this exo-planetary system fits some form of the Titius-Bode(TB) law.
We found that a simple exponential TB relation reproduces very well the five observed majorsemi-axis, provided we assign the orbital n=6 to thelargest a.This way of counting leaves empty the position n=5, a situation curiously reminiscent of TB law in our planetary system, before the discovery of Ceres. ... This equation "predicts" the existence of a fifth planet at about 2AU and, with less certainty, a seventh one at about 15AU. ...".

The physics of the Conformal Kepler picture (using its consistency with the TB law)
is discussed by Vladan Pankovic and Aleksandar-Meda Radakovic in
arXiv 0903.1732 9 [physics.gen-ph] where they said: "... we shall demonstrate that third Kepler law, or, corresponding equilibrium condition between centrifugal and Newtonian gravitational force, implies that planet orbital momentum becomes effectively a function of the planet distance ... Then,approximation of the planet distance by its first order Taylor expansion over planet orbital momentum holds an exponential form corresponding to Titius-Bode rule. ... Physically, it simply means that, in the linear approximation, "quantized" planets orbital momentums do a geometrical progression. ...".

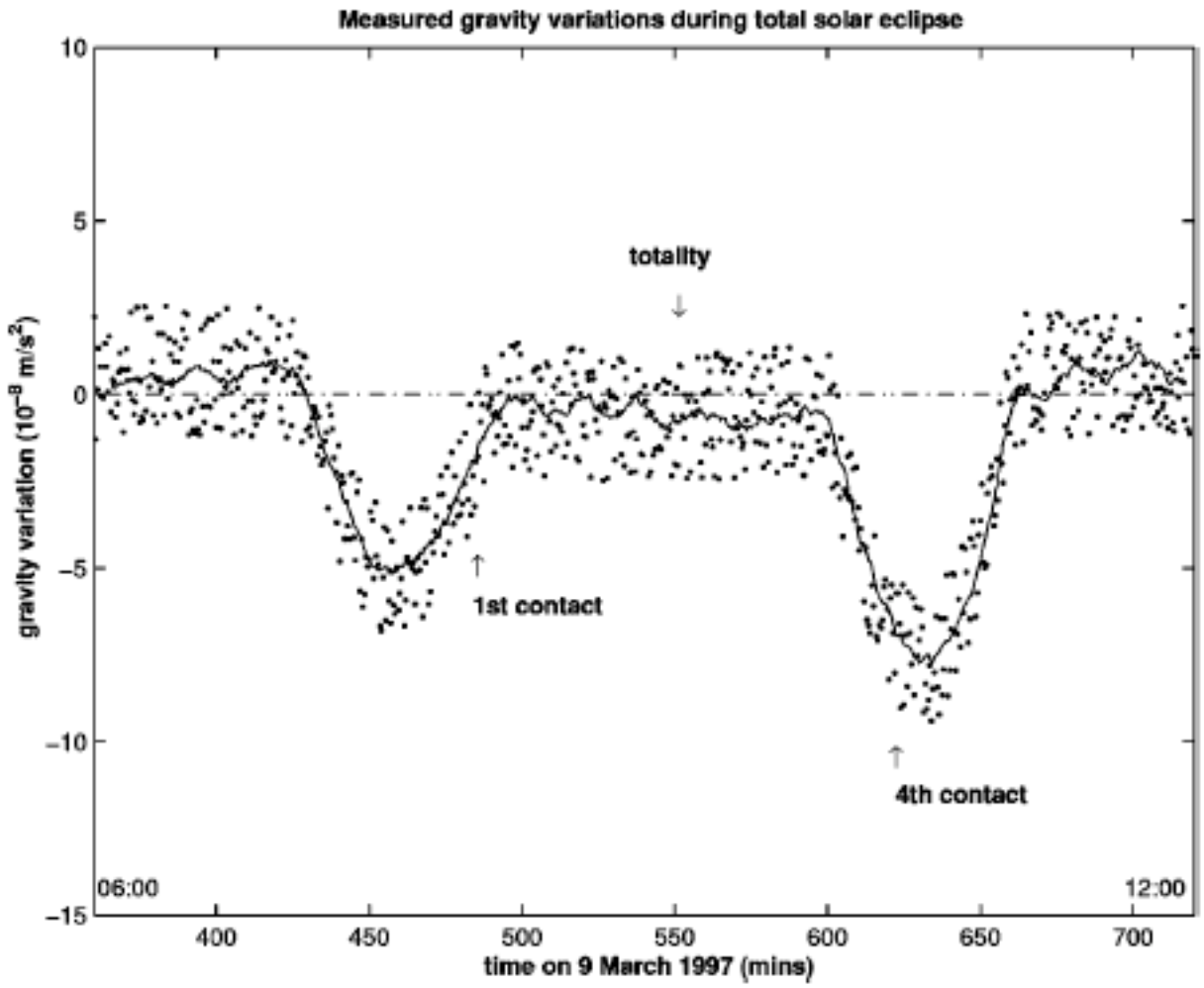
On 9 March 1997 a total solar eclipse moved fron Gora Belukha (the highest Altai Mountai, 14,783 feet, also known as Kunlun Shan (home of Xi Wang Mu, Queen of the West) and as Su Meru (home of Indra))



to near the North Pole. Looking East at dawn on 9 March 1997 from the top of Gora Belukha you would have seen



Wang, Yang, Wa, Guo, Liu, and Hua, in Phys. Rev. D 62, 04110, said:
"… we conducted a precise measurement of the vertical gravity variations during a total eclipse of the Sun on 9 March 1997 in China … in Moho, Helongjiang province, China … which lies in the center of the shadow of the totality during the eclipse. …



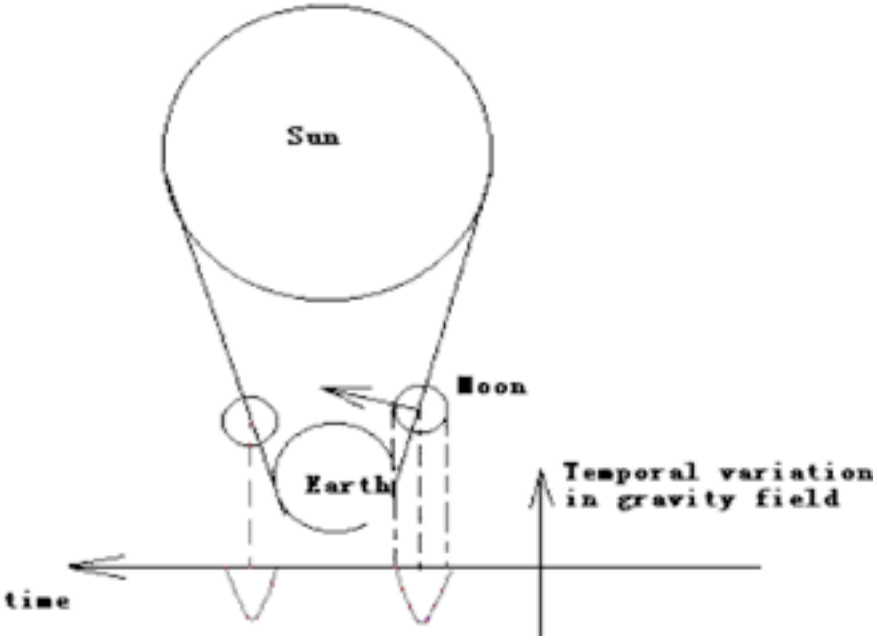
… there exist two regions with significant gravity decrease … One … with a maximum significant decrease of 6.0 +/- 2.5 mugal …and another … with a maximum change of 7.0 +/- 2.7 mugal … These two changes were quite closely related to the timing of eclipse phases of first contact and fourth (last) contact. … The changes are quite significant and they are not the effect of temperature and pressure changes. …".

Tang, Wang, Zhang, Hua, Peng, and Hu in
"Gravity Effects of Solar Eclipse and Inducted Gravitational Field",
American Geophysical Union, Fall Meeting 2003, abstract #G32A-0735, said:
"… We have tried to explain those anomalies by the induced gravitational field. …".

Chen Shouyuan in a paper
"Induction Gravity From Temporal Variations in Gravity Field During Total Solar Eclipse"
http://forrootbasic.51.net/wytk/xtwzh/chenshouyuan/inductiongravity/igtvgfdtse.htm

said:
"… The temporal variation in the gravity field durng the total eclipse of 1997 March 9 occur with the onset of the solar eclipse and the departure ….

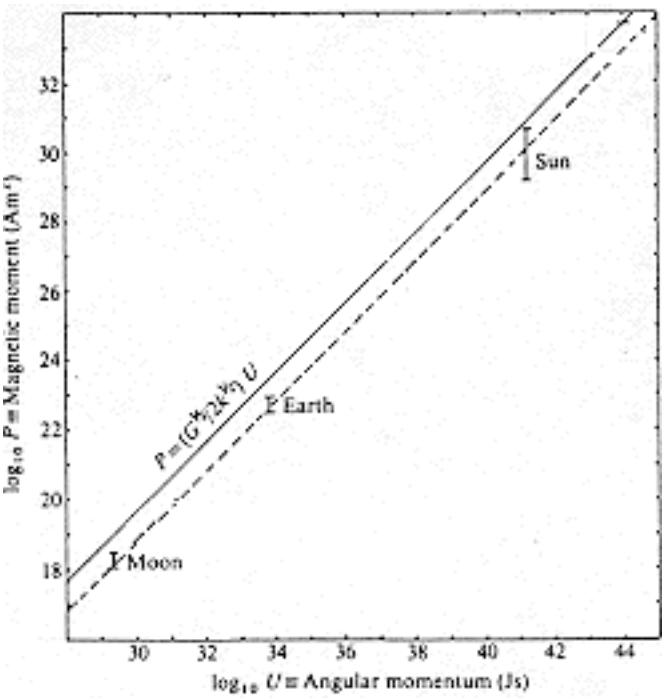
The … variation … maybe similar to Faraday's electromagnet induction …



…".

Saul-Paul Sirag in Fig. 1 of his paper "Gravitational Magnetism" (Nature 278 (5 April 1979) 535-538) showed that

the Sun, Earth, and Moon



have a log-log proportionality of Rotational Spin Angular Momentum to
Magnetic Dipole Moment which suggests
that the Sun, Earth, and Moon are all affected by the gravitational/magnetic force
carried by the Wesson particle whose mass is the Planck mass reduced by a
factor alpha = 1/137,
so that the Wesson force strength is given by
(1 / Mass_Wesson)^2 = (1/alpha)^2 x (1 / Mass_Planck)^2 =
= (1/alpha)^2 x Ordinary Static-Mass Gravity
and
that the variation in the gravity field during the total eclipse of 1997 March 9 was indeed a
Wesson force Faraday induction effect.

In “Gravitational Induction” 0803.0390 [gr-qc] Bini, Cherubini, Chicone, and Mashhoon said: “… in the linear perturbation approach to gravitoelectromagnetism (GEM), one recovers the Maxwell equations for the GEM field, but the corresponding Lorentz force is recovered, to first order in v/c, only when we deal with a stationary GEM field. ... time-varying GEM fields have been implicitly considered by many authors … some gravitational Faraday experiments were proposed … based on the existence of gravitational induction in analogy with electrodynamics. The purpose of the present paper is to show explicitly that general relativity does indeed contain induction effects; these turn out to be, despite the differences that have been mentioned, on the whole closely analogous to electromagnetic induction effects.

…

It is possible to define the gravitoelectric field $\mathbf{E}$ and the gravitomagnetic field $\mathbf{B}$ in close analogy with electrodynamics

$$\mathbf{E} = -\nabla\Phi - \frac{1}{c}\partial_t\left(\frac{1}{2}\mathbf{A}\right), \qquad \mathbf{B} = \nabla\times\mathbf{A}\,. \tag{1.6}$$

It follows from these definitions that

$$\nabla\times\mathbf{E} = -\frac{1}{c}\partial_t\left(\frac{1}{2}\mathbf{B}\right), \qquad \nabla\cdot\left(\frac{1}{2}\mathbf{B}\right) = 0, \tag{1.7}$$

while the gravitational field equations imply

$$\nabla\cdot\mathbf{E} = 4\pi G\rho, \qquad \nabla\times\left(\frac{1}{2}\mathbf{B}\right) = \frac{1}{c}\partial_t\mathbf{E} + \frac{4\pi G}{c}\mathbf{j}. \tag{1.8}$$

…

assuming that the source has gravitoelectric charge $Q_E = GM$ and gravitomagnetic charge $Q_B = 2GM$. Moreover, a test particle of inertial mass m has gravitoelectric charge $q_E = -m$ and gravitomagnetic charge $q_B = -2m$ in this convention. The signs of (q_E, q_B) are opposite to those of (Q_E, Q_B) due to the attractive nature of gravity; furthermore, the ratio of gravitomagnetic charge to the gravitoelectric charge is always 2, as the linear approximation of general relativity involves a spin-2 field. This circumstance is consistent with the fact that the ratio of the magnetic charge to the electric charge of a particle is unity in Maxwell's spin-1 electrodynamics. We note that the magnetic charge employed here is different from the magnetic monopole strength, which is always strictly zero throughout this work.

Given Maxwell's equations for the electromagnetic field, Faraday's law of induction simply follows, for instance, from the consideration of the temporal variation of the magnetic flux linking a static closed circuit. A similar approach in the GEM case would fail, however, as the line integral of $\mathbf{E}$ along the closed circuit does not in general correspond to the work done by the gravitational field of the source. This is the crucial point

…

Consider next a spacetime metric of the GEM form with potentials

$$\Phi = \frac{GM}{r}, \qquad \mathbf{A} = \frac{G}{c}(J_0 + J_1 t)\frac{\hat{\mathbf{J}} \times \mathbf{x}}{r^3}\ , \tag{2.7}$$

where the magnitude of the proper angular momentum of the source varies linearly with time, i.e. $\mathbf{J}(t) = (J_0 + J_1 t)\hat{\mathbf{J}}$. Henceforth we assume that $\hat{\mathbf{J}} = \hat{\mathbf{z}}$ and $J_0 \geq 0$.

...

The special nonstationary spacetime given by potentials 2.7 involves a static gravitoelectric field and a linearly time-varying gravitomagnetic field

$$\mathbf{E} = \frac{GM\mathbf{x}}{r^3} - \frac{G}{2c^2}\frac{\dot{\mathbf{J}} \times \mathbf{x}}{r^3}, \tag{3.1}$$

$$\mathbf{B} = \frac{G}{c}(J_0 + J_1 t)\frac{1}{r^3}[3\,(\hat{\mathbf{J}} \cdot \hat{\mathbf{x}})\hat{\mathbf{x}} - \hat{\mathbf{J}}]\ . \tag{3.2}$$

Thus the gravitational analogue of the displacement current is zero in this case. To first order in $\mathbf{v}/c$, the analogue of the Lorentz force law is given by

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -m\boldsymbol{\mathcal{E}} - \frac{2m}{c}\mathbf{v} \times \mathbf{B}, \tag{3.3}$$

where $\boldsymbol{\mathcal{E}}$ can be expressed as

$$\boldsymbol{\mathcal{E}} = \frac{GM\mathbf{x}}{r^3} - \frac{2G}{c^2}\frac{\dot{\mathbf{J}} \times \mathbf{x}}{r^3}\ . \tag{3.4}$$

The distinction between $\mathbf{E}$ and $\boldsymbol{\mathcal{E}}$ in general relativity is the root of the difference between the electromagnetic and gravitational inductions. A free particle initially at rest picks up an azimuthal speed due to the force term $(2m/c)\partial_t\mathbf{A}$ in Eq. 3.3

...

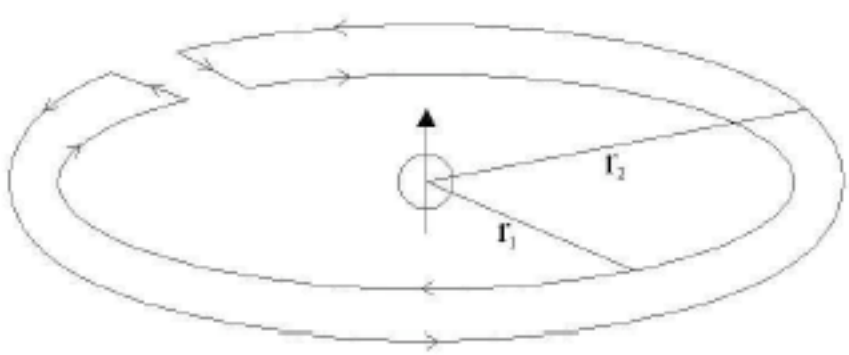


Figure 1. Schematic diagram of the closed equatorial annular loop for a thought experiment that illustrates gravitational induction. The loop is assumed to be sufficiently far from a nonstationary rotating source.

...

gravitomotive force (gmf) to be

$$\mathcal{G} = \oint \boldsymbol{\mathcal{E}} \cdot \mathrm{d}\boldsymbol{\ell} \,, \tag{3.8}$$

where $\boldsymbol{\mathcal{E}}$ is given by Eq. 3.4 since the $\mathbf{v} \times \mathbf{B}$ term does not contribute to the work. Thus the gravitomotive force is the circulation of $\boldsymbol{\mathcal{E}}$ rather than $\mathbf{E}$. Calculating the gmf for the loop in Figure 1, we find

$$\mathcal{G} = \frac{4\pi G \dot{J}}{c^2} \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \,. \tag{3.9}$$

...

Thus the gravitational analogue of Faraday's law of induction turns out to be

$$\mathcal{G} = -\frac{2}{c} \frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t} \,. \tag{3.10}$$

the actual induced current

is evident from the factor of four difference between the coefficients of $\partial_t \mathbf{A}$ in Eqs. 3.4 and 3.1 these considerations show theoretically that gravitational induction exists and — within the linear GEM framework — is closely analogous to the Faraday law of induction in electrodynamics.

...

It remains to show that the direction of the induced current is such as to oppose the change that caused it. Let us first remark that for a long straight line supporting a mass current I, Eq. 1.8 implies that the gravitomagnetic field has closed circular field lines of radius r around the line current as in electrodynamics; moreover, the magnitude of the field is $4GI/(cr)$ and its direction follows from the usual right-hand rule. In fact, this is the gravitational analogue of the Biot-Savart law of electrodynamics. For a moving test particle of mass m, the gravitoelectric charge is $-m$; hence, the direction of its current is *opposite* to its velocity. As the gravitomagnetic field increases ($\dot{J} > 0$), the fluid (in the tube in Figure 1) begins to co-rotate from rest due to the induced azimuthal acceleration $(2/c)\partial_t \mathbf{A}$. The speed of this co-rotation is larger in the inner circle than in the outer circle of the loop, so that a clockwise motion develops in the loop corresponding to an induced counter-clockwise current and hence positive flux through the loop that opposes the increasingly negative flux of the gravitomagnetic field of the source. This is the gravitational analogue of Lenz's law of electrodynamics.

...

The purpose of this work has been to provide an *explicit* treatment of gravitational induction. The main ingredients of our discussion include the acceleration term $(2/c)\partial_t \mathbf{A}$ in the GEM force law and our special ansatz 2.7 for a gravitomagnetic vector potential $\mathbf{A}$ that varies linearly with time. It has been shown that, despite the existence of certain differences in the force law between GEM and electrodynamics, it is nevertheless possible to establish a certain analogy between gravitational induction and electromagnetic induction.

The acceleration term $(2/c)\partial_t \mathbf{A}$ in the gravitational force law has been traditionally interpreted to be in essence responsible for a Machian inductive action of accelerated masses such that a test mass would accelerate in the same direction as the acceleration of neighboring masses However, our analysis of geodesic motion in the special nonstationary spacetime in this paper demonstrates explicitly that this Machian interpretation cannot be generally maintained in general relativity

…".

Some possibly useful (approximate) data for the Moon-Earth-Sun system are:

G = 6.67 x 10^(-8) cm^3 / gm sec^2

c = 3 x 10^10 cm/sec

Mass_moon = 7.35 x 10^25 gm

Mass_earth = 6 x 10^27 gm

Mass_sun = 2 x 10^33 gm

Distance_earth-moon = 3.8 x 10^10 cm

Distance_earth-sun = 1.5 x 10^13 cm