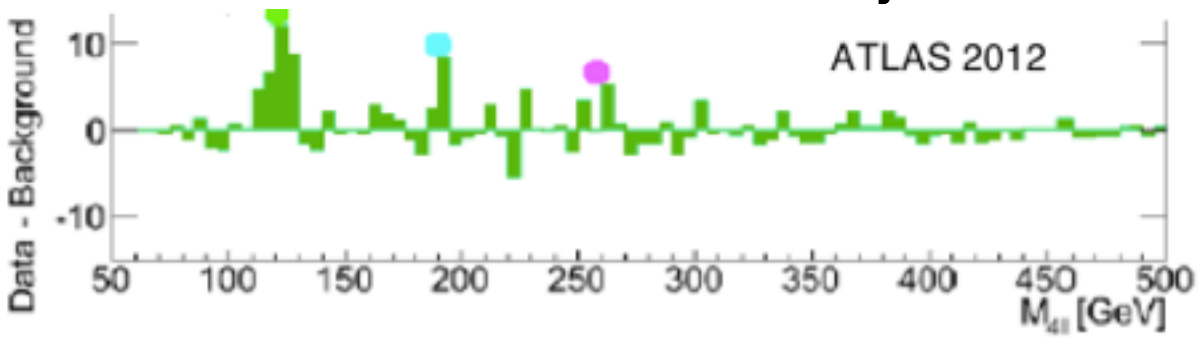
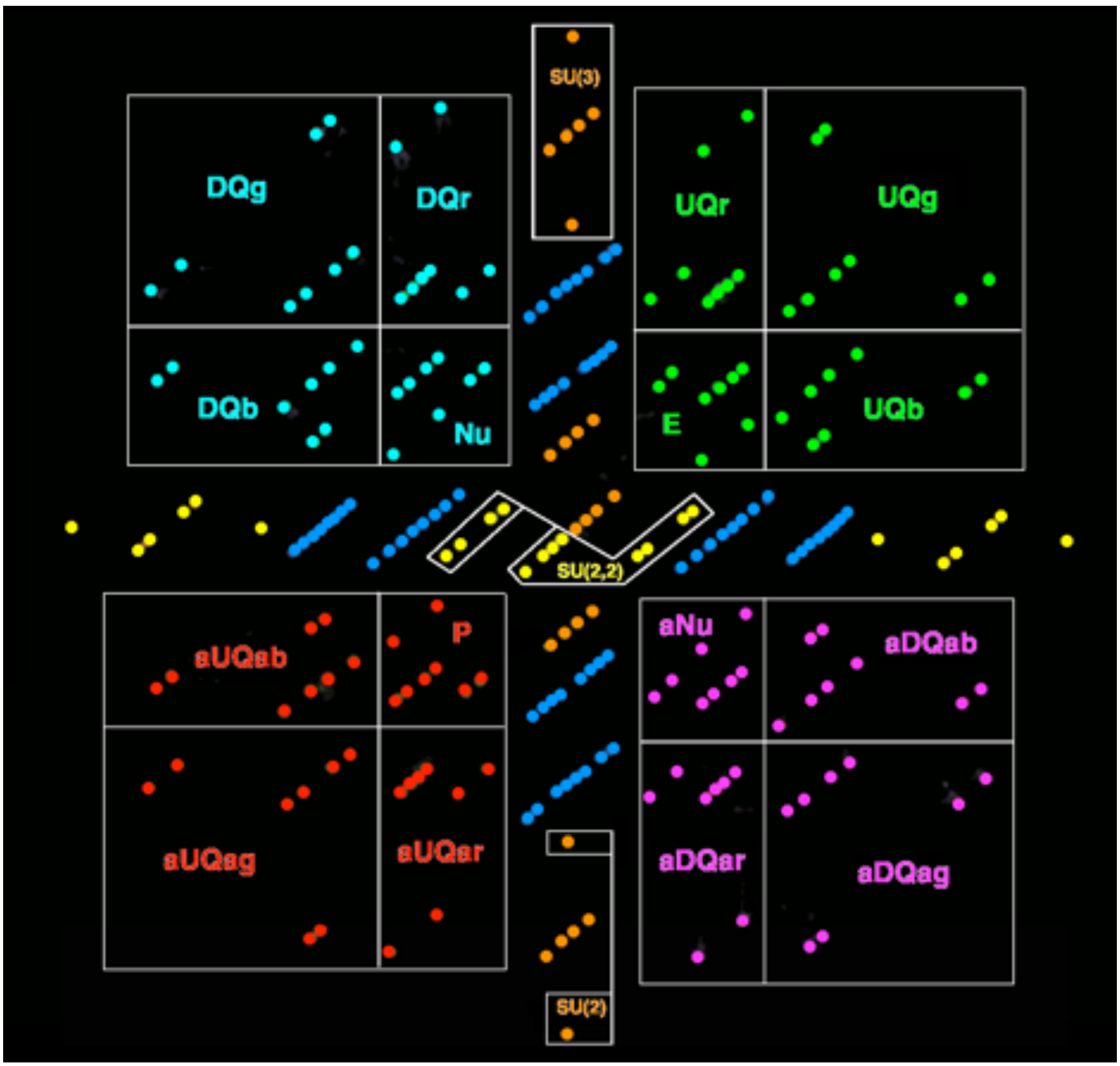
LHC 2015-16 and E8 Physics





Frank Dodd (Tony) Smith, Jr. - 2015 - viXra 1508.0157

Abstract:

Run-2 2015-16 of the LHC will probably, when enough data is collected,
show whether or not there exist two new Standard Model Higgs mass states,
around 200 GeV and around 250 GeV, each with cross sections
around 25% of the full Standard Model cross section for the 125 GeV state.

Run-1 results were consistent with the existence of those two new Higgs mass states,
but were also consistent with their peaks being statistical fluctuations that could go away
with more data from Run-2.

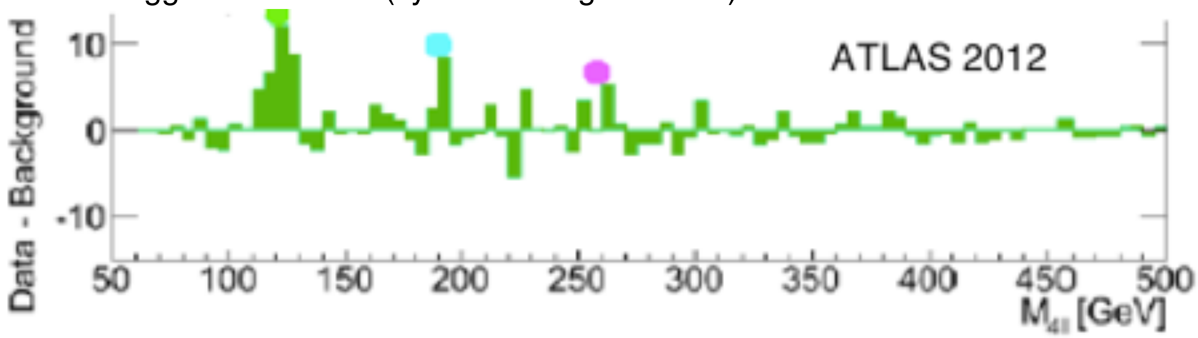
The purpose of this paper is two-fold:

1 - To describe my E8 Physics Model, which predicts the existence
of those two new Higgs mass states around 200 GeV and 250 GeV,
and which allows calculation of realistic particle masses, force strengths,
K-M parameters, Dark Energy : Dark Matter : Ordinary Matter ratio, etc,
a description of which takes up pages 6 through 105 (the end) of this paper;
and
2 - To describe the status of the LHC Run-2 data
that will show the existence or non-existence of the two new Higgs mass states
around 200 GeV and 250 GeV with cross sections about 25% or so
of the 125 GeV Higgs mass state cross section,
and therefore either support my E8 Physics Model or kill it.
Since it may be well into 2016 before Run-2 has enough data
to decide the issue, I expect to replace this paper with updated
versions throughout Run-2 until it has decided the issue.

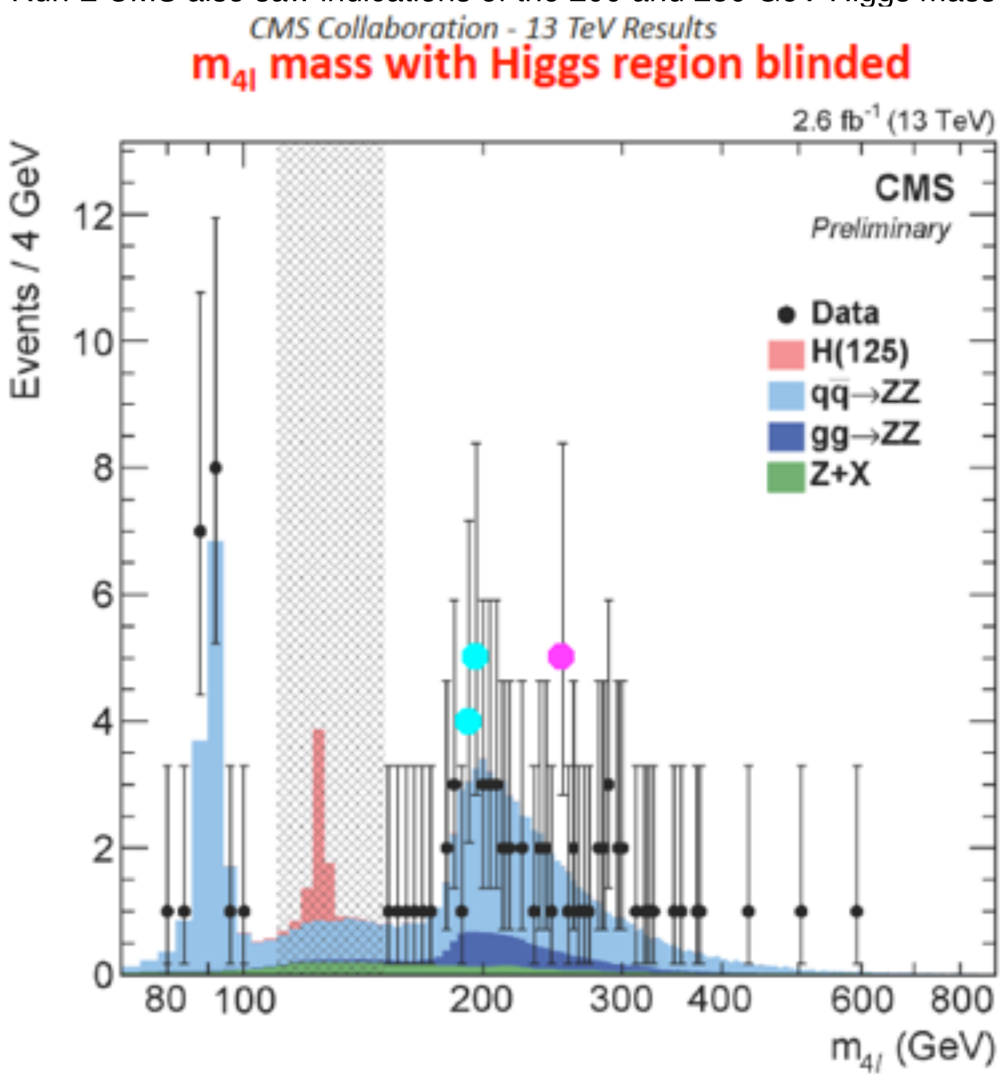
Table of Contents:

LHC Run-1 (2012) and Run-2 (2015-16) and Higgs mass states:

By the end of Run-1 in 2012 the LHC had seen clear evidence for a Higgs (green dot) with mass around 125 GeV and the expected Standard Model cross section.
LHC Run-1 saw in the Higgs -> ZZ -> 4l channel indications of
two more Higgs mass states (cyan and magenta dots) around 200 GeV and 250 GeV

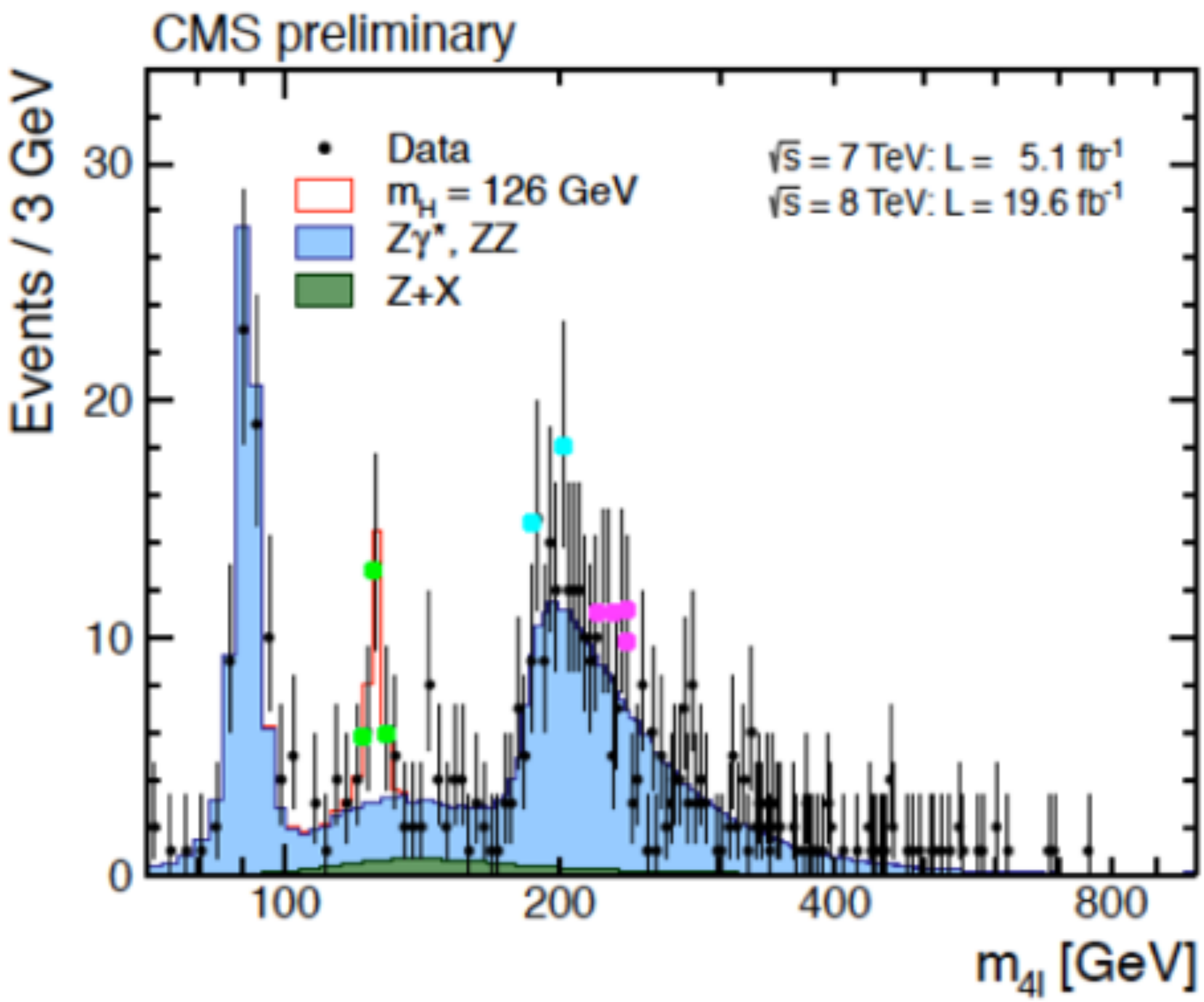


In 2015 Run-2 CMS also saw indications of the 200 and 250 GeV Higgs mass states

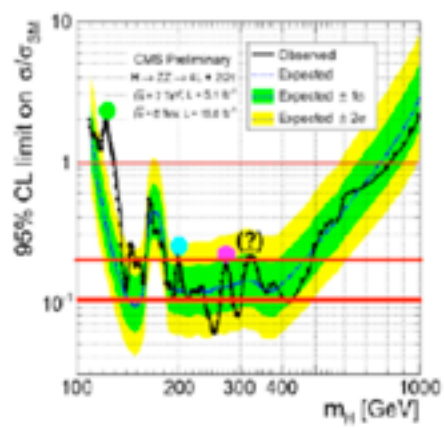


(from slide 28 by Jim Olsen for 15 Dec 2015 LPCC Special Seminar)

In Run-1 CMS had also seen indications of Higgs mass states around 200 and 250 GeV whose cyan and magenta dots coincide with their 2015 Run-2 positions

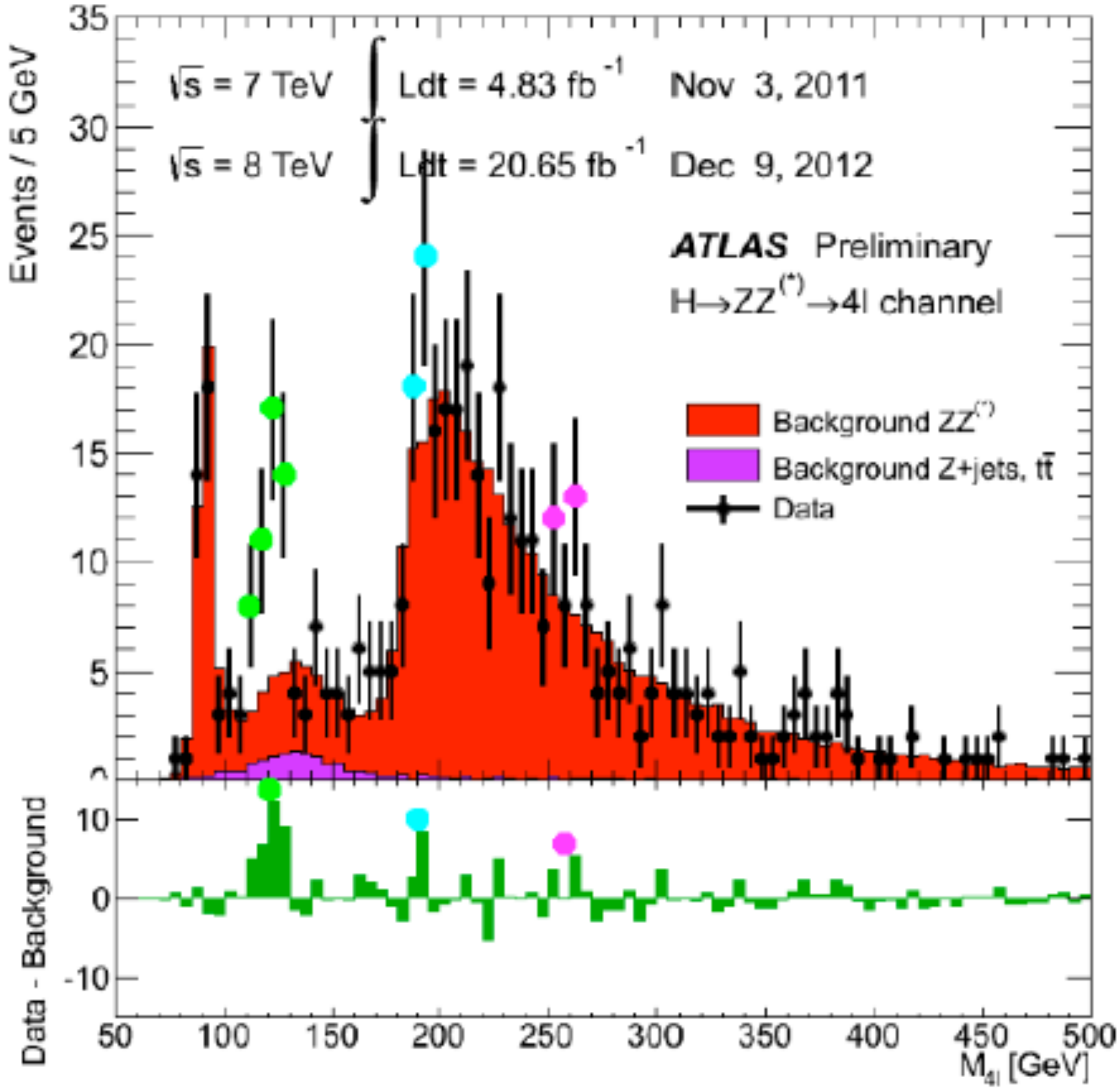


and with cross sections around 25% of SM expectation



CMS Run-1 also saw a (?) peak around 320 GeV that I expect to go away with 2016 Run-2 data. The two unmarked peaks around 160 and 180 GeV are probably due to WW and ZZ.

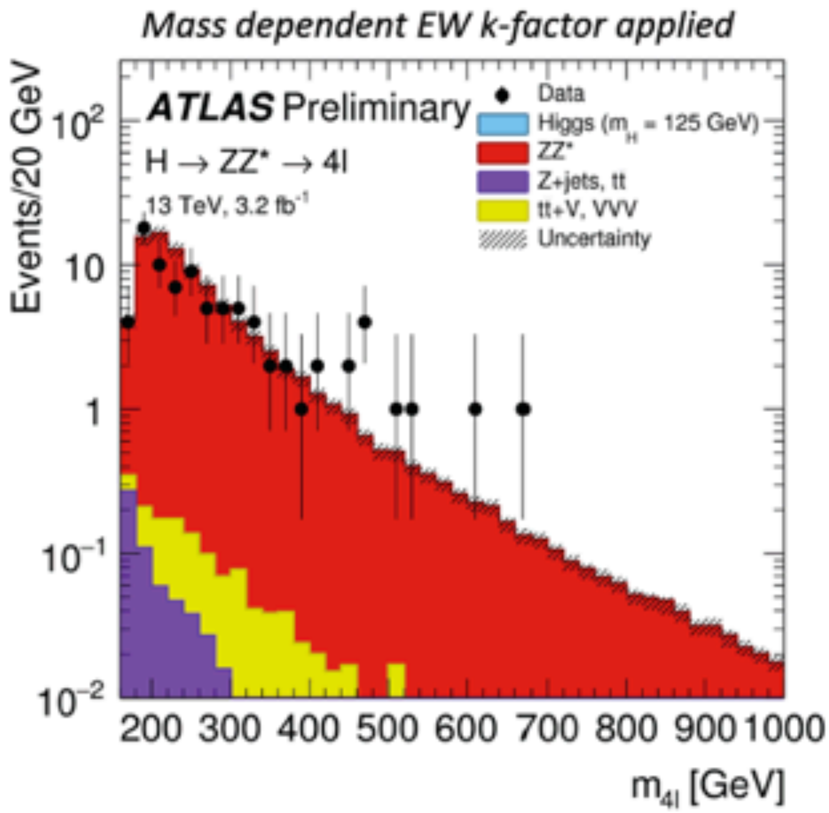
Further,
in Run-1 ATLAS had seen indications of Higgs mass states around 200 and 250 GeV whose cyan and magenta dots coincide with the CMS 2015 Run-2 positions



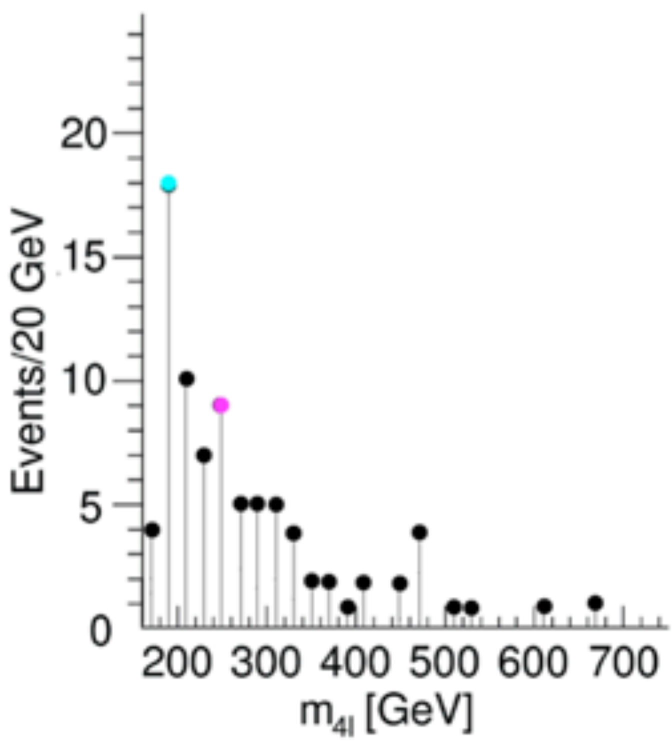
ATLAS Run-1 did not see the (?) CMS Run-1 peak around 320 GeV
as ATLAS saw an excess bin adjacent to two deficient bins.

In 2015 Run-2 did ATLAS see indications of 200 and 250 GeV Higgs mass states ?

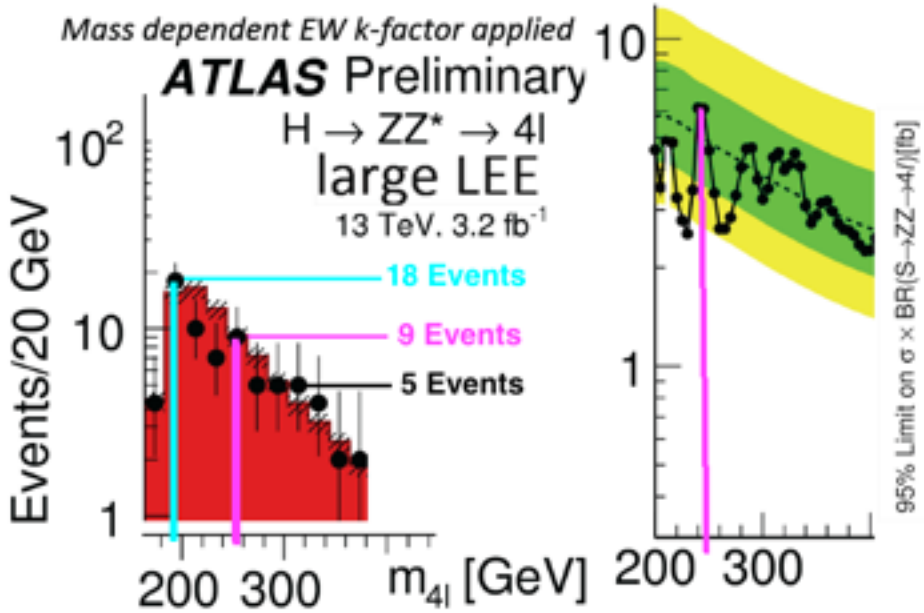
Here is what ATLAS reported (slide 22 by Marumi Kado) on 15 Dec 2015 LPCC Special Seminar:



Here is that ATLAS 2015 Higgs -> ZZ -> 4l histogram replotted with linear scale:



Here are some details (from slide 22 by Maarumi Kado for 15 Dec 2015 LPCC Special Seminar):



In my opinion the indications of 200 (cyan) and 250 (magenta) GeV Higgs mass states are there, but are obscured by:

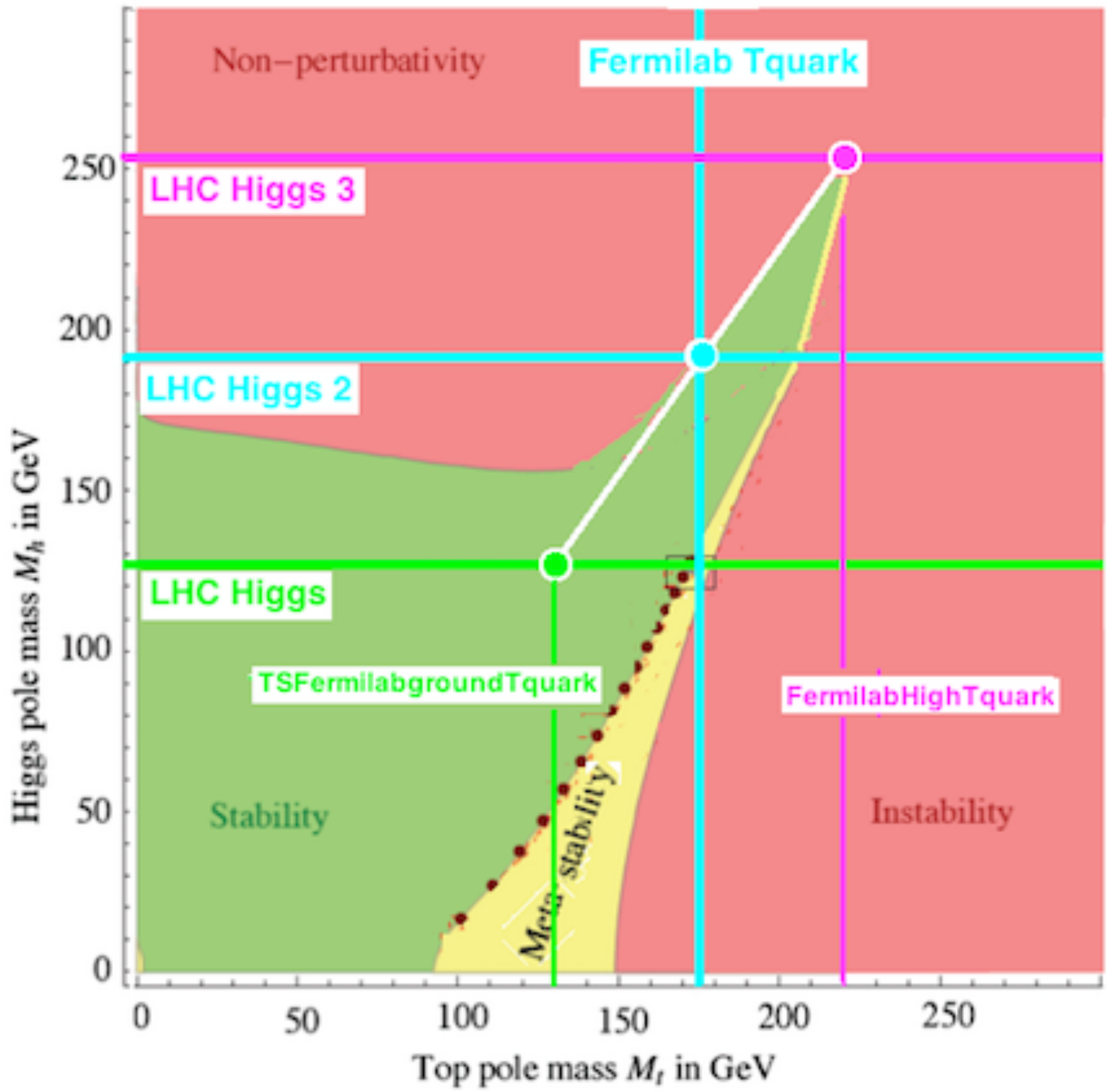
1 - a large LEE effect that is NOT appropriate for the 200 and 250 GeV Higgs mass states that were predicted by my E8 Physics model and indicated by prior Run-1 data

2 - the Brazil Band plot does NOT show the peak just below the 200 GeV line

2 - use of a log scale for the histogram of Events/20 GeV makes it hard to see the details of the Events around 200 and 250 GeV.
It seems clear to me that the linear plot indicates that the 200 GeV (cyan) peak and the 250 GeV (magenta) peak are serious candidates with over 5 Events that might well be confirmed by 2016 data as real Higgs mass states.

Why E8 Physics has two new Higgs mass states

As to **WHY** two new Higgs mass states are expected in my E8 Physics Model:
The Cl(16)-E8 model identifies the Higgs with Primitive Idempotents of the Cl(8) real Clifford algebra, whereby the Higgs is not seen as a simple-minded single fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate and effectively a 3-state Higgs-Tquark System.



The Green Dot  where the White Line originates in our Ordinary Phase is the low-mass state of a 130 GeV Truth Quark and a 125 GeV Higgs**.**

The Cyan Dot where the White Line hits the Triviality Boundary leaving the Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:
"... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top (m_t) and the Higgs (m_H) ... based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[Kaluza-Klein type]... dimension... D=8 ...
m_t = 172-175 GeV and m_H=176-188 GeV ...".

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176: "... The Higgs quartic coupling has a complicated scale dependence. It evolves according to d lambda / d t = (1 / 16 pi^2) beta_lambda where the one loop contribution is given by beta_lambda = 12 lambda^2 - ... - 4 H ... The value of lambda at low energies is related [to] the physical value of the Higgs mass according to the tree level formula m_H = v sqrt(2 lambda) while the vacuum value is determined by the Fermi constant ... for a fixed vacuum value v, let us assume that the Higgs mass and therefore lambda is large. In that case, beta_lambda is dominated by the lambda^2 term, which drives the coupling towards its Landau pole at higher energies.
Hence the higher the Higgs mass, the higher lambda is and the close[r] the Landau pole to experimentally accessible regions.
This means that for a given (large) Higgs mass, we expect the standard model to enter a strong coupling regime at relatively low energies, losing in the process our ability to calculate. This does not necessarily mean that the theory is incomplete,
only that we can no longer handle it ... it is natural to think that this effect is caused by new strong interactions, and that the Higgs actually is a composite ...
The resulting bound on lambda is sometimes called the triviality bound.
The reason for this unfortunate name (the theory is anything but trivial)
stems from lattice studies where the coupling is assumed to be finite everywhere;
in that case the coupling is driven to zero, yielding in fact a trivial theory.
In the standard model lambda is certainly not zero. ...".

Middle Mass State Cross Section:

In the Cl(16)-E8 model, the Middle-Mass Higgs has structure that is not restricted to Effective M4 Spacetime as is the case with the Low-Mass Higgs Ground State

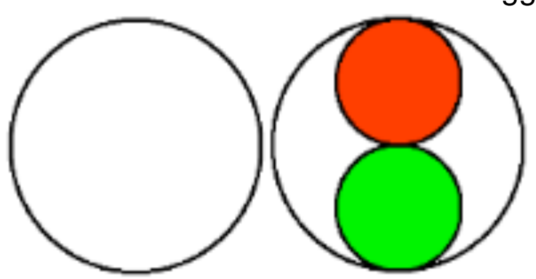
but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

```
T ----------- Tbar        in CP2 Internal Symmetry Space
 \           /
  \         /
   \       /
    \     /
     \   /
    Higgs                 in M4 Physical SpaceTime
```

Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.

The T and Tbar form a Pion-like state.
Since Tquark Mid-Mass State is 174 GeV
the Middle-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein
has mass (174+174) x (135 / (312+312) = 75 GeV.

The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein
has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.
So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein
with mass 75+125 = 200 GeV.
This is consistent with the Mid-Mass States of the Higgs and Tquark
being on the Triviality Boundary of the Higgs - Tquark System
and with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.As to the cross-section of the Middle-Mass Higgs



consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime
(left white circle)
while the Middle-Mass Higgs cross-section lives in full 4+4 = 8-dim Kaluza-Klein
(right circle with red area only in CP2 ISS and white area partly in CP2 ISS
with only green area effectively living in 4-dim M4 spacetime)
so that
our 4-dim M4 Physical Spacetime experiments only see for the Middle-Mass Higgs
a cross-section that is 25% of the full Ground State cross-section.

The 25% may also be visualized in terms of 8-dim coordinates {1,i,j,k,E,I,J,K}

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which {1,i,j,k} represent M4 and {E,I,J,K} represent CP2.

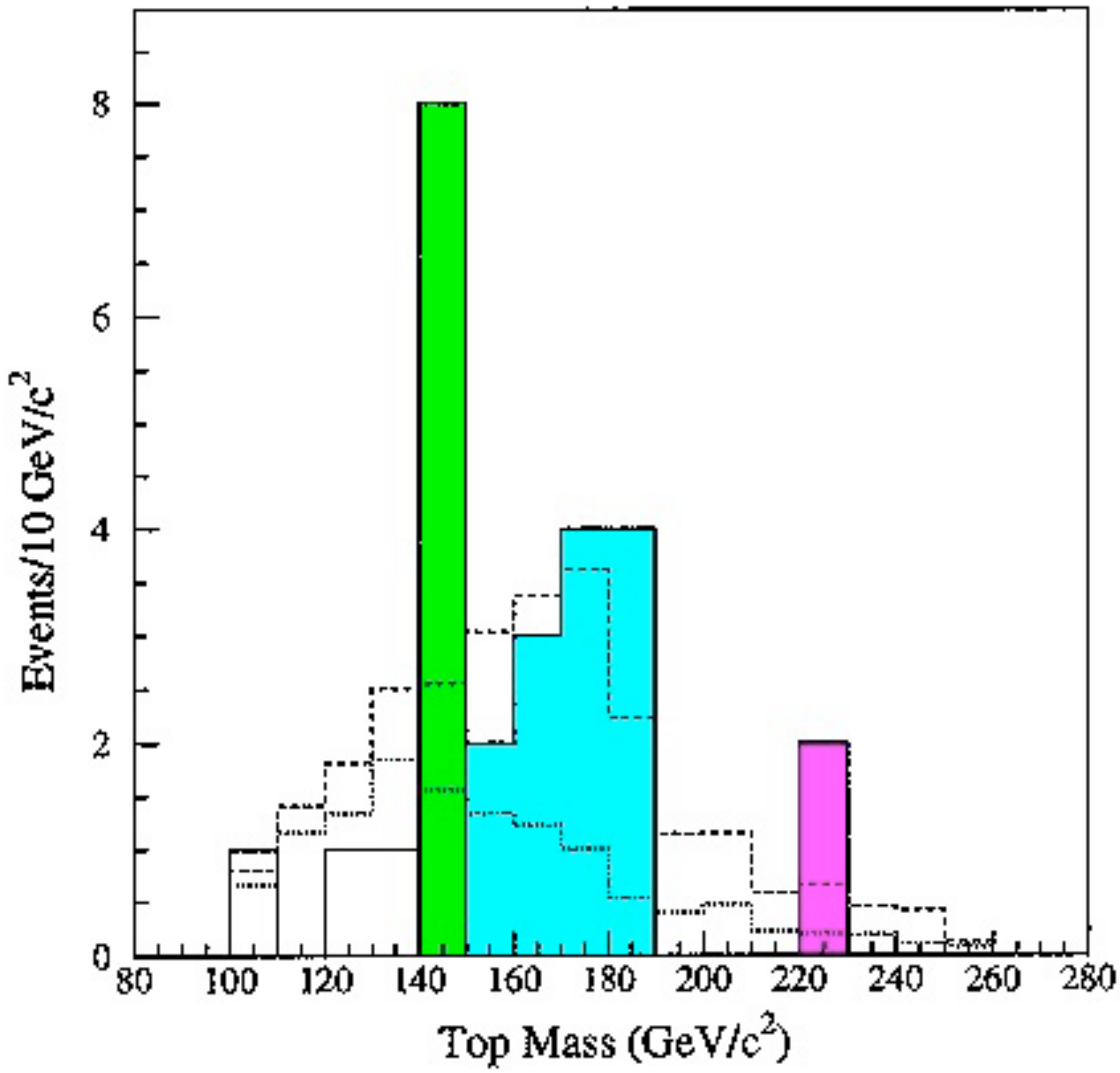
The Magenta Dot at the end of the White Line is the high-mass state of
a 220 GeV Truth Quark and a 240 GeV Higgs. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model:
"... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ...
BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale Λ ...[10^19 GeV]...
there must be a certain matching scale Λ_Matching such that
the perturbative picture (BHL) is valid for mu < Λ_Matching, while only the
nonperturbative picture (MTY) becomes consistent for mu > Λ_Matching ...
However, thanks to the presence of a quasi-infrared fixed point,
BHL prediction is numerically quite stable against ambiguity at high energy region,
namely, rather independent of whether this high energy region is replaced by
MTY or something else. ... Then we expect mt = mt(BHL) = ... = 1/(sqrt(2)) ybart v
within 1-2%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ...
the one-loop RG equation ...
The composite Higgs loop changes ybart^2 by roughly the factor Nc/(Nc +3/2) = 2/3
compared with the MTY value, i.e., 250 GeV -> 250 x sqrt(2/3) = 204 GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by mt = 218 +/- 3 GeV, at Λ = 10^19 GeV.
The Higgs boson was predicted as a tbar-t bound state
with a mass MH = 2mt based on the pure NJL model calculation.
Its mass was also calculated by BHL through the full RG equation ...
the result being ... MH / mt = 1.1) at /.\ = 10^19 GeV ...
... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki
(MTY) and by Nambu independently ... entirely replaces the standard Higgs
doublet by a composite one formed by a strongly coupled short range
dynamics (four-fermion interaction) which triggers the top quark condensate.
The Higgs boson emerges as a tbar-t bound state and hence is deeply connected
with the top quark itself. ... MTY introduced explicit four-fermion interactions
responsible for the top quark condensate in addition to the standard gauge
couplings. Based on the explicit solution of the ladder SD equation, MTY found
that even if all the dimensionless four-fermion couplings are of O(1), only the
coupling larger than the critical coupling yields non-zero (large) mass ... The model
was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL)
in the SM language, based on the RG equation and the compositenes condition.
BHL essentially incorporates 1/Nc sub-leading effects such as those of the
composite Higgs loops and ... gauge boson loops which were disregarded by the
MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY
at 1/Nc-leading order. Such effects turned out to reduce the above MTY value
250 GeV down to 220 GeV ...".

High Mass State Cross Section:

As with the Middle-Mass Higgs,
the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions
so
its cross-section is also about 25% of the Higgs Ground State cross-section.

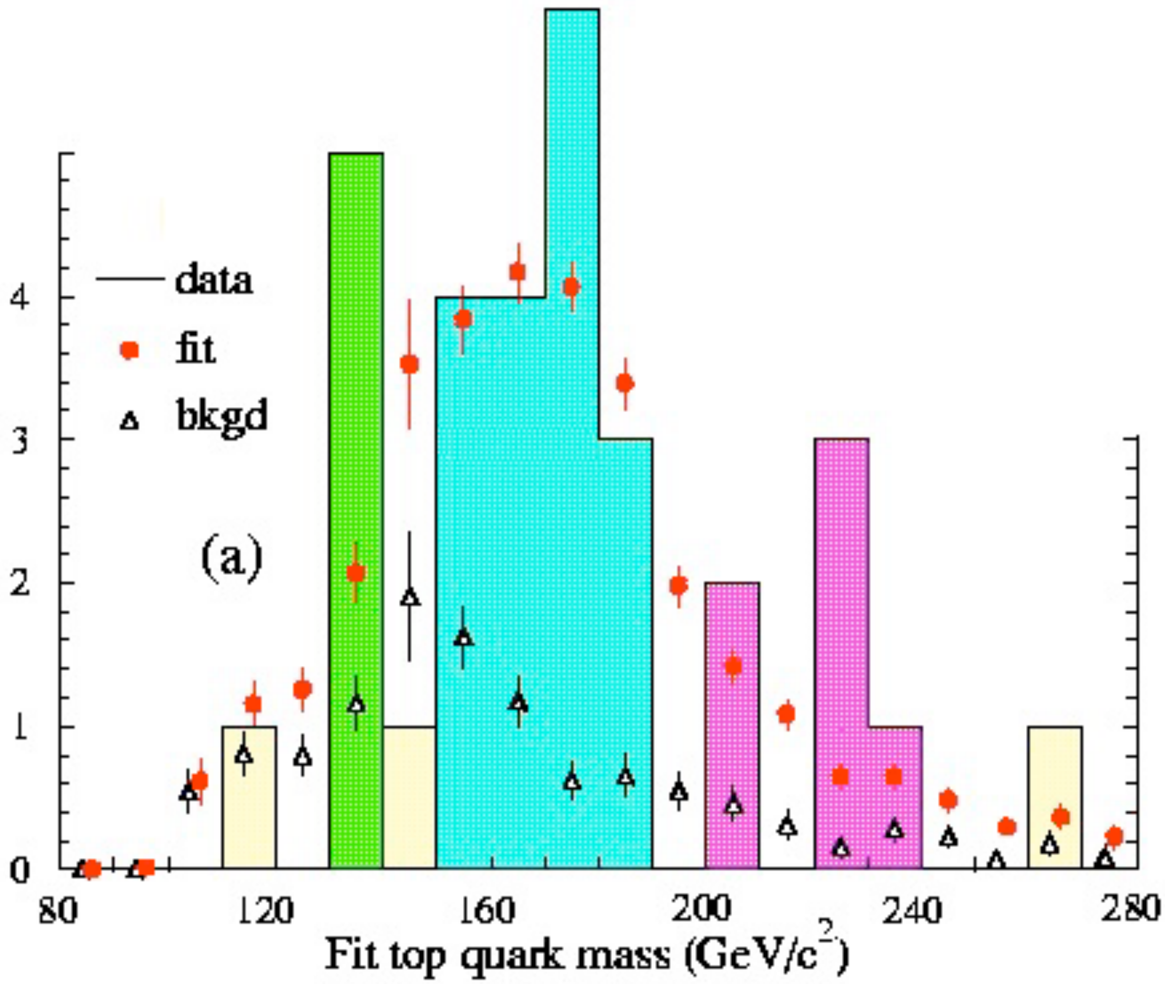
Three T-quark mass states

The 174 GeV Tquark mass state (cyan dot) is not controversial. It has been observed at Fermilab since 1994, when a semileptonic histogram from CDF (FERMILAB-PUB-94/097-E)



showed all three states of the T-quark.
In particular, the green bar represents a bin in the 140-150 GeV range containing Semileptonic events considered by me to represent the Truth Quark, but as to which CDF said "... We assume the mass combinations in the 140 to 150 GeV/c^2 bin represent a statistical fluctuation since their width is narrower than expected for a top signal. ...". I strongly disagree with CDF's "statistical fluctuation" interpretation, based on my interpretations of much Fermilab T-quark data.

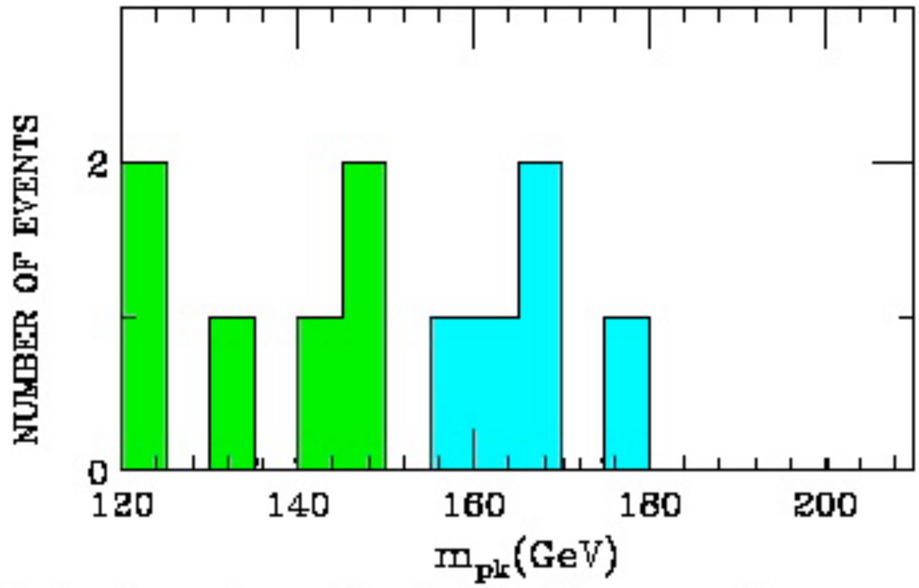
The same three Tquark mass states were seen in 1997 by D0 (hep-ex/9703008) in this semileptonic histogram:



The fact that the low (green) state showed up in both independent detectors indicates a significance of 4 sigma.

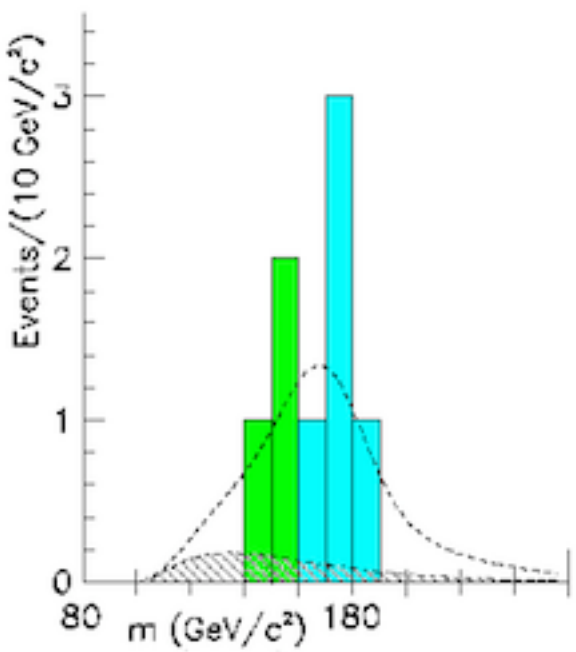
My opinion is that the middle (cyan) state is wide because
it is on the Triviality boundary
where the composite nature of the Higgs as T-Tbar condensate
becomes manifest
and
the low (cyan) state is narrow because it is in the usual non-trivial region
where the T-quark acts more nearly as a single individual particle.

Further, in February 1998 a dilepton histogram of 11 events from CDF (hep-ex/9802017)



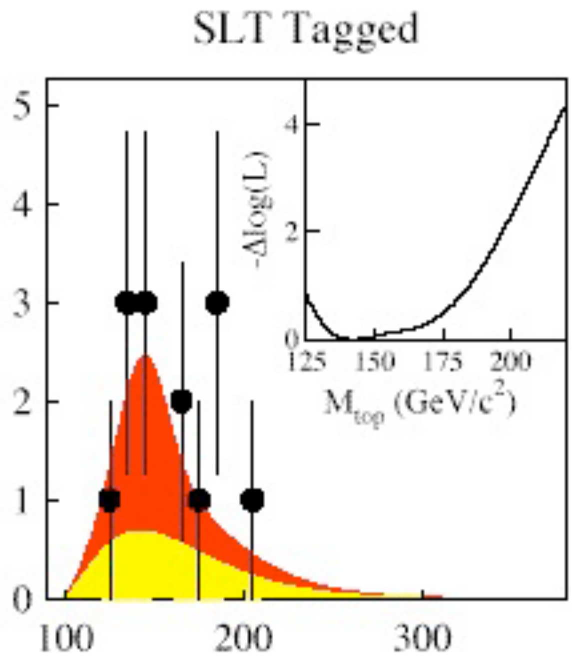
The distribution of m_{pk} values determined from 11 CDF dilepton events available empirically.

shows both the low (green) state and the middle (cyan) T-quark state
but
in October 1998 CDF revised their analysis using 8 Dilepton CDF events (hep-ex/9810029)



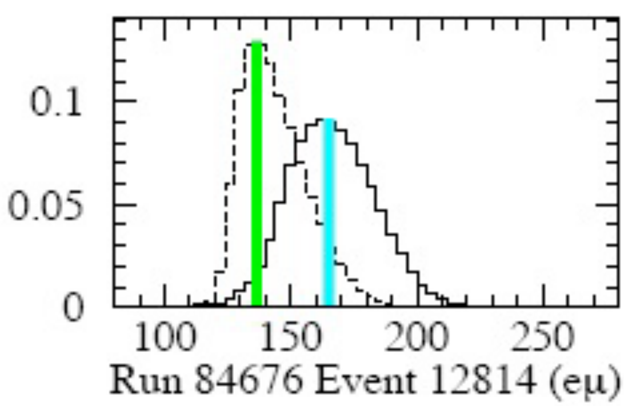
shows that CDF kept the 8 highest-mass dilepton events, and threw away the 3 lowest-mass dilepton events that were indicated to be in the 120-135 GeV range, and shifted the mass scale upward by about 10 GeV, indicating to me tthat Fermilab was attempting to discredit the low-mass T-quark state by use of cuts etc on its T-quark data.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF (hep-ex/9801014)



showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.

In his 1997 Ph.D. thesis Erich Ward Varnes (Varnes-fermilab-thesis-1997-28 at page 159) said: "... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



...” (colored bars added by me)

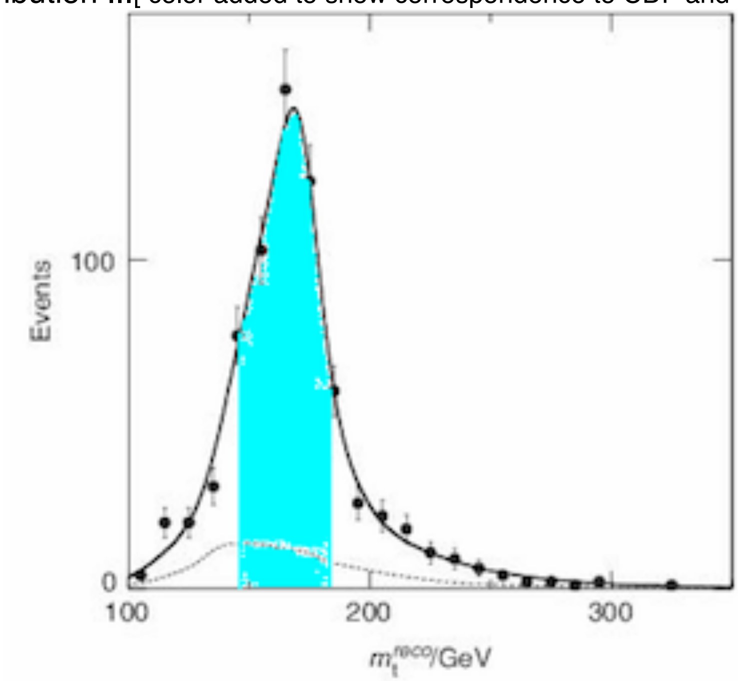
The event for all 3 jets (solid curve) seens to me to correspond to
decay of a middle (cyan) T-quark state
with one of the 3 jets corresponding to
decay from the Triviality boundary down to the low (green) T-quark state,
whose immediately subsequent decay corresponds to the 2-jet (dashed curve) event at
the low (green) energy level.

As to the T-quark width for the 174 GeV mass state, which appears
in the 1994 CDF and 1997 D0 semileptonic histograms to be about 40 GeV,
which is 4 of the 10 GeV histogram bins,
Mark Thomson, in "Modern Particle Physics" (Cambridge 2013) says:
"... Decay of the top quark ... The total decay rate is ...

$$\Gamma(t \to bW^+) = \frac{G_F m_t^3}{8\sqrt{2}\pi}\left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + \frac{2m_W^2}{m_t^2}\right),$$

... For ... mt = 173 GeV ... the lowest-order calculation of the total decay width of the top quark gives Γ t = 1.5 GeV ...

The total width of the top quark is measured to be Γ t = 2.0 +/- 0.6 GeV.
The top width is determined much less precisely than the top quark mass because the width of the distribution ...[color added to show correspondence to CDF and D0 histograms]...



... is dominated by the experimental resolution. ...".

The T-quark total width Γ t = 2 GeV is much smaller than the 40 GeV width experimentally observed at Fermilab and would, except for experimental resolution, fit well within one single bin in the 1994 CDF and 1997 D0 semileptonic histograms.

As to the T-quark width for the 130 GeV mass state,
which appears in the 1994 CDF and 1997 D0 semileptonic histograms
to be less than the 10 GeV histogram bin width,
using the total width formula from Mark Thomson's book and paraphrasing:

"... For mt = 130 GeV ... the lowest-order calculation
of the total decay width of the top quark gives Γt = about 0.5 GeV ...".

I think that the CDF explanation
for the low mass T-quark peak in a single 10 GeV bin

"... We assume the mass combinations in the ... bin represent a statistical fluctuation since their width is narrower than expected for a top signal. ...".

is highly unlikely since
a similar low mass single 10 GeV bin T-quark mass peak was observed by the independent D0 detector.

The mt = 130 GeV width of 0.5 GeV is only 1/20 of the 10 GeV bin width of that peak.
The 20:1 = 10 : 0.5 observed width : actual width ratio for mt = 130 GeV
is the same as
the 20:1 = 40 : 2.0 observed width : actual width ratio for mt = 173 GeV.

What differences between the mt = 130 GeV and mt = 173 GeV states
might affect their relative experimental resolutions ?

The mt = 130 GeV peak is in the normal Stable region
in which the T-quark is represented by a Schwinger Source in M4 Physical Spacetime
which Schwinger Source has Green's Function structure based on Kernel Functions
of Bounded Symmetric Domains whose symmetry is that of the T-quark.
Since it is a simple Schwinger Source it has simple W - b - 2 jet decay.

The mt = 173 GeV peak is on the boundary
of the Non-Perturbativity region where the composite nature of Higgs as T-quark Condensate becomes manifest, as does the 8-dim nature of Kaluza-Klein spacetime M4 x CP2 with M4 Physical Spacetime and CP2 Internal Symmetry Space where CP2 = SU(3) / SU(2)xU(1) has symmetries of the Standard Model Gauge Groups.
Its decay scheme is more complicated, with 2 stages:

175 to 130 GeV, a process of the Higgs - T-quark condensate system of E8 Physics
and
simple W - b - 2 jet decay of the 130 GeV intermediate state.

The wider width of the 173 GeV decay peak
is due to the Higgs - T-quark condensate process.

The 1997 UC Berkeley PhD thesis of Erich Ward Varnes gives details of some D0 events and analysis, based on the Standard Model view of one T-quark mass state:
"... the leptonic decays of the t tbar events are divided into two broad categories:
the lepton plus jets and dilepton channels.
The former has the advantage of a large branching ratio, accounting for about 30% of all t tbar decays, with the disadvantage that electroweak processes or detector misidentification of fina-state particle can mimic the t tbar signal relatiely frequently.
Conversely,
the dilepton channels have lower backgrounds, but account for only 5% of all decays.
...
The kinematic selection of dilepton events is summarized in Table 5.2 ...

	ee	$e\mu$	$\mu\mu$
Leptons	$E_T > 20$ GeV $\lvert\eta\rvert < 2.5$	$E_T(e) > 15$ Gev, $p_T(\mu) > 15$ GeV/c $\lvert\eta(e)\rvert < 2.5$	$p_T(\mu) > 15$ GeV/c
Jets	≥ 2 with $E_T > 20$ GeV and $\lvert\eta\rvert < 2.5$		
$\not{E}_T$	> 25 GeV	$\not{E}_T > 20$ GeV $\not{E}_T^{cal} > 10$ GeV	N/A
H_T^e	> 120 GeV	> 120 GeV	> 100 GeV

Table 5.2: Kinematic cuts for the dilepton event selection. The cut used in place of $\not{E}_T$ to reject $Z \to \mu\mu$ events is described in the text, as is the H_T^e variable. Also, the muon η cut is run-dependent, as detailed in Chapter 4.

...
In the dilepton channels, one expects the final state to consist of two charged leptons, two neutrinos, and two b jets (see Fig. 6.1)

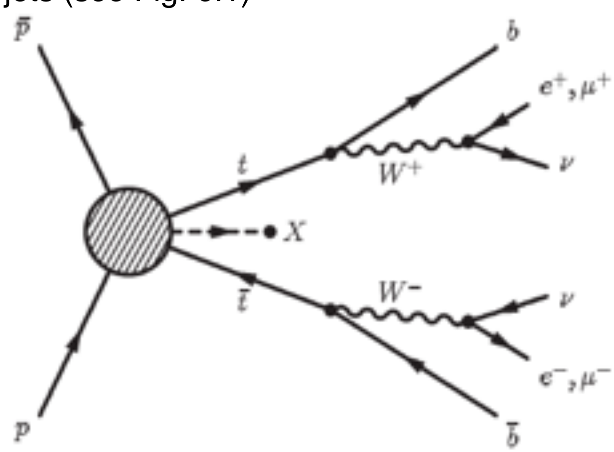


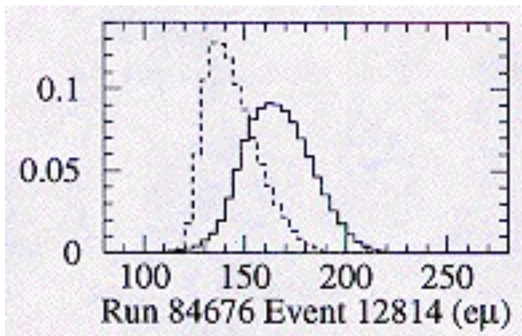
Figure 6.1: Schematic representation of $t\bar{t}$ production and decay in the dilepton channels.

so that the final state is completely specified by knowledge of the energy four-vectors of these six particles ... there are ... kinematic constraints:

The invariant mass of each lepton and neutrino pair is equal to the W mass.
The masses of the reconstructed t and tbar in the event are equal.

...
The result of reconstructing the top quark mass for a dilepton event is the distribution W(mt), which is evaluaed for 50 values of the top quark mass ... the intrinsic resolution of the dilepton mass reconstruction is much broader than the 4 GeV/c2 interval between assumed top quark masses ... the RMS of the typical W(mt) distribution ... typically lies between 35 and 40 GeV/c2 ...

...
Figure 8.1: W(mt) distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



...

Run 84676 Event 12814					z vertex: -6.17 cm		
Object	E	E_x	E_y	E_z	E_T	η	ϕ
Electron	81.3	-75.4	-1.1	-30.2	74.5	-0.39	3.16
Muon	30.2	-25.2	10.6	-12.8	27.4	-0.45	2.75
$\not{E}_T$	–	62.0	5.2	–	62.3	–	0.08
Jet 1	93.8 (95.9)	38.0 (38.9)	-83.7 (-85.6)	-15.6 (-16.0)	91.9 (94.0)	-0.17	5.14
Jet 2	37.8 (38.8)	13.9 (14.2)	32.3 (33.1)	-11.2 (-11.4)	35.2 (36.0)	-0.31	1.17
Jet 3	31.4 (32.2)	-1.6 (-1.6)	28.6 (29.3)	11.6 (11.9)	28.7 (29.4)	0.39	1.63

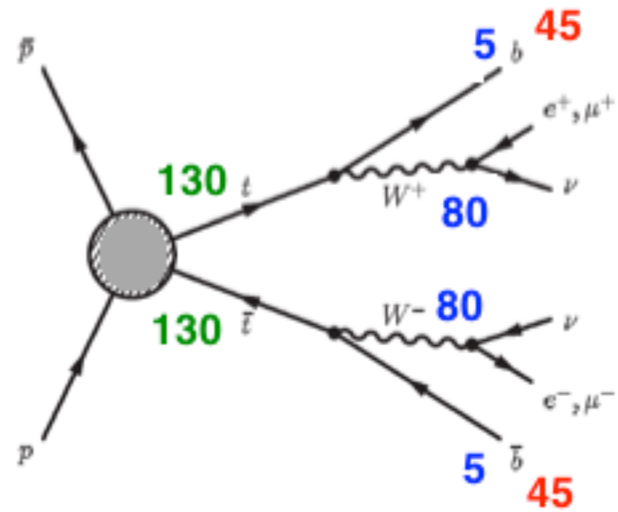
...".

In E8 Physics (viXra 1508.0157) there are, as stated above, three T-quark mass states, so in order to keep the kinematic constraint

"The masses of the reconstructed t and tbar in the event are equal"

the t and tbar must be in the same mass state, which is physically realistic because the t and tbar are created together in the same collider collision event.

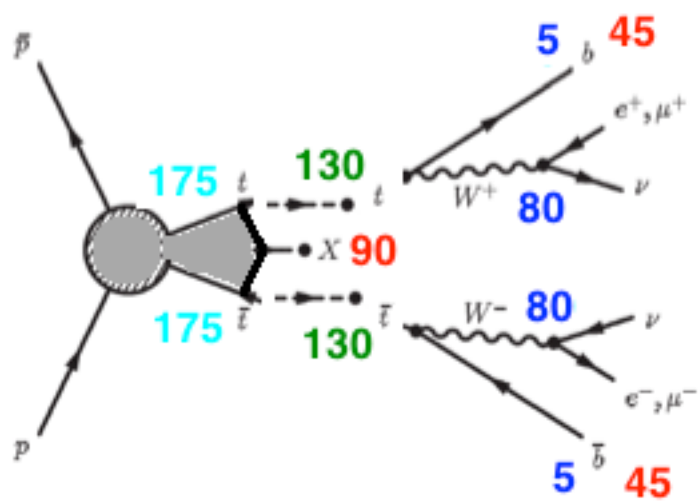
If the t and tbar are both in the 130 GeV mass state then the decay is simple with 2 jets:



and both jets are highly constrained as being related to the W - b decay process
so it is reasonable to expect that the 130 GeV decay events would fall in the narrow width of a single 10 GeV histogram bin.

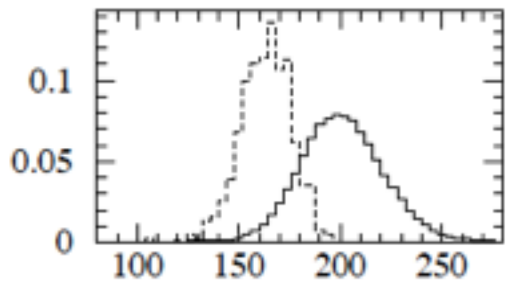
(In these two diagrams I have indicated energies only approximately for t and tbar mass states (cyan and green) and W and b-quark (blue) and jets (red). Actual kinematic data may vary from the idealized numbers on the diagrams, but they should give similar physics results.)

If the t and tbar are both in the 173 GeV mass state
(as, for example, in Run 84676 Event 12814 (e mu) described above)
the decay has two stages and 3 jets:



First, the 175 GeV t and tbar both decay to the 130 GeV state, emitting a jet.
Then, the 130 GeV t and tbar decay by the simple 2-jet process.
The first jet is a process of the Higgs - T-quark condensate system of E8 Physics and is not a W -b decay process so it is not so highly constrained
and it is reasonable to expect that the 175 GeV decay events would appear to have a larger (on the order of 40 GeV) width.

As to t and tbar being the high T-quark mass state (around 225 GeV)
there would be a third stage for decay from 225 GeV to 175 GeV
with a fourth jet carrying around 100 GeV of decay energy.
In the Varnes thesis there is one dilepton event



Run 84395 Event 15530 (μμ)

Run 84395 Event 15530					z vertex: 5.9 cm		
Object	E	E_x	E_y	E_z	E_T	η	ϕ
Muon 1	68.6	-63.9	12.7	-21.4	65.1	-0.32	2.94
Muon 2	34.9	-16.0	31.0	1.9	34.9	0.05	2.05
$\not\!\!E_T$	–	71.2	53.2	–	88.9	–	0.64
Jet 1	146.1	32.1	-98.2	-102.4	103.3	-0.88	5.03
	(153.5)	(33.8)	(-103.1)	(-107.6)	(108.5)		
Jet 2	35.1	-8.6	21.4	26.2	23.1	0.97	1.95
	(37.2)	(-9.1)	(22.7)	(27.7)	(24.5)		
Jet 3	47.1	-7.6	-16.8	43.0	18.4	1.58	4.29
	(52.3)	(-8.4)	(-18.6)	(47.8)	(20.5)		

that seems me to represent that third stage of decay from 225 GeV to 175 GeV.
Since it is described as a 3-jet event and not a 4-jet event as I would have expected,
my guess is that the third and fourth jets of my model were not distinguished by the
experiment so that they appeared to be one third jet.

What Does E8 Physics Do ?

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04

Fermions as Schwinger Sources have geometry of Complex Bounded Domains with Kerr-Newman Black Hole structure size about 10^(-24) cm.

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for nu_1
mu-neutrino	0	9 x 10^(-3) eV for nu_2
tau-neutrino	0	5.4 x 10^(-2) eV for nu_3
electron	0.5110 MeV	
down quark	312.8 MeV	charged pion = 139 MeV
up quark	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange quark	625 MeV	
charm quark	2090 MeV	
tauon	1.88 GeV	
beauty quark	5.63 GeV	
truth quark (low state)	130 GeV	(middle state) 174 GeV
		(high state) 218 GeV
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Mplanck	1.217x10^19 GeV	
Higgs VEV (assumed)	252.5 GeV	
Higgs (low state)	126 GeV	(middle state) 182 GeV
		(high state) 239 GeV
Gravity Gg (assumed)	1	
(Gg)(Mproton^2 / Mplanck^2)		5 x 10^(-39)
EM fine structure	1/137.03608	
Weak Gw	0.2535	
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))		1.05 x 10^(-5)
Color Force at 0.245 GeV	0.6286	0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The phase angle d13 is taken to be 1 radian.

How Does E8 Physics Work ?

Mathematical Prerequisites:

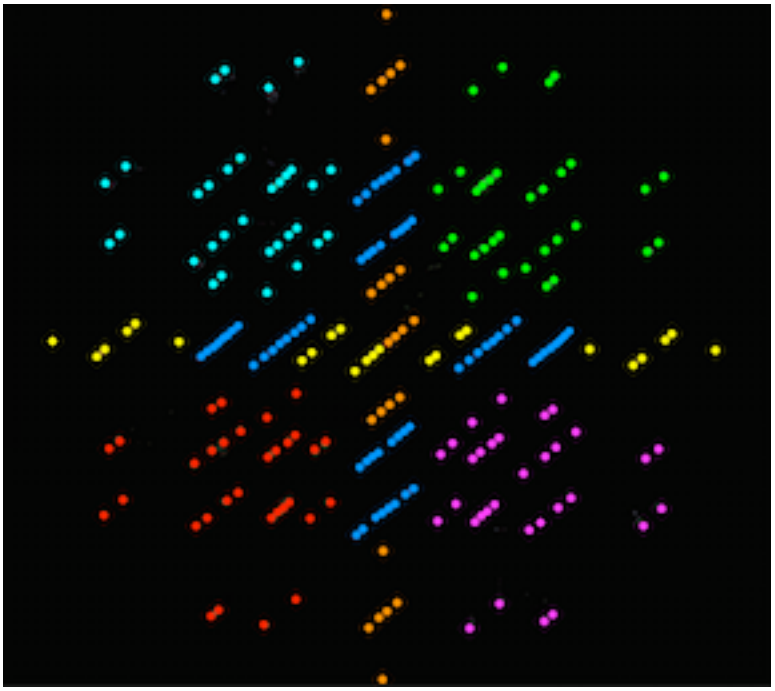
In my opinion, the best text / reference for the mathematics used in E8 Physics is the Princeton University Advanced Calculus text by H. K. Nickerson, D. C. Spencer, and N. E. Steenrod.

However, it is over 50 years old,
so I have added some Supplementary Material to produce
a 21 MB pdf file on the web at
http://www.valdostamuseum.com/hamsmith/NSS6313.pdf

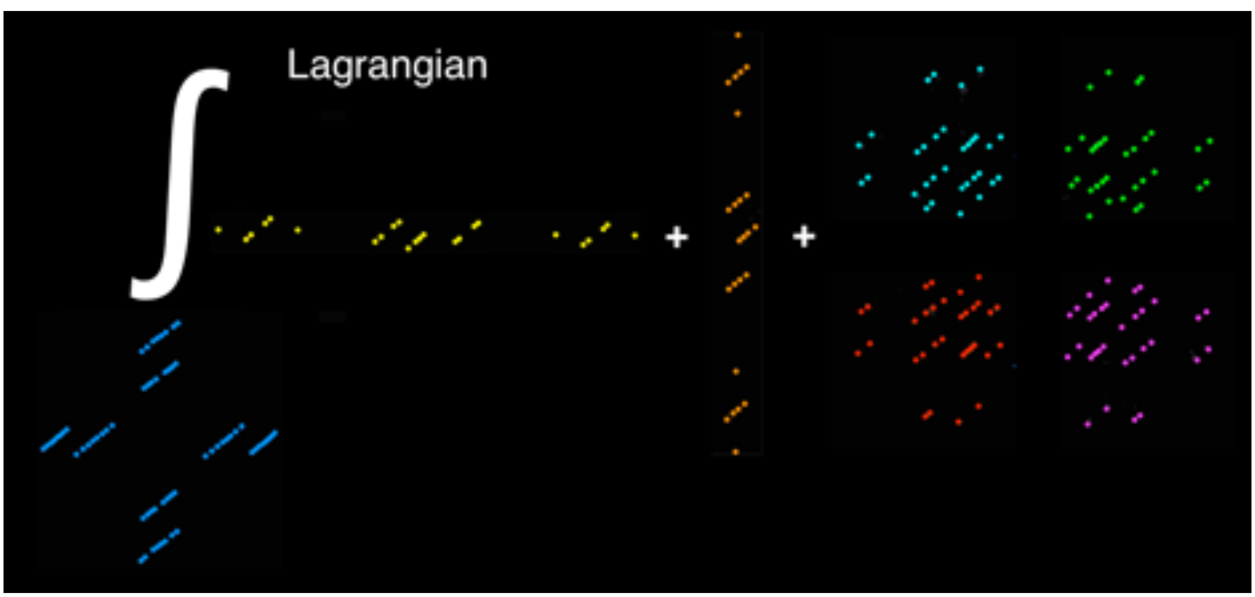
TABLE OF CONTENTS OF THE SUPPLEMENTED TEXT:
Supplementary Material in Red

Lagrangian Structure

E8 Physics is based on Lagrangian integral of gauge boson and fermion terms integrated over spacetime, all of which are represented by the 240 E8 root vectors
(this visualization uses a square/cubel type of projection of the 240 E8 root vectors to 2-dim)

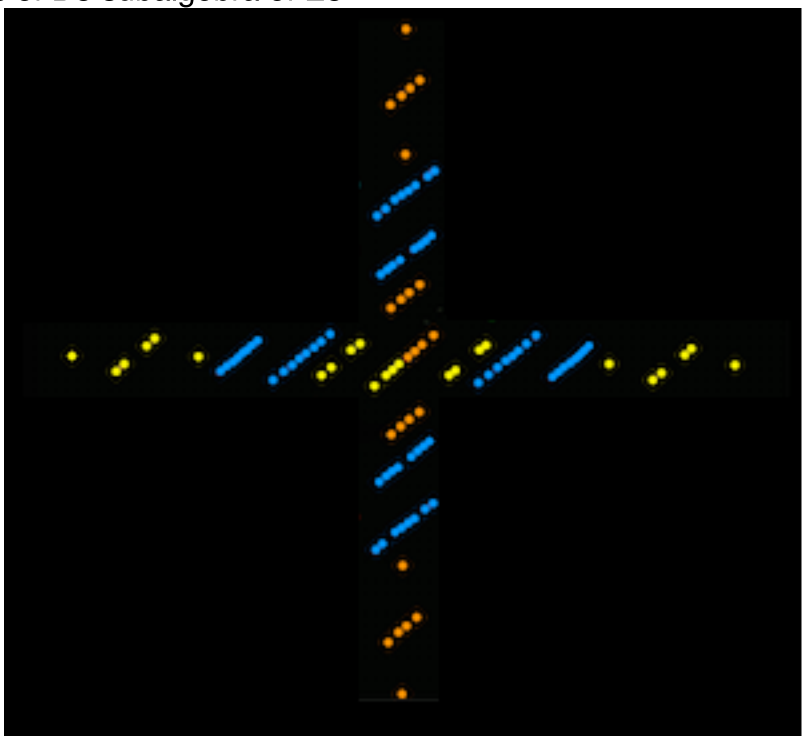


organized to produce this Lagrangian

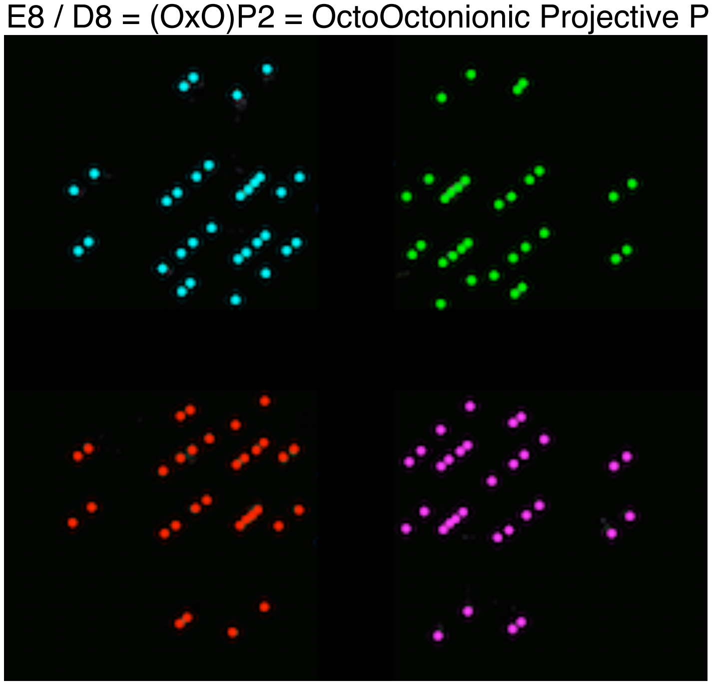


based on these E8 structures:

112 root vectors of D8 subalgebra of E8



128 root vectors of E8 / D8 = (OxO)P2 = OctoOctonionic Projective Plane

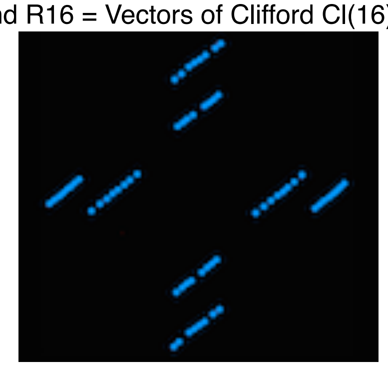


24 root vectors of D4 with D3 subalgebra representing Conformal Gravity + Dark Energy

24 root vectors of D4 with A3 subalgebra containing A2 of Standard Model Color Force

64 root vectors of D8 / D4xD4 = Gr(8,16) = 64-dim Octonionic Subspaces of R16

(Gr = Grassmanian and R16 = Vectors of Clifford Cl(16) Matrix Algebra for D8)

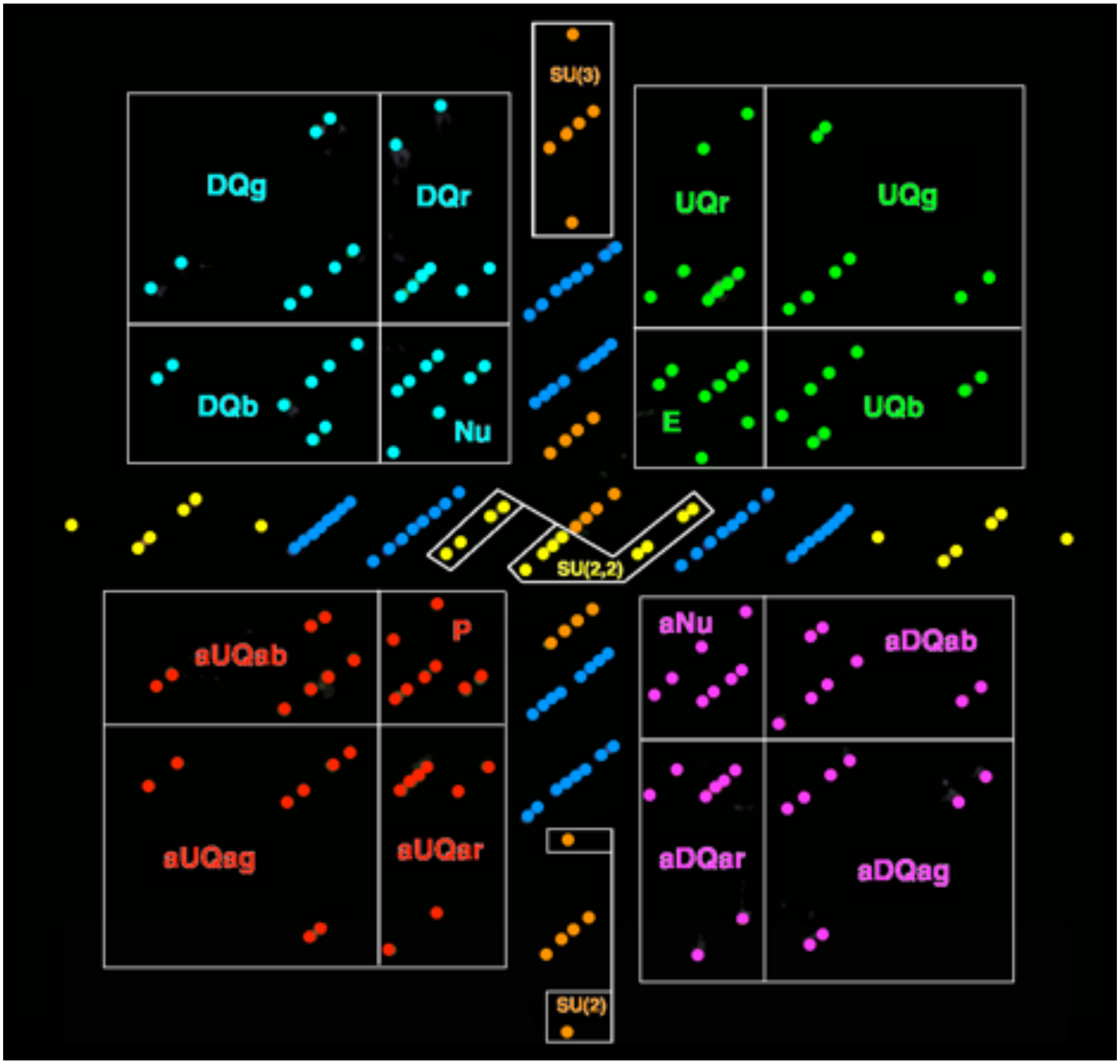


representing 8-dim Octonionic spacetime which, upon freezing out of a preferred Quaternionic structure, produces 4+4 dim Kaluza-Klein M4 x CP2

M4 being the Horizontal 16+16 corresponding to Gravity+Dark Energy

CP2 = SU(3)/SU(2)xU(1) being the vertical 16+16 corresponding to the Standard Model.

The physical interpretations of the 240 E8 root vectors are:



E = electron, UQr = red up quark, UQg = green up quark, UQb = blue up quark
Nu = neutrino, DQr = red down quark, DQg = green down quark, DQb = blue down quark
P = positron, aUQar = anti-red up antiquark,
aUQag = anti-green up antiquark, aUQab = anti-blue up antiquark
aNu = antineutrino, aDQar = anti-red down antiquark,
aDQag = anti-green down antiquark, aDQab = anti-blue down antiquark

Each Lepton and Quark has 8 components with respect to 4+4 dim Kaluza-Klein
6 orange SU(3) and 2 orange SU(2) represent Standard Model root vectors
24-6-2 = 16 orange represent U(2,2) Conformal Gravity Ghosts
12 yellow SU(2,2) represent Conformal Gravity SU(2,2) root vectors
24-12 = 12 yellow represent Standard Model Ghosts
32+32 = 64 blue represent 4+4 dim Kaluza-Klein spacetime position and momentum

Gauge Gravity and Standard Model terms of Lagrangian have total weight 28 x 1 = 28
12 generators for SU(3) and U(2) Standard Model +
+ 16 generators for U(2,2) of Conformal Gravity =
= 28 D4 Gauge Bosons each with 8-dim Lagrangian weight = 1

Fermion Particle-AntiParticle term also has total weight 8 x (7/2) = 28
8 Fermion Particle/Antiparticle types each with 8-dim Lagrangian weight = 7/2

Since Boson Weight 28 = Fermion Weight 28
the Cl(16)-E8 model has a Subtle SuperSymmetry and is UltraViolet Finite.

The Cl(16)-E8 model has 8-dim Lorentz structure satisfying Coleman-Mandula because its fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime.
(See pages 382-384 of Steven Weinberg's book "The Quantum Theory of Fields" Vol. III)

The Cl(16)-E8 model is Chiral because
E8 contains Cl(16) half-spinors (64+64) for a Fermion Generation
but does not contain Cl(16) Fermion AntiGeneration half-spinors (64+64).
Fermion +half-spinor Particles with high enough velocity are seen as left-handed.
Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

The Cl(16)-E8 model obeys Spin-Statistics because
the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number 2+1 = 3 and Atiyah-Singer index -1/8 which is not the net number of generations because
CP2 has no spin structure but you can use a generalized spin structure
(Hawking and Pope (Phys. Lett. 73B (1978) 42-44))
to get (for integral m) the generalized CP2 index n_R - n_L = (1/2) m (m+1)
Prior to Dimensional Reduction: m = 1, n_R - n_L = (1/2)x1x2 = 1 for 1 generation
After Reduction to 4+4 Kaluza-Klein: m = 2, n_R - n_L = (1/2)x2x3 = 1 for 3 generations
(second and third generations emerge as effective composites of the first)

Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...what happens in CP2 … is a two-surface K which cannot be shrunk to zero. ... However, one could replace the electromagnetic field by a Yang-Mills field whose group G had a double covering G~. The fermion field would have to occur in representations which changed sign under the non-trivial element of the kernel of the projection … G~ -> G while the bosons would have to occur in representations which did not change sign …".
For Cl(16)-E8 model gauge bosons are in the 28+28=56-dim D4 + D4 subalgebra of E8.
D4 = SO(8) is the Hawking-Pope G which has double covering G~ = Spin(8).
The 8 fermion particles / antiparticles are D4 half-spinors represented within E8
by anti-commutators and so do change sign while
the 28 gauge bosons are D4 adjoint represented within E8 by commutators
and so do not change sign.

E8 inherits from F4 the property whereby
its Spinor Part need not be written as Commutators
but can also be written in terms of Fermionic AntiCommutators. (vixra 1208.0145)

Fundamental Spinor - Clifford Algebra Origin of E8

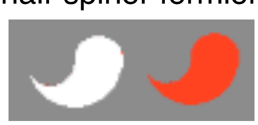
Where does the E8 of E8 Physics come from ?
Based on David Finkelstein's view of Fundamental Physics:

In the beginning there was Cl(0) spinor fermion void

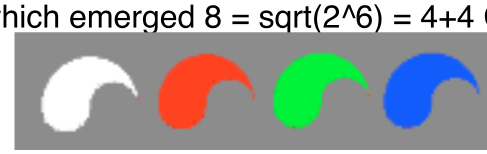
from which emerged 2 = sqrt(2^2) = 1+1 Cl(2) half-spinor fermions/antifermions

 and 

from which emerged 4 = sqrt(2^4) = 2+2 Cl(4) half-spinor fermions/antifermions

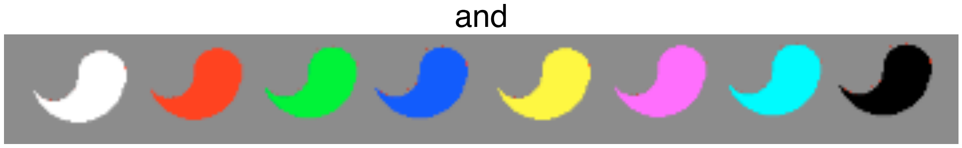
 and 

from which emerged 8 = sqrt(2^6) = 4+4 Cl(6) half-spinor fermions/antifermions

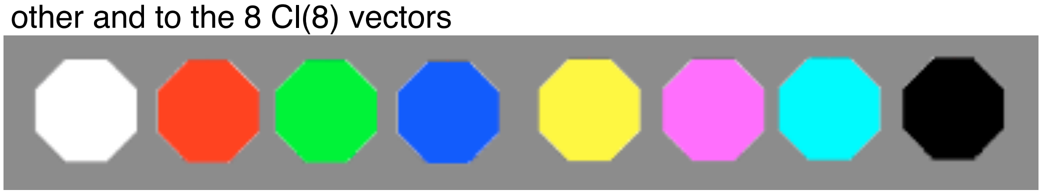
 and 

from which emerged 16 = sqrt(2^8) = 8+8 Cl(8) half-spinor fermions/antifermions

and

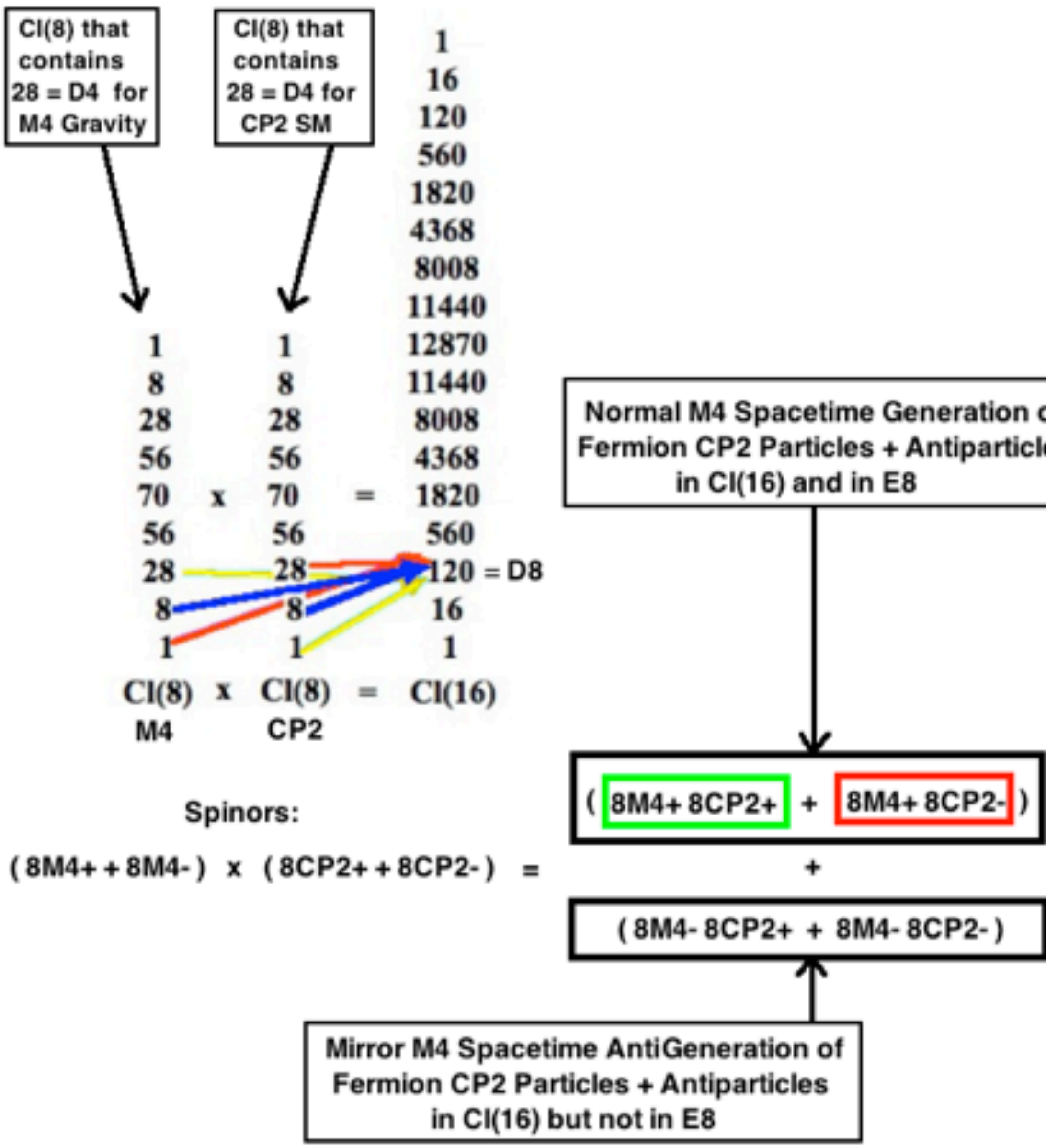


8 half-spinor fermions and 8 half-spinor antifermions are isomorphic by Cl(8) Triality to each other and to the 8 Cl(8) vectors



8-Periodicity of Real Clifford Algebras
Cl(8) x ...(N times tensor product)... x Cl(8) = Cl(8N)
shows that Cl(8) (or any tensor multiple it) is the basic building block
of ALL Real Clifford Algebras, no matter how large they may be.

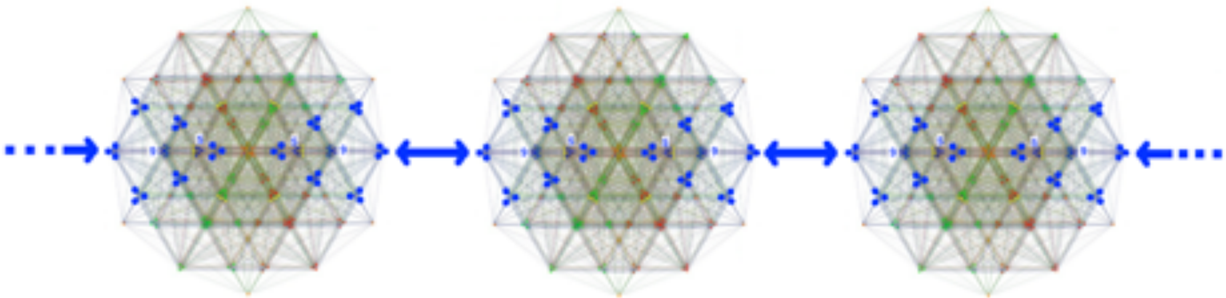
In particular, the tensor product Cl(8) x Cl(8) = Cl(16)



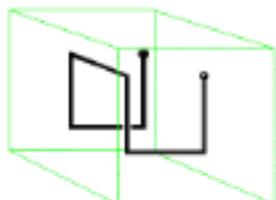
256 = sqrt(2^16) = 128 + 128 Cl(16) spinors
128 Cl(16) half-spinors = 64 + 64 fermions + antifermions
120 = Cl(16) bivectors = D8 root vectors
120 + 64 + 64 = E8 root vectors
E8 / D8 = 128-dim (OxO)P2 OctoOctonionic Projective Plane
D8 / D4xD4 = Gr(8,16) = 64-dim Octonionic Subspaces of R16
(Gr = Grassmanian and R16 = Vectors of Clifford Cl(16) Matrix Algebra for D8)
one D4 contains D3 of Conformal Gravity+Dark Energy
other D4 contains A3 of Standard Model Color Force SU(3)
(CP2 = SU(3) / SU(2)xU(1) of Kaluza-Klein contains SU(2)xU(1) of Electroweak Forces)

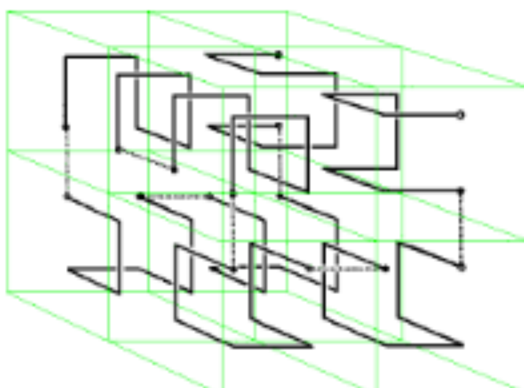
One Cl(16) containing one E8 gives a Lagrangian description of one local spacetime neighborhood. To get a realistic global spacetime structure, take the tensor product Cl(16) x ... x Cl(16) with all E8 local 8-dim Octonionic spacetimes consistently aligned as described by 64-dim D8 / D4xD4 (blue dots)

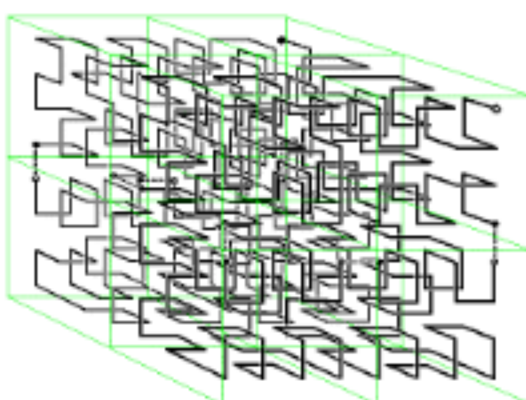
(this visualization uses a hexagonal type of projection of the 240 E8 root vectors to 2-dim)



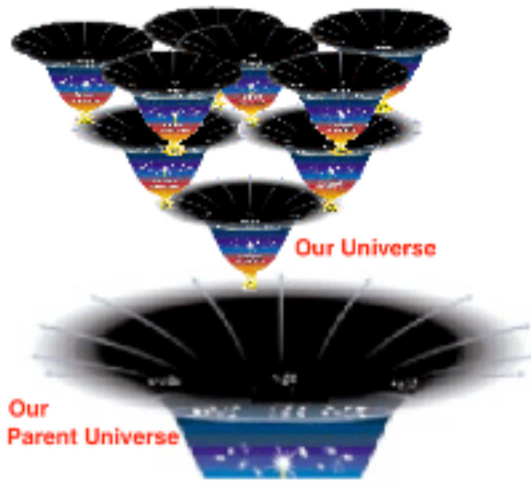
which then fill up spacetime according to Gray Code Hilbert's curves:







Our Universe emerged from its parent in Octonionic Inflation



As Our Parent Universe expanded to a Cold Thin State Quantum Fluctuations occurred.
Most of them just appeared and disappeared as Virtual Fluctuations,
but at least one Quantum Fluctuation had enough energy to produce 64 Unfoldings
and reach Paola Zizzi's State of Decoherence
thus making it a Real Fluctuation that became Our Universe.

As Our Universe expands to a Cold Thin State, it will probably give birth
to Our Child, GrandChild, etc, Universes.

Unlike "the inflationary multiverse" decribed by Andrei Linde in arXiv 1402.0526 as
"a scientific justification of the anthropic principle",
in the Cl(16)-E8 model ALL Universes (Ours, Ancestors, Descendants)
have the SAME Physics Structure as E8 Physics (viXra 1312.0036 and 1310.0182)

In the Cl(16)-E8 model,
our SpaceTime remains Octonionic 8-dimensional throughout inflation.

Stephen L. Adler in his book Quaternionic Quantum Mechanics and Quantum Fields (1995) said at pages 50-52, 561: "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) I g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]...
failure of unitarity in octonionic quantum mechanics ...".

The NonAssociativity and Non-Unitarity of Octonions
accounts for particle creation without the need for a conventional inflaton field.

E8 Physics has Representation space for 8 Fermion Particles + 8 Fermion Antiparticles on the original Cl(16) E8 Local Lagrangian Region

_ _ _ _ _ _ _ _ + _ _ _ _ _ _ _ _

where a Fermion Representation slot _ of the 8+8 = 16 slots can be filled
by Real Fermion Particles or Real Fermion Antiparticles

IF the Quantum Fluctuation(QF) has enough Energy to produce them as Real
and
IF the Cl(16) E8 Local Lagrangian Region has an Effective Path from its QF Energy to that Particular slot.

Let Cl(16) = Cl(8) x Cl(8) where
the first Cl(8) contains the D4 of Conformal Gravity with actions on M4 physical spacetime whose CPT symmetry determines the property matter - antimatter.

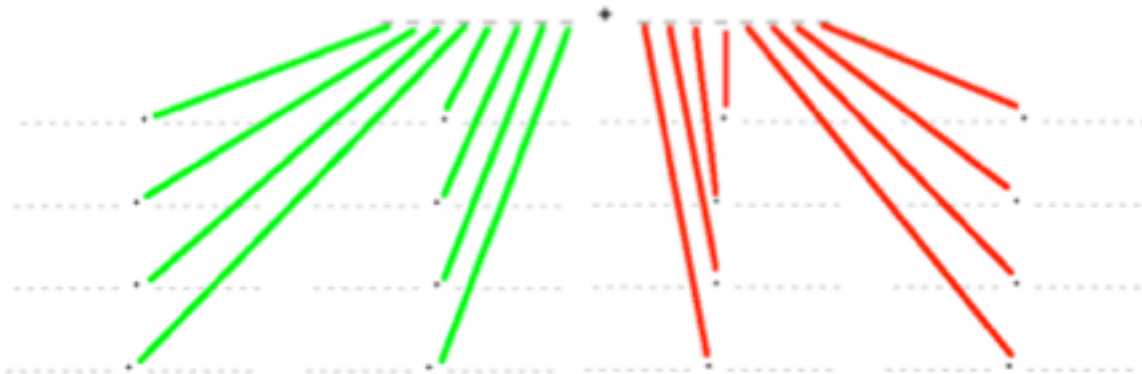
Consider, following basic ideas of Geoffrey Dixon related to his characterization of 64-dimensional spinor spaces as C x H x O (C = complex, H = quaternion, O = ocrtonion),
64-dim 64s++ = 8s+ x 8s+ of Cl(8) x Cl(8) = Cl(16)
and
64-dim 64s+- = 8s+ x 8s- of Cl(8) x Cl(8) = Cl(16)
so that
64s++ + 64s+- = 128s+ are +half-spinors of Cl(16) which is in E8

Then Cl(16) contains
128-dim +half-spinor space 64s++ + 64s+- of Cl(16) in E8 = Fermion Generation
and
128-dim -half-spinor space 64s-+ + 64s-- of Cl(16) not in E8 = Fermion AntiGeneration

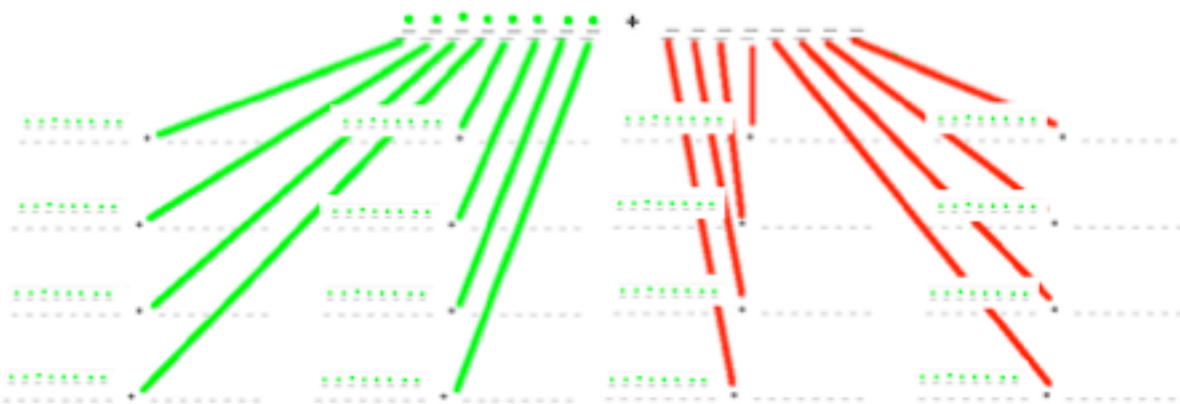
Since E8 contains only the 128 +half-spinors and none of the 128 -half-spinors of Cl(16)
and
since, due to their +half-spinor property with respect to the first Cl(8),
the 128s+ = 64s++ + 64s+- have only Effective Paths of QF Energy that go
to the Fermion Particle slots that are also of type +
that is, to the 8 Fermion Particle Representation slots

Next, consider **the first Unfolding step of Octonionic Inflation**. It is based on all 16 = 8 Fermion Particle slots + 8 Fermion Antiparticle Representation slots whether or not they have been filled by QF Energy.

7 of the 8 Fermion Particle slots correspond to the 7 Imaginary Octonions and
therefore to the 7 Independent E8 Integral Domain Lattices and
therefore to 7 New Cl(16) E8 Local Lagrangian Regions.
The 8th Fermion Particle slot corresponds to the 1 Real Octonion and
therefore to the 8th E8 Integral Domain Lattice (not independent - see Kirmse's mistake) and
therefore to the 8th New Cl(16) E8 Local Lagrangian Region.
Similarly, the 8 Fermion Antiparticle slots Unfold into 8 more New New Cl(16) E8 Local Lagrangian Regions, so that one Unfolding Step is a 16-fold multiplication of Cl(16) E8 Local Lagrangian Regions:



If the QF Energy is sufficient, the Fermion Particle content after the first Unfolding is



so it is clear that **the Octonionic Inflation Unfolding Process creates Fermion Particles with no Antiparticles, thus explaining the dominance of Matter over AntiMatter in Our Universe**.

Each Unfolding has duration of the Planck Time Tplanck
and none of the components of the Unfolding Process Components are simultaneous,
so that **the total duration of N Unfoldings is 2^N Tplanck**.
Paola Zizzi in gr-qc/0007006 said: "... **during inflation,
the universe can be described as a superposed state of quantum ... [qubits]**.
the self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... **the decoherence time ... [Tdecoh = 10^9 Tplanck = 10^(-34) sec]** ...
and **corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...".

Why decoherence at 64 Unfoldings = 2^{64} qubits ?

2^{64} qubits corresponds to the Clifford algebra Cl(64) = Cl(8x8).
By the periodicity-8 theorem of Real Clifford algebras, Cl(64) is the smallest Real Clifford algebra for which we can reflexively identify each component Cl(8) with a vector in the Cl(8) vector space. This reflexive identification/reduction causes our universe to decohere at $N = 2^{64} = 10^{19}$
which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.
The Real Clifford Algebra Cl(8) is the basic building block of Real Clifford Algebras due to 8-Periodicity whereby Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8)
An Octonionic basis for the Cl(8) 8-dim vector space is {1,i,j,k,E,I,J,K}
NonAssociativity, NonUnitarity, and Reflexivity of Octonions is exemplified by the 1-1 correspondence between Octonion Basis Elements and E8 Integral Domains

1 <=> 0E8
i <=> 1E8
j <=> 2E8
k <=> 3E8
E <=> 4E8
I <=> 5E8
J <=> 6E8
K <=> 7E8

where 1E8,2E8,3E8,4E8,5E8,6E8,7E8 are 7 independent Integral Domain E8 Lattices and 0E8 is an 8th E8 Lattice (Kirmse’s mistake) not closed as an Integral Domain.
Using that correspondence expands the basis {1,i,j,k,E,I,J,K} to

{0E8,1E8,2E8,3E8,4E8,5E8,6E8,7E8}

Each of the E8 Lattices has 240 nearest neighbor vectors so the total dimension of the Expanded Space is 240 x 240 x 240 x 240 x 240 x 240 x 240 x 240
Everything in the Expanded Space comes directly from the original Cl(8) 8-dim space so all Quantum States in the Expanded Space can be held in Coherent Superposition.
However,
if further expansion is attempted, there is no direct connection to original Cl(8) space and any Quantum Superposition undergoes Decoherence.
If each 240 is embedded reflexively into the 256 elements of Cl(8) the total dimension is

$$256 \times 256 \times 256 \times 256 \times 256 \times 256 \times 256 \times 256 = 256^8 = 2^{(8 \times 8)} = 2^{64} =$$
$$= Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) \times Cl(8) = Cl(8 \times 8) = Cl(64)$$

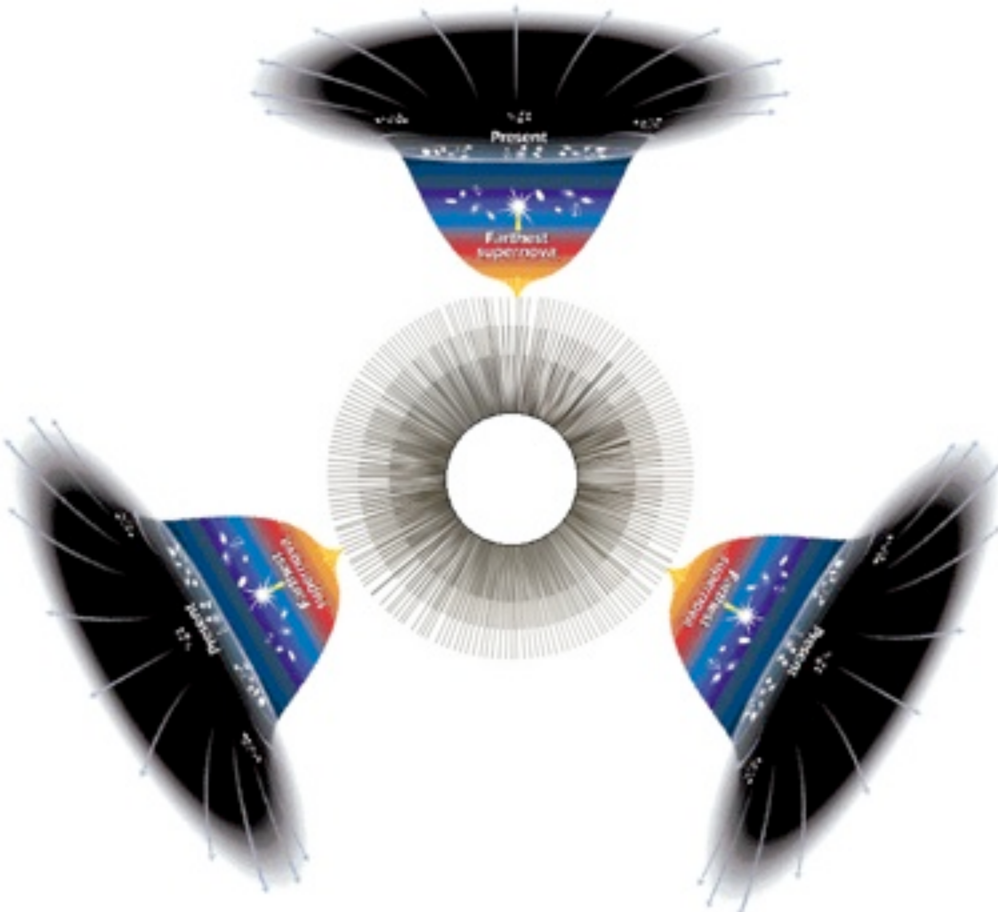
so the largest Clifford Algebra that can maintain Coherent Superposition is Cl(64) which is why Zizzi Quantum Inflation ends at the Cl(64) level.

At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about $(1/2)\, 16^{64} = (1/2)\, (2^4)^{64} = 2^{255} = 6 \times 10^{76}$ Fermion Particles

The End of Inflation time was at about 10^{-34} sec = 2^{64} Tplanck
and
the size of our Universe was then about 10^{-24} cm
which is **about the size of a Fermion Schwinger Source Kerr-Newman Cloud**.
(see viXra 1311.0088)

End of Inflation and Low Initial Entropy in Our Universe:

Roger Penrose in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:
"... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... **the low-entropy states in the past are a puzzle**. ...".
The key to solving Penrose's Puzzle is given by Paola Zizzi in gr-qc/0007006:
"... **The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [Tdecoh = 10^9 Tplanck = 10(-34) sec] ... and corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...
... **This is also the number of superposed tubulins-qubits in our brain ... leading to a conscious event**. ...".
The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of Quantum Theory,



only one of which Worlds is our World. The central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World.

Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

End of Inflation and Quaternionic Structure

In Cl(16)-E8 Physics (vixra 1405.0030) Octonionic symmetry of 8-dim spacetime is broken at the End of Non-Unitary Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2 physical spacetime x internal symmetry space.

Here are some details about that transition:

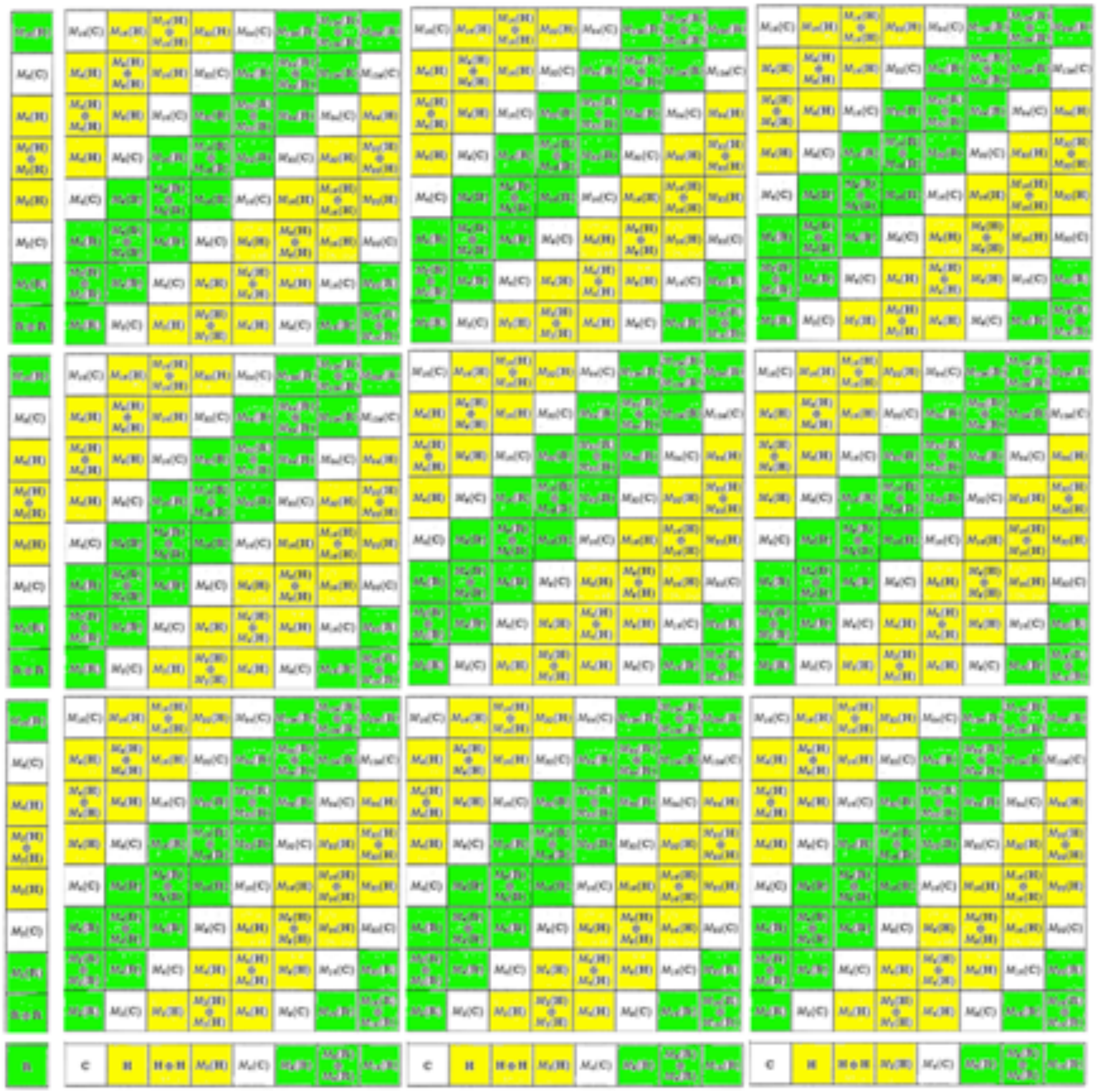
The basic local entity of Cl(16)-E8 Physics is
Cl(0,16) = Cl(1,15) = Cl(4,12) = Cl(5,11) = Cl(8,8) = M(R,256) = 256x256 Real Matrices
which contains E8 with 8-dim Octonionic spacetime
and is the tensor product Cl(0,8) x Cl(0,8) = Cl(1,7) x Cl(1,7)
where Cl(0,8) = Cl(1,7) = M(R,16) is the Clifford Algebra of the 8-dim spacetime.

Non-Unitary Octonionic Inflation is based on Octonionic spacetime structure with superposition of E8 integral domain lattices. At the End of Inflation the superposition ends and Octonionic 8-dim structure is replaced by Quaternionic (4+4)-dim structure.

Since M(R,16) = M(Q,2) x M(Q,2) and M(Q,2) = Cl(1,3) = Cl(0,4)
Cl(0,8) = Cl(1,7) can be represented as Cl(1,3) x Cl(0,4)
where
Cl(1,3) is the Clifford Algebra for M4 physical spacetime
and
Cl(0,4) is the Clifford Algebra for CP2 = SU(3) / U(2) internal symmetry space
thus
making explicit the Quaternionic structure of (4+4)-dim M4 x CP2 Kaluza-Klein.

Quaternionic structure similar to that of Cl(1,3) = Cl(0,4) = M(Q,2) is seen in
Cl(2,4) = M(Q,4) = 4x4 Quaternion matrices with grading based on 4x4 = 1 4 6 4 1

1	2	1				
	4	8	4			
		6	12	6		
			4	8	4	
				1	2	1
1	6	15	20	15	6	1



Conformal Gravity Spin(2,4) = SU(2,2) of Cl(2,4) = M(Q,4) 4x4 Quaternionic Matrices

Batakis Standard Model Gauge Groups and Mayer-Trautman Higgs

The Mayer-Trautman Mechanism reduces
the Lagrangian integral over the 8-dim SpaceTime whose 8-Position x 8-Momentum is represented by 64-dim D8 / D4xD4 where D8 is the Adjoint part of E8.

$$\int_{\text{8-dim SpaceTime}} \text{Gauge Gravity} + \text{Standard Model} + \text{Fermion Particle-AntiParticle} \quad .$$

to

a Lagrangian integral over the 4-dim M4 Minkowski Physical SpaceTime part of Kaluza-Klein M4 x CP2

$$\int_{\text{4-dim M4}} \text{GG} + \text{SM} + \text{Fermion Particle-AntiParticle} + \text{Higgs} \quad .$$

by integrating out the Lagrangian Density over the CP2 Internal Symmetry Space and so creating a new Higgs term in the Lagrangian Density integrated only over M4.

Since the D4 = U(2,2) of Gauge Gravity acts on the M4, there is no problem with it.

As to the D4 = U(4) of the Standard Model, U(4) contains as a subgroup color SU(3) which is also the global symmetry group of the CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space of M4 X CP2 Kaluza-Klein SpaceTime.

A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 said:
"... In a standard Kaluza-Klein framework, M4 x CP2 allows
the classical unified description of an SU(3) gauge field with gravity ...
[and]
the possibility of an additional SU(2) x U(1) gauge field structure is uncovered. ...".

Since the CP2 = SU(3) / U(2) has global SU(3) action,
the SU(3) can be considered as a local gauge group acting on the M4,
so there is no problem with it.

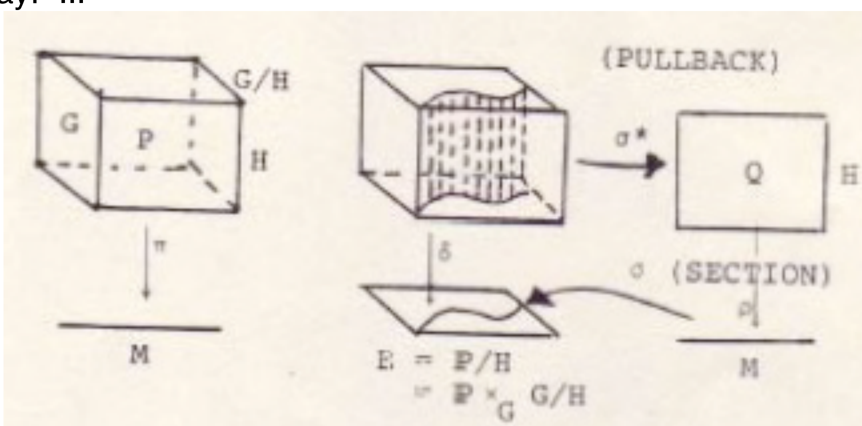
However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so **the local action of U(2) on CP2 must be integrated out to get the desired U(2) = SU(2)xU(1) local action directly on M4**.

Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4.

This is done by a mechanism due to Meinhard Mayer and A. Trautman in
“A Brief Introduction to the Geometry of Gauge Fields” and
“The Geometry of Symmetry Breaking in Gauge Theories”,
Acta Physica Austriaca, Suppl. XXIII (1981)

where they say: "...



... We start out from ... four-dimensional M [M4] ...[and]... R ...[that is]... obtained from ... G/H [CP2 = SU(3) / U(2)] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on M [M4] with values in H [SU(2)] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ...
S_YM = Integral Tr (F /\ *F)
... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2] .
We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ...
Then the integrand ... becomes
Tr(F_!! F^!! + 2 F_!? F^!? + F_?? F^??)
...

The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory
...
the middle term .. becomes, symbolically,
Tr Sum D_! PHI(?) D^! PHI(?)
where PHI(?) is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field ? , in the G/H [CP2] direction
...
the third term ... involves the contraction F_?? of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]
2 F_?? = [PHI(?) , PHI(?)] - PHI([?,?])
... Thus,
the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:
Tr F_?? F^?? = (1/4) Tr ([PHI , PHI] - PHI)^2
...
special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

See S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11: "...

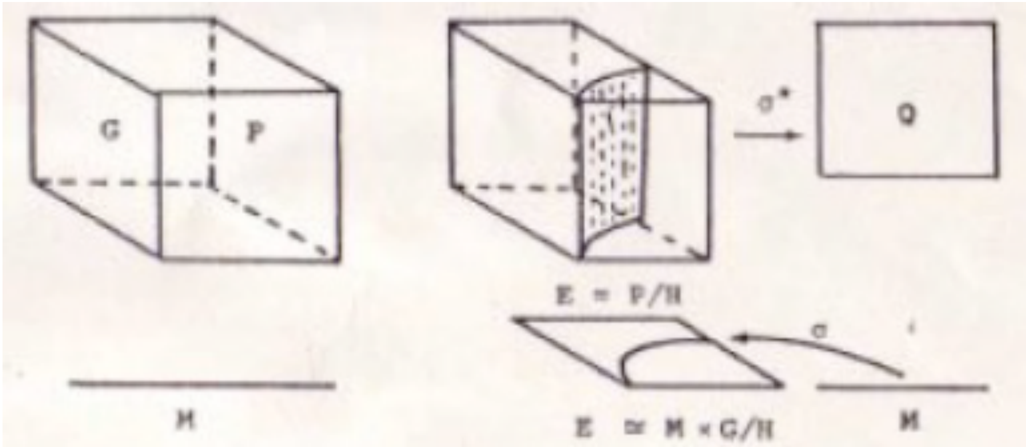
THEOREM 11.7. *Assume in Theorem* 11.5 *that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ (*direct sum*) *and* $\mathrm{ad}\,(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}\,(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then* ...

The curvature form Ω *of the* K*-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \text{for } X, Y \in \mathfrak{m},$$

...".

Along the same lines, Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): “...



... each point of ... the ... fibre bundle ... E consists of a four- dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2] ... the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4]) ...
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and a quartic functional for the “Higgs scalars”, which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...”.

Fock - Hua - Wolf - Schwinger - Wyler Quantum Theory

Fock (1931) showed that Fundamental Quantum Theory requires Linear Operators "... represented by a definite integral [of a]... kernel ... function ...".

Hua (1958) showed Kernel Functions for Complex Classical Domains.

Schwinger (1951 - see Schweber, PNAS 102, 7783-7788) "... introduced a description in terms of Green's functions, what Feynman had called propagators ... The Green's functions are vacuum expectation values of time-ordered Heisenberg operators, and the field theory can be defined non-perturbatively in terms of these functions ...[which]... gave deep structural insights into QFTs; in particular ... the structure of the Green's functions when their variables are analytically continued to complex values ...".

Wolf (J. Math. Mech 14 (1965) 1033-1047) showed that the Classical Domains
(complete simply connected Riemannian symmetric spaces)
representing 4-dim Spacetime with Quaternionic Structure are:
S1 x S1 x S1 x S1 = 4 copies of U(1)
S2 x S2 = 2 copies of SU(2)
CP2 = SU(3) / SU(2)xU(1)
S4 = Spin(5) / Spin(4) = Euclidean version of Spin(2,3) / Spin(1,3)

Armand Wyler (1971 - C. R. Acad. Sc. Paris, t. 271, 186-188) showed how to use
Green's Functions = Kernel Functions of Classical Domain structures characterizing
Sources = Leptons, Quarks, and Gauge Bosons,
to calculate Particle Masses and Force Strengths (see also viXra 1405.0030).

Schwinger (1969 - see physics/0610054) said: "... operator field theory ... replace[s] the particle with ... properties ... distributed througout ... small volumes of three-dimensional space ... particles ... must be created ... even though we vary a number of experimental parameters ... The properties of the particle ... remain the same ... We introduce a quantitative description of the particle source in terms of a source function ...
we do not have to claim that we can make the source arbitrarily small ...
the experimeter... must detect the particles ...[by]... collision that annihilates the particle ... the source ... can be ... an abstraction of an annilhilation collision, with the source acting negatively, as a sink ... The basic things are ... the source functions ... describing the intermediate propagation of the particle ...".

Creation and Annihilation operators indicate a Clifford Algebra, and 8-Periodicity shows that the basic Clifford Algebra is formed by tensor products of 256-dim Cl(8) such as Cl(8) x Cl(8) = Cl(16) containing 248-dim E8 = 120-dim D8 + 128-dim D8 half-spinor whose maximal contraction is a realistic generalized Heisenberg Algebra

h92 x A7 = 5-graded 28 + 64 + ((SL(8,R)+1) + 64 + 28

(see viXra 1507.0069 and 1405.0030)

Schwinger Sources with inherited Monster Group Symmetry
have
Kerr-Newman Black Hole structure size about 10^(-24) cm
and
Geometry of Bounded Complex Domains and Shilov boundaries

The Cl(16)-E8 model Lagrangian over 4-dim Minkowski SpaceTime M4 is

$$\int_{\text{4-dim M4}} GG + SM + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

Consider the Fermion Term.

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
The Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively themass term is, in the conventional picture, an ad hoc inclusion.

The Cl(16)-E8 model does not put in the mass m in an ad hoc way,
but constructs the Lagrangian integral such that
the mass m emerges naturally from the geometry of the spinor fermions by setting the spinor fermion mass term as the volume of the Schwinger Source Fermions.

Effectively the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion gives the volume of the Schwinger Source fermion and defines its mass,
which, since it is dressed with the particle/antiparticle pair cloud,
gives quark mass as constituent mass.

The Cl(16)-E8 model constructs the Lagrangian integral such that
the mass m emerges as the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion so that the volume of the Schwinger Source fermion defines its mass, which, being dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space
Spin(10) / Spin(8)xU(1)
which has local symmetry of the Spin(8) gauge group
from which the first generation spinor fermions are formed as +half-spinor and -half-spinor spaces
and
Bounded Complex Domain D8 of type IV8 and Shilov Boundary Q8 = RP1 x S7

Consider the GG + SM term from Gauge Gravity and Standard Model Gauge Bosons.
The process of breaking Octonionic 8-dim SpaceTime
down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein
creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime

There 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces
with Quaternionic structure consistent with 4-dim Physical SpaceTime:

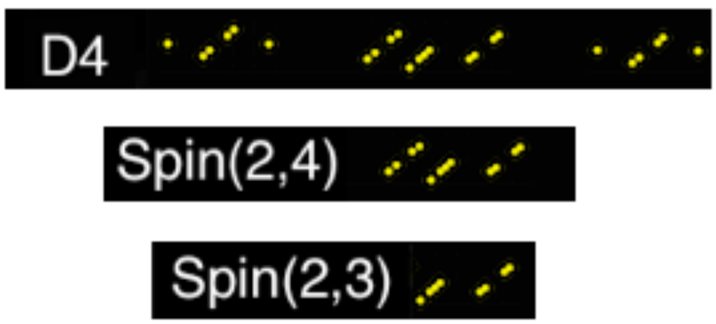
S4 = 4-sphere = Spin(5) / Spin(4) where Spin(5) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives MacDowell-Mansouiri Gravity

CP2 = complex projective 2-space = SU(3) / U(2) with the SU(3) of the Color Force

S2 x S2 = SU(2)/U(1) x SU(2)/U(1) with two copies of the SU(2) of the Weak Force

S1 x S1 x S1 x S1 = U(1) x U(1) x U(1) x U(1) = 4 copies of the U(1) of the EM Photon
(1 copy for each of the 4 covariant components of the Photon)

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in
a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8).

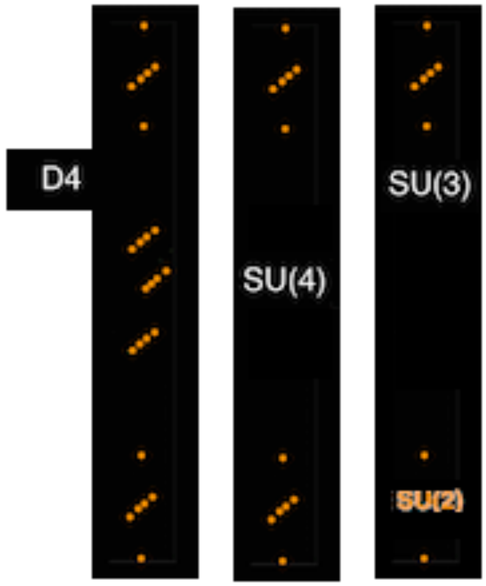


They "see" M4 Physical spacetime as the 4-sphere S4
so that their part of the Physical Lagrangian is

$$\int_{S4} \text{Gravity Gauge Boson Term} \quad .$$

an integral over SpaceTime S4.
The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons.
However, due to Stabilization of Condensate SpaceTime
by virtual Planck Mass Gravitational Black Holes,
for Gravity, the effective force strength that we see in our experiments
is not just composed of the S4 volume and the Spin(5) Schwinger Source volume,
but is suppressed by the square of the Planck Mass.
The unsuppressed Gravity force strength is the Geometric Part of the force strength.

The Standard Model SU(3) Color Force bosons live in
a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8).



They "see" M4 Physical spacetime as the complex projective plane CP2
so that their part of the Physical Lagrangian is

$$\int_{CP2} \text{SU(3) Color Force Gauge Boson Term} \quad .$$

an integral over SpaceTime CP2.
The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.
The Color Force Strength is given by
the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.
Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is
for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in
a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as two 2-spheres S2 x S2
so that their part of the Physical Lagrangian is

$$\int_{S2xS2} \text{SU(2) Weak Force Gauge Boson Term} \quad .$$

an integral over SpaceTime S2xS2.
The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.
However, due to the action of the Higgs mechanism,
for the Weak Force, the effective force strength that we see in our experiments
is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume,
but is suppressed by the square of the Weak Boson masses.
The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in
a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4
(T4 = 4-torus) so that their part of the Physical Lagrangian is

$$\int_{T4} \text{(U(1) Electromagnetism Gauge Boson Term} \quad .$$

an integral over SpaceTime T4.
The Schwinger Sources for U(1) photons
are the Complex Bounded Domains and Shilov Boundaries for U(1) photons.
The Electromagnetic Force Strength is given by
the SpaceTime T4 volume and the U(1) Schwinger Source volume.

Schwinger Sources as described above are continuous manifold structures
of Bounded Complex Domains and their Shilov Boundaries
but
the Cl(16)-E8 model at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines
with 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice is the Monster Group of order about 8 x 10^53.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single Planck-scale entity becauseTachyons create a cloud of particles/antiparticles.
The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.

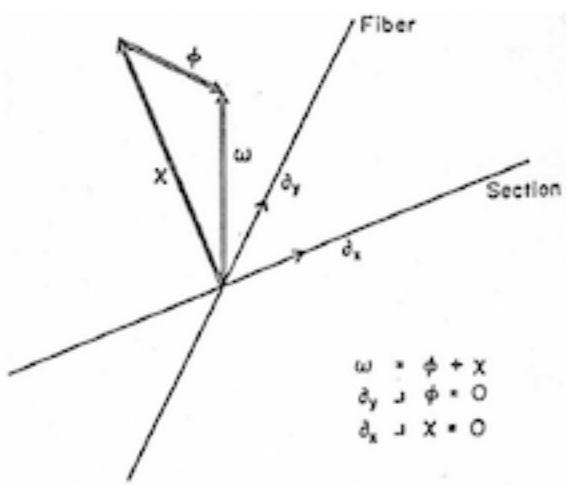
That cloud constitutes the Schwinger Source.
Its structure comes from the 24-dim Leech lattice part of the Monster Group which is 2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain lattices there are 7 (or 8 if you include a non-integral domain E8 lattice)mdistinct Leech lattices.
The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of 10^27 x Planck scale,
so the Kerr-Newman Cloud should contain about 10^27 particle/antiparticle pairs
and its size should be about 10^(27/3) x 1.6 x 10^(-33) cm =
= roughly 10^(-24) cm.

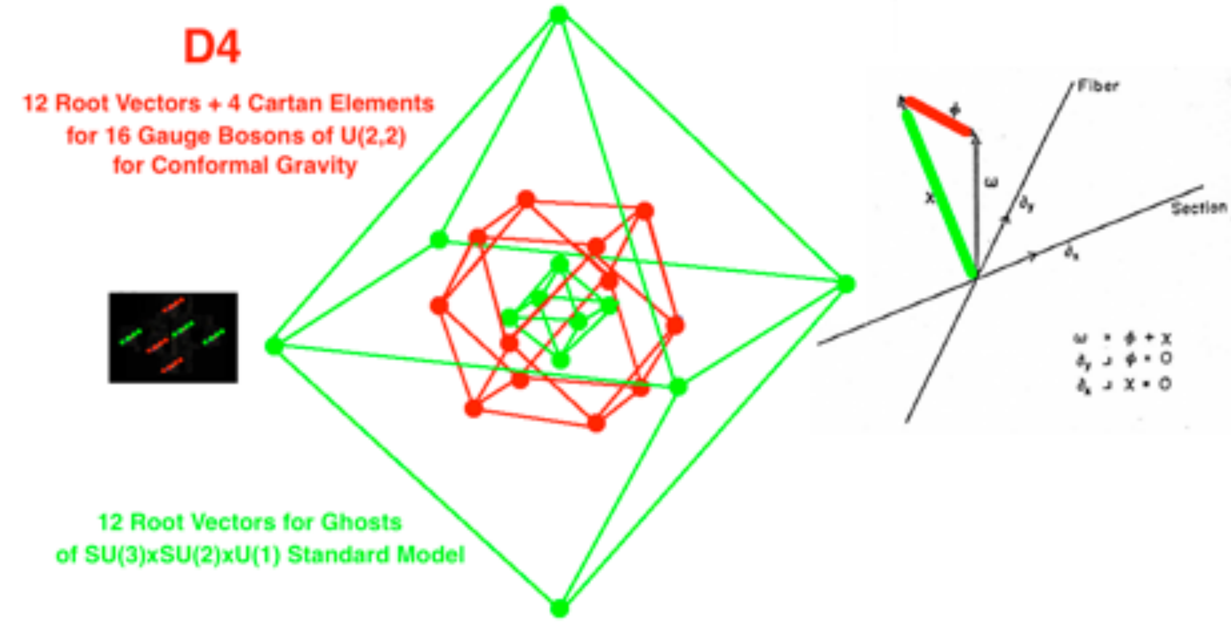
Ghosts

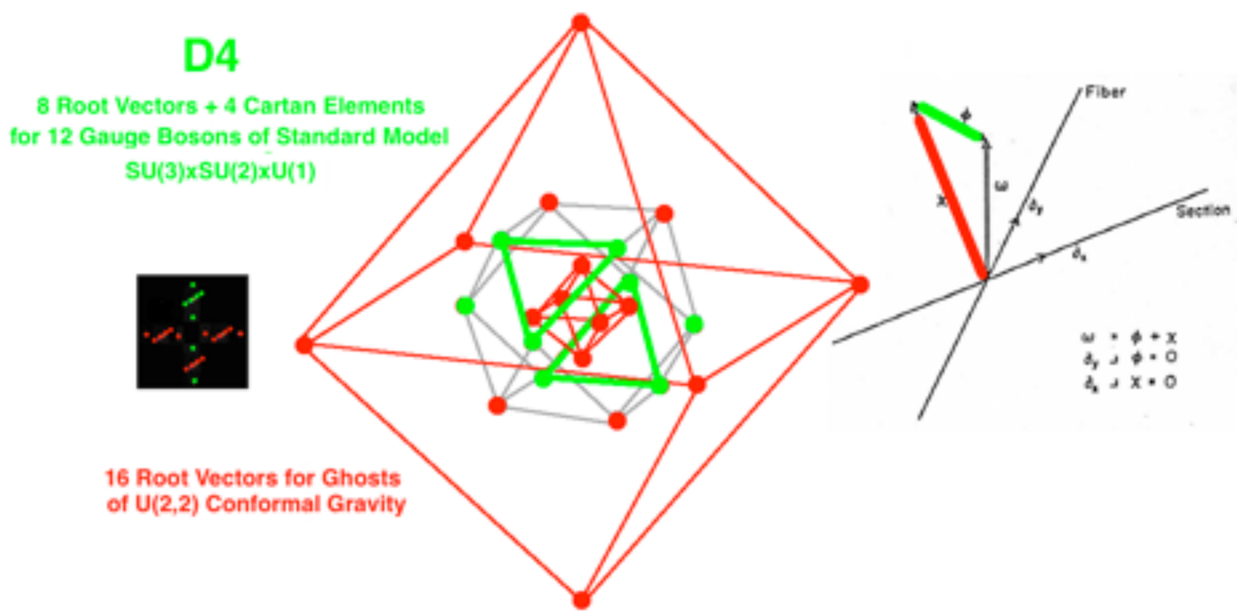
AQFT of Cl(16)-E8 Physics comes from the generalized von Neumann factor algebra constructed by completion of the union of all tensor products of Cl(16) Clifford Algebra where each Cl(16) contains E8 and a local Lagrangian constructed from E8.
The tensor product structure of Cl(16)-E8 AQFT is analogous to the sum-over-histories structure of Path Integral Quantization.
Jean Thierry-Mieg in J. Math. Phys. 21 (1980) 2834-2838 said: "... Because of gauge invariance, the classical Yang-Mills Lagrangian does not define a propagator for the gauge field. Using the path integral formulation of quantum field theory, Faddeev and Popov attributed this effect to the overcounting of gauge equivalent configurations. By fixing the gauge, Feynman diagrams are generated but unitarity is lost unless additional quantum fields are introduced: the ghost particles ...



... FIG. 1. The ghost and the gauge field: The single lines represent a local coordinate system of a principal fiber bundle of base space-time. The double lines are 1 forms. The connection of the principle bundle w is assumed to be vertical. Its contravariant components PHI and X are recognized, respectively, as the Yang-Mills gauge field and the Faddeev-Popov ghost form ... By assumption, the ghost does not contribute to the description of motions tangent to the section. The exterior differential over ... the principal bundle ... of a function also splits, and its component normal to the section is recognized as the BRS operator ... the Cartan-Maurer structural theorem, which states the compatibility of the connection with the fibration, implies the BRS transformation rules of the gauge and ghost fields ... the ghost does not contribute to the curvature 2 form (field strength) and may thus be eliminated from the description of the classical theory. ... In ... the construction of the effective Lagrangian by using the generating functional ... No infinite constant has to be extracted, as the differential of the volume element of the group is actually lifted into the effective Lagrangian in the form of the ghost. The nongeometric transformation of the antighost, a Lagrange multiplier, is not recovered. However, the proof of renormalizability is not altered by the noninvariance of the effective Lagrangian, as one usually cancels the antighost variation via its equations of motion. On the contrary, the renormalized BRS operator is shown, as geometry suggests, not to act on the antighost ...".

There are two D4 in D8 in E8 in Cl(16): D4 Gravity and D4 Standard Model





Force Strength and Boson Mass Calculation

Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the Cl(16)-E8 model Lagrangian (as the D4xD4 subalgebra of the D8 subalgebra of E8)

$$\int_{\text{4-dim M4}} GG + SM + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

with the Higgs term coming from integrating over the CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein by the Mayer-Trautman Mechanism

This shows that the **Force Strength is made up of two parts**:

the relevant spacetime manifold of gauge group global action
and
the relevant symmetric space manifold of gauge group local action.

The 4-dim spacetime Lagrangian GG SM gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally
of the gauge force term of the gauge boson acting locally
for the gauge bosons of each of the four forces:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) subgroup of Conformal Spin(2,4) for gravity by the MacDowell-Mansouri mechanism.

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit (like g IFI^2) or an implicit (incorporated into the IFI^2) form.
Either way, the conventional picture is that the force strength g is an ad hoc inclusion.

The Cl(16)-E8 model does not put in force strength g ad hoc,
but constructs the integral such that
the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as T^4 = S1 x S1 X S1 x S1
the SU(2) weak boson sees 4-dim spacetime as S2 x S2
the SU(3) weak boson sees 4-dim spacetime as CP2
the Spin(5) of gravity sees 4-dim spacetime as S4

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson.
The nontrivial Shilov boundaries are:

for SU(2) Shilov = RP^1xS^2
for SU(3) Shilov = S^5
for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space

S1 for U(1)
S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
CP2 = SU(3)/SU(2)xU(1) for SU(3)
S4 = Spin(5)/Spin(4) for Spin(5)

Each gauge group is the local symmetry of a symmetric space

U(1) for itself
SU(2) for Spin(5) / SU(2)xU(1)
SU(3) for SU(4) / SU(3)xU(1)
Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces
correspond to bounded complex domains

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

Very roughly, think of the force strength as
integral over global symmetry space of physical (ie Shilov Boundary) volume =
= strength of the force.

That is:
the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson means
that the highest force strength or probability should be 1
the gravity Volume product is normalized to be 1, and so (approximately):

Volume product for gravity = 1
Volume product for color = 2/3
Volume product for weak = 1/4
Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

1 - for massive gauge bosons, a suppression by a factor of $1 / M^2$
2 - renormalization running (important for color force)

Consider Massive Gauge Bosons:

Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of
Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes,
so that the geometric strength of gravity should be reduced by $1/Mp^2$

The weak force is carried by weak bosons,
so that the geometric strength of the weak force should be reduced by $1/MW^2$

That gives the result (approximate):

gravity strength = G (Newton's G)
color strength = 2/3
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

Consider Renormalization Running for the Color Force:: That gives the result:

gravity strength = G (Newton's G)
color strength = 1/10 at weak boson mass scale
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

he use of compact volumes is itself a calculational device,
because it would be more nearly correct,
instead of the integral over the compact global symmetry space of
the compact physical (ie Shilov Boundary) volume=strength of the force
to use
the integral over the hyperbolic spacetime global symmetry space
of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios,
and the compact volumes (finite and easy to look up in the book by Hua)
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices,
and
that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Here are less approximate more detailed force strength calculations:

The force strength of a given force is

alphaforce = (1 / Mforce^2) (Vol(MISforce)) (Vol(Qforce) / Vol(Dforce)^(1 / mforce))

where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the relevant part of the target Internal Symmetry Space;

Vol(MISforce) stands for volume of MISforce and is sometimes also denoted by Vol(M);

Qforce represents the link from the origin to the relevant target for the gauge boson;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain
of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce, which is
4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to have two copies of QW for SpaceTime),
1 for Electromagnetism (which therefore is considered to have four copies of QE for SpaceTime)

Vol(Dforce)^(1 / mforce) stands for a dimensional normalization factor
(to reconcile the dimensionality of the Internal Symmetry Space of the target vertex
with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space, and Dforce manifolds for the four forces are:

Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk

4-dim T^4 = S^1 x S^1 x S^1 x S^1 = topological boundary of 8-dim 4-polydisk
Shilov Boundary of 8-dim 4-polydisk = S^1 + S^1 + S^1 + S^1 =
= 1-dim wire frame of 4-dim T^4

Note (thanks to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.
Note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
Weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Note (thanks to Carlos Castro for noticing this) that the volume listed for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.
The effect is particularly pronounced with the color force.
The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

Higgs:

As with forces strengths, the calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.

In the Cl(16)-E8 model, the value of the fundamental mass scale vacuum expectation value v = <PHI> of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, W+, W-, and Z0, whose tree-level masses will then be shown by ratio calculations to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively, and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and v is given by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, i
n the notation of quant-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:

lambda = 3 (M_H / PHI)^2

which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2 (a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165) we have

M_H^2 / v^2 = (cos(pi / 6))^2 / 3

In the Cl(16)-E8 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and v is set to be 252.514 GeV so that

M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

This is the value of the Low Mass State of the Higgs observed by the LHC.
MIddle and High Mass States come from a Higgs-Tquark Condensate System.
The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length represents the Higgs mass

```
Higgs     Higgs in CP2 Internal Symmetry Space
  |          |
  |          |
  |          |
  | mass = 145 | Non-Condensate Higgs Mass = 145
  |          |
  |          |
  |          |
Higgs     Higgs in M4 spacetime
```

and the value of lambda is 1 = 1^2
so that the Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in the Cl(16)-E8 model, the Higgs has structure of a Tquark condensate

```
mass = 145
T ---------Tbar     Effective Higgs in CP2 Internal Symmetry Space
 \    |    /             |
  \   |   /              |
   \  |  /mass = 145     | Higgs Effective Mass =
    \ | /                | = 145 x cos(pi/6)= 145 x 0.866 = 126
     \|/                 |
   Higgs              Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected
by lines forming an equilateral triangle composed of 2 right triangles
(one from the CP2 origin to the T and to the M4 Higgs and
another from the CP2 origin to the Tbar and to the M4 Higgs).
In the T-quark condensate picture
lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi / 6))^2 + (cos(pi / 6))^2
and
lambda(H) = (cos(pi / 6))^2

Therefore the Effective Higgs mass observed by LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257 GeV.

To get W-boson masses,
denote the 3 SU(2) high-energy weak bosons
(massless at energies higher than the electroweak unification)
by W+, W-, and W0,
corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair,
so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar,
which carries the Higgs mechanism by which the W0 becomes the physical Z0,
so that the total mass of the triplet { W+, W-, Z0 }
is equal to the vacuum expectation value v of the Higgs scalar field, v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 } which can be represented by the 3-sphere S^3.
The Hopf fibration of S^3 as

S^1 --> S^3 --> S^2

gives a decomposition of the W bosons into the neutral W0 corresponding to S^1
and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0
should be the volume ratio of the S^2 in S^3 to the S^1 in S3.
The unit sphere S^3 in R^4 is normalized by 1 / 2.
The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).
The ratio of the sum of the W+ and W- masses to the W0 mass should then be
(2 / sqrt3) V(S^2) / (2 / sqrt2) V(S^1) = 1.632993

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV,
the total mass of a T - Tbar pair, and the charged weak bosons have equal mass,
we have

M_W+ = M_W- = 80.326 GeV and M_W0 = 98.379 GeV.

The charged W+/- neutrino-electron interchange must be symmetric
with the electron-neutrino interchange, so that the tree-level absence
of right-handed neutrino particles requires that

the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector,
so that the
charged W+/- SU(2) weak bosons act only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral W0 must be equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-,
there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction is proportional to
(M_W+/-^2 / M_W0^2) acting on left-handed fermions
and
(1 - (M_W+/-^2 / M_W0^2)) acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2 and denoted by K,
and if the strength of the W+/- charged weak force
(and of the custodial SU(2) symmetry) is denoted by T,
then the W0 neutral weak interaction can be written as W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force
and as W0LR with respect to the parity conserving U(1) electromagnetic force,
the W0 mass mW0 has two components:
the parity violating SU(2) part mW0L that is equal to M_W+/-
and the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR,
and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using the volume V(S^2) of an S^2 \subset R^3,
normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,
while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2,
each of which contains one 1-sphere of the *alphaE force.

Therefore

*alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),

*e = e / (4th root of 6) = e / 1.565 ,

and

the mass mW0LR must be reduced to an effective value

M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV

for the *alphaE force to act like an electromagnetic force in the E8 model:

*e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct Cl(16)-E8 model values for weak boson masses and the Weinberg angle theta_w are:

M_W+ = M_W- = 80.326 GeV;

M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree- level values somewhat.

2nd and 3rd Generation Fermions

The 8 First Generation Fermion Particles
can each be represented by the 8 basis elements {1,i,j,k,E,I,J,K} of the Octonions O

1 <=> e-neutrino
i <=> red down quark
j <=> green down quark
k <=> blue down quark

E <=> electron
I <=> red up quark
J <=> green up quark
K <=> blue up quark with AntiParticles being represented similarly.

The Second and Third Generations can be represented by
Pairs of Octonions OxO and Triples of Octonions OxOxO
respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the
Kaluza-Klein M4 x CP2 at the End of Inflation,
there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in M4,
so its path can be represented by the single O;

2 - Either A or B, but not both, is in CP2,
so its path must be augmented by one projection from CP2 to M4,
which projection can be represented by a second O,
giving a second generation OxO;

3 - Both A and B are in CP2,
so its path must be augmented by two projections from CP2 to M4,
which projections can be represented by a second O and a third O,
giving a third generation OxOxO.

Combinatorics contributes to Fermion mass ratios. For example:

Blue Down Quark is 1 out of 8 and Blue Up Quark is 1 out of 8
so the Down Quark : Up Quark mass ratio is 1 : 1

Blue Strange Quark is 3 out of 8x8 = 64 and Blue Charm Quark is 17 out of 8x8 = 64
so the Strange Quark : Charm Quark mass ratio is 3 : 17

Blue Beauty Quark is 7 out of 8x8x8 = 512 and Blue Truth Quark is 161 out of 8x8x8 = 512
so the Beauty Quark : Truth Quark mass ratio is 7 : 161

Fermion Mass Calculations

In the Cl(16)-E8 model, the first generation spinor fermions are seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8).
Due to Triality,
Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime.

Take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:
the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is taken in the same way (unlike what happens for the force strength calculation), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.
Cl(16)-E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes, so the quark mass in the Cl(16)-E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

V(Qfermion) x N(Graviton) x N(octonion) x Sym

V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold S^7 x RP^1 related to the fermion particle by photon, weak boson, or gluon interactions.

N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion.
The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2).
2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons.
One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.

N(octonion) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

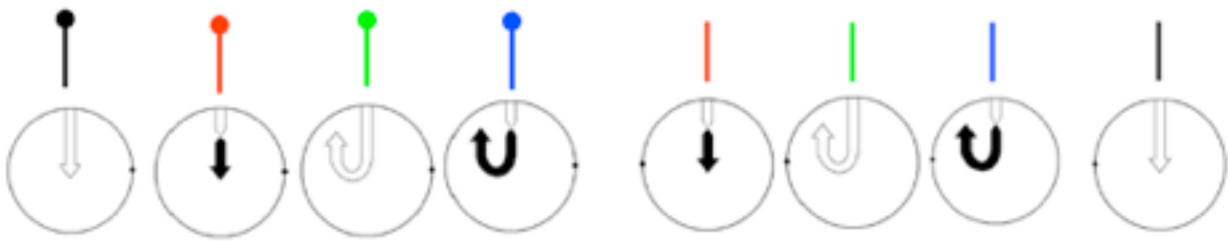
Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

3 Generation Fermion Combinatorics

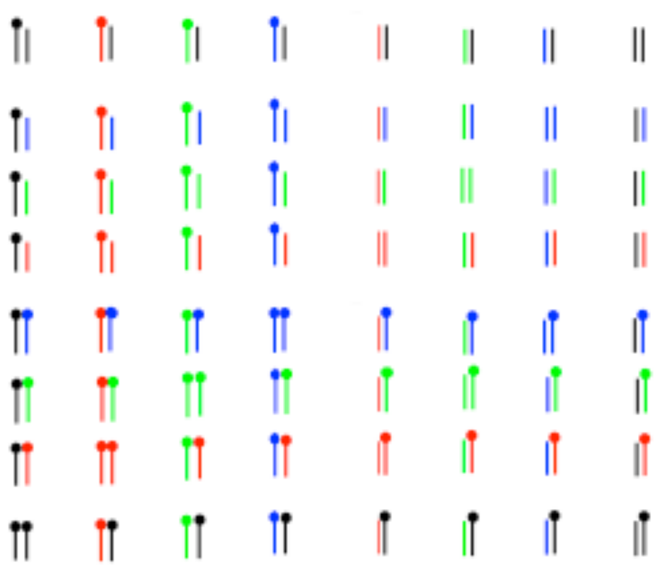
First Generation (8)

(geometric representation of Octonions is from arXiv 1010.2979)

electron	red up quark	green up quark	blue up quark	red down quark	green down quark	blue down quark	neutrino
E	I	J	K	i	j	k	1



Second Generation (64)

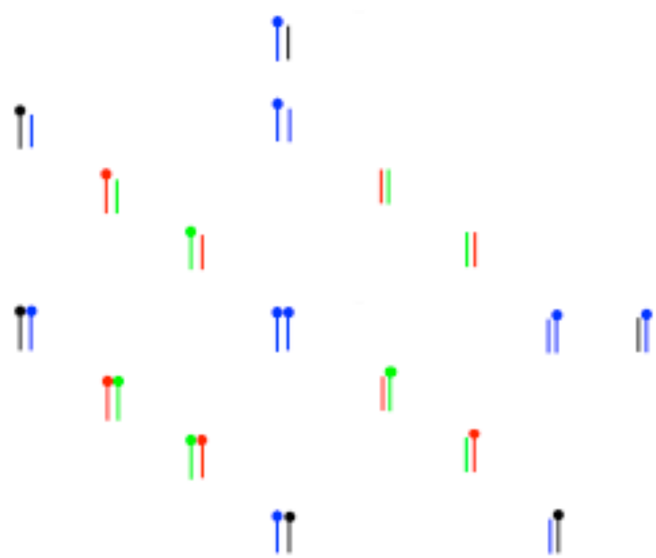


Mu Neutrino (1)
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element) .

Muon (3)
Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative
(that is, is E which is the only Colorless NonAssociative element).

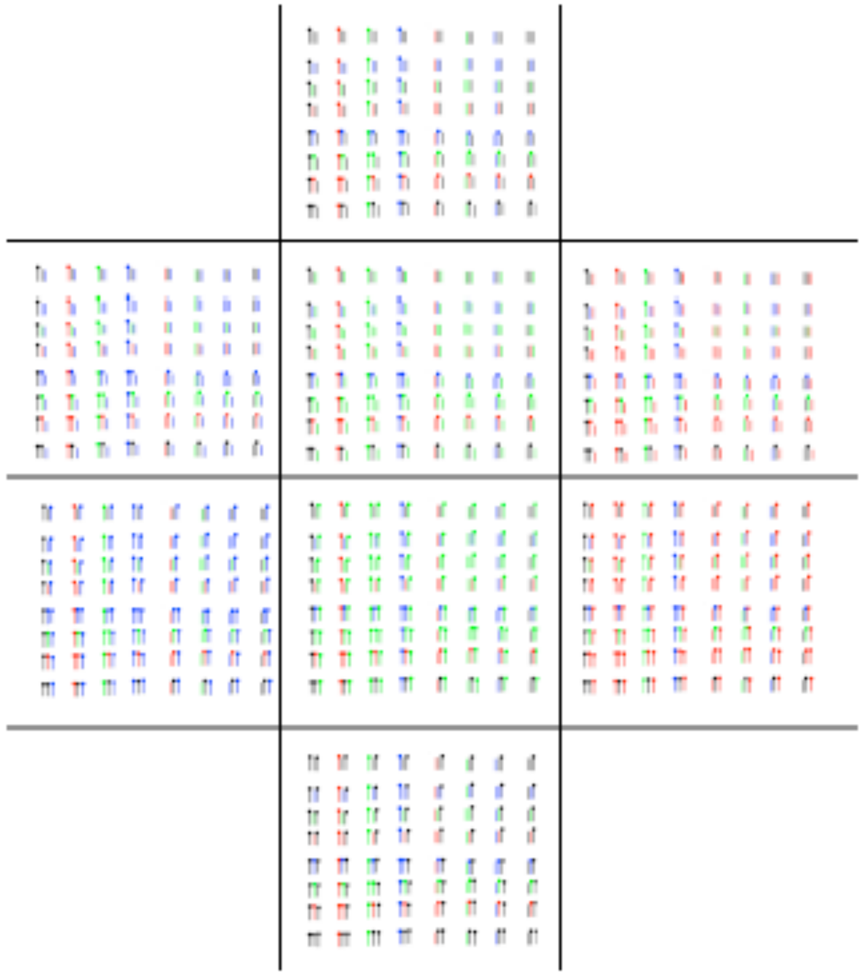
Blue Strange Quark **(3)**
Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Charm Quark **(17)**
Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element (Red x Green = Blue).



(Red and Green Strange and Charm Quarks follow similar rules)

Third Generation (512)

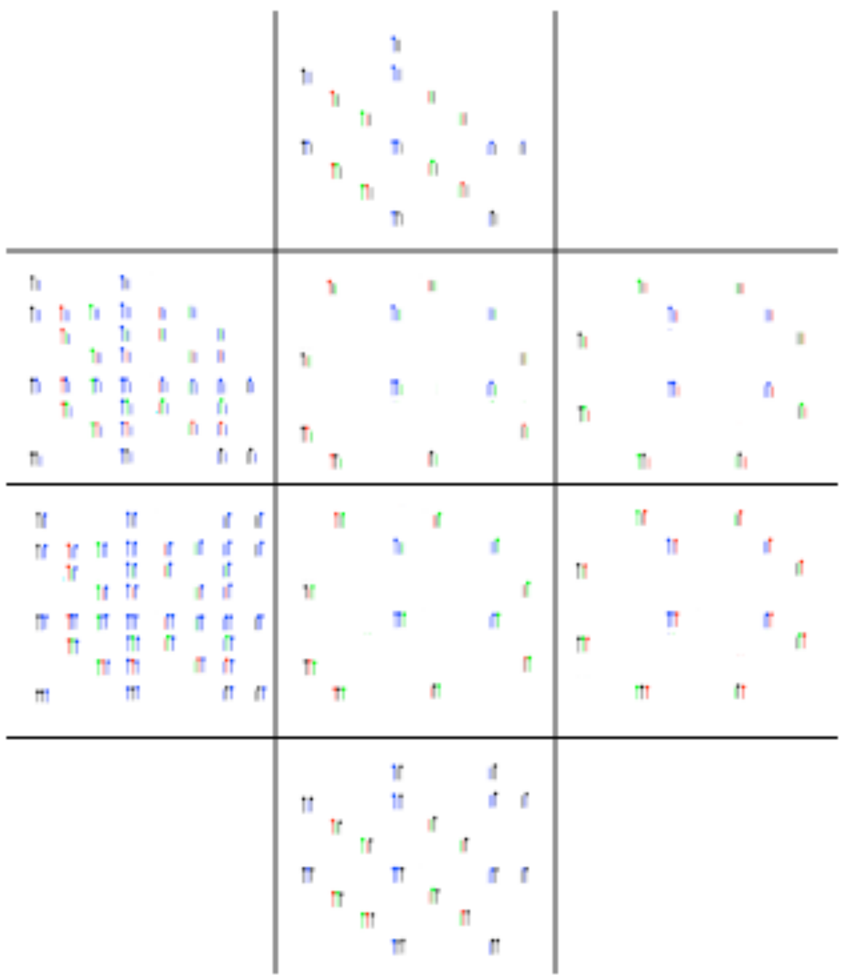


Tau Neutrino (1)
Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element)

Tauon (7)
Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)

Blue Beauty Quark (7)
Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Truth Quark (161)
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative and must be Blue.



(Red and Green Beauty and Truth Quarks follow similar rules)

The first generation down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions.
The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.
The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.
Since the electron cannot be related to any other massive Dirac fermion,
its volume V(Qelectron) is taken to be 1.

Next consider a red down quark i.
By gluon interactions, i can be taken into j and k, the blue and green down quarks.
By also using weak boson interactions,
it can also be taken into I, J, and K, the red, blue, and green up quarks.
Given the up and down quarks, pions can be formed from quark-antiquark pairs,
and the pions can decay to produce electrons and neutrinos.
Therefore the red down quark (similarly, any down quark)
is related to all parts of S^7 x RP^1,
the compact manifold corresponding to { 1, i, j, k, E, I, J, K }
and therefore
a down quark should have
a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.

The ratio of the down quark spinor manifold volume factor
to the electron spinor manifold volume factor is

V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md / me = 6 V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to I, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.
Since the model only gives ratios of masses,
the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:

Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

which is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions. There are 8^3 = 512 such triples.

The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

{ E, E, E }
{ E, E, 1 }
{ E, 1, E }
{ 1, E, E }
{ 1, 1, E }
{ 1, E, 1 }
{ E, 1, 1 }

The symmetry of the 7 tauon triples is the same
as the symmetry of the first generation tree-level-massive fermions,
3 down, quarks, the 3 up quarks, and the electron,
so by the Sym factor the tauon mass should be the same as
the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV,
the effective tauon mass should be renormalized
from the energy level of 1 GeV at which the mass is 1.88 GeV
to the energy level of 2 GeV.
Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent,
the effective tauon mass at 2 GeV would be about 1.78 GeV.
The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples involving 1 and E,
but for 1 and I, 1 and J, and 1 and K,
which correspond to the red, green, and blue beauty quarks,
respectively.

The seven red beauty quark triples correspond to the seven tauon triples,
except that
the beauty quark interacts with 6 Spin(0,5) gravitons
while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times
the third generation graviton factor 6/2 = 3,
so the red beauty quark mass is mb = 5.63111 GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.
Therefore, the calculated beauty quark mass of 5.63 GeV
corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives
a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass
if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 5 GeV is about 0.22.

Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV
gives an MSbar 1-loop beauty quark mass of 4.6 GeV,
and
an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV,
the MSbar mass decreases by about 1.3 GeV,
giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark
and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV,
with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

The theoretical model calculated Beauty Quark mass of 5.63 GeV
corresponds to a pole mass of 5.32 GeV,
which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value
of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value
of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and
the theoretical model calculated value
of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value
of the color force strength constant alpha_s at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that
is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV),
and a color force strength alpha_s at 5 GeV (0.166)
such that 1 + alpha_s = 1.166 is about 4 percent lower
than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Triples of the type { 1, I, J } , { I, J, K }, etc.,
do not correspond to the beauty quark, but to the truth quark.
The truth quark corresponds to those 512 - 1 - 7 - 21 = 483 triples,
so the constituent mass of the red truth quark
is 161 / 7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in the Cl(16)_E8 model.
The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994.
The Low and High Mass States of the Truth Quark have, in my opinion, also been
observed by Fermilab (see Chapter 17 of this paper) but the Fermilab and CERN
establishments disagree.

All other masses than the electron mass
(which is the basis of the assumption of the value of the Higgs scalar field vacuum
expectation value v = 252.514 GeV),
including the Higgs scalar mass and Truth quark mass,
are calculated (not assumed) masses in the Cl(16)-E8 model.
These results when added up give a total mass of third generation fermion
particles:

Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.
There are 8^2 = 64 such pairs.

The pair { 1,1 } corresponds to the mu-neutrino.

The pairs { 1, E }, { E, 1 }, and { E, E } correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs
to the symmetries of the first generation fermion particles:
The pair { E, E } should correspond to the E electron.
The other two muon pairs have a symmetry group S2,
which is 1/3 the size of the color symmetry group S3
which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of
the { E, E } electron mass and
the { 1, E }, { E, 1 } symmetry mass, which is 1/3 of the up or down quark mass.
Therefore, mmu = 104.76 MeV .

According to the 1998 Review of Particle Physics of the Particle Data Group,
the experimental muon mass is about 105.66 MeV which may be consistent with
radiative corrections for the calculated tree-level mmu = 104.76 MeV as
Bailin and Love, in “Introduction to Gauge Field Theory”, IOP (rev ed 1993), say:
"... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to mmu^5 the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.

The red, blue and green strange quark each corresponds
to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs { 1, i }, { i, 1 }, { i, i }
because i is the red down quark.
Its mass should be the sum of two parts:
the { i, i } red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.

Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.

Therefore the graviton factor for the second and third generations is 6/2 = 3.

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and
the red strange quark constituent mass is ms = 312.75 MeV + 312.75 MeV = 625.5 MeV

The blue strange quarks correspond to the three pairs involving j,
the green strange quarks correspond to the three pairs involving k,
and their masses are similarly determined to also be 625.5 MeV.
The charm quark corresponds to the remaining 64 - 1 - 3 - 9 = 51 pairs.

Therefore, the mass of the red charm quark should be the sum of two parts:
the { i, i }, red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor 51 / 9,
which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is
mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a
conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the
Charm Quark pole mass from 1.2 to 1.9 GeV.

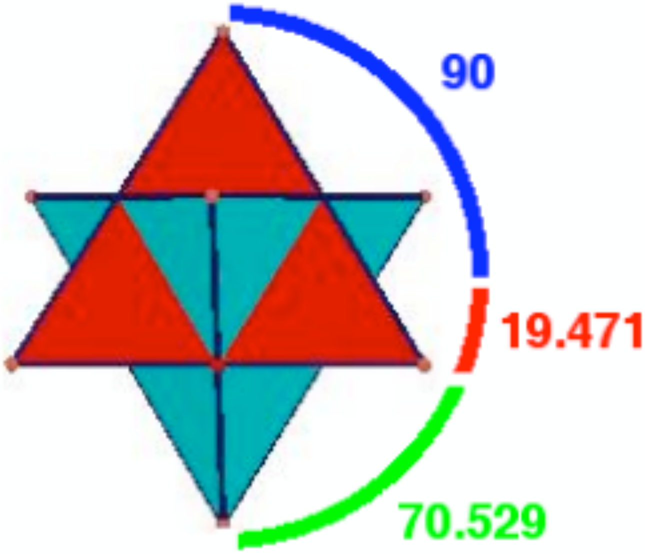
The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 2 GeV is about 0.39,
which is somewhat lower than the theoretical model value.
Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV
gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

These results when added up give a total mass of second generation fermion particles:

Sigmaf2 = 32.9 GeV

Kobayashi-Maskawa Parameters

In E8 Physics the KM Unitarity Triangle angles can be seen on the Stella Octangula



The Kobayashi-Maskawa parameters are determined in terms of
the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted by

Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The resulting KM matrix is:

d		s	b
u	0.975	0.222 0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Below the energy level of ElectroWeak Symmetry Breaking the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current $W\pm$ interactions couple to the ... quarks with couplings given by ...

Vud	Vus	Vub
Vcd	Vcs	Vcb
Vtd	Vts	Vtb

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix.
It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from
$Vud\ Vub{*} + Vcd\ Vcb{*} + Vtd\ Vtb{*} = 0$,
by dividing each side by the best-known one, $Vcd\ Vcb{*}$
...
$\bar{\rho} + i\bar{\eta} = -(Vud\ Vub{*})/(Vcd\ Vcb{*})$ is phase-convention- independent ...

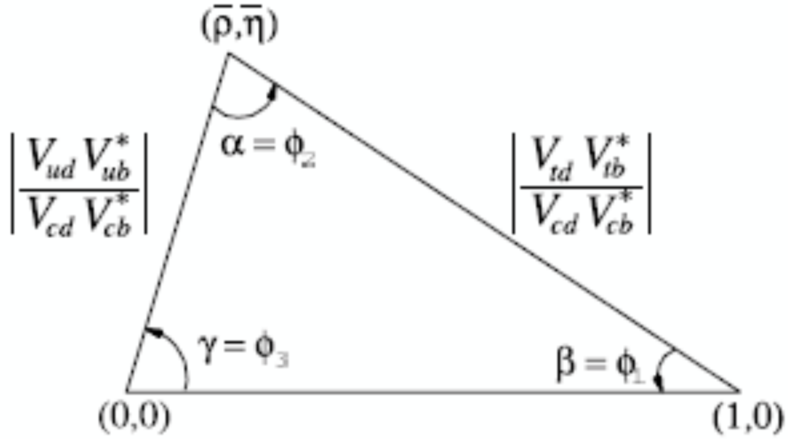


Figure 11.1: Sketch of the unitarity triangle.

... $\sin 2\beta = 0.673 \pm 0.023$ ... $\alpha = 89.0 +4.4 -4.2$ degrees ... $\gamma = 73 +22 -25$ degrees ...
The sum of the three angles of the unitarity triangle, $\alpha + \beta + \gamma = (183 +22 -25)$ degrees, is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J, which is a phase-convention-independent measure of CP violation, defined by Im Vij Vkl Vil$*$ Vkj$*$ = J SUM(m,n) ε_ikm ε_jln

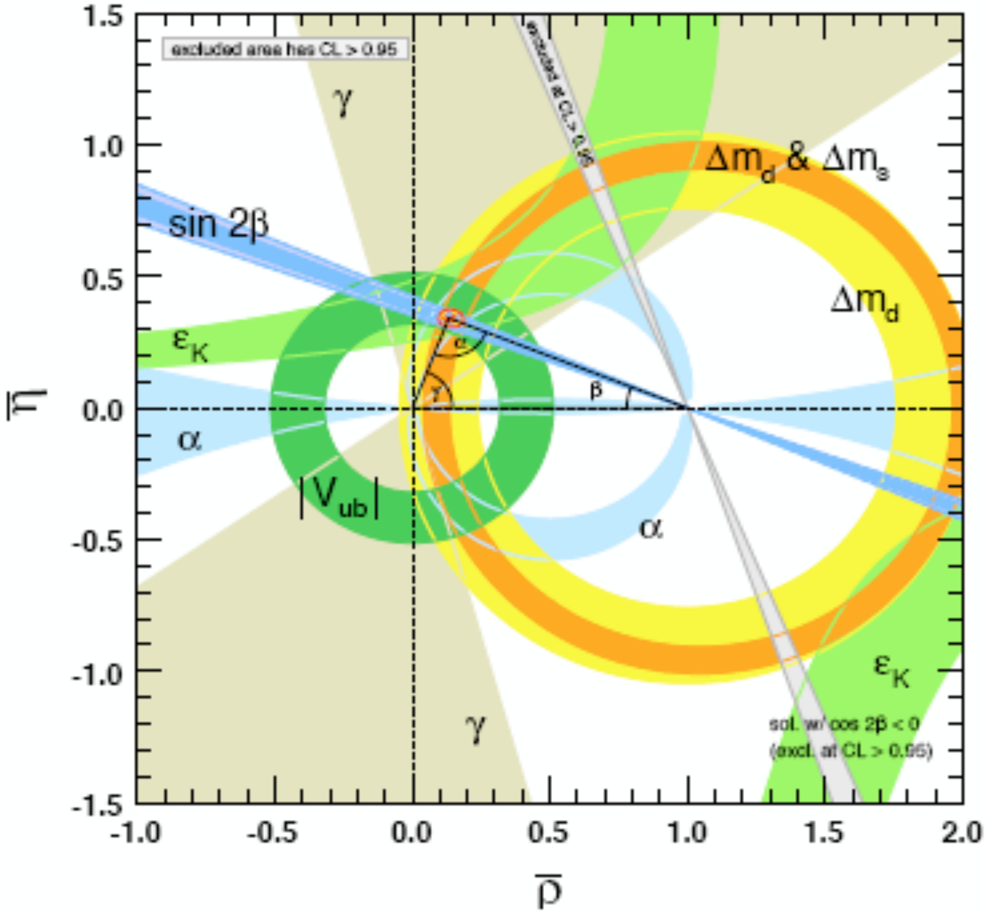


Figure 11.2: Constraints on the $\bar{\rho}, \eta$ plane.
The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015	0.2253 ± 0.0007	0.00347 +0.00016 −0.00012
0.2252 ± 0.0007	0.97345 +0.00015 −0.00016	0.0410 +0.0011 −0.0007
0.00862 +0.00026 −0.00020	0.0403 +0.0011−0.0007	0.999152 +0.000030−0.000045

and the Jarlskog invariant is J = (2.91 +0.19-0.11)x10−5. ...".

Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed
that in the Massless Realm the mixing matrix might be democratic.
In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...
the mass matrix ... MD ... of the type ... 1/3 x m x

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

... has name... "democratic" family mixing ...
the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

$$\begin{matrix} 1/\mathrm{sqrt}(2) & -1/\mathrm{sqrt}(2) & 0 \\ 1/\mathrm{sqrt}(6) & 1/\mathrm{sqrt}(6) & -2/\mathrm{sqrt}(6) \\ 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) & 1/\mathrm{sqrt}(3) \end{matrix}$$

as A MD At =

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m \end{matrix}$$

...".

Up in the Massless Realm you might just say that there is no mass matrix,
just a democratic mixing matrix of the form 1/3 x

$$\begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix}$$

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking
then you might as a first approximation use m = 1
so that all the mass first goes to the third generation as

$$\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$$

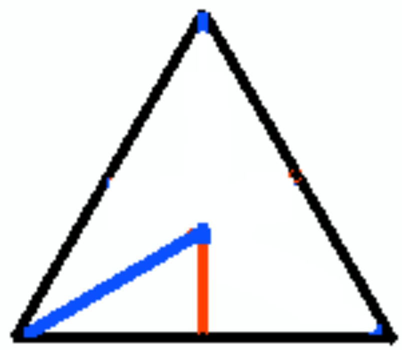
which is physically like the Higgs being a T-Tbar quark condensate.

Consider a 3-dim Euclidean space of generations:

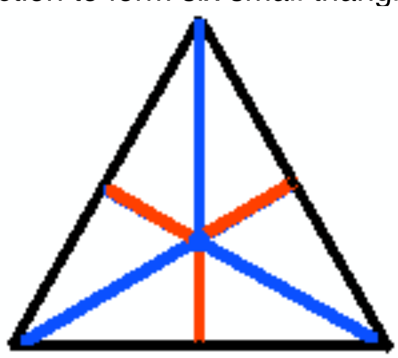
The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

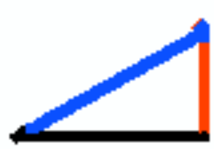
If mass only goes to one other generation
that can be represented by a red line extending to a second dimension
forming a small blue-red-black triangle



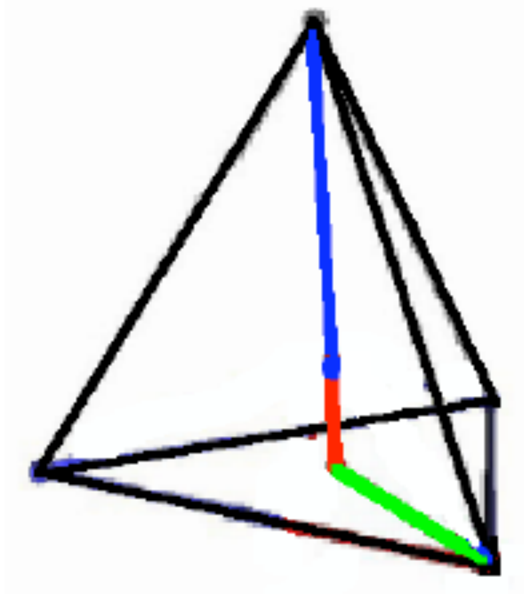
that can be extended by reflection to form six small triangles making up a large triangle



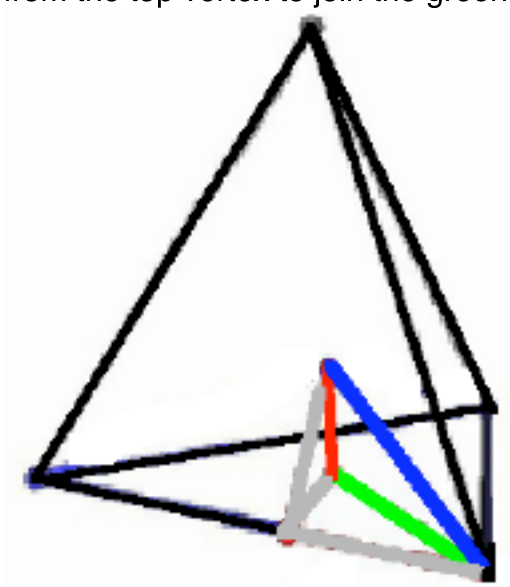
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations
that can be represented by a green line extending to a third dimension



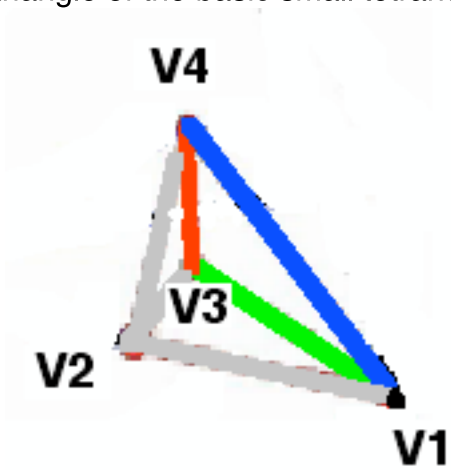
If you move the blue line from the top vertex to join the green vertex



you get a small blue-red-green-gray-gray-gray tetrahedron
that can be extended by reflection to form 24 small tetrahedra
making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds
to the 12+12 = 24 elements of the Binary Tetrahedral Group.

The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module"
with lengths

V1.V2 = (1/2) EL ≡ Half of the regular Tetrahedron's edge length.
V1.V3 = (1 / sqrt(3)) EL ≅ 0.577 350 269 EL
V1.V4 = 3 / (2 sqrt(6)) EL ≅ 0.612 372 436 EL
V2.V3 = 1 / (2 sqrt(3)) EL ≅ 0.288 675 135 EL
V2.V4 = 1 / (2 sqrt(2)) EL ≅ 0.353 553 391 EL
V3.V4 = 1 / (2 sqrt(6)) EL ≅ 0.204 124 145 EL

the Unitarity Triangle angles are:

β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin $2\beta = 0.6285$

α = V1.V3.V4 = 90 degrees

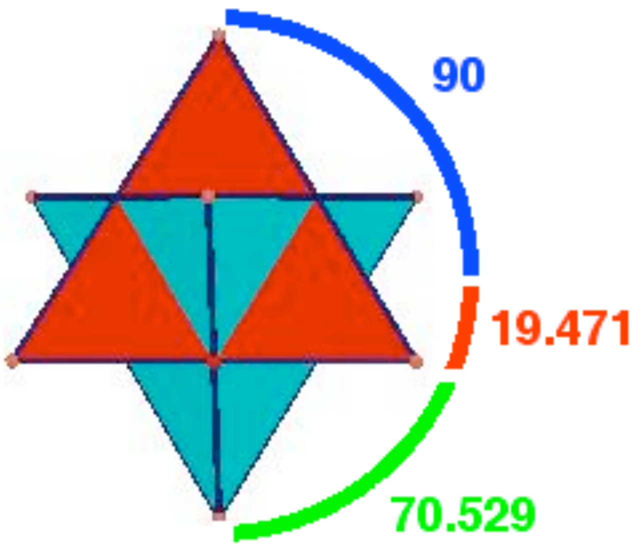
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

which is substantially consistent with the 2010 Review of Particle Properties

sin $2\beta = 0.673 \pm 0.023$ so $\beta = 21.1495$ degrees
$\alpha = 89.0 \; {+4.4} \; {-4.2}$ degrees
$\gamma = 73 \; {+22} \; {-25}$ degrees

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In the Cl(16)-E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses
to calculate Kobayashi-Maskawa parameters:

phase angle d13 = gamma = 70.529 degrees

sin(theta12) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(theta13) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*theta23 = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(theta23) = s23 = sin(*theta23) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(theta23)	sin(theta23)
0	-sin(theta23)	cos(theta23)

cos(theta13)	0	sin(theta13)exp(-i d13)
0	1	0
-sin(theta13)exp(i d13)	0	cos(theta13)

cos(theta12)	sin(theta12)	0
-sin(theta12)	cos(theta12)	0
0	0	1

The resulting Kobayashi-Maskawa parameters
for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output,
or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level,
the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model,
the only source of CP violation is the Kobayashi-Maskawa (KM) phase ...
The study of CP violation is, at last, experiment driven. ...
The CKM matrix provides a consistent picture
of all the measured flavor and CP violating processes. ...
There is no signal of new flavor physics. ...
Very likely,
the KM mechanism is the dominant source of CP violation in flavor changing processes.
... The result is consistent with the SM predictions. ...".

Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry
space,lying entirely therein. According to the Cl(16)-E8 model
the mass of nu_3 is zero at tree-level
but it picks up a first-order correction
propagating entirely through internal symmetry space by merging
with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that

M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
= 7.35 / 137 = 5.4 x 10^(-2) eV.

The neutrino-plus-electron loop can be anchored by weak force
action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in M4 physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.

According to the Cl(16)-E8 model the mass of nu_2 is zero at
tree-level but it picks up a first-order correction at only one
(but not both) of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so

the first-order corrected mass of nu_2 is
M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
= 9 x 10^(-3)eV.

The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski
spacetime.
thus having only one anchoring to CP2 interna symmetry space.

According to the Cl(16)-E8 model the mass of nu_1 is zero at
tree-level but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3
first-order corrections
or the 6 different possible anchorings for nu_2 first-order
corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so

the first-order corrected mass of nu_1 is
M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
= 1.5 x 10^(-3)eV.

Therefore:

```
the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
                            = ( 2916 - 81 ) x 10^(-6) eV^2 =
                            = 2.8 x 10^(-3) eV^2
```

and

```
the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
                               = ( 81 - 2 ) x 10^(-6) eV^2 =
                               = 7.9 x 10^(-5) eV^2
```

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

```
     c12 c13                      s12 c13                      s13 e-id

U = - s12 c23 - c12 s23 s13 eid     c12 c23 - s12 s23 s13 eid    s23 c13

     s12 s23 - c12 c23 s13 eid   - c12 s23 - s12 c23 s13 eid    c23 c13
```

where cij = cos(theta_ij) , sij = sin(theta_ij)

The angles are

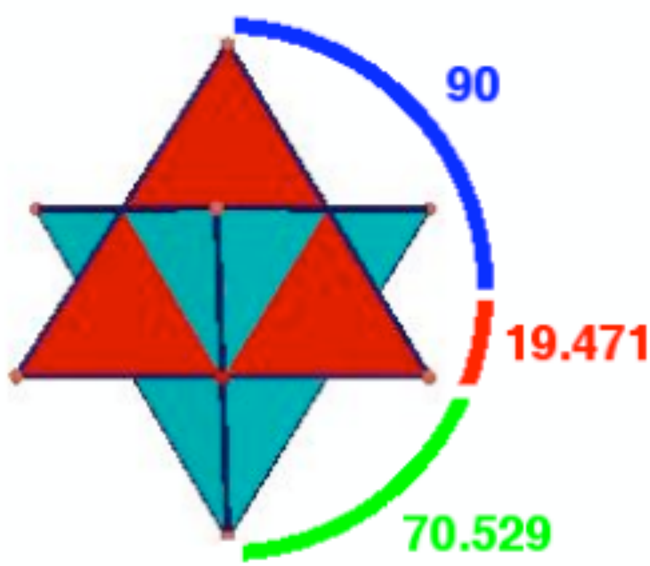
theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.

Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev,
does not often swap places with a 2.09 Gev charm sea quark,
but
a 313 Mev down valence quark
can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force
constituent mass of the down valence quark is heavier by about

$(ms - md) (md/ms)^2 a(w) |Vds| = 312 \times 0.25 \times 0.253 \times 0.22$ Mev = 4.3 Mev,

(where $a(w) = 0.253$ is the geometric part of the weak force strength
and $|Vds| = 0.22$ is the magnitude
of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass Qmd of the down quark is

$$Qmd = 312.75 + 4.3 = 317.05 \text{ MeV}.$$

Similarly, the up quark Quantum color force mass increase is about

$(mc - mu) (mu/mc)^2 a(w) |V(uc)| = 1777 \times 0.022 \times 0.253 \times 0.22$ Mev = 2.2 Mev,

(where $|Vuc| = 0.22$ is the magnitude
of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass Qmu of the up quark is

$$Qmu = 312.75 + 2.2 = 314.95 \text{ MeV}.$$

Therefore, the Quantum color force Neutron-Proton mass difference is

$mN - mP = Qmd - Qmu = 317.05$ Mev $- 314.95$ Mev $= 2.1$ Mev.

Since the electromagnetic Neutron-Proton mass difference is roughly

$$mN - mP = -1 \text{ MeV}$$

the total theoretical Neutron-Proton mass difference is

$$mN - mP = 2.1 \text{ Mev} - 1 \text{ Mev} = 1.1 \text{ Mev},$$

an estimate that is comparable to the experimental value of 1.3 Mev.

Pion as Sine-Gordon Breather

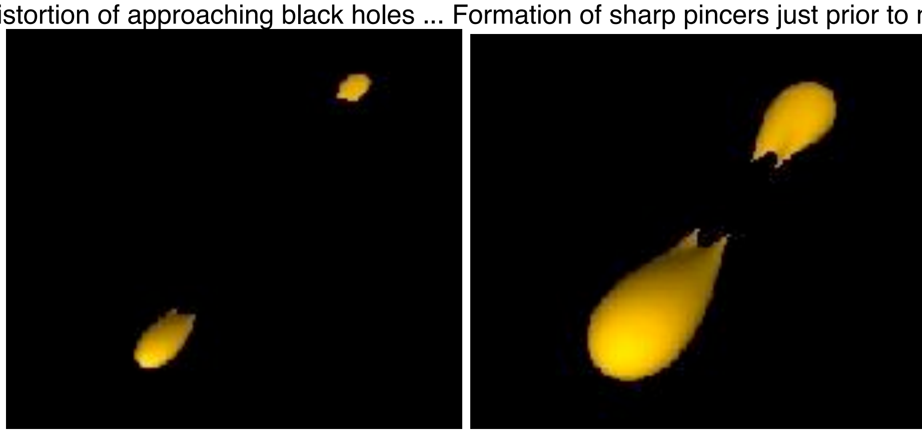
The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Schwinger Source Kerr-Newman Black Hole
with constituent mass M 312 MeV.

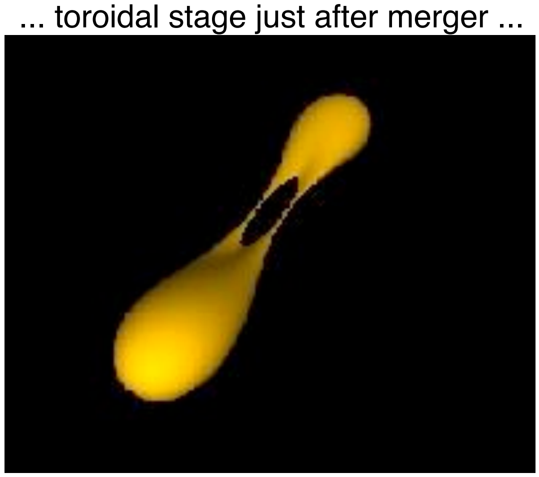
The antiquark is also a Schwinger Source Kerr-Newman Black Hole,
with constituent mass M 312 MeV.

According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):
"... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...

Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion,
the toroidal black hole remains a torus.
The torus is an event horizon and therefore is not a 2-spacelike dimensional torus,
but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):
"... The surface r = r+ is ... the event horizon ... and is a null surface ...

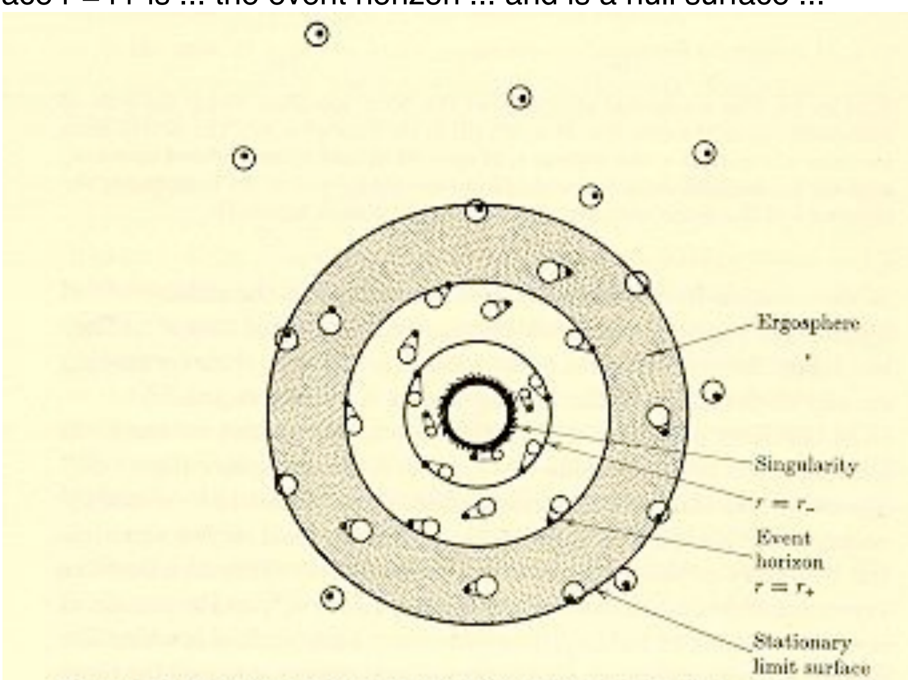


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather.
The soliton and antisoliton of a Sine-Gordon Breather correspond
to the quark and antiquark that make up the pion,
analagous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985),
where he writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

$$L = (1 / B^2) ((1/2) (df)^2 + A (\cos(f) - 1))$$

Coleman says: "... We see that, in classical physics, B is an irrelevant parameter:
if we can solve the sine-Gordon equation for any non-zero B,
we can solve it for any other B.
The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [eq. 4.4]

$$L / \hbar = (1 / (B^2 \hbar)) ((1/2) (df)^2 + A (\cos(f) - 1))$$

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ...
the classical limit, vanishing hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...
... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

$$f(x, t) = (4 / B) \arctan((n \sin(w t) / \cosh(n w x))$$

where [eq. 4.60] $n = \sqrt{A - w^2} / w$ and w ranges from 0 to A.
This solution has a simple physical interpretation ... a soliton far to the left ...[and]...
an antisoliton far to the right. As sin(w t) increases, the soliton and antisoliton move farther apart from each other. When sin(w t) passes through one,
they turn around and begin to approach one another. As sin(w t) comes down to zero ... the soliton and antisoliton are on top of each other ...
when sin(w t) becomes negative .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

$$E = 2 M \sqrt{1 - (w^2 / A)}$$

where [eq. 4.65] $M = 8 \sqrt{A} / B^2$ is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass,
as we would expect from a soliton-antisoliton pair. ...

Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974).
...[found that]... there is only a single series of bound states, labeled by the integer N ...
The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]
B'^2 = B^2 / (1 - (B^2 / 8 pi)) and M is the soliton mass.
M is not given by Eq. (4.65), but is the soliton mass corrected by the DHN formula,
or, equivalently, by the first-order weak coupling expansion. ...
I have written the equation in this form .. to eliminate A,
and thus avoid worries about renormalization conventions.
Note that the DHN formula is identical to the Bohr-Sommerfeld formula,
except that B is replaced by B'. ...
Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter
family of periodic motions, labeled by the period, T,
then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula,
but it is much more general;
it is always the leading approximation for any dynamical system ...
Dashen et al speculate that Eq. (4.82) is exact. ...
the sine-Gordon equation is equivalent ... to the massive Thirring model.
This is surprising,
because the massive Thirring model is a canonical field theory
whose Hamiltonian is expressed in terms of fundamental Fermi fields only.
Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent
to a free massive Dirac theory, in one spatial dimension. ...
Furthermore, we can identify the mass term in the Thirring model
with the sine-Gordon interaction, [eq. 5.13]

M = - (A / B^2) N_m cos(B f)

.. to do this consistently ... we must say [eq. 5.14]

B^2 / (4 pi) = 1 / (1 + g / pi)

....[where]... g is a free parameter, the coupling constant [for the Thirring model]...
Note that if B^2 = 4 pi , g = 0 ,
and the sine-Gordon equation is the theory of a free massive Dirac field. ...
It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.
Certainly this could not happen in three dimensions,
where it would be forbidden by the spin-statistics theorem.
However, there is no spin-statistics theorem in one dimension,
for the excellent reason that there is no spin. ...
the lowest fermion-antifermion bound state of the massive Thirring model
is an obvious candidate for the fundamental meson of sine-Gordon theory. ...
equation (4.82) predicts that
all the doublet bound states disappear when B^2 exceeds 4 pi .

This is precisely the point where
the Thirring model interaction switches from attractive to repulsive. ...
these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ...
define identical physics. ...
I have computed the predictions of ...[various]... approximation methods
for the ration of the soliton mass to the meson mass for three values of B^2 :
4 pi (where the qualitative picture of the soliton as a lump totally breaks down),
2 pi, and pi . At 4 pi we know the exact answer ...
I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b]

E = 8 sqrt(A) / B^2

...[is the]... energy of the lump ... of sine-Gordon theory ...
frequently called 'soliton...' in the literature ...
[Zeroth-order is the classical case, or classical limit.] ...
... Coherent-state variation always gives
the same result as the ... Zeroth-order weak coupling expansion ... .
The ... First-order weak-coupling expansion ...
explicit formula ... is (8 / B^2) - (1 / pi). ...".

Using the Cl(16)-E8 model constituent mass of the Up and Down quarks and
antiquarks, about 312.75 MeV, as the soliton and antisoliton masses,
and setting B^2 = pi and using the DHN formula,
the mass of the charged pion is calculated to be (312.75 / 2.25) MeV = 139 MeV
which is close to the experimental value of about 139.57 MeV.

Why is the value B^2 = pi the special value that gives the pion mass ?

(or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi)

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula. In other words,

The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex.

Once a Planck-mass black hole is formed, it is stable in the E8 model.
Less mass would not be gravitationally bound at the vertex.
More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles
for a total of 64 particle-antiparticle pairs.
Of the 64 particle-antiparticle pairs, 12 are bosonic pions.

A typical combination should have about 6 pions so
it should have a mass of about .14x6 GeV = 0.84 GeV.

Just as the pion mass of .14 GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses.

If such a bound state of oppositely charged pions has a mass as small as .1 GeV,
and if the typical combination has one such pair and 4 other pions,
then the typical combination could have a mass in the range of 0.66 GeV.

Summing over all 2^64 combinations,
the total mass of a one-vertex universe should give a Planck mass roughly around
0.66 x 2^64 = 1.217 x 10^19 GeV.

The value for the Planck mass given in by the 1998 Particle Data Group is 1.221 x 10^19 GeV.

Conformal Gravity ratio
Dark Energy : Dark Matter : Ordinary Matter

MacDowell-Mansouri Gravity is described by Rabindra Mohapatra in section 14.6 of his book "Unification and Supersymmetry":

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators X_A of a group

$$[X_A, X_B] = f_{AB}^{C} X_C, \tag{14.6.1}$$

where f_{AB}^{C} are structure constants of the group. We can then introduce a gauge field connection h_μ^A as follows:

$$h_\mu = h_\mu^A X_A. \tag{14.6.2}$$

Let us denote the parameter associated with X_A by ε^A. The gauge transformations on the fields h_μ^A are given as follows:

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + h_\mu^B \varepsilon^C f_{CB}^A \equiv (D_\mu \varepsilon)^A. \tag{14.6.3}$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^A. \tag{14.6.4}$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \varepsilon^C f_{CB}^A. \tag{14.6.5}$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x \, Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \tag{14.6.6}$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_m e_\mu^m + M_{mn} \omega_\mu^{mn} + K_m f_\mu^m + D b_\mu. \tag{14.6.7}$$

The various $R_{\mu\nu}$ are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^m - \partial_\mu e_\nu^m + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^m + b_\nu e_\mu^m, \tag{14.6.8}$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{mp} \omega_{\mu,p}^n - \omega_\mu^{mp} \omega_{\nu,p}^n - 4(e_\mu^m f_\nu^n - e_\nu^m f_\mu^n), \tag{14.6.9}$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^m - \partial_\mu f_\nu^m - b_\mu f_\nu^m + b_\nu f_\mu^m + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \tag{14.6.10}$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2e_\mu^m f_\nu^m - 2e_\nu^m f_\mu^m. \tag{14.6.11}$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x \, \varepsilon_{mnrs} \varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{mn}(M) R_{\rho\sigma}^{rs}(M). \tag{14.6.12}$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \tag{14.6.13}$$

which expresses ω_μ^{mn} as a function of (e, b). The reason for imposing this constraint has to do with the fact that P_m transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein e_μ^m. Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^m = \partial_\mu \xi^\lambda e_\lambda^m + \xi^\lambda \partial_\lambda e_\mu^m. \tag{14.6.14}$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^m = \delta_P(\xi e^n)e_\mu^m + \delta_M(\xi\omega^{mn})e_\mu^m + \delta_D(\xi b)e_\mu^m + \xi^\nu R_{\mu\nu}^m(P),$$

where

$$\delta_P(\xi^n)e_\mu^m = \partial_\mu \xi^m + \xi^n \omega_\mu^{mn} + \xi^m b_\mu. \tag{14.6.15}$$

If $R^{\mu\nu}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \partial_\mu - iX_A h_\mu^A$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{P}$ such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_{A'} \delta_{A'}(\xi^m h_m^A), \tag{14.6.16}$$

where A' goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^m)\phi = \xi^m D_m^C \phi. \tag{14.6.17}$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to D_m^C and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to f_μ^m to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\tfrac{1}{4}[e_m^\lambda e_{n\nu} R_{\mu\lambda}^{mn} - \tfrac{1}{6} g_{\mu\nu} R], \tag{14.6.18}$$

where f_μ^m has been set to zero in R written in the right-hand side.

This eliminates (from the theory the degrees of freedom) ω_μ^{mn} and f_μ^m and we are left with e_μ^m and b_μ. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field ϕ. It has conformal weight $\lambda = 1$. So we can write a convariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \tag{14.6.19}$$

We note that the conformal charge of ϕ can be assumed to be zero since $K_m = x^2 \partial$ and is the dimension of inverse mass. In order to calculate $\Box^c \phi$ we start with the expression for d'Alambertian in general relativity

$$\frac{1}{e}\partial_\nu(g^{\mu\nu}eD^C_\mu\phi). \tag{14.6.20}$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi^m_K e_{m\mu}, \qquad \delta(\phi b_\mu) = \phi\delta b_\mu = -2\phi f^m_\mu e^\mu_m = +\tfrac{2}{12}\phi R, \tag{14.6.21}$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\Box^C\phi = \frac{1}{e}\partial_\nu(g^{\mu\nu}eD^c_\mu\phi) + b_\mu D^c_\mu\phi + \tfrac{2}{12}\phi R. \tag{14.6.22}$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e\, d^4x\, \tfrac{1}{2}\phi\Box^C\phi. \tag{14.6.23}$$

Now we can use conformal transformation to gauge $b_\mu = 0$ and local scale transformation to set $\phi = \kappa^{-1}$ leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that, $\Box^C$, the conformal d'Alambertian contains R, which for constant ϕ, leads to gravity. We may call ϕ the auxiliary field.

After the scale and conformal gauges have been fixed,
the conformal Lagrangian becomes a de Sitter Lagrangian.

Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian,
as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739).
(Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri "... approach
to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ...
S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... ,
and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977);
also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".)

The minimal group required to produce Gravity,
and therefore **the group that is used in calculating Force Strengths**,
is the [anti] de Sitter group, as is described by
Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) (chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ...
Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab
(n_ab = Minkowski metric).
What happened to the w^ab_u ?
The field equations obtained from the Hilbert-Einstein action by varying the w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in terms of the e^a_u ... The w do not propagate ...
We start from the four-dimensional de-Sitter algebra ... so(3,2).
Technically this is the anti-de-Sitter algebra ...
We envision space-time as a four-dimensional manifold M.
At each point of M we have a copy of SO(3,2) (a fibre ...) ...
and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.
From these potentials h^A_mu we calculate the field-strengths
(curvature components) [let @ denote partial derivative]
R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu
...[where]... the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....
We now wish to write down the action S as an integral over
the four-manifold M ... S(Q) = INTEGRAL_M R^A /\ R^B Q_AB
where Q_AB are constants ... to be chosen ... we require
... the invariance of S(Q) under local Lorentz transformations
... the invariance of S(Q) under space inversions ...
...[AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE]...
we shall see ...[that]... the action becomes invariant
under all local [anti]de-Sitter transformations ...[and]... we recognize ... t
he familiar Hilbert-Einstein action with cosmological term in vierbein notation ...
Variation of the vierbein leads to the Einstein equations with cosmological term.
Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.
So at this level full sp(4) invariance has been checked.
... Were it not for the assumed space-inversion invariance ...
we could have had a parity violating gravity. ...
Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.
Parity violation and torsion go hand-in-hand.
Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether
the universe as a whole is in a space-inversion symmetric configuration. ...".

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:

"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological constant limit can be considered as limiting cases of the fundamental symmetry. ...

... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations. Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...

... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.

The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the conespace N. The points at infinity of M are concentrated in the vertex of the conespace N, and those on the light-cone of M becomes the infinity of N. It is concepts of space isotropy and equivalence between inertial frames in the conespace N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

3 Rotations
3 Boosts
4 Translations
4 Special Conformal transformations
1 Dilatation

The **Cosmological Constant / Dark Energy** comes from
the **10 Rotation, Boost, and Special Conformal generators**
of the Conformal Group Spin(2,4) = SU(2,2),
so the fractional part of our Universe of the Cosmological Constant
should be **about 10 / 15 = 67% for tree level**.

Black Holes, including **Dark Matter Primordial Black Holes**, are curvature singularities in our 4-dimensional physical spacetime,
and since Einstein-Hilbert curvature comes from the **4 Translations**
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2)
through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate),
the fractional part of our Universe of Dark Matter Primordial Black Holes
should be **about 4 / 15 = 27% at tree level**.

Since **Ordinary Matter** gets mass from the Higgs mechanism
which is related to the **1 Scale Dilatation**
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our universe of Ordinary Matter
should be **about 1 / 15 = 6% at tree level**.

However,
as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates,
so that the differences in evolution must be taken into account
from the initial End of Inflation to the Present Time.

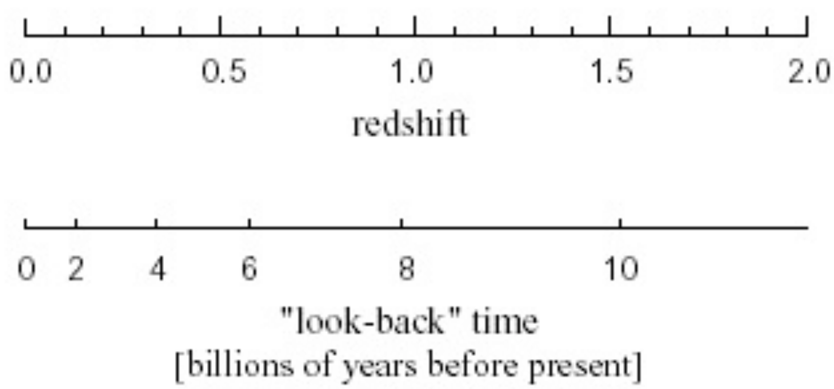
Without taking into account any evolutionary changes with time,
our Flat Expanding Universe should have roughly:

67% Cosmological Constant
27% Dark Matter - possilbly primordial stable Planck mass black holes
6% Ordinary Matter

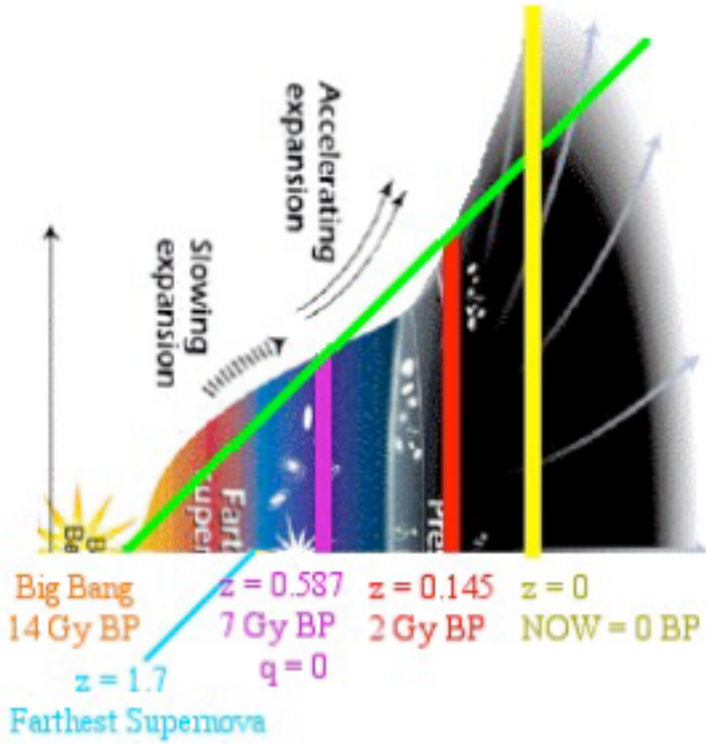
As Dennis Marks pointed out to me,
since density rho is proportional to (1+z)^3(1+w) for red-shift factor z
and a constant equation of state w:
w = -1 for Λ and the average overall density of Λ Dark Energy remains constant with time and the expansion of our Universe;
and
w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands;
and
w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;
so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.
Therefore,
the above calculated ratio 0.67 : 0.27 : 0.06 is valid
only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,
the time when Ordinary Matter was 6% would be
at redshift z such that
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145.
To translate redshift into time,
in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf to see that
the time when Ordinary Matter was 6%
would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7 provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe
of Linear Expansion (green line) with the Third Intersection
(at red vertical line at z = 0.145 and about 2 Gy BP),
which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.
Those four Special Times define four Special Epochs:
The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).
The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.
The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.
The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant Λ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.
Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy Λ and Cold Dark Matter terms have evolved
during the past 2 Gy, a rough estimate analysis would be:

Λ and CDM would be effectively created during expansion in their natural ratio
67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;
CDM Black Hole decay would be ignored; and
pre-existing CDM Black Hole density would decline by the same 1 / R^3 factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the Λ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21
for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

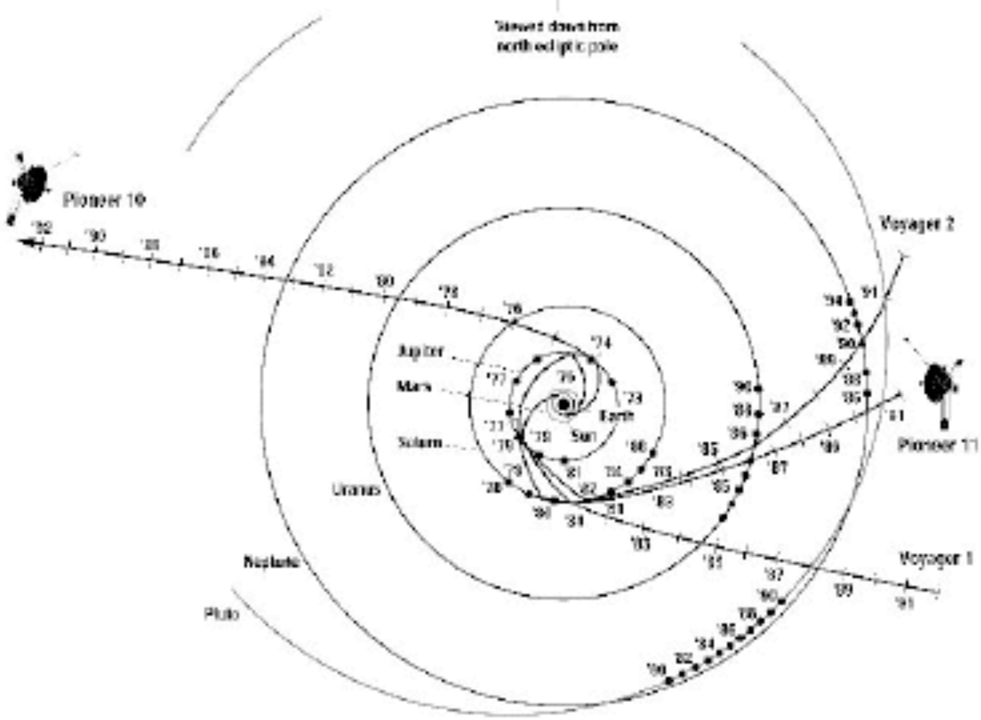
2013 Planck Data (arxiv 1303.5062) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05
WMAP: DE : DM : OM = 73 : 23 : 04
Planck: DE : DM : OM = 69 : 26 : 05
basic unevolved E8 Conformal calculation: DE : DM : OM = 67 : 27 : 06

Since uncertainties are substantial, I think that there is reasonable consistency.

Dark Energy: Pioneer and Uranus; BSCCO Josephson Junctions

After the Inflation Era and our Universe began its current phase of expansion, some regions of our Universe become Gravitationally Bound Domains (such as, for example, Galaxies) in which the 4 Conformal GraviPhoton generators are frozen out, forming domains within our Universe like IceBergs in an Ocean of Water. On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.



Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.
In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about 68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....
[the above image is] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager trajectories. Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.
... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...

The last communication from Pioneer 11 was received in November 1995, when the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should pass close to the nearest star in the constellation Aquila in about 4 million years ...
... Calculations of the motion of a spacecraft are made on the basis of the range time-delay and/or the Doppler shift in the signals. This type of data was used to determine the positions, the velocities, and the magnitudes of the orientation maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ... analyses of radio Doppler ... data ... indicated that an apparent anomalous acceleration is acting on Pioneer 10 and 11 ... The data implied an anomalous, constant acceleration with a magnitude a_P = 8 x 10^(-8) cm/cm/s^2, directed towards the Sun ...
... the size of the anomalous acceleration is of the order c H, where H is the Hubble constant ...
... Without using the apparent acceleration, CHASMP shows a steady frequency drift of about -6 x 10^(-9) Hz / s, or 1.5 Hz over 8 years (one-way only). ... This equates to a clock acceleration, -a_t, of -2.8 x 10^(-18) s / s^2 . The identity with the apparent Pioneer acceleration is a_P = a_t c. ...
... Having noted the relationships
a_P = c a_t
and that of ...
a_H = c H -> 8 x 10^(-8) cm / s^2
if H = 82 km / s / Mpc ...
we were motivated to try to think of any ... "time" distortions that might ... fit the CHASMP Pioneer results ... In other words ...
Is there any evidence that some kind of "time acceleration" is being seen?
... In particular we considered ... Quadratic Time Augmentation. This model adds a quadratic-in-time augmentation to the TAI-ET (International Atomic Time - Ephemeris Time) time transformation, as follows
ET -> ET + (1/2) a_ET ET^2
The model fits Doppler fairly well
...
There was one [other] model of the ...[time acceleration]... type that was especially fascinating. This model adds a quadratic in time term to the light time as seen by the DSN station:
delta_TAI = TAI_received - TAI_sent ->
-> delta_TAI + (1/2) a_quad (TAI_received^2 - TAI_sent^2)
It mimics a line of sight acceleration of the spacecraft, and could be thought of as an expanding space model.
Note that a_quad affects only the data. This is in contrast to the a_t ... that affects both the data and the trajectory. ... This model fit both Doppler and range very well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant and a_P ... remains an interesting conjecture. ...".

In his book “Mathematical Cosmology and Extragalactic Astronomy” (Academic Press 1976) (pages 61-62 and 72), Irving Ezra Segal says:
"... Temporal evolution in ... Minkowski space ... is
$H \to H + s I$
... unispace temporal evolution ... is ...
$H \to (H + 2 \tan(a/2)) / (1 - (1/2) H \tan(a/2)) = H + a I + (1/4) a H^2 + O(s^2)$
...".

Therefore,
the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:
"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates;
its constant value is ... H c ...
This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.
The Rosales and Sanchez-Gomez paper very nicely unites:

the physical 2-phase (bounded and unbounded orbits) view;
the Foucault pendulum idea; and the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez, can be summarized as a 2-phase model based on Segal's work which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands;
and
a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.

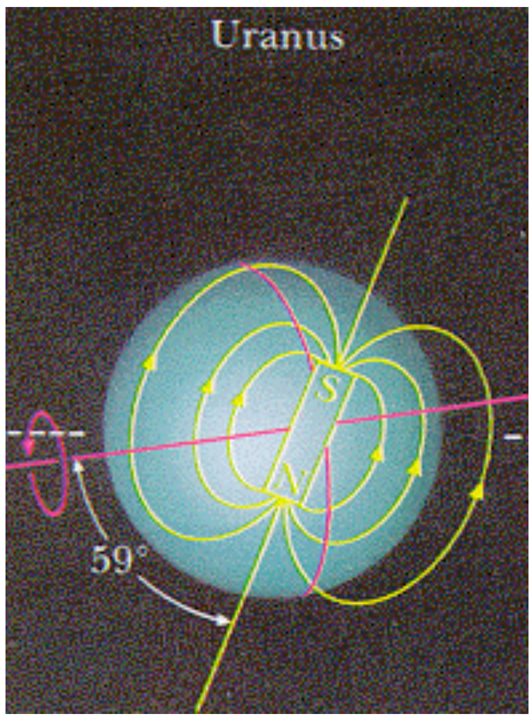
If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where c^2 Λ dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU) whereby ordinary matter dominates inside that distance from the sun
and exotic dark energy / dark matter appears at greater distances,
then Jack's model could also explain the Pioneer anomaly
and it may be that Jack's model with ordinary and exotic phases
and my model with deSitter/Poincare and Conformal phases
may be two ways of looking at the same thing.
As to what might be the physical mechanism of the phase transition, Jack says "... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".
In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe).
That physical interpretation is consistent with my view.

Transition at Orbit of Uranus:
It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than nuclear energy.

In gr-qc/0104064 Anderson et al say:
"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

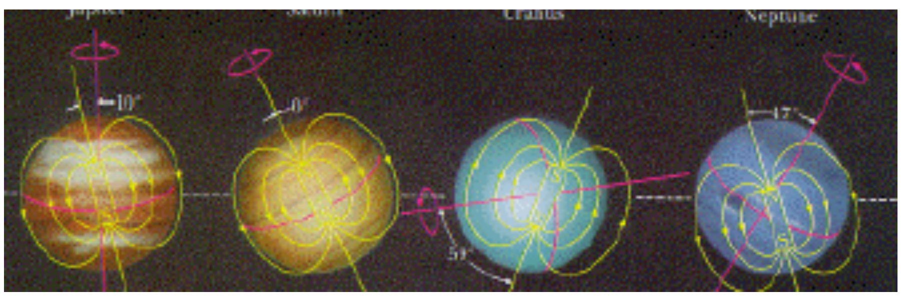
so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.
The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.

Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now.
However, such an effect may have been been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from “Universe”, 4th ed, by William Kaufmann, Freeman 1994).



According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:
"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [or 2 x 10^11 cm] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... . [C]omets ... suffer ... from outgassing ... [and their nuclei are hard to locate precisely] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

Dark Energy experiment by BSCCO Josephson Junctions and geometry of 600-cell

I. E. Segal proposed a MInkowski-Conformal 2-phase Universe
and
Beck and Mackey proposed 2 Photon-GraviPhoton phases:
Minkowski/Photon phase locally Minkowski with ordinary Photons and
Gravity weakened by 1 / (M_Planck)^2 = 5 x 10^(-39).
so that we see Dark Energy as only 3.9 GeV/m^3
Conformal/GraviPhoton phase with GraviPhotons and Conformal symmetry
(like the massless phase of energies above Higgs EW symmetry breaking)
With massless Planck the 1 / M_Planck^2 Gravity weakening goes away
and the Gravity Force Strength becomes the strongest possible = 1
so Conformal Gravity Dark Energy should be enhanced by M_Planck^2
from the Minkowski/Photon phase value of 3.9 GeV/m^3.

The Energy Gap of our Universe as superconductor condensate spacetime
is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length).
Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.
Neutrino masses are involved because their mass is zero at tree level
and their masses that we observe come from virtual graviphotons becoming virtual neutrino-antineutrino pairs.

BSCCO superconducting crystals are by their structure natural Josephson Junctions. Dark Energy accumulates (through graviphotons) in the superconducting layers of BSCCO.
Josephson Junction control voltage acts as a valve for access to the BSCCO Dark Energy, an idea due to Jack Sarfatti.

Christian Beck and Michael C. Mackey in astro-ph/0703364 said: "... Electromagnetic dark energy .... is based on a Ginzburg-Landau ... phase transition for the gravitational activity of virtual photons ... in two different phases:
gravitationally active [GraviPhotons] ...
and gravitationally inactive [Photons]
...
Let IPI^2 be the number density of gravitationally active photons ...
start from a Ginzburg-Landau free energy density ...

F = a IPI^2 + (1/2) b IPI^4

... The equilibrium state Peq is ... a minimum of F ... for T > Tc ...

Peq = 0 [and] Feq = 0

... for T< Tc

IPeqI^2 = - a / b [and] Fdeq = -(1/2) a^2 / b

... temperature T [of] virtual photons underlying dark energy ... is ..

h v = ln3 k T

... dark energy density ...[is]...

$$\rho_{dark} = (1/2) (\pi h / c^3) (\nu_c)^4$$

... The currently observed dark energy density in the universe of about 3.9 GeV/m^3 implies that the critical frequency ν_c is ...

$$\nu_c = 2.01 \text{ THz}$$

... BCS Theory yields ... for Fermi energy ... in copper ... 7.0 eV and the critical temperature of ... YBCO ... around 90 K ...

$$h \nu_c = 8 \times 10^{-3} \text{ eV}$$

... Solar neutrino measurements provide evidence for a neutrino mass of about $m_\nu c^2 = 9 \times 10^{-3}$ eV ...

[the Cl(16)-E8 model has first-order masses for the 3 generations of neutrinos as
1 x 10^(-3) and 9 x 10^(-3) and 5.4 x 10(-2) eV]

... in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor ... (i.e. the energy obtained when a Cooper pair forms out of two electrons) ...
for [graviphotons] ... at low temperatures (frequencies) Cooper-pair like states [of neutrino-antineutrino pairs] can form in the vacuum ... the ... energy gap would be of the order of typical neutrino mass differences ...".

Clovis Jacinto de Matos and Christian Beck in arXiv 0707.1797 said: "...
Tajmar's experiments ... at Austrian Research Centers Gmbh-ARC ...
with ... rotating superconducting rings ... demonstrated ...
a clear azimuthal acceleration ... directly proportional to the superconductive ring angular acceleration, and
an angular velocity orthogonal to the ring's equatorial plane ...
In 1989 Cabrera and Tate, through the measurement of the London moment magnetic trapped flux, rekported an anomalous Cooper pair mass excess in thin rotating Niobium supeconductive rings ...
A non-vanishing cosmological constant (CC) Λ can be interpreted in terms of a non-vanishing vacuum energy density

$$\rho_{vac} = (c^4 / 8 \pi G) \Lambda$$

which corresponds to dark energy with equation of state w = -1.
The ... astronomically observed value [is]... $\Lambda = 1.29 \times 10^{-52}$ [$1/m^2$] ...
Graviphotons can form weakly bounded states with Cooper pairs, increasing their mass slightly from m to m' .
The binding energy is $E_c = u c^2$:

$$m' = m + my - u$$

... Since the graviphotons are bounded to the Cooper pairs, their zeropoint energies form a condensate capable of the gravitoelectrodynamic properties of superconductive cavities. ...
Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff ν_c of vacuum fluctuations:

$$\rho^* = (1/2) (\pi h / c^3) (\nu_c)^4$$

in vacuum one may put rho* = rho_vac from which the cosmological cutoff frequency v_cc is estimated as

v_cc = 2.01 THz

The corresponding "cosmological" quantum of energy is:

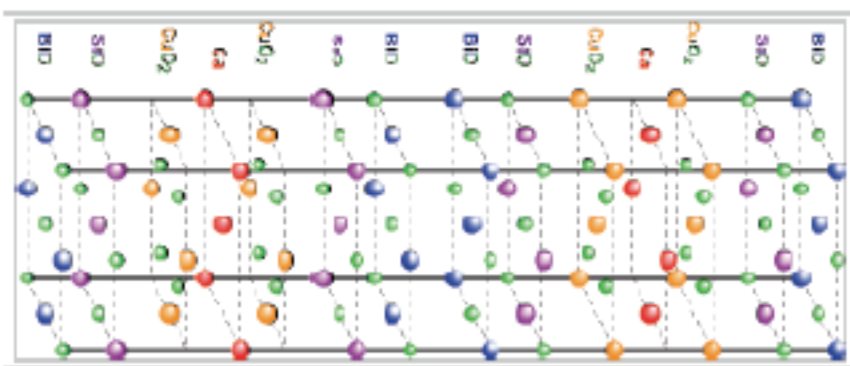
Ecc = h v_cc = 8.32 MeV

... In the interior of superconductors ... the effective cutoff frequency can be different ... h v = ln 3 k T ... we find the cosmological critical temprature Tcc

Tcc = 87.49 K

This temperature is characteristic of the BSCCO High-Tc superconductor. ...".

Xiao Hu and Shi-Zeng Lin in arXiv 0911.5371 said: "... The Josephson effect is a phenomenon of current flow across two weakly linked superconductors separated by a thin barrier, i.e. Josephson junction, associated with coherent quantum tunneling of Cooper pairs. ... The Josephson effect also provides a unique way to generate high-frequency electromagnetic (EM) radiation by dc bias voltage ... The discovery of cuprate high-Tc superconductors accelerated the effort to develop novel source of EM waves based on a stack of atomically dense-packed intrinsic Josephson junctions (IJJs), since the large superconductivity gap covers the whole terahertz (THz) frequency band. Very recently, strong and coherent THz radiations have been successfully generated from a mesa structure of Bi2Sr2CaCu2O8+d single crystal ...[BSCCO image from Wikipedia



]

which works both as the source of energy gain and as the cavity for resonance. This experimental breakthrough posed a challenge to theoretical study on the phase dynamics of stacked IJJs, since the phenomenon cannot be explained by the known solutions of the sine-Gordon equation so far. It is then found theoretically that, due to huge inductive coupling of IJJs produced by the nanometer junction separation and the large London penetration depth ... of the material, a novel dynamic state is stabilized in the coupled sine-Gordon system, in which +/- pi kinks in phase differences are developed responding to the standing wave of Josephson plasma and are

stacked alternately in the c-axis. This novel solution of the inductively coupled sine-Gordon equations captures the important features of experimental observations.
The theory predicts an optimal radiation power larger than the one observed in recent experiments by orders of magnitude ...".

What are some interesting BSCCO JJ Array configurations ?

Christian Beck and Michael C. Mackey in astro-ph/0605418 describe
"... the AC Josephson effect ...
a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory each superconductor is described by a complex wave function whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions ... by P(t).
...
at zero external voltage a superconductive current given by Is = Ic sin(P) flows between the two superconducting electrodes ... Ic is the maximum superconducting current the junction can support.
...
if a voltage difference V is maintained across the junction, then the phase difference P evolves according to

d P / dt = 2 e V / hbar

i.e. the current ... becomes an oscillating curent
with amplitude Ic and frequency v = 2 e V / h
This frequency is the ... Josephson frequency ... The quantum energy h v ... can be interpreted as the energy change of a Cooper pair that is transferred across the junction ...".

Xiao Hu and Shi-Zeng Lin in arXiv 1206.516 said:
"... to enhance the radiation power in teraherz band based on the intrinsic Josephson Junctions of Bi2Sr2CaCu2O8+d single crystal ...
we focus on the case that the Josephson plasma is uniform along a long crystal as established by the cavity formed by the dielectric material. ...
A ... pi kink state ... is characterized by static +/- pi phase kinks in the lateral directions of the mesa, which align themselves alternatingly along the c -axis. The pi phase kinks provide a strong coupling between the uniform dc current and the cavity modes, which permits large supercurrent flow into the system at the cavity resonances, thus enhances the plasma oscillation and radiates strong EM wave ...
The maximal radiation power ... is achieved when the length of BSCCO single crystal at c-axis equals the EM wave length. ...".

Each long BSCCO single crystal looks geometrically like a line so configure the JJ Array using BSCCO crystals as edges.

The simplest polytope, the **Tetrahedron, is made of 6 edges**:
Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 said:
"... Superconducting tetrahedral quantum bits ...

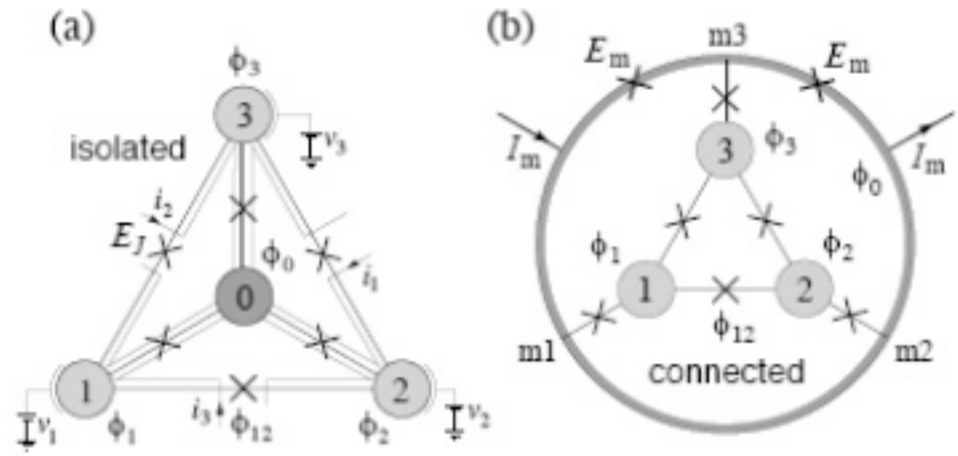
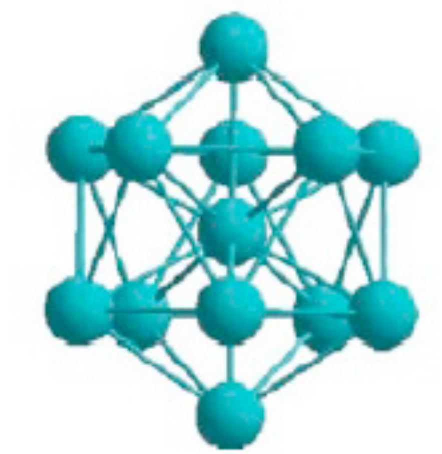


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling E_J and charging energy E_C); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages v_i and bias currents i_i. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_{\mathrm{m}} \gg E_J$ on the outer ring which are driven by external currents I_{m} (schematic, see Fig. 6 for details); the large coupling E_{m} effectively binds the ring segments into one island.

... tetrahedral qubit design ... emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit.
Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin;
the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space. ...".

42 edges make an Icosahedron plus its center
(image from Physical Review B 72 (2005) 115421 by Rogan et al)

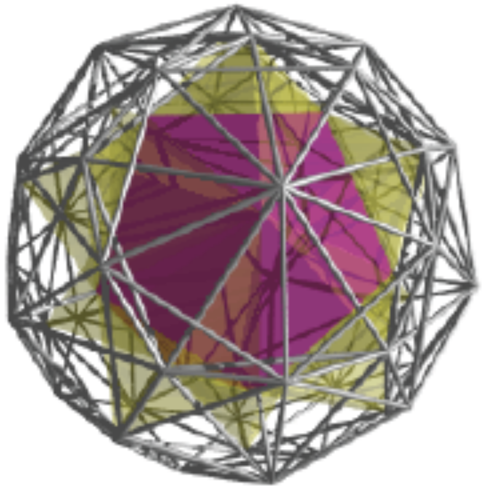


with 30 exterior edges and 12 edges from center to vertices.
It has 20 cells which are approximate Tetrahedra in flat 3-space but become exact regular Tetrahedra in curved 3-space.

Could an approximate-20Tetrahedra-Icosahedron configuration of 42 BSCCO JJ tap into Dark Energy so that the Dark Energy might regularize the configuration to exact Tetrahedra and so curve/warp spacetime from flat 3-space to curved 3-space ?

720 edges make a 4-dimensional 600-cell

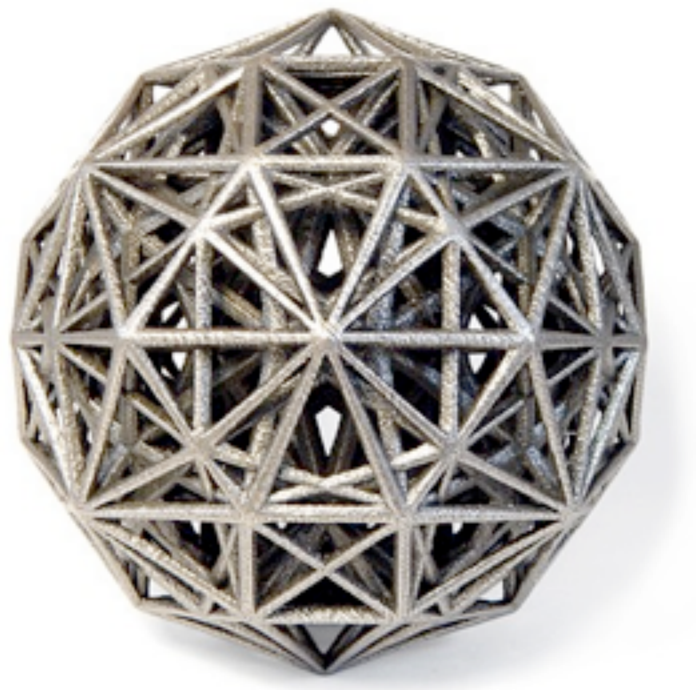
(image from Wikipedia)



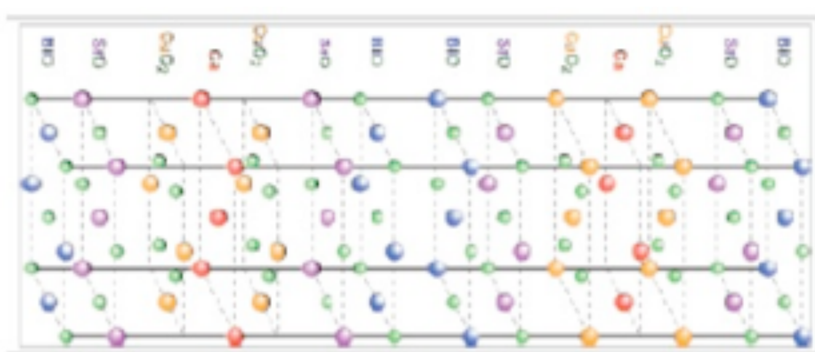
At each vertex 20 Tetrahedral faces meet forming an Icosahedron
which is exact because the 600-cell lives on a curved 3-shere in 4-space.
It has 600 Tetrahedral 3-dim faces and 120 vertices

Could a 600 approximate-Tetrahedra configuration of 720 BSCCO JJ
approximating projection of a 600-cell into 3-space
tap into Dark Energy so that the Dark Energy might regularize
the configuration to exact Tetrahedra and an exact 600-cell
and so curve/warp spacetime from flat 3-space to curved 3-space ?

The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell as follows: Consider 3-dim models of 600-cell such as metal sculpture from Bathsheba Grossman who says:
"… for it I used an orthogonal projection rather than the Schlegel diagrams of the other polytopes I build.
… In this projection all cells are identical, as there is no perspective distortion. …".



For the Dark Energy experiment each of the 720 lines would be made of a single BSCCO crystal



whose layers act naturally to make the BSCCO crystal an intrinsic Josephson Junction. (see Wikipedia and arXiv 0911.5371)

Each of the 600 tetrahedral cells of the 600-cell has 6 BSCCO crystal JJ edges.

Since the 600-cell is in flat 3D space the tetrahedra are distorted.

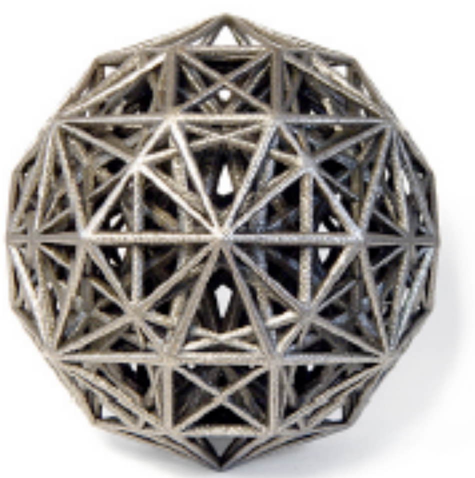
According to the ideas of Beck and Mackey (astro--ph/0703364) and
of Clovis Jacinto de Matos (arXiv 0707.1797) the superconducting
Josephson Junction layers of the 720 BSCCO crystals will bond with
Dark Energy GraviPhotons that are pushing our Universe to expand.

My idea is that the Dark Energy GraviPhotons will not like being
configured as edges of tetrahedra that are distorted in our flat 3D space
and
they will use their Dark Energy to make all 600 tetrahedra to be exact and
regular by curving our flat space (and space-time).

My view is that the Dark Energy Graviphotons will have enough strength
to do that because their strength will NOT be weakened by
the (1 / M_Planck)^2 factor that makes ordinary gravity so weak.

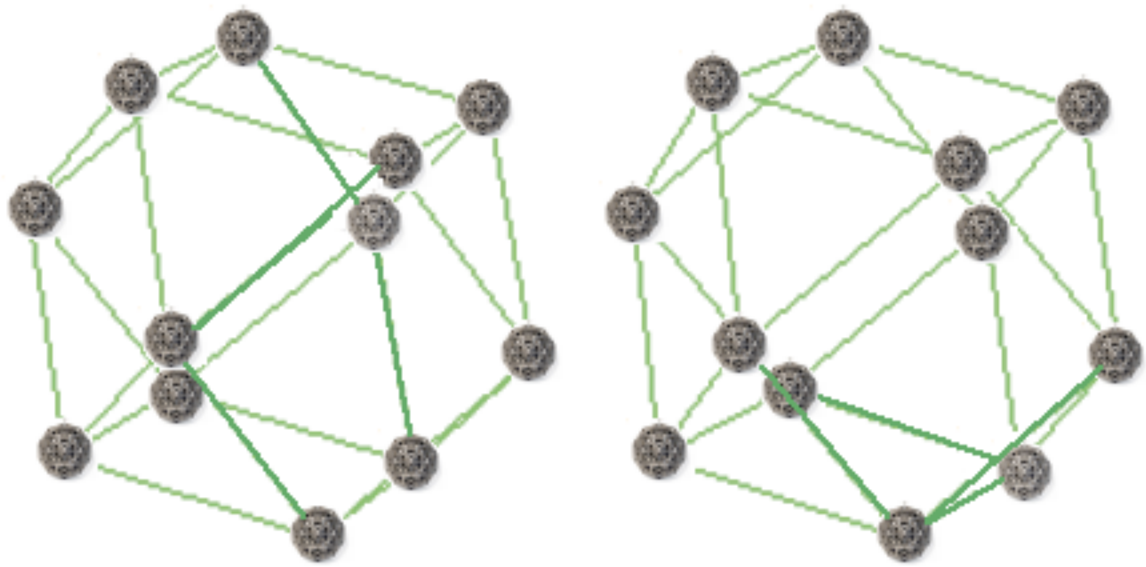
It seems to me to be a clearly designed experiment that will either
1- not work and show my ideas to be wrong or
2 - work and open the door for humans to work with Dark Energy.

Consider BSCCO JJ 600-cells

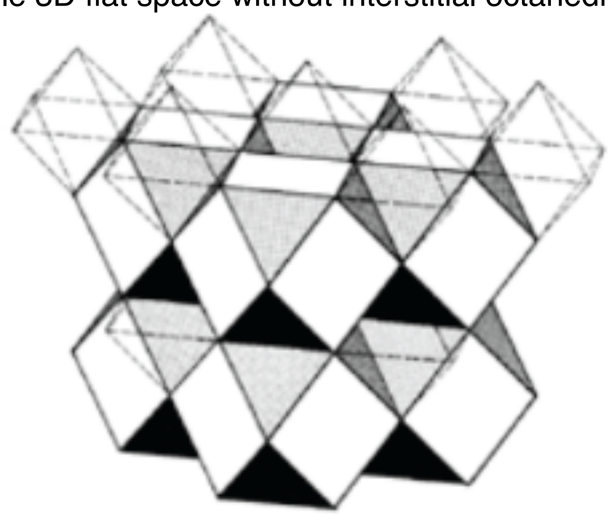


in this configuration:

First put 12 of the BSCCO JJ 600-cells at the vertices of a cuboctahedron shown here as a 3D stereo pair:

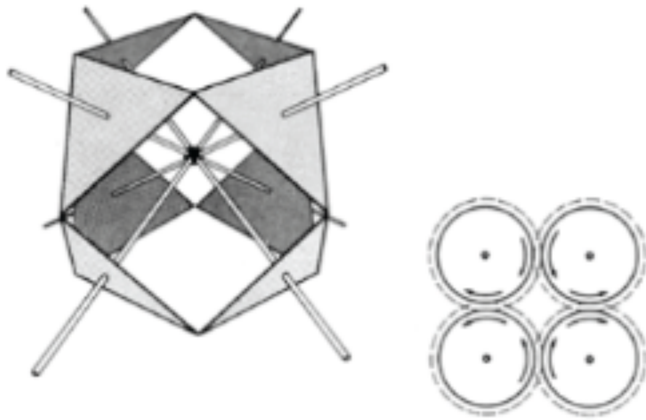


Cuboctahedra do not tile 3D flat space without interstitial octahedra

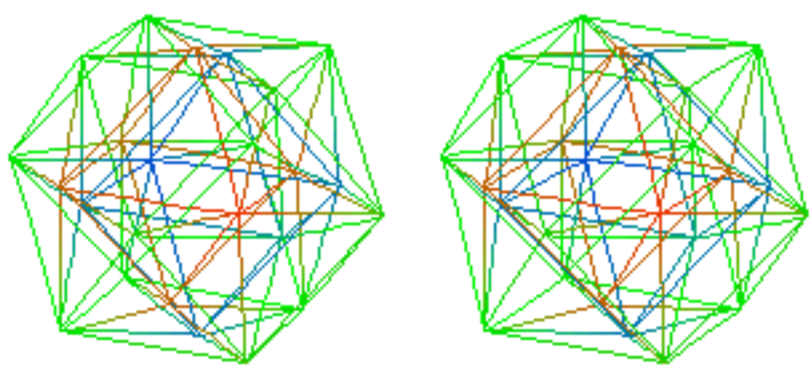


but BSCCO JJ 600-cell cuboctahedra can be put together square-face-to-square-face in flat 3D configurations including flat sheets.

As Buckminster Fuller described, the 8 triangle faces of a cuboctahedron



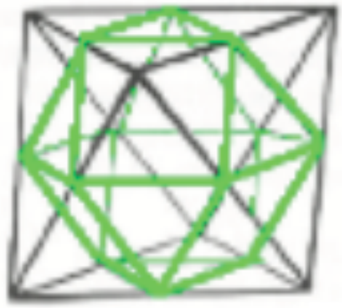
give it an inherently 4D structure consistent with the green cuboctahedron



central figure of a 24-cell (3D stereo 4thD blue-green-red color)
that tiles flat Euclidean 4D space.

So, cuboctahedral BSCCO JJ 600-cell structure likes flat 3D and 4D space
but
if BSCCO JJ Dark Energy act to transform flat space into curved space
like a 720-edge 600-cell with 600 regular tetrahedra
then
Dark Energy should transform cuboctahedral BSCCO JJ 600-cell structure
into
a 720-edge BSCCO JJ 600-cell structure that likes curved space.

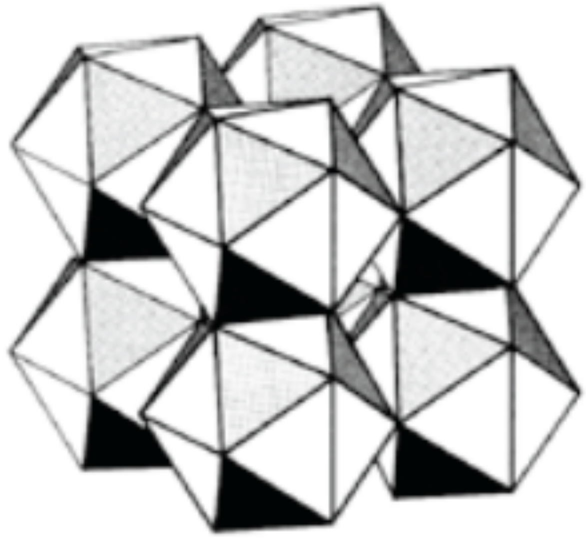
There is a direct JItterbug transformation of
the 12-vertex cuboctahedron to the 12-vertex icosahedron



whereby the 12 cuboctahedron vertices as midpoints of octahedral edges
are mapped to
12 icosahedron vertices as Golden Ratio points of octahedral edges.
There are two ways to map a midpoint to a Golden Ratio point.
For the Dark Energy experiment the same choice of mapping
should be made consistently throughout the BSCCO JJ 600-cell structure.

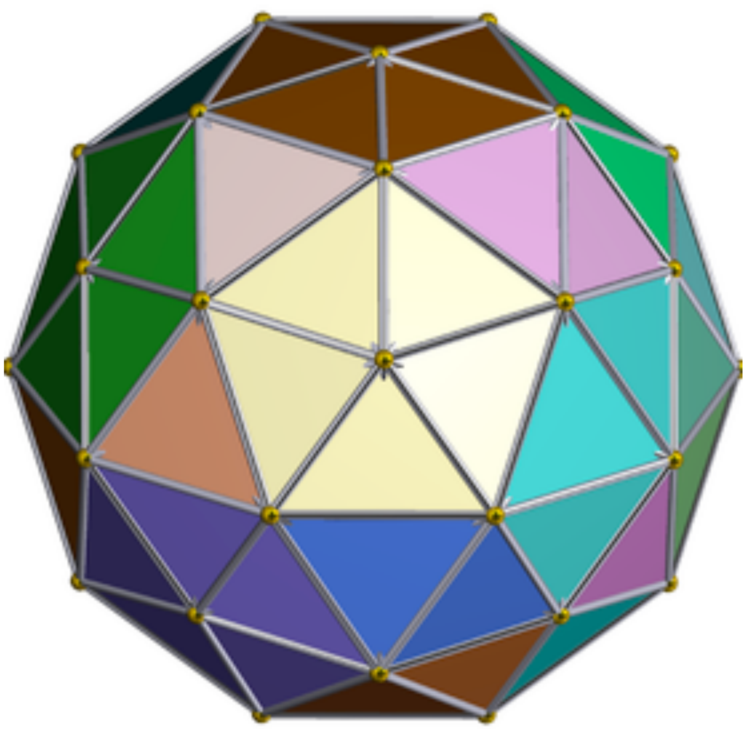
The result of the Jitterbug mapping is that
each cuboctahedron in the BSCCO JJ 600-cell structure
with its 12 little BSCCO JJ 600-cells at its 12 vertices
is mapped to
an icosahedron with 12 little BSCCO JJ 600-cells at its 2 vertices

and the overall cuboctahedral BSCCO JJ 600-cell structure
is transformed into
an overall icosahedral BSCCO JJ 600-cell structure



does not fit in flat 3D space in a naturally characteristic way
(This is why icosahedral QuasiCrystal structures do not extend as simply throughout flat 3D space as do cuboctahedral structures).

However, the BSCCO JJ 600-cell structure JItterbug icosahedra
do live happily in 3-sphere curved space within the icosahedral 120-cell



which has the same 720-edge arrangement as the 600-cell (see Wikipedia).
The icosahedral 120-cell is constructed by 5 icosahedra around each edge.
It has:

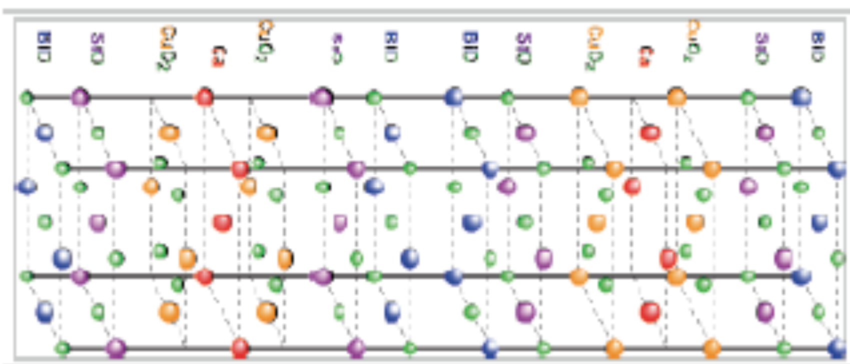
cells - 120 {3,5}
faces - 1200 {3}
edges - 720
vertices - 120
vertex figure - {5,5/2}
symmetry group H4,[3,3,5]
dual - small stellated 120-cell

In summary,

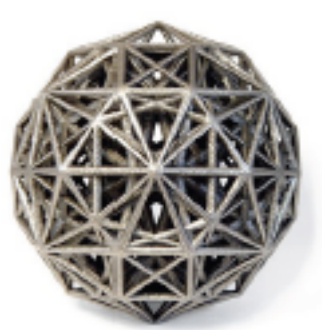
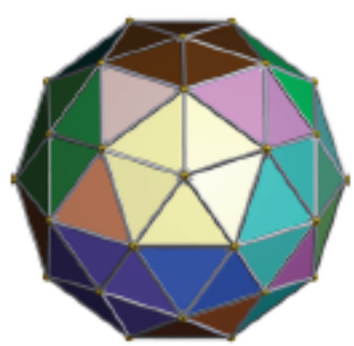
**Jitterbug transformations and BSCCO Josephson Junctions
may be the Geometric Key to controlling Dark Energy**

(as were Chain Reactions for Nuclear Fission and Ellipsoidal Focussing for H-Bombs)

The Energy Gap of our Universe as superconductor condensate spacetime is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length).
Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.

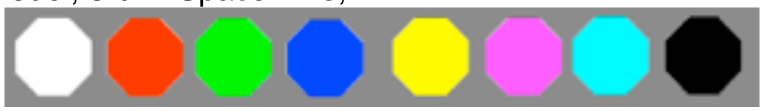


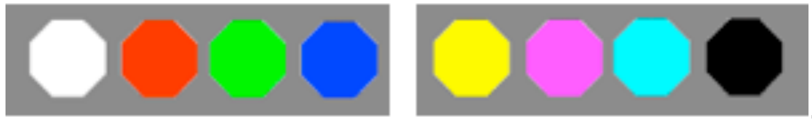
BSCCO superconducting crystals are natural Josephson Junctions.
Dark Energy accumulates in the superconducting layers of BSCCO.
The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell each of whose 720 edge-lines would be made of a single BSCCO crystal. It may be useful to use a Jitterbug-type transformation between a 600-cell configuration and a configuration based on icosahedral 120-cells which also have 720 edge-lines:

Strong CP Problem

In the Cl(16)-E8 model, 8-dim SpaceTime,

both Octonionic 

and Quarternionic ,

is represented by the 64-dim Adjoint D8 / D4xD4 part of E8
which is the A7 x R grade-0 part of the Maximal Contraction A7 x h92 with 5-grading

28 + 64 + (SL(8,R) + 1) + 64 + 28

In the Cl(16)-E8 model Gravity is most often written as in Chapter 18 of this paper in terms of the MacDowell-Mansouri Conformal Group Spin(2,4) which is the 15-dimensional Conformal BiVector Group of the 64-dim Cl(2,4) Clifford Algebra
but
it can also be written in terms of 64-dim grade-0 Maximal Contraction term SL(8,R) + 1 in which case it is known as Unimodular SL(8,R) Gravity which effectively describes a generalized checkerboard of 8-dim SpaceTime HyperVolume Elements and, with respect to Cl(16) = Cl(8)xCl(8), is the tensor product of the two 8v vector spaces of the two Cl(8) factors of Cl(16). If those two Cl(8) factors are regarded as Fourier Duals, then 8v x 8v describes Position x Momentum in 8-dim SpaceTime.

Conformal Spin(2,4) = SU(2,2) Gravity and Unimodular SL(4,R) = Spin(3,3) Gravity seem to be effectively equivalent since, as Bradonjic and Stachel in arXiv 1110.2159 said: "... in ... Unimodular relativity ... the symmetry group of space-time is ... the special linear group SL(4,R) ... the metric tensor ... break[s up] ... into the conformal structure represented by a conformal metric ... with det = -1 and a four-volume element ... at each point of space-time ...[that]... may be the remnant, in the ... continuum limit,
of a more fundamental discrete quantum structure of space-time itself ...".
Further,
Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:
"... Because of the existence of topologically nontrivial solutions, instantons, of the classical field equations associated with quantum chromodynamics (QCD), the quantized theory contains a dimensionless parameter ø (0 < ø < 2π) not explicit in the classical lagrangian. Since ø multiplies an expression odd in CP, QCD predicts violation of that symmetry unless the phase ø takes one of the special values ... 0 (mod π) ... this fine tuning is the strong CP problem ... the quantum dynamics of ... unimodular gravity ... may lead to the relaxation of ø to ø = 0 (mod π)
without the need ... for a new particle ... such as the axion ...".

Grothendieck Universe Quantum Theory

The First Grothendieck Universe is the Empty Set.

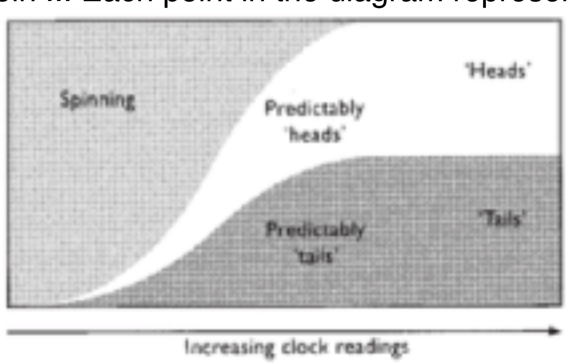
The Second Grothendieck Universe is Hereditarily Finite Sets such as a Generalized Feynman Checkerboard Quantum Theory based on E8 Lattices and Discrete Cl(16) Clifford Algebra.
(viXra 1501.0078)

The Third Grothendieck Universe is the Completion of Union of all tensor products of Cl(16) Real Clifford algebra

Since the Cl(16)-E8 Lagrangian is Local and Classical,
it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global Cl(16)-E8 Algebraic Quantum Field Theory (AQFT).

The usual Hyperfinite II1 von Neumann factor for creation and annihilation operators on Fermionic Fock Space over C^(2n) is constructed by completion of the union of all tensor products of 2x2 Complex Clifford algebra matrices, which have Periodicity 2,
so
for the Cl16)-E8 model based on Real Clifford Algebras with Periodicity 8,
whereby any Real Clifford Algebra, no matter how large,
can be embedded in a tensor product of factors of Cl(8) and of Cl(8)xCl(8) = Cl(16),
the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8)
produces a generalized Hyperfinite II1 von Neumann factor
that gives the Cl(16)-E8 model a natural Algebraic Quantum Field Theory.

The overall structure of Cl(160-E8 AQFT is similar to the Many-Worlds picture described by David Deutsch in his 1997 book "The Fabric of Reality" said (pages 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot



... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. …
in some regions of the multiverse, and in some places in space,

the snapshots of some physical objects do fall, for a period, into chains, each of whose members determines all the others to a good approximation …".

The Real Clifford Algebra Cl(16) containing E8 for the Local Lagrangian of a Region is equivalent to a " snapshot" of the Deutsch "multiverse".
The completion of the union of all tensor products of all Cl(16)-E8 Local Lagrangian Regions forms a generalized hyperfinite II1 von Neumann factor AQFT and emergently self-assembles into a structure = Deutsch multiverse.

For the Cl(16)-E8 model AQFT to be realistic, it must be consistent with **EPR entanglement relations**. Joy Christian in arXiv 0904.4259 said:
"... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings ... The alleged non-localities ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations [such as those] among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...".
Since E8 is a Lie Group and therefore parallelizable and lives in Clifford Algebra Cl(16), **the Cl(16)-E8 model is consistent with EPR**.

The Creation-Annihilation Operator structure of Cl(16)-E8 AQFT is given by the

Maximal Contraction of E8 = semidirect product A7 x h92

where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)
The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra

28 + 64 + (SL(8,R) + 1) + 64 + 28

Central Even Grade 0 = SL(8,R) + 1

The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices, so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.

Odd Grades -1 and +1 = 64 + 64

Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

Even Grades -2 and +2 = 28 + 28

Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int$ Gauge Gravity + Standard Model + Fermion Particle-AntiParticle
8-dim SpaceTime .

The Cl(16)-E8 generalized Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory is based on the Completion of the Union of all Tensor Products of the form

Cl(16) x ...(N times tensor product)... x Cl(16) = Cl(16N)

For N = 2^8 = 256 the copies of Cl(16) are on the 256 vertices of the 8-dim HyperCube

For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube

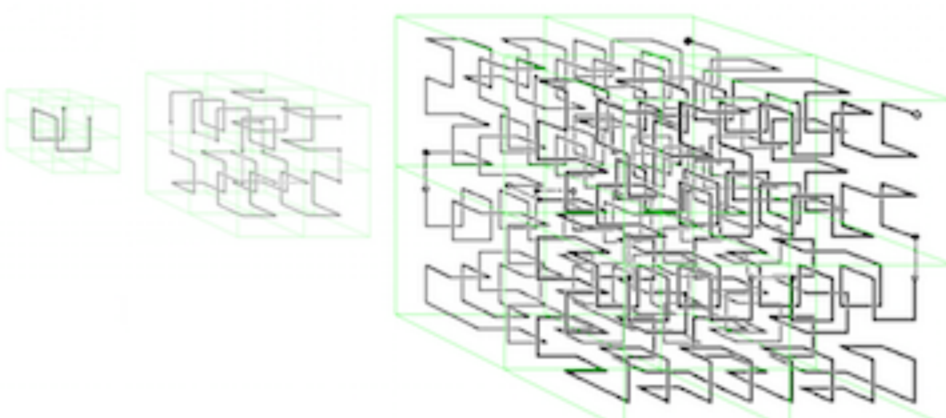
As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond

similarly to the way shown by this 3-HyperCube example for N = 2^3, 4^3, 8^3

from Wiliam Gilbert's web page:



The Union of all Cl(16) tensor products is
the Union of all subdivided 8-HyperCubes
and
their Completion is a huge superposition of 8-HyperCube Continuous Volumes
which Completion belongs to the Third Grothendieck Universe.

AQFT Quantum Code

Cerf and Adami in quantum-ph/9512022 describe virtualqubit-anti-qubit pairs (they call them ebit-anti-ebitpairs) that are related to negative conditional entropies for quantum entangled systems and are similar to fermion particle-antiparticle pairs. Therefore quantum information processes can be described by particle-antiparticle diagrams much like particle physics diagrams and **the Algebraic Quantum Field Theory of the E8 - Cl(16) = Cl(8)xCl(8) Physics Model should be equivalent to a Quantum Code Information System.**

Quantum Reed-Muller code [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8)

Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8) x Cl(8) = Cl(16) containing E8

Completion of the Union of All Tensor Products of [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
AQFT (Algebraic Quantum Field Theory) hyperfinite von Neumann factor algebra that is Completion of the Union of All Tensor Products of Cl(16)

Quantum Reed-Muller code [[256 , 0 , 24]] is described in quantum-ph/9608026 by Steane as mapping a quantum state space of 256 qubits into 256 qubits, correcting [(24-1)/2] = 11 errors, and detecting 24/2 = 12 errors.
Let C(n,t) = n! / t! (n-t)!
Then

[[256, 0, 24]] is of the form

```
[[ 2^n,   2^n - C(n,t) - 2 SUM(0 k t-1) C(n,k),   2^t + 2^(t-1) ]]
[[ 2^8,   2^8 - C(8,4) - 2 SUM(0 k 3) C(8,k),     2^4 + 2^(4-1) ]]
[[ 2^8,   2^8 - 70 - (1+8+28+56) -  (1+8+28+56),  16 + 8 ]]
[[ 256,   256 - (1+8+28+56+70+56+28+8+1),         16 + 8 ]]
[[ 256,   16x16 - SUM(0 k 8) 8/\8/\..(k)../\8,    16 + 8 ]]
```

The quantum code [[256, 0, 24]] can be constructed from the classical Reed-Muller code (256, 93, 32) of the form

```
(  2^8,   2^8 - SUM(0 k t) C(n,k),              2^(t+1) )
(  2^8,   2^8 - SUM(0 k 4) C(n,k),              2^5 )
(  2^8,   2^8 - (70+56+28+8+1),                 32 )
(  2^8,   1+8+28+56,                            32 )
```

To construct the quantum code [[256, 0, 24]] :

First, form a quantum code generator matrix
from the 128x256 generator matrix G of the classical code (256, 93, 32) :

```
|         |         |
|    G    |    0    |
|         |         |
|    0    |    G    |
|         |         |
```

Second, form the generator matrix of a quantum code of distance 16
by adding to the quantum generator matrix a matrix Dx such that
G and Dx together generate the classical Reed-Muller code (256, 163, 16) :

```
(   2^8,    1+8+28+56+70,                              16  ) :
```

```
|         |         |
|    G    |    0    |
|         |         |
|    0    |    G    |
|         |         |
|    Dx   |    0    |
|         |         |
```

This quantum code has been made by combining the classical codes
(256, 93, 32) and (256, 163, 16), so that it is of the form

```
[[ 256, 93 + 163 - 256, min(32,16) ]]  =  [[ 256, 0, 16 ]].
```

It is close to what we want, but has distance 16.
For the third and final step, increase the distance to 16+8 = 24
by adding Dz to the quantum generator matrix:

```
|         |         |
|    G    |    0    |
|         |         |
|    0    |    G    |
|         |         |
|    Dx   |    Dz   |
|         |         |
```

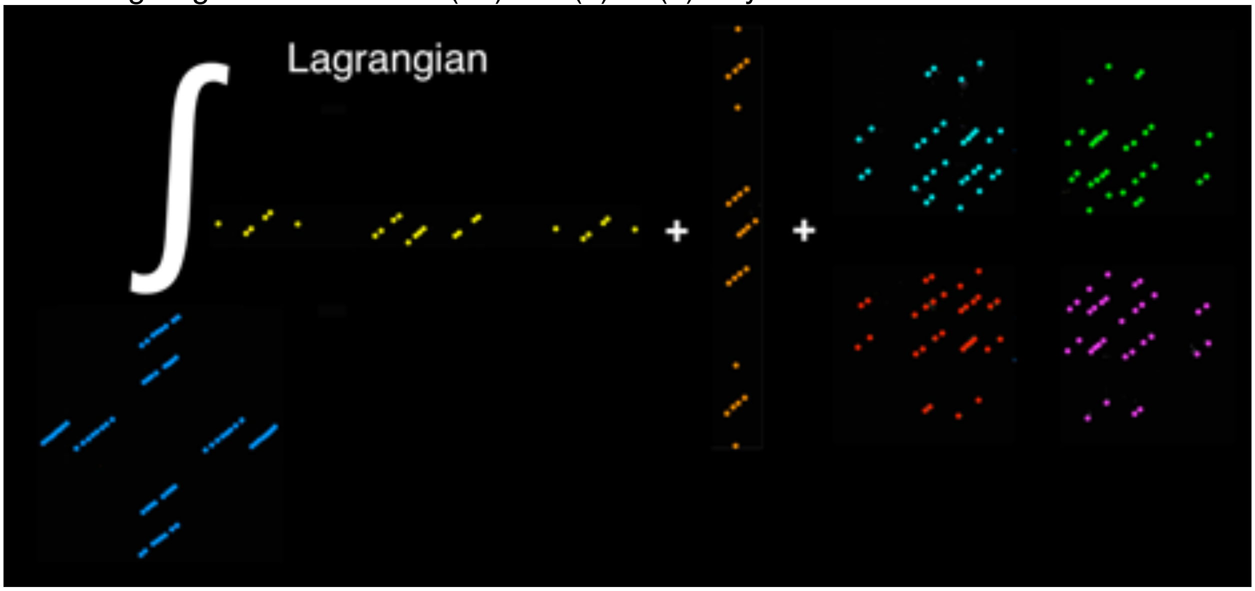
This is the generator matrix of the quantum code [[256, 0, 24]]
as constructed by Steane.

The two classical Reed-Muller codes used to build [[256, 0, 24]]
are (256, 163, 32) and (256, 93, 16),
classical Reed-Muller codes of orders 4 and 3, which are dual to each other.
Due to the nested structure of Reed-Muller codes,
they contain the Reed-Muller codes of orders 2, 1, and 0 :

```
        Classical Reed-Muller Codes                   Order
             of Length 2^8 = 256

(   256,    1+8+28+56+70+56+28+8+1,       1 )           8
(   256,    1+8+28+56+70+56+28+8,         2 )           7
(   256,    1+8+28+56+70+56+28,           4 )           6
(   256,    1+8+28+56+70+56,              8 )           5
(   256,    1+8+28+56+70,                16 )           4
(   256,    1+8+28+56,                   32 )           3
(   256,    1+8+28,                      64 )           2
(   256,    1+8,                        128 )           1
(   256,    1,                          256 )           0
```

In the Lagrangian of the E8 - Cl(16) = Cl(8)xCl(8) Physics Model

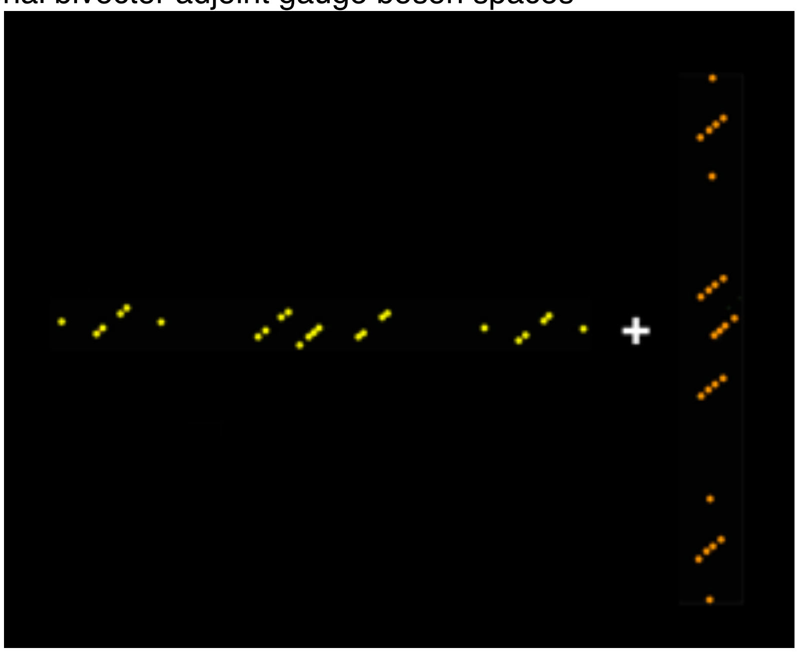


the Higgs scalar prior to dimensional reduction corresponds to the
0th order classical Reed-Muller code (256, 1, 256), the classical repetition code;

the 8-dimensional vector spacetime

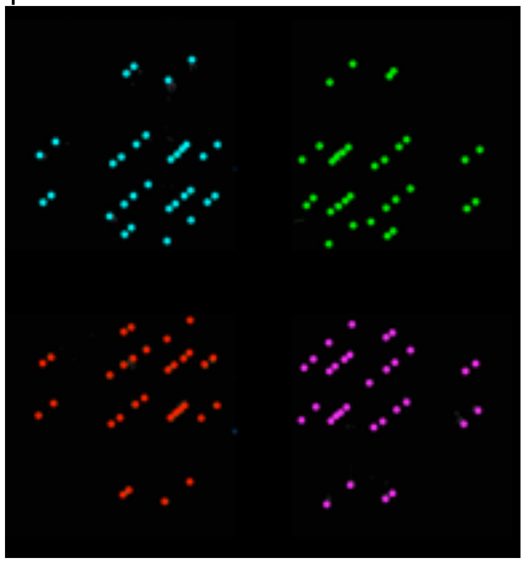
prior to dimensional reduction corresponds to non-0th-order part of the 1st order classical Reed-Muller code (256, 9, 128),
which is dual to the 6th order classical Reed-Muller code (256, 247, 4),
which is the extended Hamming code,
extended from the binary Hamming code (255, 247, 3),
which is dual to the simplex code (255, 8, 128) ;

the 28-dimensional bivector adjoint gauge boson spaces



prior to dimensional reduction correspond to the non-1st-order part of the 2nd order classical Reed-Muller code (256, 37, 64) .

The 8 first generation fermion particles and 8 first generation fermion antiparticles of the 16-dim full spinor representation of the 256-dimensional Cl(0,8) Clifford algebra



correspond to the distance of the classical Reed-Muller code (256, 93, 16),
and to the 16-dimensional Barnes-Wall lattice Λ16,
which lattice comes from the (16,5,8) Reed-Muller code.
Each Λ16 vertex has 4320 nearest neighbors.

The other 8 of the 16+8 = 24 distance of the quantum Reed-Muller code [[256, 0, 24]]
corresponds to the 8-dimensional vector spacetime, and to the 8-dimensional E8 lattice
which comes from the (8,4,4) Hamming code, with weight distribution 0(1) 4(14) 8(1).
It can also be constructed from the repetition code (8,1,1).
The dual of (8,1,1) is (8,7,2), a zero-sum even weight code,
containing all binary vectors with an even number of 1s.
Each E8 lattice vertex has 240 nearest neighbors. In Euclidean R8,
there is only one way to arrange 240 spheres so that they all touch one sphere,
and only one way to arrange 56 spheres so that they all touch
a set of two spheres in contact with each other,
and so forth, giving the following classical spherical codes:
(8,240,1/2), (7,56,1/3), (6,27,1/4), (5,16,1/5), (4,10,1/6), and (3,6,1/7).

(If you use an Octonion Integral Domain instead of Euclidean R8 without multiplication then there are 7 algebraically independent ways to arrange the 240 spheres.)

The total 24 distance of the quantum Reed-Muller code [[256, 0, 24]]
corresponds to the 24-dimensional Leech lattice,
and to the classical extended Golay code (24, 12, 8)
in which lattice each vertex has 196,560 nearest neighbors. In Euclidean R24,
there is only one way to arrange 196,560 spheres so that they all touch one sphere,
and only one way to arrange 4600 spheres so that they all touch a set of two spheres in
contact with each other, and so forth, giving the following classical spherical codes:
(24,196560,1/2), (23,4600,1/3), (22,891,1/4), (21,336,1/5), (20,170,1/6), ...

World-Line String Bohm Quantum Potential and Consciousness

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int_{\text{8-dim SpaceTime}}$ Gauge Gravity + Standard Model + Fermion Particle-AntiParticle .

whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes 8v = 8-dim SpaceTime Dimensions + + 8s+ = 8 Fermion Particle Types + 8s- = 8 Fermion AntiParticle Types combined in the traceless part J(3,O)o of the 3x3 Octonion Hermitian Jordan Algebra

a	8s+	8v
8s+*	b	8s-
8v*	8s-*	-a-b

which has total dimension 8v + 8s+ + 8s- + 2 = 26 and is the space of a
26D String Theory with Strings seen as World-Lines.

Slices of 8v SpaceTime are represented as D8 branes. Each D8 brane has Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice
denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8

Stack D8 branes to get SpaceTime with Strings = World-Lines
with
a and b representing
ordering of D8 brane stacks and Bohm-type Quantum Potential

Let Oct16 = discrete mutiplicative group { +/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K}.
Orbifold by Oct16 the 8s+ to get 8 Fermion Particle Types
Orbifold by Oct16 the 8s- to get 8 Fermion AntiParticle Types

Gauge Bosons from 1E8 and EE8 parts of a D8 give U(2) Electroweak Force
Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give SU(3) Color Force
Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give U(2,2) Conformal Gravity

The 8x8 matrices for collective coordinates linking one D8 to the next D8
give Position x Momentum

Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)

"... For the ... closed ... bosonic string .... The first excited level ... consists of ... the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ... SO(24) ... little group of a ...[26-dim]... massless particle ... and ... a ... massless ... spin two state ...".
Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analagous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The massless spin two state is what I call the Bohmion:
the carrier of the Bohm Force of the Bohm-Sarfatti Quantum Potential.
Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993)
"... the total force ... from the quantum potential ... does not ... fall off with distance ... because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Penrose-Hameroff-type Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms.
The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its 10^{18} to 10^{19} Tubulin Dimers with math description in terms of a large Real Clifford Algebra.
First consider Superposition of States involving one tubulin with one electron of mass m and two different position states separated by a .
The Superposition Separation Energy Difference is the gravitational energy

$$E_{electron} = G m^2 / a$$

For any single given tubulin $a = 1$ nanometer $= 10^{-7}$ cm so that for a single Electron

$$T = h / E_{electron} = (\text{Compton} / \text{Schwarzschild}) (a / c) = 10^{26} \text{ sec} = 10^{19} \text{ years}$$

Now consider the case of N Tubulin Electrons in Coherent Superposition
Jack Sarfatti defines coherence length L by $L^3 = N a^3$ so that the Superposition Energy E_N of N superposed Conformation Electrons is

$$E_N = G M^2 / L = N^{5/3} E_{electron}$$

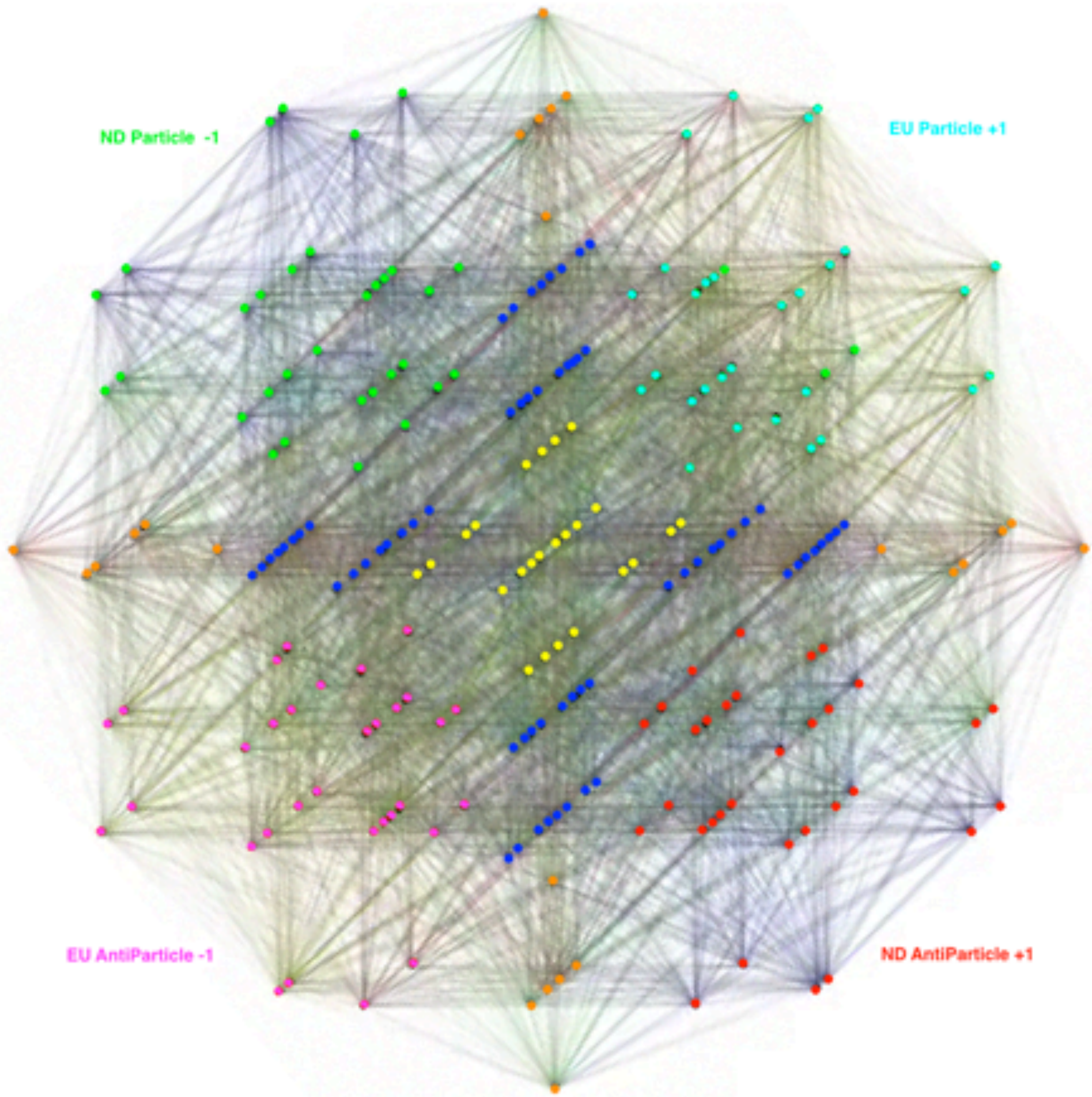
The decoherence time for the system of N Tubulin Electrons is

$$T_N = h / E_N = h / N^{5/3} E_{electron} = N^{-5/3} 10^{26} \text{ sec}$$

So we have the following rough approximate Decoherence Times T_N

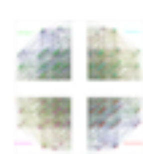
Time T_N	Number of Involved Tubulins
10^{-5} sec	10^{18}
25×10^{-3} sec (40 Hz)	10^{16}

Cl(16) - E8 Lagrangian - AQFT



240

112



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Frank Dodd (Tony) Smith, Jr. - 2014

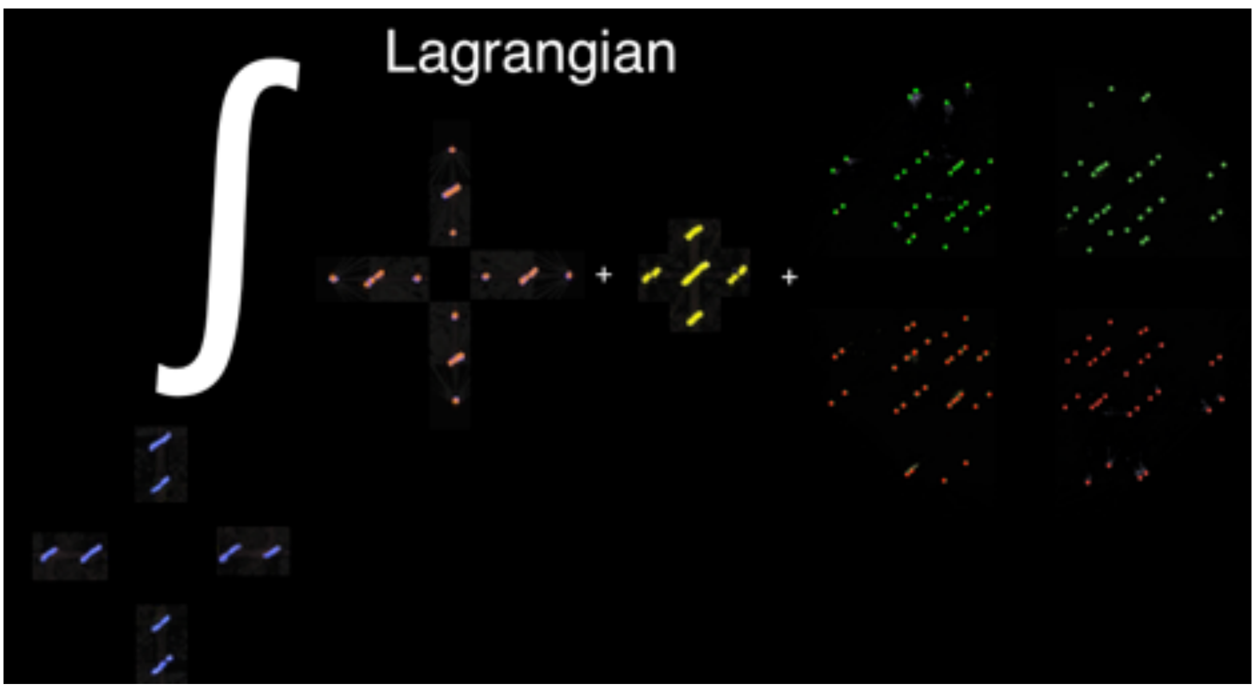
for HISTORY see pages 213ff

viXra 1405.0030

for TETRAHEDRA see pages 253ff

Abstract:

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at www.valdostamuseum.com/hamsmith/ www.tony5m17h.net/ and on viXra - list at vixra.org/author/frank_dodd_tony_smith_jr
Due to experimental observations and my learning new techniques over those 30 years my Physics Models have been in a state of evolving flux - for example, 30 years ago their basis was the Lie Algebra Spin(8), then to contain vectors and spinors it was F4, then to contain the geometry of bounded complex domains it was E6,
then Real Clifford Algebras were used to describe evolution from a Void Empty Set ø, then Periodicity showed the importance of Cl(8) and tensor product Cl(8)xCl(8) = Cl(16), then E8 emerged from Cl(16) to give the structure of a realistic local E8 Lagrangian, then completion of the union of all tensor products of Cl(16) local structures produced a realistic Algebraic Quantum Field Theory (AQFT). Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate. Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient. The Cl(16)-E8 model must be consistent with experimental observations. As of now, given that most calculations are tree-level, the model is substantially so consistent. An interesting test over the 2015-2016 time frame will be whether or not the LHC sees two additional Higgs mass states with cross section about 20% of that of a full Standard Model Higgs.



Preface

Over the past 30 years or so I have been constructing Physics Models and writing about them as can be seen on my web sites at http://www.valdostamuseum.com/hamsmith/ http://www.tony5m17h.net/ and on viXra - list at http://vixra.org/author/frank_dodd_tony_smith_jr
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produced a realistic Algebraic Quantum Field Theory (AQFT).

Since my works over those 30 years have been written from various points of view it is not easy to navigate among them. This paper is being written from a single point of view (that of May 2014) in the hope that it might be easier for readers to navigate.

A lot of math is used in my Cl(16)-E8 model, some of which may be unfamiliar to many.
My efforts to find a single volume for the math of Cl(16) - E8 Lagrangian - AQFT
led me to my Princeton University Advanced Calculus text by H. K. Nickerson,
D. C. Spencer, and N. E. Steenrod. However, it is over 50 years old, so I have added some Supplementary Material to produce a 21 MB pdf file on the web at

http://www.valdostamuseum.com/hamsmith/NSS6313.pdf

TABLE OF CONTENTS OF THE SUPPLEMENTED TEXT:

Supplementary Material in Red

Although the nice math of my Cl(16)-E8 model is necessary, it is not sufficient.
My Cl(16)-E8 model must be, and is, consistent with experimental observations .

Here is a summary of E8 Physics model calculation results. Since ratios are calculated, values for one particle mass and one force strength are assumed. Quark masses are constituent masses. Most of the calculations are tree-level, so more detailed calculations might be even closer to observations.

Dark Energy : Dark Matter : Ordinary Matter = 0.75 : 0.21 : 0.04
Inflationary Gravitational Wave (IGW) tensor-to-scalar ratio r = 7/28 = 0.25
Fermions as Schwinger Sources have geometry of Complex Bounded Domains
with Kerr-Newman Black Hole structure size about 10^(-24) cm.

Particle/Force	Tree-Level	Higher-Order
e-neutrino	0	0 for nu_1
mu-neutrino	0	9 x 10^(-3) eV for nu_2
tau-neutrino	0	5.4 x 10^(-2) eV for nu_3
electron	0.5110 MeV	
down quark	312.8 MeV	charged pion = 139 MeV
up quark	312.8 MeV	proton = 938.25 MeV
		neutron - proton = 1.1 MeV
muon	104.8 MeV	106.2 MeV
strange quark	625 MeV	
charm quark	2090 MeV	
tauon	1.88 GeV	
beauty quark	5.63 GeV	
truth quark (low state)	130 GeV	(middle state) 174 GeV
		(high state) 218 GeV
W+	80.326 GeV	
W-	80.326 GeV	
W0	98.379 GeV	Z0 = 91.862 GeV
Mplanck	1.217x10^19 GeV	
Higgs VEV (assumed)	252.5 GeV	
Higgs (low state)	126 GeV	(middle state) 182 GeV
		(high state) 239 GeV
Gravity Gg (assumed)	1	
(Gg)(Mproton^2 / Mplanck^2)		5 x 10^(-39)
EM fine structure	1/137.03608	
Weak Gw	0.2535	
Gw(Mproton^2 / (Mw+^2 + Mw-^2 + Mz0^2))		1.05 x 10^(-5)
Color Force at 0.245 GeV	0.6286	0.106 at 91 GeV

Kobayashi-Maskawa parameters for W+ and W- processes are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The phase angle d13 is taken to be 1 radian.

The 3-state system of Higgs and Tquark masses is a property of the Cl(16)-E8 model that can be tested at the **LHC 2015-2016 run by searching for Higgs middle and high mass states with cross section about 20% of that of a full SM Higgs**.

TABLE OF CONTENTS OF THIS PAPER:

The math of the Cl(16)-E8 model is based on Three Grothendieck Universes

Bohm Quantum Potential from 26D String World Lines

The Cosmology of the Cl(16)-E8 model begins with Octonionic Inflation

Standard Model

Gravity, Dark Energy, and Post-Inflation Cosmology

1. The First Grothendieck Universe is the Empty Set ø which grows by Clifford Iteration to Cl(16) which contains E8

1					= Cl(0) = 1
ø					

1	1				= Cl(1) = 2
ø	(ø)				

1	2	1			= Cl(2) = 4
ø	(ø)	(ø (ø))			
	((ø))				

1	4	6	4	1	= Cl(4) = 16
ø	(ø)	(ø (ø))	((ø) ((ø)) (ø (ø)))	(ø (ø) ((ø)) (ø (ø)))	
	((ø))	(ø ((ø)))	(ø ((ø)) (ø (ø)))		
	(((ø)))	(ø (ø (ø)))	(ø (ø) (ø (ø)))		
	((ø (ø)))	((ø) ((ø)))	(ø (ø) ((ø)))		
		((ø) (ø (ø)))			
		(((ø)) (ø (ø)))			

1 16 120 560 1820 4368 8008 11440 12870 11440 8008 4368 1820 560 120 16 1

= Cl(16) = 2^16 = 65,536 =

= ((64+64) + (64+64)) x ((64+64) + (64+64))

Cl(16) BiVectors = D8 = 120 = 28 + 28 + 64

Cl(16) Spinors = (64+64) + (64+64)

28 + 28 + 64 + 64 + 64 = E8

From Cl(1,3)) = 16 to Cl(Cl(1,3)) = 65,536 with 16 $\wedge$ 16 = 120

(**Color Scheme on this page for Cl(1,3) is not the same used for Cl(16) and E8**)

1 4 6 4 1 $\wedge$ 1 4 6 4 1

1 $\wedge$ 4 = 4
4 $\wedge$ 6 = 24

1 $\wedge$ 4 = 4
6 $\wedge$ 4 = 24

1 $\wedge$ 6 = 6
1 $\wedge$ 1 = 1
6 $\wedge$ 6 = 15
6 $\wedge$ 1 = 6

4 $\wedge$ 4 = 6
4 $\wedge$ 4 = 16
4 $\wedge$ 4 = 6

4 $\wedge$ 1 = 4
4 $\wedge$ 1 = 4

28 D4 for Gravity +
28 D4 for Standard Model +
28 AntiSymmetric D4 rotations in 8-dim SpaceTime +
28 8x8 Symmetric Off-Diagonal +
8 8x8 Symmetric Diagonal for 4 + 4 Klauza-Klein M4 x CP2 = 120

E8 structure gives a Fundamental Local Lagrangian

E8 Root Vectors = 112 + 64 + 64 = 24 + 24 + 64 + 64 + 64

Fundamental Local Lagrangian =

$$= \int_{\text{8-dim SpaceTime}} \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle}$$

8-dim SpaceTime .

where E8 structure of the Lagrangian Terms is given by:

E8 / D8 = 64 + 64

64 = 8 Components of 8 Fermion Particles (first generation)

64 = 8 Components of 8 Fermion AntiParticles (first generation)

D8 / D4xD4 = 64

64 = 8-dim SpaceTime Position and Momentum

(Triality Automorphisms: 64 = 64 = 64)

D4xD4 = 24 + 4 + 24 + 4

24 + 4 = 28 = D4 for Gravity Gauge Bosons

24 + 4 = 28 = D4 for Standard Model Gauge Bosons

Standard Model Gauge Gravity term has total weight 28 x 1 = 28

12 generators for SU(3) and U(2) Standard Model

\+

16 generators for U(2,2) of Conformal Gravity

=

28 D4 Gauge Bosons

each with 8-dim Lagrangian weight = 1

Fermion Particle-AntiParticle term also has total weight 8 x (7/2) = 28

8 Fermion Particle/Antiparticle types

each with 8-dim Lagrangian weight = 7/2

Since Boson Weight 28 = Fermion Weight 28

the Cl(16)-E8 model has a Subtle SuperSymmetry and is UltraViolet Finite.

The Cl(16)-E8 model has 8-dim Lorentz structure satisfying Coleman-Mandula because its fermionic fundamental spinor representations are built with respect to spinor representations for 8-dim Spin(1,7) spacetime.

(See pages 382-384 of Steven Weinberg's book "The Quantum Theory of Fields" Vol. III)

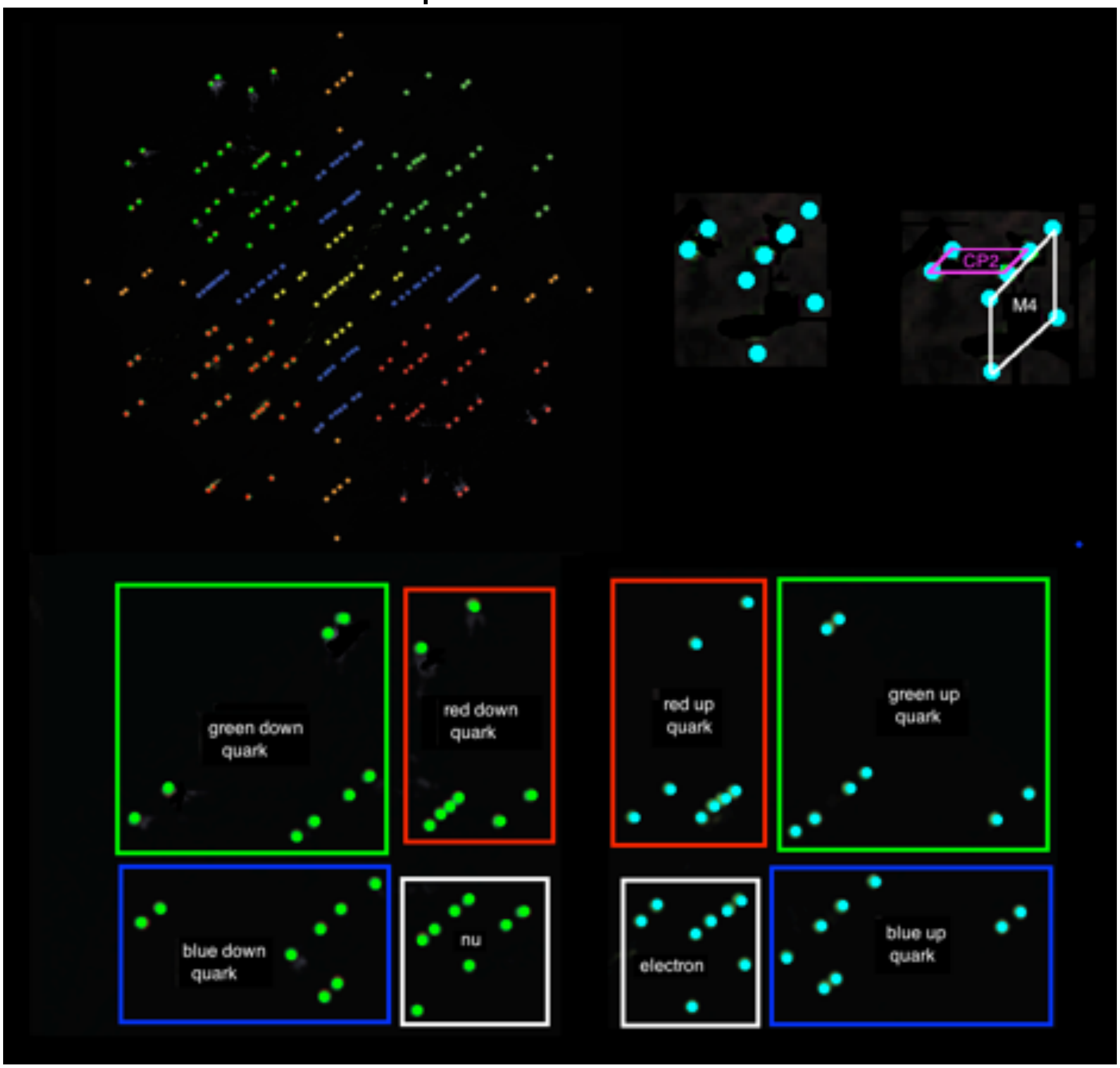
The Cl(16)-E8 model is Chiral because
E8 contains Cl(16) half-spinors (64+64) for a Fermion Generation
but does not contain Cl(16) Fermion AntiGeneration half-spinors (64+64).
Fermion +half-spinor Particles with high enough velocity are seen as left-handed.
Fermion -half-spinor AntiParticles with high enough velocity are seen as right-handed.

The Cl(16)-E8 model obeys Spin-Statistics because
the CP2 part of M4xCP2 Kaluza-Klein has index structure Euler number 2+1 = 3 and Atiyah-Singer index -1/8 which is not the net number of generations because
CP2 has no spin structure but you can use a generalized spin structure
(Hawking and Pope (Phys. Lett. 73B (1978) 42-44))
to get (for integral m) the generalized CP2 index n_R - n_L = (1/2) m (m+1)
Prior to Dimensional Reduction: m = 1, n_R - n_L = (1/2)x1x2 = 1 for 1 generation
After Reduction to 4+4 Kaluza-Klein: m = 2, n_R - n_L = (1/2)x2x3 = 1 for 3 generations
(second and third generations emerge as effective composites of the first)

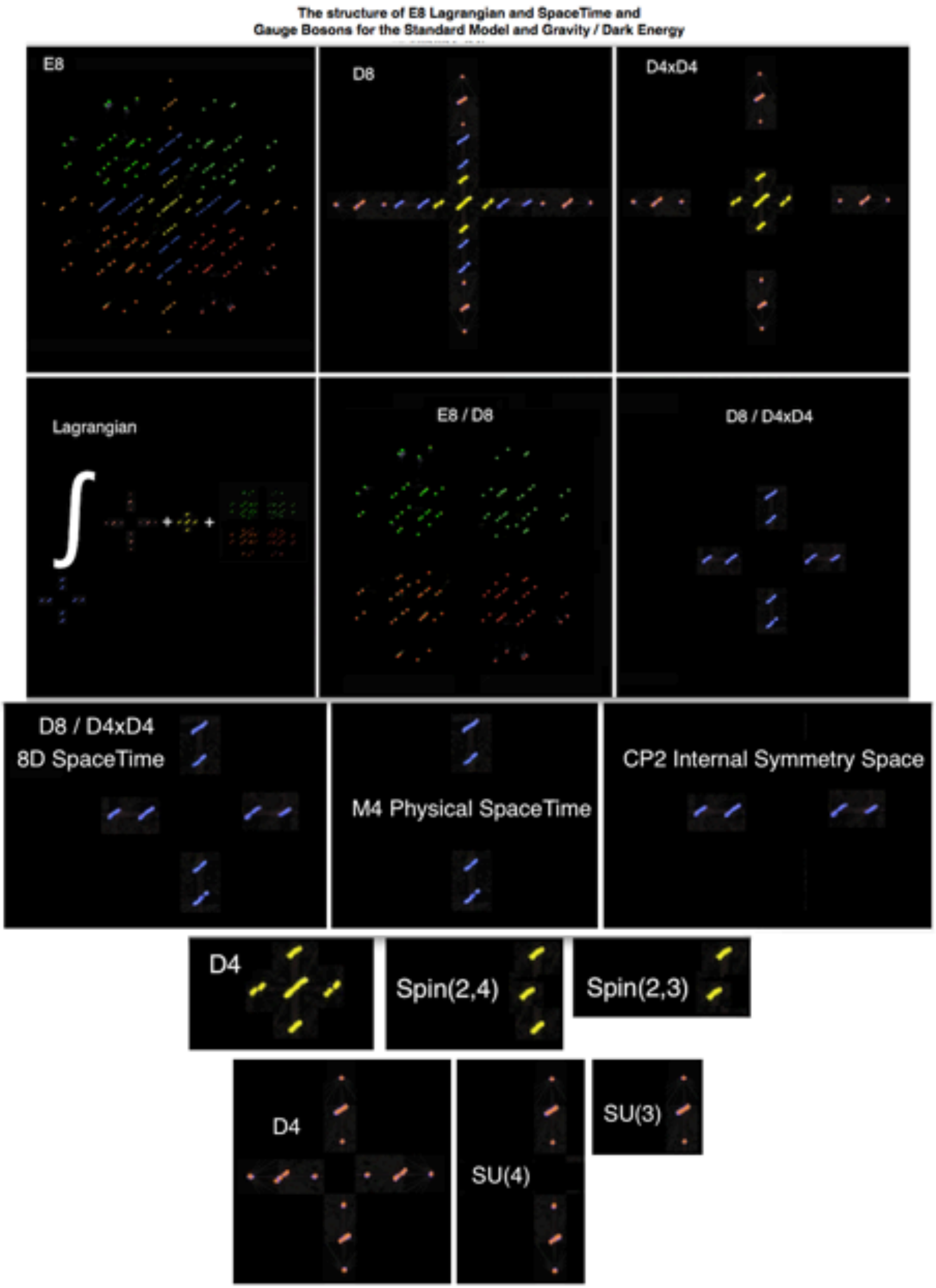
Hawking and Pope say: "Generalized Spin Structures in Quantum Gravity ...
what happens in CP2 … is a two-surface K which cannot be shrunk to zero. ...
However, one could replace the electromagnetic field by
a Yang-Mills field whose group G had a double covering G~.
The fermion field would have to occur in representations which changed sign
under the non-trivial element of the kernel of the projection … G~ -> G
while the bosons would have to occur in representations which did not change sign …".
For Cl(16)-E8 model gauge bosons are in the 28+28=56-dim D4 + D4 subalgebra of E8.
D4 = SO(8) is the Hawking-Pope G which has double covering G~ = Spin(8).

The 8 fermion particles / antiparticles are D4 half-spinors represented within E8 by anti-commutators and so do change sign while
the 28 gauge bosons are D4 adjoint represented within E8 by commutators and so do not change sign. Further,
E8 inherits from F4 the property whereby its Spinor Part need not be written as Commutators but can also be written in terms of Fermionic AntiCommutators. (vixra 1208.0145)

The structure of E8 Spinor Fermions of the First Generation is:



Spinor 128 of 240 E8 Root Vectors are 64 red/magenta and 64 green/cyan dots.
64 Green Dots represent Fermion Particles. 64 Red Dots represent AntiParticles.

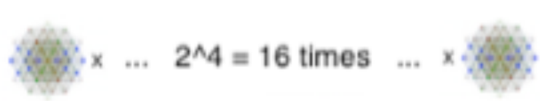


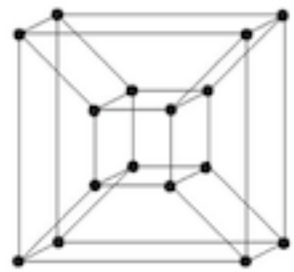
The D8 112 of the 240 E8 Root Vectors are 24 orange and 24 yellow and 64 blue dots.

2. The Second Grothendieck Universe is Hereditarily Finite Sets such as Discrete Clifford Algebras and Discrete Lattices.

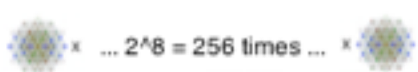
Each Local Lagrangian with Creation / Annihilation density terms
lives in an E8 which in turn lives in a Cl(16) Real Clifford Algebra.

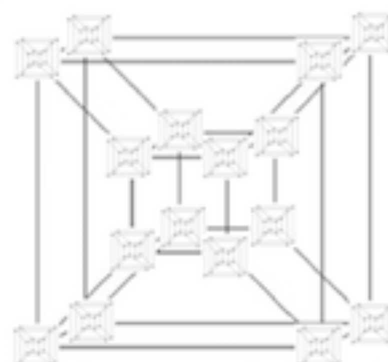
By 8-Periodicity of Real Clifford Algebras tensor products of N copies of Cl(16) form a Clifford Algebra **Cl(16N) = Cl(16) x ...(N times tensor product)... x Cl(16)** .
For N = 2^4 = 16 the 16 copies of Cl(16) form E8 Physics of a 4-dim HyperCube





corresponding to 4-dim M4 Physical SpaceTime.
For N = 2^8 = 256 the 256 copies of Cl(16) form E8 Physics of an 8-dim HyperCube





corresponding to 8-dim E8 SpaceTime (image by Conrad Schneiker in 1987 paper by Hameroff) and to M4 x CP2 Kaluza-Klein SpaceTime with each vertex of the 4-dim M4 HyperCube having a 4-dim CP2 Internal Symmetry Space.
For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".
As N grows, the copies of Cl(16) continuue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes. If the edges of the sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond.

The Union of all Cl(16) tensor products is the Union of all subdivided 8-HyperCubes and their Completion is a huge superposition of 8-HyperCube Continuous Volumes which Completion belongs to the Third Grothendieck Universe.

H. S. M. Coxeter in his paper Regular and Semi-Regular Polyotpes III (Math. Z. 200, 3-45, 1988)

about the 240 units of an E8 Integral Domain said: "... "... the 16 + 16 + 16 octaves
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K,   (±1 ±I ±J ±K)/2,   (±E ±i ±j ±k)/2, and the 192 others derived from the last two expressions by ... the cyclic permutation (E, i, j, I, K, k, J), which preserves the integral domain ... the permutation (e I J i k K j), which is an automorphism of the whole ring of octaves (and of the finite [Fano] plane ...) transforms this particular integral domain into another one of R. H. Bruck's cyclic of seven such domains. ...". An 8th E8 Lattice (not a closed Integral Domain, Kirmse's mistake) can be taken to correspond the the 1 Real Element of the Octonion Basis { 1, i, j, k, E, I, J, K}.

```
There are 7 independent E8 Integral Domain Lattices
corresponding to the 7 Octonion Imaginary Basis Elements {i,j,k,E,I,J,K}

     Associative   Coassociative  Heptaverton
      Triangle         Square
                                      k J
        I             J---j           |/
i  -->  / \  ---------->  |   | -->    I--i--E
      E---i             K---k          /|
                                     K j

                                      k I
        J             K---k           |/
j  -->  / \  ---------->  |   | -->    J--j--E
      E---j             I---i          /|
                                     K i

                                      i J
        K             I---i           |/
k  -->  / \  ---------->  |   | -->    K--k--E
      E---k             J---j          /|
                                     I j

                                      I k
        j             I---J           |/
E  -->  / \  ---------->  |   | -->    J--E--j
      i---k             K---E          /|
                                     K i

                                      E j
        J             I---j           |/
I  -->  / \  ---------->  |   | -->    J--I--k
      i---K             k---E          /|
                                     K i

                                      E k
        j             J---i           |/
J  -->  / \  ---------->  |   | -->    K--J--i
      I---K             k---E          /|
                                     I i

                                      E i
        J             K---i           |/
K  -->  / \  ---------->  |   | -->    I--K--j
      I---k             j---E          /|
                                     J k
```

E8 Lattices

E8 Lattices are based on Octonions, which have 480 different multiplication products. E8 Lattices can be combined to form 24-dimensional Leech Lattices and 26-dimensional Bosonic String Theory, which describes E8 Physics when the strings are physically interpreted as World-Lines. A basic String Theory Cell has as its automorphism group the Monster Group whose order is
2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71 = about 8 x 10^53.

For more about the Leech Lattice and the Monster and E8 Physics,
see viXra 1210.0072 and 1108.0027 .

E8 Root systems and lattices are discussed by Robert A. Wilson in his 2009 paper "Octonions and the Leech lattice":
"... The (real) octonion algebra is an 8-dimensional (non-division) algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 } labeled
by the projective line PL(7) = { oo } u F7
...
The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u
... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) ...[with]... an odd number of minus signs.
Denote by L the lattice spanned by these 240 octonions
...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A
... Writing **B = (1/2) (1 + i0) A (1 + i0)** ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
...
the roots of B are
[**16 octonions**]... +/- it for t in PL(7)
... together with
[**112 octonions**]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7
... and ...
[**112 octonions**]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7
...
B is not closed under multiplication ... Kirmse's mistake
...[but]... as Coxeter ... pointed out ...
... **there are seven non-associative rings At** = (1/2) (1 + it) B (1 + it) ,
obtained from B by swapping 1 with it ... for t in F7
...
LR = 2B and BL = L ...[which]... appear[s] not to have been noticed before ... some work ... by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I sometimes denote by {1,i,j,k,E,I,J,K}: "...

$$\begin{aligned}
\Xi_0 = &\quad \{\pm e_a\}, \\
\Xi_2 = &\quad \{(\pm e_a \pm e_b \pm e_c \pm e_d)/2 : a, b, c, d \text{ distinct}, \\
&\quad e_a(e_b(e_c e_d)) = \pm 1\},
\end{aligned}$$

$$\begin{aligned}
\Xi^{even} = &\quad \Xi_0 \cup \Xi_2, \\
\mathcal{E}_8^{even} = &\quad \mathrm{span}\{\Xi^{even}\},
\end{aligned}$$

$$\begin{aligned}
\Xi_1 = &\quad \{(\pm e_a \pm e_b)/\sqrt{2} : a, b \text{ distinct}\}, \\
\Xi_3 = &\quad \{(\textstyle\sum_{a=0}^{7} \pm e_a)/\sqrt{8} : \text{ even number of +'s}\},
\end{aligned}$$

$$\begin{aligned}
\Xi^{odd} = &\quad \Xi_1 \cup \Xi_3, \\
\mathcal{E}_8^{odd} = &\quad \mathrm{span}\{\Xi^{odd}\}
\end{aligned}$$

(spans over integers)

Ξeven has 16+224 = 240 elements ... Ξodd has 112+128 = 240 elements ...

E8even does not close with respect to our given octonion multiplication
...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's Ξeven corresponds to Wilson's B which I denote as 1E8.

Geoffrey Dixon's Ξeven[0-a] correspond to Wilson's seven At
which I denote as iE8, jE8, kE8, EE8, IE8, JE8, KE8.

Geoffrey Dixon's Ξodd corresponds to Wilson's L.

My view is that **the E8 domains 1E8 = Ξeven = B is fundamental**
because
E8 domains iE8, jE8, kE8, EE8, IE8, JE8, KE = Ξeven[0-a] are derived from 1E8
and L and L s are also derived from 1E8 = Ξeven = B.

Using the notation {1,i,j,k,E,I,J,K} for Octonion basis
notice that in the Cl(16)-E8 model introduction of Quaternionic substructure to produce (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime requires breaking Octonionic light-cone elements
(+/- 1 +/- i +/- j +/- k +/- E +/- I +/- J +/- K) / 2
into Quaternionic 4-term forms like (+/- A +/- B +/- C +/- D) / 2.

To do that, consider that there are (8l4) = 70 ways to choose 4-term subsets of the 8 Octonionic basis element terms. Using all of them produces 224 4-term subsets in each of the 7 Octonion Imaginary E8 lattices iE8,jE8,kE8,EE8,IE8,JE8,KE8 each of which also has 16 1-term first-shell vertices.

56 of the 70 4-term subsets appear as 8 in each of the 7 Octonion Imaginary E8 lattices.

The other 70-56 = 14 4-term subsets occur in sets of 3 among 7x6 = 42 4-term subsets as indicated in the following detailed list of the 7 Octonion Imaginary E8 lattices:

```
EE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±K ±E ±k)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8
(±1 ±J ±j ±E)/2 (±I ±K ±k ±i)/2 jE8 , EE8 , JE8
(±1 ±E ±I ±i)/2 (±K ±k ±J ±j)/2 iE8 , EE8 , IE8

128 of D8 half-spinors appear only in EE8
(±1 ±I ±J ±K)/2 (±E ±i ±j ±k)/2
(±1 ±k ±i ±J)/2 (±j ±I ±K ±E)/2
(±1 ±i ±K ±j)/2 (±k ±J ±E ±I)/2
(±1 ±j ±k ±I)/2 (±J ±E ±i ±K)/2
```

iE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±I ±i ±E)/2 (±j ±k ±J ±K)/2 iE8 , EE8 , IE8
(±1 ±K ±J ±i)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8
(±1 ±i ±k ±j)/2 (±E ±I ±J ±K)/2 iE8 , jE8 , kE8

128 of D8 half-spinors appear only in iE8
(±1 ±k ±K ±I)/2 (±i ±j ±E ±J)/2
(±1 ±E ±j ±K)/2 (±i ±k ±I ±J)/2
(±1 ±j ±I ±J)/2 (±i ±k ±E ±K)/2
(±1 ±J ±E ±k)/2 (±i ±j ±I ±K)/2

jE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±k ±j ±i)/2 (±E ±I ±J ±K)/2 iE8 , jE8 , kE8
(±1 ±I ±K ±j)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8
(±1 ±j ±E ±J)/2 (±i ±k ±I ±K)/2 jE8 , EE8 , JE8

128 of D8 half-spinors appear only in jE8
(±1 ±E ±I ±k)/2 (±i ±j ±J ±K)/2
(±1 ±i ±J ±I)/2 (±j ±k ±E ±K)/2
(±1 ±J ±k ±K)/2 (±i ±j ±E ±I)/2
(±1 ±K ±i ±E)/2 (±j ±k ±I ±J)/2

kE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±J ±k ±I)/2 (±i ±j ±E ±K)/2 kE8 , IE8 , JE8
(±1 ±j ±i ±k)/2 (±E ±I ±J ±K)/2 iE8 , jE8 , kE8
(±1 ±k ±K ±E)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8

128 of D8 half-spinors appear only in kE8
(±1 ±K ±j ±J)/2 (±i ±k ±E ±I)/2
(±1 ±I ±E ±j)/2 (±i ±k ±J ±K)/2
(±1 ±E ±J ±i)/2 (±j ±k ±I ±K)/2
(±1 ±i ±I ±K)/2 (±j ±k ±E ±J)/2

IE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±j ±I ±K)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8
(±1 ±i ±E ±I)/2 (±j ±k ±J ±K)/2 iE8 , EE8 , IE8
(±1 ±I ±J ±k)/2 (±i ±j ±E ±K)/2 kE8 , IE8 , JE8

128 of D8 half-spinors appear only in IE8
(±1 ±J ±i ±j)/2 (±k ±E ±I ±K)/2
(±1 ±K ±k ±i)/2 (±j ±E ±I ±J)/2
(±1 ±k ±j ±E)/2 (±i ±I ±J ±K)/2
(±1 ±E ±K ±J)/2 (±i ±j ±k ±I)/2

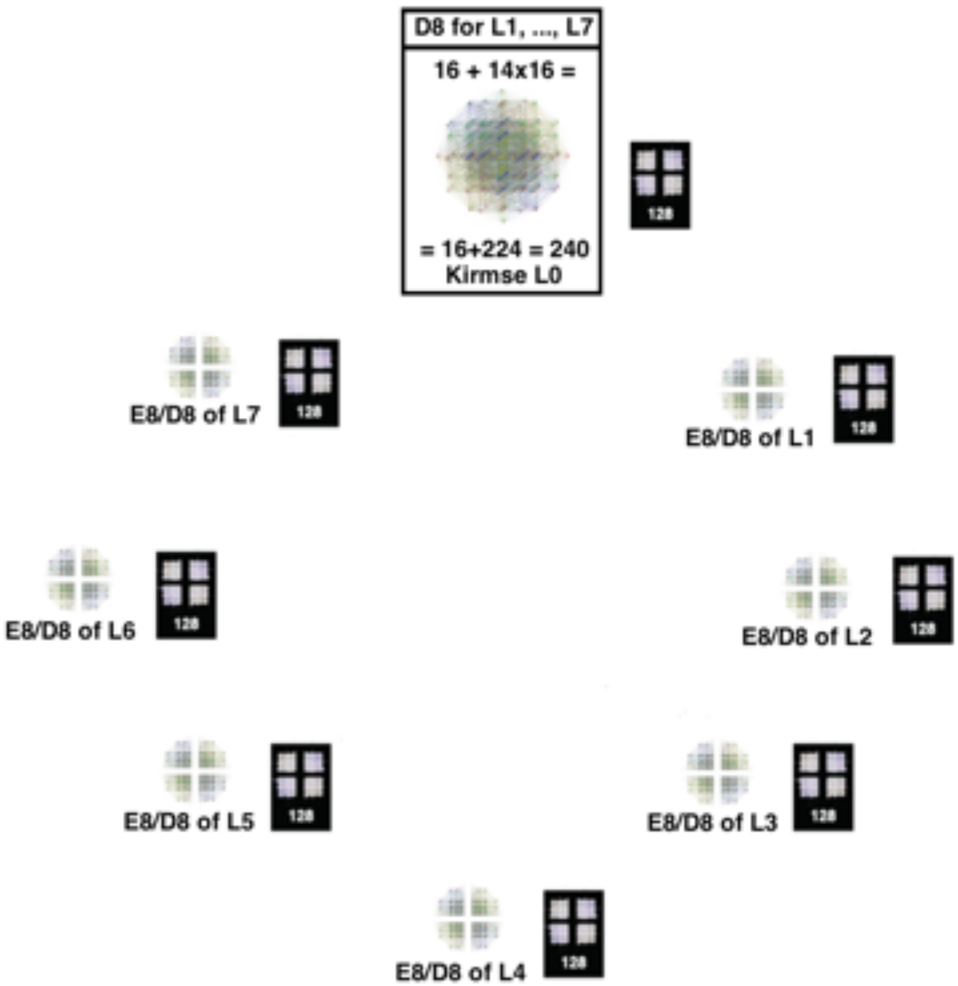
JE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±E ±J ±j)/2 (±i ±k ±I ±K)/2 jE8 , EE8 , JE8
(±1 ±k ±I ±J)/2 (±i ±j ±E ±I)/2 kE8 , IE8 , JE8
(±1 ±J ±i ±K)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8

128 of D8 half-spinors appear only in JE8
(±1 ±i ±k ±E)/2 (±j ±I ±J ±K)/2
(±1 ±j ±K ±k)/2 (±i ±E ±I ±J)/2
(±1 ±K ±E ±I)/2 (±i ±j ±k ±J)/2
(±1 ±I ±j ±i)/2 (±k ±E ±J ±K)/2

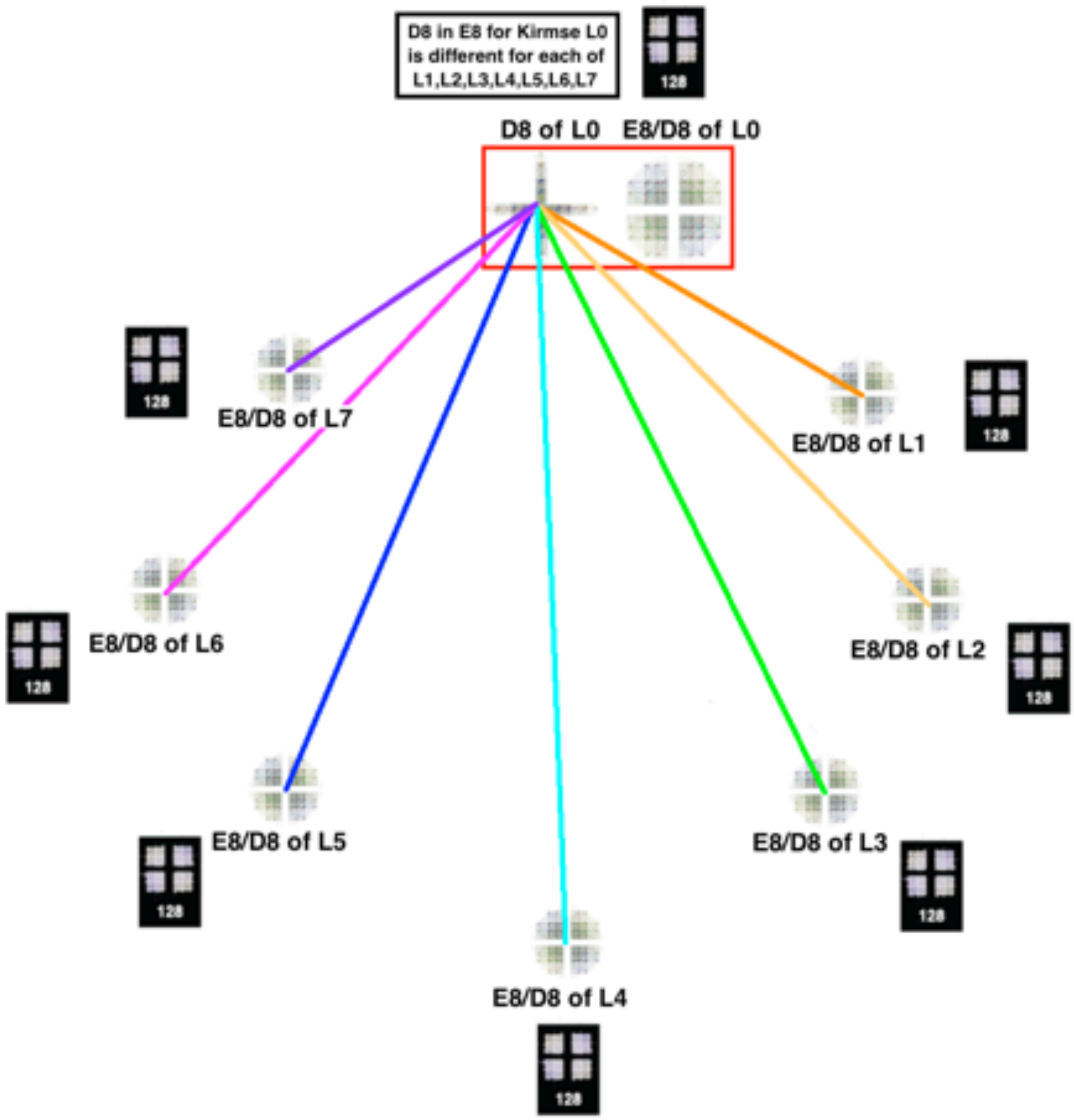
KE8:
112 of D8 Root Vectors
16 appear in all 7 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
±1, ±i, ±j, ±k, ±E, ±I, ±J, ±K
96 appear in 3 of iE8,jE8,kE8,EE8,IE8,JE8,KE8
(±1 ±i ±K ±J)/2 (±j ±k ±E ±I)/2 iE8 , JE8 , KE8
(±1 ±E ±k ±K)/2 (±i ±j ±I ±J)/2 kE8 , EE8 , KE8
(±1 ±K ±j ±I)/2 (±i ±k ±E ±J)/2 jE8 , IE8 , KE8

128 of D8 half-spinors appear only in KE8
(±1 ±j ±E ±i)/2 (±k ±I ±J ±K)/2
(±1 ±J ±I ±E)/2 (±i ±j ±k ±K)/2
(±1 ±I ±i ±k)/2 (±j ±E ±J ±K)/2
(±1 ±k ±J ±j)/2 (±i ±E ±I ±K)/2

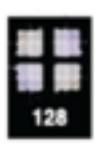
The vertices that appear in more than one lattice are:
16:
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke   in all of them;
6x16 = 96 (1 and e or neither 1 nor e):
(±1±i±e±ie)/2 and (±j±k±je±ke)/2   in 7E8, 1E8, and 3E8 ;
(±1±j±e±je)/2 and (±i±k±ie±ke)/2   in 7E8, 2E8, and 6E8 ;
(±1±k±e±ke)/2 and (±i±j±ie±je)/2   in 7E8, 4E8, and 5E8 ;
8x16 = 128 (either 1 or e singly):
(±1±i±j±k)/2 and (±e±ie±je±ke)/2   in 3E8, 4E8, and 6E8 ;
(±1±i±je±ke)/2 and (±j±k±e±ie)/2   in 2E8, 3E8, and 5E8 ;
(±1±j±ie±ke)/2 and (±i±k±e±je)/2   in 1E8, 5E8, and 6E8 ;
(±1±k±ie±je)/2 and (±i±j±e±ke)/2   in 1E8, 2E8, and 4E8 .
These 16 + 14x16 = 16 + 224 = 240 vertices
form the Kirmse E8 lattice L0 which is not an Integral Domain
because it is not closed under the chosen Octonion multiplication
but is the part of the 2160 vertices of the 2-shell of all E8 lattices
that provides the D8 part of the 7 distinct Integral Domains L1, ... , L7
whose E8/D8 half-spinor parts are in the 2-shell.



The identification of 1-shell E8 Lattices with their appearance in L0 2-shell
is up to a scale factor. The 2160-vertex 2-shell is effectively a
superposition of the 7 Integral Domains L1, ... , L7 plus the Kirmse L0.

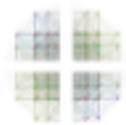


An E8 lattice 2-shell has 2160 vertices,

of which 8 x 

= 8x128 = 1024 are mirror D8 half-spinors that
are not in the E8 Lie algebra as they are not in any E8 1-shell E8 Root Vectors
and

7 x  = 7x128 = 896 are E8/D8 half-spinors that are in the E8 Lie Algebras of the
E8 1-shell Root Vectors of one of the 7 Integral Domain E8 Lattices L1, ... , L7
and

112 + 128   = 240 correspond to the D8 and E8/D8 half-spinors of the E8 1-shell Root Vectors of the Kirmse E8 Lattice L0.

For each of the 7 Integral Domain E8 Lattices L1, ... , L7 there is a different way to select 112  of the 240 of Kirmse L0 to add to the 128  of that Li and produce the 112 + 128 = 240 Root Vectors (up to scaling) of that Li E8 Lattice.

The 8 pairs  + 

of 128-element D8 half-spinors + mirror half-spinors would represent the 128 + 128 = 256-dim full spinors of the Cl(16) Clifford Algebra if the 128-dim E8/D8 half-spinors of the 8th pair, of Kirmse L0, were well-defined
but
since there are 7 different D8 sets of 112 and so 7 different E8/D8 sets of 128 for L0 depending on which of L1, ... , L7 E8/D8 is completed by the 112 D8 of Kirmse L0 the 128 E8/D8 of Kirmse L0 is not well-defined in the conventional 2160 E8 2-shell.
Therefore
the 2160 E8 2-shell should be considered to be a superposition of 7 shells
one for each of the 7 E8/D8 L1, ... , L7 that are completed to E8 by action of D8 of L0.
The superposition is expandable to an 8-shell superposition
by adding an 8th shell corresponding to Kirmse L0.

This is consistent with and justifies the superposition structures used in viXra 1210.0072 v4 and in viXra 1301.0150 v4 and in viXra 1405.0030 vF

Since the 1024 mirror D8 half-spinors are not in any E8,
the only part of the 2160 E8 2-shell that is directly relevant to E8 Physics
is the superposition structure
of its 2160 - 1024 = 1136 = 112 + 1024 = 112 + 128 + 896 vertex part.

If you look at the 2160 vertices of the second shell as expanded from the first shell,
then you see that the 2160 is made up of

112 that you might regard as a basic set of D8 root vectors plus

8 x 128 = 1024 as 8 sets of 128, each being a semi-8-hypercube that can be combined with the 112 to make the 112+128 = 240 of an E8 such that the 8 E8s that can be so formed correspond to the real 1 octonion (Kirmse) + 7 imaginary octonions (integral domains) plus

the remaining 2160 - 112 - 1024 = 1024 = 8 x 128 which are the other halves of the 8-hypercubes and so correspond to the mirror half-spinors that are NOT used in the E8s already constructed. They might in some sense be seen as the half-spinor parts of a mirror set of 8 E8s.

Given the framework of the basic D8 112 root vectors, there are 8 ways that you can fit a set of 128 into them so as to form 1 (Kirmse) + 7 (integral domain) E8 lattices.

7 E8 lattice integral domains E8i E8j E8k E8e E8ie E8je E8ke
correspond to the 7 imaginary octonions i j k e ie je ke

Scale them so that their Inner Shells have Unit Radius because as Geoffrey Dixon said "... the inner shell should ... consist of unit elements ...[since]... O multiplication of ... unit elements is closed ...".

Consider the 240-vertex Unit Radius Inner Shells of E8 Lattice Integral Domains corresponding to the algebraic generators i j e of the imaginary octonions with coordinates of the form (E8i , E8j , E8e)
(there are 3 E8s representing **8** in the 24-dim Leech Lattice)

E8i itself has 240 vertices (x , 0 , 0)
E8j itself has 240 vertices (0 , x , 0)
E8e itself has 240 vertices (0 , 0 , x)

Then, consider the 2x240 + 3840 = 4320 vertices of Unit Radius Inner Shells of Λ16 Barnes-Wall Lattices constructed from pairs of E8 Lattices using Dixon's XY-product
(there are 3 ways to choose a Barnes-Wall Λ16 representing **8**+**8** in Leech)

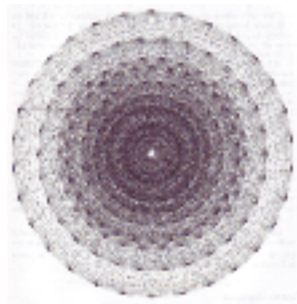
E8i x E8j = E8k with 16x240 = 3840 new vertices (x , y , 0)
E8i x E8e = E8ie with 16x240 = 3840 new vertices (x , 0 , y)
E8j x E8e = E8je with 16x240 = 3840 new vertices (0 , x , y)

Then, consider Unions of Unit Radius E8 Inner Shell with Unit Radius Λ16 Inner Shells,
rescaled by 1 / sqrt(2) so that the Unions have Unit Radius
(there are 3 ways to choose an E8 for (E8 + Barnes-Wall Λ16) to form Leech)

E8i x E8j x E8e x E8ie x E8je x E8k = E8ije x E8(-ijeek) = E8ke x E8ijk = E8ke x E8(-1)
= E8(-ke) = 3x16x3840 = 3 x 61,440 = 184,320 vertices (x , y , z)
where 61,440 = vertices are in second shell of Barnes-Wall Λ16

**The total number of inner vertices = 3 x (240 + 3840 + 61,440) = 196,560
which is the number of inner-shell vertices of the 24-dim Leech Lattice**

Coxeter said in "Integral Cayley Numbers" (Duke Math. J. 13 (1946) 561-578 and
in "Regular and Semi-Regular Polytopes III" (Math. Z. 200 (1988) 3-45):
"... the 240 integral Cayley numbers of norm1 ... are the vertices of 4_21

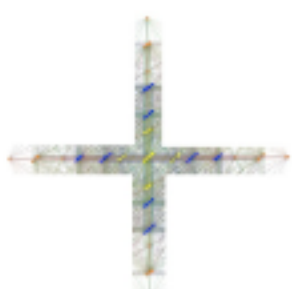


The polytope 4_21 ... has cells of two kinds ...
a seven-dimensional "cross polytope" (or octahedron-analogue) B_7
... there are ... 2160 B_7's ...
and ...
a seven-dimensional regular simplex A_7
... there are 17280 A_7's
...
the 2160 integral Cayley numbers of norm 2 are
the centers of the 2160 B_7's of a 4_21 of edge 2
...
the 17280 integral Cayley numbers of norm 4 (other than the doubles
of those of norm 1) are the centers of the 17280 A_7's of a 4_21 of edge 8/3 ...
[Using notation of {a1,a2,a3,a4,a5,a6,a7,a8} for Octonion basis elements we have]

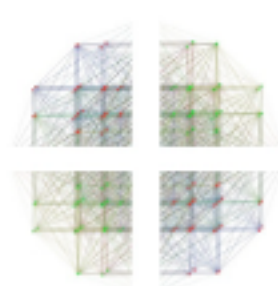
norm 1

112 like (+/- a1 +/- a2)
[which correspond to 112 = 16 + 96 = 16 + 6x16 in each of the 7 E8 lattices]

128 like (1/2) (- a1 + a2 + a3 + ... + a8) with an odd number of minus signs
[which correspond to 128 = 8x16 in each of the 7 E8 lattices]



112



128

norm 2

16 like +/- 2 a1
[which correspond to 16 fo the 112 in each of the 7 E8 lattices]

1120 like +/- a1 +/- a2 +/- a3 +/- a4
[which correspond to 70x16 = (56+14)x16 that appear in the 7 E8 lattices

with each of the 14 appearing in three of the 7 E8 lattices so that
the 14 account for (14/7)x3x16 = 6x16 = 96 in each of the 7 E8 lattices
and for 14x16 = **224 of the 1120**
and
with each of the 56 appearing in only one of the 7 E8 lattices so that
the 56 account for (56/7)x16 = 128 in each of the 7 E8 lattices
and for 56x16 = **896 = 7x128 of the 1120**]

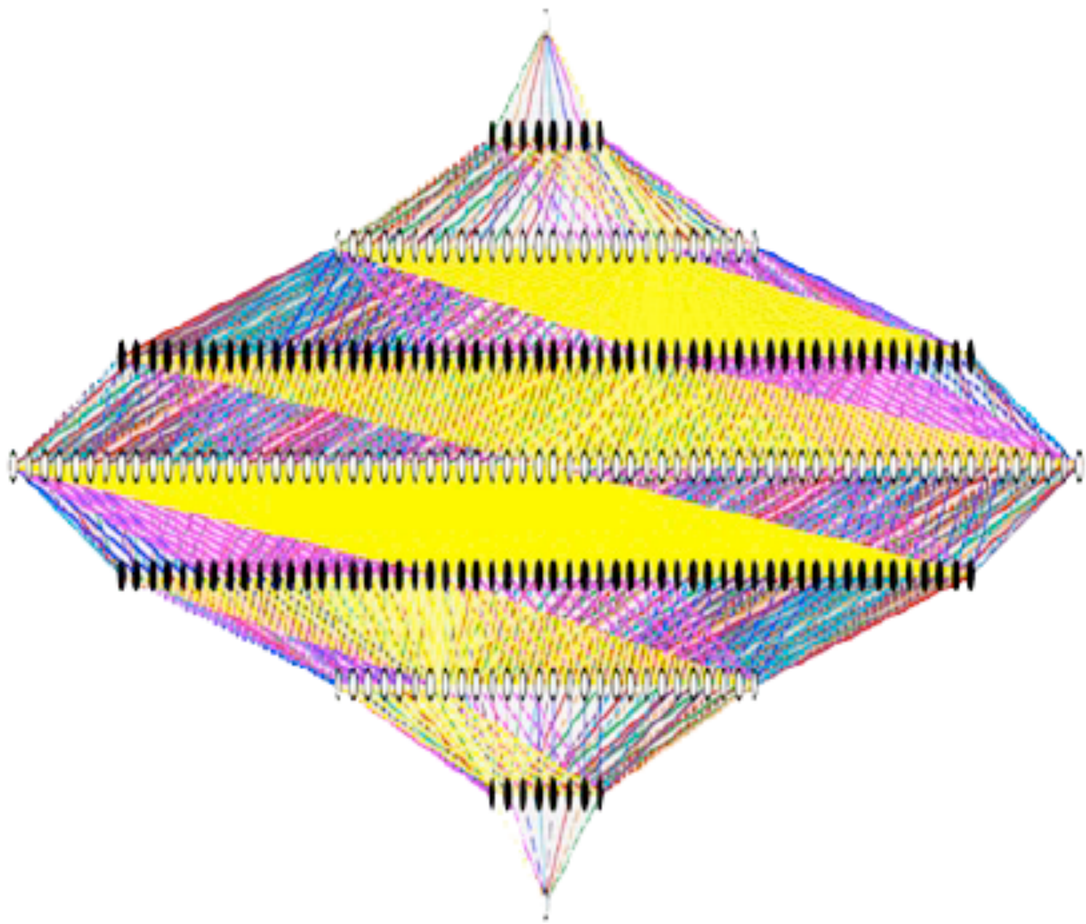
1024 like (1/2)(3a1 + 3a2 + a3 + a4 + ... + a8) with an even number of minus signs
[which correspond to **8x128** = 8 copies of the 128-dim Mirror D8 half-spinors that
are not used in the 7 E8 lattices. ...] ...".

One of the 128-dimensional Mirror D8 half-spinors from the 1024
combines with
the 128 from the 1120 corresponding to the one of the 7 E8 lattices that corresponds
to the central norm 1 240 = 112+128
and
the result is formation of a 128+128 = 256 corresponding to the Clifford Algebra Cl(8)
so that
the norm 2 second layer contains 7 copies of 256-dimensional Cl(8)

so the 2160 norm 2 vertices can be seen as

7(128+128) + 128 + 16 + 224 = 2160 vertices.

The 256 vertices of each pair 128+128 form an 8-cube with 1024 edges, 1792 square faces, 1792 cubic cells, 1120 tesseract 4-faces, 448 5-cube 5-faces, 112 6-cube 6-faces, and 16 7-cube 7-faces. The image format of African Adinkra for 256 Odu of IFA



shows Cl(8) graded structure 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 of 8-cube vertices. Physically they represent **Operators in H92 x SL(8)** Generalized Heisenberg Algebra that is the Maximal Contraction of E8:

Odd-Grade Parts of Cl(8) =
= **128 D8 half-spinors** of one of iE8, jE8, kE8, EE8, IE8, JE8, KE8
8+56 grades-1,3 = Fermion Particle 8-Component Creation (AntiParticle Annihilation)
56+8 grades-5,7 = Fermion AntiParticle 8-Component Creation (Particle Annihilation)

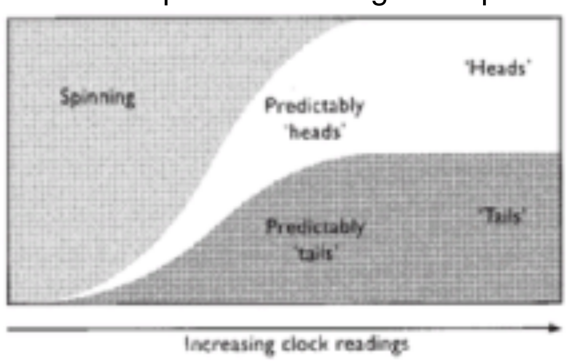
Even-Grade Subalgebra of Cl(8) = **128 Mirror D8 half-spinors** =
28 grade-2 = Gauge Boson Creation (16 for Gravity, 12 for Standard Model)
28 grade-6 = Gauge Boson Annihilation (16 for Gravity , 12 for Standard Model)
(each 28 = 24 Root Vectors + 4 of Cartan Subalgebra)
64 of grade-4 = 8-dim Position x Momentum
1+(3+3)+1 grades-0,4,8 = Primitive Idempotent:
(1+3) = Higgs Creation; (3+1) = Higgs Annihilation
= **112 D8 Root Vectors + 8 of E8 Cartan Subalgebra + 8 Higgs Operators**

3. The Third Grothendieck Universe is the Completion of Union of all tensor products of Cl(16) Real Clifford algebra

Since the Cl(16)-E8 Lagrangian is Local and Classical,
it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global Cl(16)-E8 Algebraic Quantum Field Theory (AQFT).

The usual Hyperfinite II1 von Neumann factor for creation and annihilation operators on Fermionic Fock Space over C^(2n) is constructed by completion of the union of all tensor products of 2x2 Complex Clifford algebra matrices, which have Periodicity 2,
so
for the Cl16)-E8 model based on Real Clifford Algebras with Periodicity 8,
whereby any Real Clifford Algebra, no matter how large,
can be embedded in a tensor product of factors of Cl(8) and of Cl(8)xCl(8) = Cl(16),
the completion of the union of all tensor products of Cl(16) = Cl(8)xCl(8)
produces a generalized Hyperfinite II1 von Neumann factor
that gives the Cl(16)-E8 model a natural Algebraic Quantum Field Theory.

The overall structure of Cl(160-E8 AQFT is similar to the Many-Worlds picture described by David Deutsch in his 1997 book "The Fabric of Reality" said (pages 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ... Suppose ... we toss a coin ... Each point in the diagram represents one snapshot



... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. …
in some regions of the multiverse, and in some places in space,
the snapshots of some physical objects do fall, for a period, into chains,
each of whose members determines all the others to a good approximation …".

The Real Clifford Algebra Cl(16) containing E8 for the Local Lagrangian of a Region is equivalent to a " snapshot" of the Deutsch "multiverse".
The completion of the union of all tensor products of all Cl(16)-E8 Local Lagrangian Regions forms a generalized hyperfinite II1 von Neumann factor AQFT
and emergently self-assembles into a structure = Deutsch multiverse.

For the Cl(16)-E8 model AQFT to be realistic, it must be consistent with **EPR entanglement relations**. Joy Christian in arXiv 0904.4259 said: "... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings ... The alleged non-localities ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations [such as those] among the points of a 3 or 7-sphere ... S3 and S7 ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...".
Since E8 is a Lie Group and therefore parallelizable and lives in Clifford Algebra Cl(16), **the Cl(16)-E8 model is consistent with EPR**.

The Creation-Annihilation Operator structure of Cl(16)-E8 AQFT is given by the

Maximal Contraction of E8 = semidirect product A7 x h92

where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)
The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra

28 + 64 + (SL(8,R) + 1) + 64 + 28

Central Even Grade 0 = SL(8,R) + 1

The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices, so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.

Odd Grades -1 and +1 = 64 + 64

Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.

Even Grades -2 and +2 = 28 + 28

Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int$ Standard Model Gauge Gravity + Fermion Particle-AntiParticle

8-dim SpaceTime

The Cl(16)-E8 generalized Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory is based on the Completion of the Union of all Tensor Products of the form

Cl(16) x ...(N times tensor product)... x Cl(16) = Cl(16N)

For N = 2^8 = 256 the copies of Cl(16) are on the 256 vertices of the 8-dim HyperCube

For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube

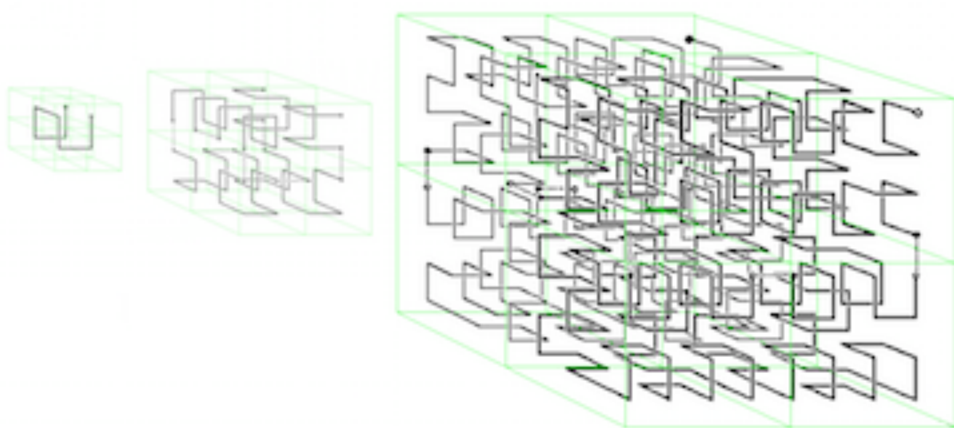
As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond

similarly to the way shown by this 3-HyperCube example for N = 2^3, 4^3, 8^3

from Wiliam Gilbert's web page:



The Union of all Cl(16) tensor products is
the Union of all subdivided 8-HyperCubes
and
their Completion is a huge superposition of 8-HyperCube Continuous Volumes
which Completion belongs to the Third Grothendieck Universe.

AQFT Possibility Space for E8 Physics must include, for each vertex of each E8 lattice, the 2^240 possible ways that each of the 240 vertices of its First Shell Root Vectors can be either 0 or 1 (off or on, inactive or active, etc).
Since 2^240 = Cl(240) = Cl(8x30) = Cl(8) x...(30 times tensor product)...x Cl(8)
the First Shell Possibility Space is the tensor product of 30 copies of Cl(8) or, equivalently since Cl(8)xCl(8) = Cl(16) which contains E8 as bivectors + half-spinors, the First Shell Possibility Space is the tensor product of 15 copies of Cl(16)

Since the Second Shell has 2160 vertices, its Possibility Space must include
Cl(2160) = Cl(8x270) = Cl(8) x...(270 times tensor product)...x Cl(8)
so the Second Shell Possibility Space has 135 copies of Cl(16).

Here (from Conway and Sloane, Sphere Packings, Lattices, and Groups, 3rd ed, Springer 1999) are

Table 4.10. The first 8 shells of E_8.

Norm	Number	Vectors
0	1	0^8.
2	240	$1^2 0^6$, $E(1/2)^8$.
4	2160	$2 0^7$, $1^4 0^4$, $D(3/2)(1/2)^7$.
6	6720	$2 1^2 0^5$, $1^6 0^2$, $E(3/2)^2(1/2)^6$.
8	17520	$2^2 0^6$, $2 1^4 0^3$, 1^8, $D(3/2)^3(1/2)^5$, $E(5/2)(1/2)^7$.
10	30240	$3 1 0^6$, $2^2 1^2 0^4$, $2 1^6 0$, $D(5/2)(3/2)(1/2)^6$, $E(3/2)^4(1/2)^4$.
12	60480	$3 1^3 0^4$, $2^3 0^5$, $2^2 1^4 0^2$, $E(5/2)(3/2)^2(1/2)^5$, $D(3/2)^5(1/2)^3$.
14	82560	$3 2 1 0^5$, $3 1^5 0^2$, $2^3 1^2 0^3$, $2^2 1^6$, $D(7/2)(1/2)^7$, $E(5/2)^2(1/2)^6$, $D(5/2)(3/2)^3(1/2)^4$, $E(3/2)^6(1/2)^2$.
16	140400	$4 0^7$, $3 2 1^3 0^3$, $3 1^7$, $2^4 0^4$, $2^3 1^4 0$, $E(7/2)(3/2)(1/2)^6$, $D(5/2)^2(3/2)(1/2)^5$, $E(5/2)(3/2)^4(1/2)^3$, $D(3/2)^7(1/2)$.

so Possibility Space for each E8 lattice contains tensor products of MANY Cl(16) copies
15 + 135 + 420 + 1,095 + 1,890 + 3,780 + 5,160 + 8,775 + ... more from beyond 8 shells

If you take the Union of all those Tensor Products of copies of Cl(16)
(each of which contains E8 = bivectors + half-spinors)
and then take the Completion of that,
you will get a generalization of the Hyperfinite II1 von Neumann factor algebra
but
even that is only 1 part (corresponding to one E8 Lattice) of the realistic AQFT of E8 Physics.

To get the full realistic Algebraic Quantum Field Theory of E8 Physics
you need
the Superposition of 8 Cl(16)-E8 generalized Hyperfinite II1 von Neumann factors:
7 for the 7 independent E8 Integral Domain Lattices
+ 1 Kirmse E8 Lattice (not Integral Domain)

4. World-Line String Bohm Quantum Potential and Quantum Consciousness

Leech Lattice and E8 Bosonic String Theory

A Single Cell of E8 26-dimensional Bosonic String Theory,
in which Strings are physically interpreted as World-Lines,
can be described by taking the quotient of
its 24-dimensional O+, O-, Ov subspace modulo the 24-dimensional Leech lattice.
Its automorphism group is the largest finite sporadic group,
the Monster Group, whose order is
8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000 =
= 2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71
or about 8 x 10^53.
A Leech lattice construction is described by Robert A. Wilson in his 2009 paper
"Octonions and the Leech lattice": "... The (real) octonion algebra is
an 8-dimensional ... algebra with an orthonomal basis { 1=ioo , i0 , i1 , i2 , i3 , i4 , i5 , i6 }
labeled by the projective line PL(7) = { oo } u F7
... **The E8 root system embeds in this algebra ... take the 240 roots to be ...
112 octonions ... +/- it +/- iu for any distinct t,u ... and ...
128 octonions (1/2)(+/- 1 +/- i0 +/- ... +/- i6) which have an odd number of minus signs.nDenote by L the lattice spanned by these 240 octonions** ...
Let s = (1/2)(- 1 + i0 + ... + i6) so s is in L ... write R for Lbar ...
(1/2) (1 + i0) L = (1/2) R (1 + i0) is closed under multiplication ... Denote this ...by A ...
Writing B = (1/2) (1 + i0) A (1 + i0) ...from ... Moufang laws ... we have
L R = 2 B , and ... B L = L and R B = R ...[also]... 2 B = L sbar
... **the roots of B are
[16 octonions]... +/- it for t in PL(7) ... together with
[112 octonions]... (1/2) (+/- 1 +/- it +/- i(t+1) +/- i(t+3)) ...for t in F7 ... and ...
[112 octonions]... (1/2) (+/- i(t+2) +/- i(t+4) +/- i(t+5) +/- i(t+6)) ...for t in F7** ...

the octonionic Leech lattice ... contains the following 196560 vectors of norm 4 ,
where M is a root of L and j,k are in J = { +/- it l t in PL(7) },
and all permutations of the three coordinates are allowed:

(2 M, 0 , 0)
(M sbar, +/- (M sbar) j , 0)
((M s) j , +/- M k , +/- (M j) k) ...

Number: 3x240 = 720 Number: 3x240 x 16 = 11520: Number: 3x240 x 16 x 16 = 184320

The key to the simple proofs above is the observation that LR = 2B and BL = L:
these remarkable facts appear not to have been noticed before ... some work ...
by Geoffrey Dixon ...".

Geoffrey Dixon says in his book "Division Algebras, Lattices, Physics, Windmill Tilting" using notation {e0,e1,e2,e3,e4,e5,e6,e7} for the Octonion basis elements that Robert A. Wilson denotes by {1=ioo,i0,i1,i2,i3,i4,i5 ,i6} and I often denote by {1,i,j,k,E,I,J,K}:
"... (spans over integers) ...

Ξeven has 16+224 = 240 elements ...

Ξodd has 112+128 = 240 elements ...

E8even does not close with respect to our given octonion multiplication ...[but]...
the set Ξeven[0-a], derived from Ξeven by replacing each occurrence of e0 ... with ea, and vice versa, is multiplicatively closed. ...".

Geoffrey Dixon's Ξeven corresponds to B
Geoffrey Dixon's **Ξeven[0-a] corresponds to the seven At**
Geoffrey Dixon's Ξodd corresponds to L

Ignoring factors like 2 , j , k , and +/-1 the Leech lattice structure is:

(L , 0 , 0)
(B , B , 0)
(L s , L , L)

(Ξodd , 0 , 0)
(Ξeven , Ξeven , 0)
(Ξodd s , Ξodd , Ξodd)

Number: 3x240 = 720 Number: 3x240 x 16 = 11520: Number: 3x240 x 16 x 16 = 184320

Dixon Octonion XY, Fibrations, and 24-dim Leech Lattice

Frank Dodd (Tony) Smith, Jr. - 2014

Geoffrey Dixon, in his book “Division Algebras, Lattices, Physics, Windmill Tilting”, defines an Octonionic XY-product and uses it to construct the $\Lambda 16$ Barnes-Wall Lattice whose S15 Unit Sphere Inner Shell contains 2x240 +16x240 = 4320 vertices.
His construction makes use of the Last Hopf Fibration S7 -> S15 -> S8
and its lattice version E8 -> $\Lambda 16$ -> Z9

Ian Porteous, in his book “Clifford Algebras and the Classical Groups”, notes that Spin(9) is transitive on S15 but Spin(10) is not transitive on S31 and constructs the non-Hopf Fibration S15 -> B24 -> S9
where S15 = Spin(9) / Spin(7) and S9 = Spin(10) / Spin(9)
and B24 = Spin(10) / Spin(7) = 24-dim orbit of 1
in the S31 Unit Sphere of the 32-dim Spinor Space of Spin(10)

Mikhail Postnikov, in “Lectures in Geometry Semester V Lie Groups and Lie Algebras”, says Lie Algebra f4 = spin(8) + Mo24 where Tangent Space of B24 = Mo24 with basis

$$Y_1(\eta)=\begin{pmatrix}0&0&0\\0&0&8\\0&-\bar{8}&0\end{pmatrix},\quad Y_2(\eta)=\begin{pmatrix}0&0&-\bar{8}\\0&0&0\\8&0&0\end{pmatrix},\quad Y_3(\eta)=\begin{pmatrix}0&8&0\\-\bar{8}&0&0\\0&0&0\end{pmatrix}.$$

where f4 inherits structure from the Real Clifford Algebra Cl(8) with grading
1 + 8 + **28** + 56 + 70 + 56 +28 + 8 + 1 = 256 = (8+8) x (8+8)

The resulting structure (here B24 = Mo24 means Mo24 is Tangent Space of B24):

$$\begin{tikzcd}
\textbf{Spin(7)} \arrow[r] \arrow[d] & \textbf{Spin(8)} \arrow[r] \arrow[d] & \textbf{S7} \\
\textbf{Spin(10)} \arrow[d] & \textbf{F4} \arrow[d] & \\
\textbf{B24} & = \begin{matrix}0&8&-\bar{8}\\-\bar{8}&0&8\\8&-\bar{8}&0\end{matrix} &
\end{tikzcd}$$

shows that B24 and Mo24, through F4 and its relationship to E8, correspond to Cl(16)-E8 Physics. (see viXra 1405.0030)

So:

f4 = spin(8) + Mo24 = **28** + (**8** + (**8**+**8**))

where Spin(9) / Spin(8) = Spin(9) / **28** = OP1 = **8** with 8-dim E8 Lattice (**8** of B24)
and F4 / Spin(9) = OP2 = (**8**+**8**) with 16-dim Barnes-Wall Lattice Λ16 (Y and Z of B24)

All this is related to E8 Physics because the Leech Lattice B24 lives in F4
and F4 lives in Cl(8) and (by Real Clifford 8-Periodicity) Cl(16) = Cl(8) x Cl(8)
and **248-dim E8 lives in Cl(16) as 120-dim bivector D8 + 128-dim D8 half-spinor**
with F4 relationship coming from:

120 = **28**x1 + 1x**28** + **8**x**8**
128 = **8**x**8** + **8**x**8**

Note that the F4 **8**+**8** which represents Fermions can be written in terms of anti-commutators as well as commutators, so the same is true of E8 **8**x**8** + **8**x**8**
(see viXra 1208.0145)

Unit Sphere Inner Shell of B24 Λ24 Leech Lattice has 196,560 vertices
based on the fibration S15 -> B24 -> S9 where S9 is Unit Sphere in 10-dim.
Conway and Sloan in "Sphere Packings, Lattices, and Groups" 3rd ed),
Table 6.4, page 180, list a series of lattices:

8-dim E8 and 9-dim E8 A1 and 10-dim E8 A2

which series terminates with E8 A2.
(analogous to Minkowski Spin(1,3) and Spin(2,3) and Conformal Spin(2,4))

Leech Inner Shell Lattice Fibration Λ16 -> Postnikov Mo24 -> E8 A2
(Λ16 contains S15 and E8 A2 contains Unit Sphere of E8 A1 and
Tangent B24 = Mo24 contains S23 Leech Lattice Unit Sphere Inner Shell)

240 from the **8** E8 Inner Shell Unit Sphere of 9-dim E8 A1 of 10-dim E8 A2 lattice

2 x 240 = 720 from the Barnes-Wall E8 Inner Shell of **8**+**8** in Λ16

16 x 240 = 3840 from the XY-product, X and Y in **8** of E8 A2

16 x 240 = 3840 from the XY-product, X and Y in **8** of Λ16
16 x 240 = 3840 from the XY-product, X and Y in **8** of Λ16

16 x (3 x 16 x 240) = 184,320 from XY-product
of things outside **8** and **8**+**8**.

now all the Leech Lattice Unit Sphere Inner Shell points are accounted for:
3x240 + 3x16x240 + 16x3x16x240 = 196560

If you look at the 2160 vertices of the second shell as expanded from the first shell,
then you see that the 2160 is made up of

112 that you might regard as a basic set of D8 root vectors plus

8 x 128 = 1024 as 8 sets of 128, each being a semi-8-hypercube that can be combined with the 112 to make the 112+128 = 240 of an E8 such that the 8 E8s that can be so formed correspond to the real 1 octonion (Kirmse) + 7 imaginary octonions (integral domains) plus

the remaining 2160 - 112 - 1024 = 1024 = 8 x 128 which are the other halves of the 8-hypercubes and so correspond to the mirror half-spinors that are NOT used in the E8s already constructed. They might in some sense be seen as the half-spinor parts of a mirror set of 8 E8s.

Given the framework of the basic D8 112 root vectors, there are 8 ways that you can fit a set of 128 into them so as to form 1 (Kirmse) + 7 (integral domain) E8 lattices.

7 E8 lattice integral domains E8i E8j E8k E8e E8ie E8je E8ke
correspond to the 7 imaginary octonions i j k e ie je ke

Scale them so that their Inner Shells have Unit Radius because as Geoffrey Dixon said "... the inner shell should ... consist of unit elements ...[since]... O multiplication of ... unit elements is closed ...".

Consider the 240-vertex Unit Radius Inner Shells of E8 Lattice Integral Domains corresponding to the algebraic generators i j e of the imaginary octonions with coordinates of the form (E8i , E8j , E8e)
(there are 3 E8s representing **8** in the 24-dim Leech Lattice)

E8i itself has 240 vertices (x , 0 , 0)
E8j itself has 240 vertices (0 , x , 0)
E8e itself has 240 vertices (0 , 0 , x)

Then, consider the 2x240 + 3840 = 4320 vertices of Unit Radius Inner Shells of Λ16 Barnes-Wall Lattices constructed from pairs of E8 Lattices using Dixon's XY-product
(there are 3 ways to choose a Barnes-Wall Λ16 representing **8**+**8** in Leech)

E8i x E8j = E8k with 16x240 = 3840 new vertices (x , y , 0)
E8i x E8e = E8ie with 16x240 = 3840 new vertices (x , 0 , y)
E8j x E8e = E8je with 16x240 = 3840 new vertices (0 , x , y)

Then, consider Unions of Unit Radius E8 Inner Shell with Unit Radius Λ16 Inner Shells,
rescaled by 1 / sqrt(2) so that the Unions have Unit Radius
(there are 3 ways to choose an E8 for (E8 + Barnes-Wall Λ16) to form Leech)

E8i x E8j x E8e x E8ie x E8je x E8k = E8ije x E8(-ijeek) = E8ke x E8ijk = E8ke x E8(-1)
= E8(-ke) = 3x16x3840 = 3 x 61,440 = 184,320 vertices (x , y , z)
where 61,440 = vertices are in second shell of Barnes-Wall Λ16

The total number of inner vertices = 3 x (240 + 3840 + 61,440) = 196,560
which is the number of inner-shell vertices of the 24-dim Leech Lattice

Ian Porteous, in his book “Clifford Algebras and the Classical Groups” Chapter 24 “Triality”, says (quoted/paraphrased):

“... The induced Clifford or spinor actions of

Spin(1)	on S0
Spin(2)	on S1
Spin(3) and Spin(4)	on S3
Spin(5), Spin(6), Spin(7) and Spin(8)	on S7
Spin(9)	on S15

are … all transitive although the Clifford action of Spin(10) on S31 is not ...

The action of $Spin(10)$ ***on*** S^{31}

All the Clifford actions on spheres discussed up until now have been transitive. By contrast, the Clifford action of $Spin(10)$ on S^{31} is not, for the isotropy subgroup at 1 at least contains a Clifford copy of $Spin(7)$ as a subgroup, from which it follows that the dimension of the orbit of 1 is at most equal to

$$\dim Spin(10) - \dim Spin(7) = 45 - 21 = 24.$$

In fact the space of orbits, assigned the quotient topology, can be shown to be homeomorphic to a closed interval of the real line, one end-point of which represents an orbit A_{21} of dimension 21, homeomorphic both to $Spin(9)/Spin(6)$ and to $Spin(10)/SU(5)$, the embedding of $Spin(6)$ in $Spin(9)$ in the former case being a Clifford one, while the other end-point represents an orbit B_{24} of dimension 24, homeomorphic to $Spin(10)/Spin(7)$, the embedding of $Spin(7)$ in $Spin(10)$ being Clifford, $Spin(7) = H_0$ being indeed the isotropy subgroup at 1. Each of the interior points of the interval represents an orbit of dimension 30, homeomorphic to

$$C_{30} = Spin(10)/Spin(6) \cong A_{21} \times S^9$$

(the embedding of $Spin(6)$ in $Spin(10)$ being Clifford).

$$\begin{array}{ccccc}
Spin(6) \cong SU(4) & = & Spin(6) & & \\
\downarrow & & \downarrow & & \\
SU(5) & \rightarrow & Spin(10) & \rightarrow & A_{21} \\
\downarrow & & \downarrow & & = \\
S^9 & \rightarrow & C_{30} & \rightarrow & A_{21}
\end{array}$$

$$\begin{array}{ccccc}
Spin(6) & = & Spin(6) & & \\
\downarrow & & \downarrow & & \\
Spin(7) & \rightarrow & Spin(10) & \rightarrow & B_{24} \\
\downarrow & & \downarrow & & = \\
S^6 & \rightarrow & C_{30} & \rightarrow & B_{24}
\end{array}
\qquad
\begin{array}{ccccc}
Spin(7) & = & Spin(7) & & \\
\downarrow & & \downarrow & & \\
Spin(9) & \rightarrow & Spin(10) & \rightarrow & S^9 \\
\downarrow & & \downarrow & & = \\
S^{15} & \rightarrow & B_{24} & \rightarrow & S^9
\end{array}$$

...".

J. F. Adams, in "Lectures on Exceptional Lie Groups", said: "...

F_4 contains $Spin(9)$ as a subgroup of maximal rank:

$$F_4 \supset Spin(9) \supset Spin(8) \supset T.$$

the roots are

$\pm x_i \pm x_j$, $1 \leq i < j \leq 4$: there are 24 of these, all long;
$\pm x_i$, $1 \leq i \leq 4$: there are 8 of these short roots;

(These 32 roots come from $Spin(9)$.)

$\frac{1}{2}(\pm x_1 \pm x_2 \pm x_3 \pm x_4)$: there are 16 of these short roots, all from Δ.

Theorem 16.7. $\quad F_4 \cong Aut(J)$

The subgroup of $Aut(J)$ fixing $e_1 = diag(1,0,0)$ is $Spin(9)$.

$$\theta(e_2 - e_3) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \lambda & x_1 \\ 0 & \bar{x}_1 & -\lambda \end{pmatrix} \quad \text{where } \lambda^2 + \bar{x}_1 x_1 = 2.$$

The subgroup of $Aut(J)$ fixing e_1, $e_2 = diag(0,1,0)$ and $e_3 = diag(0,0,1)$ is $Spin(8)$.

Its $\frac{1}{2}$-eigenspace, of dimension 16, is $V_2 + V_3$,

the space of matrices of the form $\begin{pmatrix} 0 & x_3 & \bar{x}_2 \\ \bar{x}_3 & 0 & 0 \\ x_2 & 0 & 0 \end{pmatrix}$

...".

Here are some details about Dixon's XY-product:

Geoffrey Dixon, in his book "Division Algebras, Lattices, Physics, Windmill Tilting", says:
"... use the product ...

$$\mathbf{O}^{+3}: \quad e_a e_{a+1} = e_{a+3};$$
$$(124),\ (235),\ (346),\ (457),\ (561),\ (672),\ (713),$$

the X-product could be used to generate all the 480 renumberings of the e_a, $a = 1, ..., 7$, which leave $e_0 = 1$ fixed as the identity (starting from either $\mathbf{O}^{\pm 3}$). There are 7680 renumberings of the entire collection, e_a, $a = 0, ..., 7$, and the XY-product plays exactly the same role in this context. In addition, in the X-product case the 480 renumberings arose from a pair of octonion Λ_8 lattices. The XY-product renumberings are related in a similar fashion to the pair of octonion Λ_{16} lattices.

there are 16 possible values for $\pm e_o$

$16 \times 240 = 3840$ form part of the inner shell of a Λ_{16} lattice (of radius $\sqrt{2}$).

an additional $2 \times 240 = 480$ elements for inner shell

bring the total to $3840 + 480 = 4320$ which is the kissing number of Λ_{16}.

Let $A, B, X \in \mathbf{O}$, with X a unit octonion:

$$XX^\dagger = 1 \Longrightarrow X \in S^7,$$

where S^7 is the 7-sphere in the 8-dimensional space of $\mathbf{O}$ Define

$$\boxed{A \circ_X B = (AX)(X^\dagger B) = (A(BX))X^\dagger = X((X^\dagger A)B),}$$

the X-product of A and B.

Because of the nonassociativity of $\mathbf{O}$, $A \circ_X B \neq AB$ in general.

The X-product changes $\mathbf{O}$ to $\mathbf{O}_X$, which is isomorphic to $\mathbf{O}$.
the identities of $\mathbf{O}$ and $\mathbf{O}_X$ are both $e_0 = 1$.

However, it is possible to modify the octonion product in such a way that e_0 is not the identity of the result.

In particular, define

$$\boxed{\begin{aligned} A \circ_{X,Y} B &= (AX)(Y^\dagger B) \\ &= A \circ_X ((XY^\dagger) \circ_X B) \\ &= (A \circ_Y (XY^\dagger)) \circ_Y B, \end{aligned}}$$

where as usual we assume that both $X, Y \in S^7$ (the X-product is obtained by setting $X = Y$). Let $\mathbf{O}_{X,Y}$ be $\mathbf{O}$ with this modified product.

let

$$E_a, \; a = 0, ..., 7,$$

be a basis for $\mathbf{O}_{X,Y}$, for arbitrarily chosen $X, Y \in S^7$.

$YX^\dagger = Z$ is the identity of $\mathbf{O}_{X,Y}$.

We will establish $e_a \longrightarrow E_a$, $a = 0, ..., 7$, as an isomorphism from $\mathbf{O}$ to $\mathbf{O}_{X,Y}$.

...". Lattice Fibrations he uses include

For E8:
inner shell lattice fibration D4 -> E8 -> Z5 (D4 contains S3 and Z5 contains S4 and E8 contains S7 inner shell)

For Barnes-Wall:
inner shell lattice fibration E8 -> /\16 -> Z9 (E8 contains S7 and Z9 contains S8 and /\16 contains S15 inner shell)

Spheres and Physics

Frank Dodd (Tony) Smith, Jr. - 2014

Parallelizable Spheres:

S0 = equator of S1 = RP1 (… infinite sequence of RPn … = useless Category stuff)

S1 = equator of S2 = CP1 (… infinite sequence of CPn … = useless Category stuff)

S3 = equator of S4 = HP1 (… infinite sequence of HPn … = useless Category stuff)

S7 = equator of S8 = OP1 G2 / SU(3) = S6 = equator of S7

OP2 = F4 / Spin(9)

(CxO)P2 = E6 / Spin(10) x U(1)

(HxO)P2 = E7 / Spin(12) x SU(2)

(OxO)P2 = E8 / Spin(16) (finite exceptional stuff stops here = God says is important)

Spin(16) = bivector of real Clifford Algebra Cl(16) = Cl(8) x Cl(8)

F4 = 8 + 28 + (8+8) of Cl(8) = 1+8+28+56+70+56+28+8+1 = 256 = (8+8) x (8+8)

F4 = (28 = Spin(8)) + (8 + (8+8) = E8 Lattice + /\16 Lattice = /\24 Leech)

also

F4 acts on 26-dim rep = J3(O)o = traceless part of Jordan J3(O) = /\25,1 Lorentz Leech

F4 + J3(O)o = E6

from string theory (with strings physically interpreted as World-Lines) point of view
26-dim J3(O)o = 16-dim orbifold fermions + 10-dim spacetime

10-dim spacetime = 4+6
where 4 = CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space of Kaluza-Klein
and
6 = Conformal (2,4) space over Minkowski (1,3) physical spacetime of Kaluza-Klein

Exceptional Groups and Physics

hep-th/0301050 **Pierre Ramond**

Introduce the sixteen (256×256) Dirac matrices

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab}, \qquad a, b = 1, 2 \ldots 16,$$

with vector indices transforming as the $SO(9)$ spinor(recall the anomalous Dynkin embedding). These are not to be confused with the (16×16) nine Dirac matrices which transform as $SO(9)$ vectors

$$\{\gamma^i, \gamma^j\} = 2\delta^{ij}, \qquad i, j = 1, 2 \ldots, 9.$$

Together they allow for a neat way of writing the $SO(9)$ generators

$$S^{ij} = -\frac{i}{4}\left(\gamma^{ij}\right)_{ab}\Gamma^{ab},$$

where in the usual notation $\gamma^{ij} = \gamma^i\gamma^j$, $i \neq j$, $\Gamma^{ab} = \Gamma^a\Gamma^b$, $a \neq b$. The 52 F_4 parameters split into the 36 S^{ij} which generate $SO(9)$, and sixteen $SO(9)$ spinors, T^a. Algebraic closure is given by

$$[T^a, T^b] = \frac{i}{2}\left(\gamma^{ij}\right)^a S^{ij},$$

there are three equivalent ways to embed $SO(9)$ inside F_4

This is the octonionic equivalent I-spin, U-spin and V-spin which label three equivalent ways to embed $SU(2)$ inside $SU(3)$. The F_4 Weyl chamber is 1/3 that of $SO(9)$. Take a highest weight in the F_4 Weyl chamber, λ. Let ρ be the sum of the fundamental weights. There exist two Weyl reflections C, which map λ outside the F_4 Weyl chamber, but stay inside that of $SO(9)$. Hence there is a unique way to associate one F_4 representation to three $SO(9)$ irreps. The mapping is

$$C \bullet \lambda = C(\lambda + \rho_{F_4}) - \rho_{SO(9)}.$$

This mapping associates with each F_4 irrep, a set of three $SO(9)$ representations called Euler triplets. Equality betwen its Dynkin indices is guaranteed by the character formula

$$V_\lambda \otimes S^+ - V_\lambda \otimes S^- = \sum_C sgn(C)\,\mathcal{U}_{C\bullet\lambda},$$

where V_λ is any F_4 representation written in terms of its $SO(9)$ content, $S^\pm$ are the two spinor irreps of $SO(16)$

S^+ and S^- have different Pfaffian invariants

One recognizes the "trivial" Euler triplet as

akin to an index formula for Kostant's operator associated with the coset $F_4/SO(9)$, the sixteen-dimensional projective Cayley-Moufang plane. Euler triplets are solutions of Kostant's equation

$$\not{K}\,\Psi \;\equiv\; \Gamma^a T^a \Psi \;=\; 0\,,$$

where the T^a generate the $F_4/SO(9)$ tranformations.

$$[T^a\,,T^b] \;=\; i\,f^{[ij]\,ab}\,T^{ij}\,.$$

Kostant's operator commutes with the generalized $SO(9)$ generator made up of an "orbital" and the previously defined "spin" part

$$L^{ij} \;\equiv\; T^{ij} + S^{ij}\,.$$

The solutions to Kostant's equation are the Euler triplets

The number of representations in each Euler set is the ratio of the order of the F_4 and $SO(9)$ Weyl groups. It is also the Euler number of the coset manifold, hence the name.

It is convenient to express the F_4 in terms of three sets of 26 real coordinates: u_i which transform as transverse space vectors, u_0 as scalars, and ζ_a as space spinors. This enables us to write the Euler triplets as chiral superfields of the form

$$\Phi(y^-,\vec{x}\,,\theta^\alpha) \;=\; \theta^1\theta^8\Big(h(y^-,\vec{x}\,,u_i\,,\zeta_a) + \theta^4\,\psi(y^-,\vec{x}) + \theta^4\theta^5\,A(y^-,\vec{x})\Big)\,,$$

where now the components h, ψ and A are the highest weight components of the three irreps with definite polynomial dependence on the new coordinates. For the proper spin-statistics interpretation, the twistor-like variables ζ_a must appear quadratically. It turns out that the ζ's appear in even powers only for those Euler triplets that have the same number of bosons and fermions!

hep-th/0112261 **Pierre Ramond**

sixteen (256×256) matrices, Γ^a satisfy the Dirac algebra

$$\{\Gamma^a, \Gamma^b\} = 2\delta^{ab} .$$

This leads to an elegant representation of the $SO(9)$ generators

$$S^{ij} = -\frac{i}{4}(\gamma^{ij})^{ab}\,\Gamma^a\,\Gamma^b \equiv -\frac{i}{2}f^{\,ij\,ab}\Gamma^a\,\Gamma^b .$$

The coefficients

$$f^{\,ij\,ab} \equiv \frac{1}{2}(\gamma^{ij})^{ab} ,$$

naturally appear in the commutator between the generators of $SO(9)$ and any spinor operator T^a, as

$$[\,T^{ij}\,,\,T^a\,] = \frac{i}{2}\left(\gamma^{ij}\,T\right)^a = i f^{\,ij\,ab}\,T^b .$$

the $(\gamma^{ij})^{ab}$ can also be viewed as structure constants of a Lie algebra. Manifestly antisymmetric under $a \leftrightarrow b$, they can appear in the commutator of two spinors into the $SO(9)$ generators

$$[\,T^a\,,\,T^b\,] = \frac{i}{2}(\gamma^{ij})^{ab}\,T^{ij} = f^{\,a\,b\,ij}\,T^{ij} ,$$

and one easily checks that they satisfy the Jacobi identities.

the 52 operators T^{ij} and T^a generate the exceptional Lie algebra F_4

3.3 The Kostant Operator

This character formula can be viewed as the index formula of a Dirac-like operator formed over the coset $F_4/SO(9)$. This coset is the sixteen-dimensional Cayley projective plane, over which we introduce the previously considered Clifford algebra

$$\{\Gamma^a\,,\Gamma^b\} \;=\; 2\,\delta^{ab}\,,\quad a,b=1,2,\ldots,16\;,$$

generated by (256×256) matrices. The Kostant equation is defined as

$$\not{K}\,\Psi \;=\; \sum_{a=1}^{16}\Gamma^a T^a\,\Psi \;=\; 0\;,$$

where T_a are F_4 generators not in $SO(9)$, with commutation relations

$$[T^a\,,T^b] \;=\; i\,f^{\,ab\,ij}\,T^{ij}\;.$$

Although it is taken over a compact manifold, it has non-trivial solutions. To see this, we rewrite its square as the difference of positive definite quantities,

$$\not{K}\not{K} \;=\; C^2_{F_4} - C^2_{SO(9)} + 72\;,$$

where

$$C^2_{F_4} \;=\; \frac{1}{2}T^{ij}\,T^{ij} + T^a\,T^a\;,$$

is the F_4 quadratic Casimir operator, and

$$C^2_{SO(9)} \;=\; \frac{1}{2}\left(T^{ij} - i\,f^{ab\,ij}\,\widetilde{\Gamma}^{ab}\right)^2\;,$$

is the quadratic Casimir for the sum

$$L^{ij} \;\equiv\; T^{ij} + S^{ij}\;,$$

where S^{ij} is the previously defined $SO(9)$ generator

We have also used the quadratic Casimir on the spinor representation

$$\frac{1}{2} S^{ij} S^{ij} = 72 .$$

Kostant's operator commutes with the sum of the generators,

$$[\not{K}, L^{ij}] = 0 ,$$

allowing its solutions to be labelled by $SO(9)$ quantum numbers.

The same construction of Kostant's operator applies to all equal rank embeddings

In particular we note the cases $E_6/SO(10) \times SO(2)$, with Euler number 27, $E_7/SO(12) \times SO(3)$ with Euler number 63, and $E_8/SO(16)$, where the Euler triplets contain 135 representations These cosets with dimensions 32 ,64, and 128 could be viewed as complex, quaternionic and octonionic Cayley plane

3.4 Oscillator Representation of F_4

Schwinger's celebrated representation of $SU(2)$ generators of in terms of one doublet of harmonic oscillators can be extended to other Lie algebras

The generalization involves several sets of harmonic oscillators, each spanning the fundamental representation. For example, $SU(3)$ is generated by two sets of triplet harmonic oscillators, $SU(4)$ by two quartets. In the same way, all representations of the exceptional group F_4 are generated by three sets of oscillators transforming as **26**. We label each copy of 26 oscillators as $A_0^{[\kappa]}$, $A_i^{[\kappa]}$, $i = 1, \cdots, 9$, $B_a^{[\kappa]}$, $a = 1, \cdots, 16$, and their hermitian conjugates, and where $\kappa = 1, 2, 3$. Under $SO(9)$, the $A_i^{[\kappa]}$ transform as **9**, $B_a^{[\kappa]}$ transform as **16**, and $A_0^{[\kappa]}$ is a scalar. They satisfy the commutation relations of ordinary harmonic oscillators

$$[A_i^{[\kappa]}, A_j^{[\kappa']\dagger}] = \delta_{ij}\, \delta^{[\kappa]\,[\kappa']} , \qquad [A_0^{[\kappa]}, A_0^{[\kappa']\dagger}] = \delta^{[\kappa\,\kappa']} .$$

Note that the $SO(9)$ spinor operators satisfy Bose-like commutation relations

$$[\, B_a^{[\kappa]} \,,\, B_b^{[\kappa']\,\dagger} \,] \;=\; \delta_{ab}\,\delta^{[\kappa]\,[\kappa']} \;.$$

The generators T_{ij} and T_a

$$\begin{aligned}
T_{ij} \;&=\; -i\sum_{\kappa=1}^{4}\left\{\left(A_i^{[\kappa]\dagger}A_j^{[\kappa]} - A_j^{[\kappa]\dagger}A_i^{[\kappa]}\right) + \frac{1}{2}\,B^{[\kappa]\dagger}\,\gamma_{ij}B^{[\kappa]}\right\} \;, \\
T_a \;&=\; -\frac{i}{2}\sum_{\kappa=1}^{4}\left\{(\gamma_i)^{ab}\left(A_i^{[\kappa]\dagger}B_b^{[\kappa]} - B_b^{[\kappa]\dagger}A_i^{[\kappa]}\right) - \sqrt{3}\left(B_a^{[\kappa]\dagger}A_0^{[\kappa]} - A_0^{[\kappa]\dagger}B_a^{[\kappa]}\right)\right\} \;,
\end{aligned}$$

satisfy the F_4 algebra,

$$\begin{aligned}
[\,T_{ij}\,, T_{kl}\,] \;&=\; -i\,(\delta_{jk}\,T_{il} + \delta_{il}\,T_{jk} - \delta_{ik}\,T_{jl} - \delta_{jl}\,T_{ik}) \;, \\
[\,T_{ij}\,, T_a\,] \;&=\; \frac{i}{2}\,(\gamma_{ij})_{ab}\,T_b \;, \\
[\,T_a\,, T_b\,] \;&=\; \frac{i}{2}\,(\gamma_{ij})_{ab}\,T_{ij} \;,
\end{aligned}$$

so that the structure constants are given by

$$f_{ij\,ab} \;=\; f_{ab\,ij} \;=\; \frac{1}{2}\,(\gamma_{ij})_{ab} \;.$$

The last commutator requires the Fierz-derived identity

$$\frac{1}{4}\,\theta\,\gamma^{ij}\,\theta\,\chi\,\gamma^{ij}\,\chi \;=\; 3\,\theta\,\chi\,\chi\,\theta + \theta\,\gamma^i\,\chi\,\chi\,\gamma^i\theta \;,$$

from which we deduce

$$3\,\delta^{ac}\delta^{db} + (\gamma^i)^{ac}\,(\gamma^i)^{db} - (a \leftrightarrow b) \;=\; \frac{1}{4}\,(\gamma^{ij})^{ab}\,(\gamma^{ij})^{cd} \;.$$

To satisfy these commutation relations, we have required both A_0 and B_a to obey Bose commutation relations

(Curiously, if both are anticommuting, the F_4 algebra is still satisfied).

The traceless Jordan matrices span the **26** representations of F_4. One can supplement the F_4 transformation by an additional 26 parameters, and define

$$\mathcal{D}_X\,J \;\equiv\; X \circ J \;,$$

leading to a group with 78 parameters. These extra transformations are non-compact, and close on the F_4 transformations, leading to the exceptional group $E_{6(-26)}$. The subscript in parenthesis denotes the number of non-compact minus the number of compact generators.

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

$\int$ Standard Model Gauge Gravity + Fermion Particle-AntiParticle
8-dim SpaceTime

whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes 8v = 8-dim SpaceTime Dimensions + + 8s+ = 8 Fermion Particle Types + 8s- = 8 Fermion AntiParticle Types combined in the traceless part J(3,O)o of the 3x3 Octonion Hermitian Jordan Algebra

a	8s+	8v
8s+*	b	8s-
8v*	8s-*	-a-b

which has total dimension 8v + 8s+ + 8s- + 2 = 26 and is the space of a 26D String Theory with Strings seen as World-Lines.

Slices of 8v SpaceTime are represented as D8 branes. Each D8 brane has Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8

Stack D8 branes to get SpaceTime with Strings = World-Lines
with
a and b representing
ordering of D8 brane stacks and Bohm-type Quantum Potential

Let Oct16 = discrete mutiplicative group { +/-1, +/-i, +/-j, +/-k, +/-E, +/-I, +/-J, +/-K}.
Orbifold by Oct16 the 8s+ to get 8 Fermion Particle Types
Orbifold by Oct16 the 8s- to get 8 Fermion AntiParticle Types

Gauge Bosons from 1E8 and EE8 parts of a D8 give U(2) Electroweak Force
Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give SU(3) Color Force
Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give U(2,2) Conformal Gravity

The 8x8 matrices for collective coordinates linking one D8 to the next D8
give Position x Momentum

Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)
"... For the ... closed ... bosonic string .... The first excited level ... consists of ...
the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ...
SO(24) ... little group of a ...[26-dim]... massless particle ... and ...
a ... massless ... spin two state ...".
Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analagous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The massless spin two state is what I call the Bohmion:
the carrier of the Bohm Force of the Bohm-Sarfatti Quantum Potential.
Peter R. Holland says in his book "The Quantum Theory of Motion" (Cambridge 1993)
"... the total force ... from the quantum potential ... does not ... fall off with distance ... because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms. The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its 10^{18} to 10^{19} Tubulin Dimers with math description in terms of a large Real Clifford Algebra.

First consider Superposition of States involving one tubulin
with one electron of mass m and two different position states separated by a .
The Superposition Separation Energy Difference is the gravitational energy

$$E_{electron} = G m^2 / a$$

For any single given tubulin $a = 1$ nanometer $= 10^{-7}$ cm so that for a single Electron

$$T = h / E_{electron} = (\text{Compton} / \text{Schwarzschild}) (a / c) = 10^{26} \text{ sec} = 10^{19} \text{ years}$$

Now consider the case of N Tubulin Electrons in Coherent Superposition
Jack Sarfatti defines coherence length L by $L^3 = N a^3$ so that
the Superposition Energy E_N of N superposed Conformation Electrons is

$$E_N = G M^2 / L = N^{5/3} E_{electron}$$

The decoherence time for the system of N Tubulin Electrons is

$$T_N = h / E_N = h / N^{5/3} E_{electron} = N^{-5/3} 10^{26} \text{ sec}$$

So we have the following rough approximate Decoherence Times T_N

Time T_N	Number of Involved Tubulins
10^{-5} sec	10^{18}
25×10^{-3} sec (40 Hz)	10^{16}

Quantum Resonant States in Superposition

A Quantum Resonant Consciousness (QRC) Superposition State is
a Tubulin Configuration of up to 2^64 = 10^19 Tubulins (each Tubulin = 1 qubit)
with each QRC State in the Superposition being organized
with respect to the E8 inside Cl(16) Clifford Algebras.

Each QRC State, analagous to a Possible Conscious Thought,
is represented by a Chain of Local E8-Cl(16) Deutsch-type Multiverse Snapshots
in which each Link in the Chain is
a Central Local E8-Cl(16) Multiverse Shapshot
connected to
a Past Local E8-Cl(16) Multiverse Snapshot
and
a Future Local E8-Cl(16) Multiverse Snapshot.

Since Cl(16) is 2^16 = 65,536-dimensional each Link in the QRC State Chain requires
the information of 2^16 x 2^16 x 2^16 = 2^48 Tubulin qubits.

The remaining 2^(64-48) = 2^16 = 2^6 x 2^10 = 64 x 1024 Tubulin qubits represent:
64 Links in each Chain of a Possible Conscious Thought
and
1024 Possible Conscious Thoughts in the QRC Superposition.

After Decoherence of the QRC Superposition there emerges the One Actual Thought.

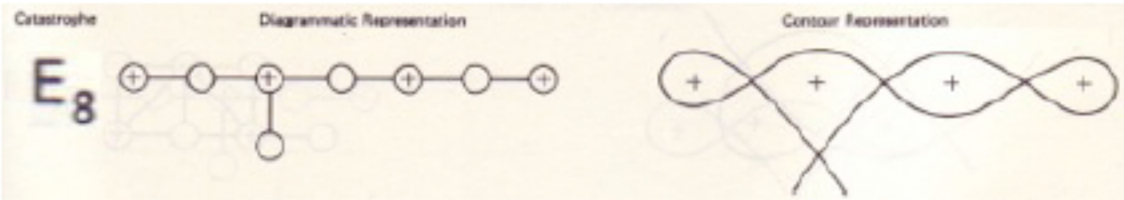
Each of the Local E8-Cl(16) Multiverse Snapshots is described by an E8 State.
Since E8 has 240 Root Vectors
and
the 240 Root Vectors correspond to the 240-Polytope (see "Geometric Frustration" by Sadoc and Mosseri (Cambridge 2006) where they say "The polytope 240 ...[is]... not a regular polytope ... but ... an ordered structure on a hypersphere ... S3 ... which is chiral ... generated by adding two replicas of the {3,3,5}, displaced along a screw axis of S3 ...".)
each Local E8-Cl(16) Multiverse Snapshot is represented by a pair of {3,3,5} 600-cells.

Each of the 600-cells has 120 vertices corresponding to the 120-dimensional Icosahedral Double (ID) group which in turn corresponds to E8 (John McKay said on usenet sci.math in 1993:
"... For each finite subgroup of SU2, we get an affine Dynkin diagram ...

```
               3
               |
E[8] 1-2-3-4-5-6-4-2
```

... The [McKay] correspondence is ...
E[8] ...[corresponds to] 2.Alt[5] = SL(2,5) binary icosahedral [ID group] ...
There are [8+1 = 9 balance numbers for E8]...
The sum of the numbers [1+2+3+4+5+6+4+2+3 = 30 is] h = Coxeter number.
The sum of the squares is the order of ...[120-element ID for E8]...
They are the periods of products of pairs of Fischer involutions mod centre ... E[8] ...[for]... Monster
... for the E8 - icosahedral ... case, the singularity is x^2+y^3+z^5=0 ...".

Robert Gilmore, in his book “Catastrophe Theory” (Dover 1981) said:
"...[The Icosahedral Double Group Catastrophe]... E8 ...[has]... Catastrophe Germ ... X^3 + Y^5 ...[with]... Perturbation ... a1 Y + a2 Y^2 + a3 Y^3 + a4 X + a5 X Y + a6 X Y^2 + a7 X Y^3



...

The germs E_6, E_8 are

$$E_6: \quad f(x, y) = x^3 + y^4$$

$$E_8: \quad f(x, y) = x^3 + y^5$$

The rules for determinacy and unfolding are particularly easy to carry out for E_6 and E_8 because both $\partial f/\partial x$ and $\partial f/\partial y$ are monomials. These calculations are summarized diagrammatically in Fig. 23.4.

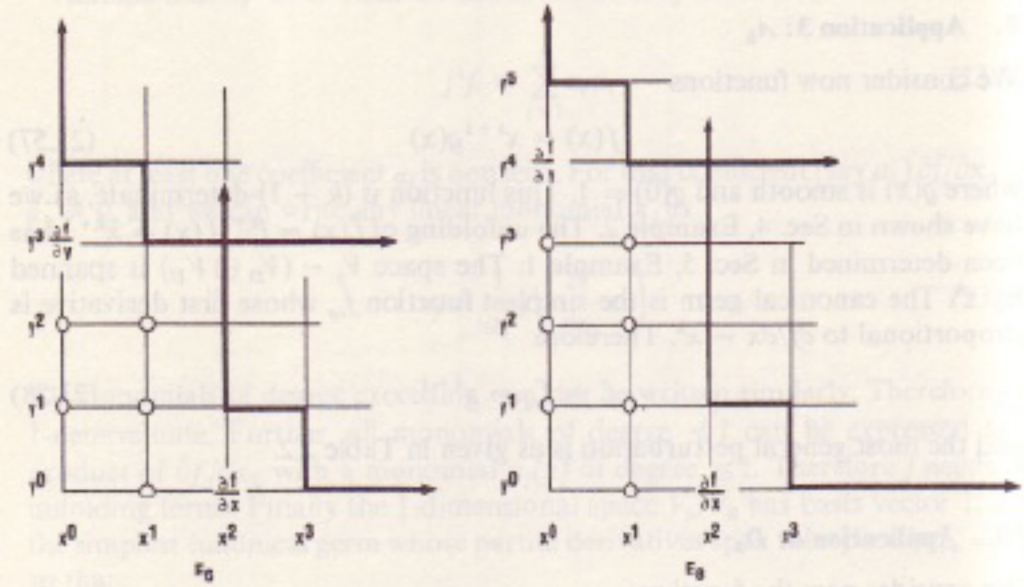


Figure 23.4 For E_6 and E_8 all monomials of degree 4 and 5 can be expressed in the form $(\partial f/\partial x_i)m_j$. The unfolding terms are represented by open circles. We exclude the constant term.

for E8 ...[with]... control parameter space R7 ...[basis {a1,a2,a3,a4,a5,a6,a7}]...
the maximum number ... of isolated critical points ..[is]... 8 ...”.

In his Appendix to Jeffrey Mishlove’s book “Roots of Consciousness”, Saul-Paul Sirag did not “exclude the constant term” as Robert Gilmore did, so, if we add a control parameter a0, we see that the ID E8 Catastrophe Control Parameter Space is R8 with basis {a0,a1,a2,a3,a4,a5,a6,a7}.
Adding two basis elements {X,Y} of ID Catastrophe Germ space whose polynomials are invariant under the Icosahedral Double Group ID results in 8+2 = 10 dimensions.

David Ford and John McKay wrote in the book "The Geometric Vein"
(Springer-Verlag 1981):
"... The columns of the character tables of ... the binary icosahedral group
...[ID Icosahedral Double Group]... of order 120
are the (suitably normalized) eigenvectors of the Cartan matrices of type ... E8 ...
[Let gr = (1/2) (- 1 - sqrt(5)) and GR = (1/2) (- 1 + sqrt(5)) and note that gr + GR = -1]
...

1	1	1	1	1	1	1	1	1
2	-2	0	-1	1	GR	gr	-gr	-GR
2	-2	0	-1	1	gr	GR	-GR	-gr
3	3	-1	0	0	-gr	-GR	-GR	-gr
3	3	-1	0	0	-GR	-gr	-gr	-GR
4	4	0	1	1	-1	-1	-1	-1
4	-4	0	1	-1	-1	-1	1	1
5	5	1	-1	-1	0	0	0	0
6	-6	0	0	0	1	1	-1	-1

...".

A Cartan matrix for E8 is

2	0	-1	0	0	0	0	0
0	2	0	-1	0	0	0	0
-1	0	2	-1	0	0	0	0
0	-1	-1	2	-1	0	0	0
0	0	0	-1	2	-1	0	0
0	0	0	0	-1	2	-1	0
0	0	0	0	0	-1	2	-1
0	0	0	0	0	0	-1	2

Note that E8 can be constructed from the representations of E6 and D8.
The grade-1 vector representation of D8 is 120-dimensional.
The half-spinor representation of D8 is 128-dimensional.
The adjoint representation of E8 is 120 + 128 = 248-dimensional.
E6 has 27-dimensional and 78-dimensional representations.

E8 Dynkin representations are:

```
                                                147,250
                                                   |
248 - 30,380 - 2,450,240 - 146,325,270 - 6,899,079,264 - 6,696,000 - 3,875
```

To construct them:
First, construct the exterior/wedge products of the E8 adjoint 248:
The grade-1 part has dimension 248.
The grade-2 part has dimension 248/\248 = 30,628.
The grade-3 part has dimension 248/\248/\248 = 2,511,496.
The grade-4 part has dimension 248/\248/\248/\248 = 153,829,130.
The grade-5 part has dimension 248/\248/\248/\248/\248 = 7,506,861,544.
Now:
Keep the grade-1 part of dimension 248.
Subtract off 248 from 248/\248 = 30,628 to get 30,380.
Subtract off 2 x 248/\248 = 2 x 30,628 from 248/\248/\248 = 2,511,496 to get 2,450,240.
Subtract off 2 x 2,511,496 and 2,450,240 and 30,628
from 248/\248/\248/\248 = 153,829,130 to get 146,325,270.
Subtract off 2 x 153,829,130 and 2 x 146,325,270 and 2 x 2,511,496 and 2,450,240 and 248
from 248/\248/\248/\248/\248 = 7,506,861,544 to get 6,899,079,264.
These are 5 of the 8 fundamental representations of E8.

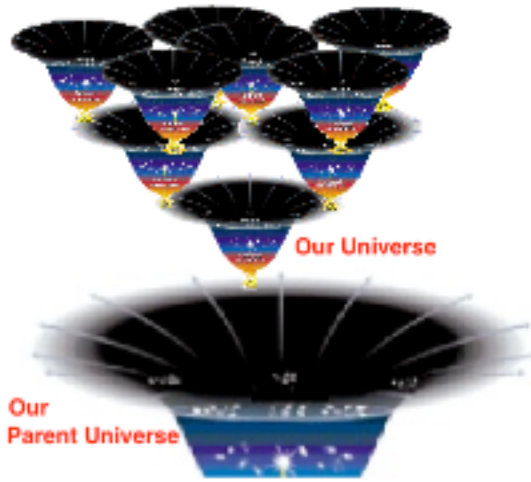
They, like the D(N) and A(N) series constructions,
are all in the same exterior algebra (of /\248),
and so can be represented as the vertices of a pentagon

What about the 6th and 7th fundamental representations of E8?
Consider the 27-dimensional E6 representation space. Add 32 copies of the 128-dimensional D8 half-spinor space, and subtract off one copy of the 248-dimensional E8 representation space
to get a 27 + 32 x 128 - 248 = 3,875-dimensional representation space.
Now, consider the antisymmetric exterior wedge algebra of that 3,875-dimensional space.
The grade-1 part has dimension 3,875. The grade-2 part has dimension 3,875/\3,875 = 7,505,875.
Now:
Keep the grade-1 part of dimension 3,875.
From the grade-2 part, subtract off 5 x 147,250 and 2 x 30,628 and 3 x 3,875 and 3 x 248
from 3,875/\3,875 = 7,505,875 to get 6,696,000. They are the 6th and 7th fundamental representations.
Since they are not in the same /\248 exterior algebra as the 5 pentagon-vertex fundamental representations of E8, they should not be vertices in the same plane as the pentagon.
However, since they are in the same /\3,875 exterior algebra, they should be collinear, one above and one below the pentagon, thus forming a pentagonal bipyramid.

What about the 8th fundamental representation of E8?
Consider 2 x 24x24 - 1 = 2 x 576 - 1 = 1,151 copies of the 128-dimensional D8 half-spinor space,
and subtract off one copy of the 78-dimensional E6 representation space
to get a representation space of dimension 1,151 x 128 - 78 = 147,328 - 78 = 147,250.
Now, consider the antisymmetric exterior wedge algebra of that 147,250-dimensional space.
The grade-1 part has dimension 147,250. It is the 8th fundamental representation of E8.
Since it is not in the same /\248 exterior algebra as the 5 pentagon-vertex fundamental representations of E8, it should not be a vertex in the same plane as the pentagon.
Also, since it is not in the same /\3,875 exterior algebra as the two bipyramid-peak-vertex fundamental representations of E8, it should not be a vertex on the same line as the pentagonal bipyramid axis.
It should represent a vertex creating a triangle whose base is one of the sides of the pentagon
and whose top is near one of the bipyramid-peak-vertices, to which it is connected by a line.
To produce a symmetric figure, the vertex must be reproduced in 5 copies,
one over each of the 5 sides of the pentagon.
Then, for the entire figure to be symmetric, it must form an icosahedron.
The binary icosahedral group {2,3,5} is of order 120.

Another way to look at it is:
The graded sequence 248 248/\248 248/\248/\248 248/\248/\248/\248 248/\248/\248/\248/\248
has symmetry Cy(5) of order 5 for cyclic permutations,but do not use Hodge duality
since 248/\248/\248/\248/\248 is fixed by its relation to 3,875/\3,875/\3,875.
The graded sequence 3,875 3,875/\3,87 3,875/\3,875/\3,875
has symmetry Cy(3) of order 3 for cyclic permutations, but do not use Hodge duality
since 3,875/\3,875/\3,875 is fixed by its relation to 248/\248/\248/\248/\248.
The graded sequence 147,250 147,250/\147,250
has symmetry Cy(2) of order 2 for cyclic permutations, but do not use Hodge duality
since 147,250/\147,250 is fixed by its relation to 248/\248/\248/\248/\248.
The +/- signs for the D5 half-spinors inherited from E6 through E7 have symmetry of order 2.
Since E8 is the sum of the 120-dimensional adjoint representation of D8
plus ONE of the 128-dimensional half-spinor representations of D8,
there is a choice to be made as to which of the two half-spinor representations of D8 are used.
As they are mirror images of each other, that choice has a symmetry of order 2.
Therefore:
the total symmetry group is of order 5x3x2x2x2 = 120,
the symmetry of the binary icosahedral group {2,3,5} corresponding by McKay to the E8 Lie Algebra.

5. Our Universe emerged from its parent in Octonionic Inflation



As Our Parent Universe expanded to a Cold Thin State Quantum Fluctuations occurred.
Most of them just appeared and disappeared as Virtual Fluctuations,
but at least one Quantum Fluctuation had enough energy to produce 64 Unfoldings
and reach Paola Zizzi's State of Decoherence
thus making it a Real Fluctuation that became Our Universe.

As Our Universe expands to a Cold Thin State, it will probably give birth
to Our Child, GrandChild, etc, Universes.

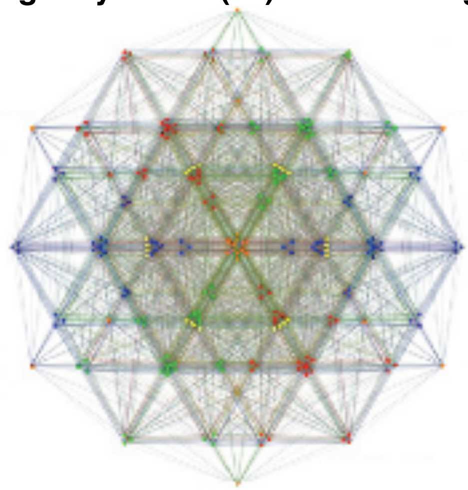
Unlike "the inflationary multiverse" decribed by Andrei Linde in arXiv 1402.0526 as
"a scientific justification of the anthropic principle",
in the Cl(16)-E8 model ALL Universes (Ours, Ancestors, Descendants)
have the SAME Physics Structure as E8 Physics (viXra 1312.0036 and 1310.0182)

In the Cl(16)-E8 model,
our SpaceTime remains Octonionic 8-dimensional throughout inflation.

Stephen L. Adler in his book Quaternionic Quantum Mechanics and Quantum Fields (1995) said at pages 50-52, 561: "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) I g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so there is a]...
failure of unitarity in octonionic quantum mechanics ...".

The NonAssociativity and Non-Unitarity of Octonions
accounts for particle creation without the need for a conventional inflaton field.

Inflation begins in Octonionic Cl(16)-E8 Physics with a Quantum Fluctuation initially containing only one Cl(16) E8 Local Lagrangian Region



The Fermion Representation Space for a Cl(16) E8 Local Lagrangian Region
is E8 / D8 = the 64+64 = 128-dim +half-spinor space 64s++ + 64s+- of Cl(16)

64s++ = 8 components of 8 Fermion Particles

64s+- = 8 components of 8 Fermion Antiparticles

By 8-Periodicity of Real Clifford Algebras Cl(16) = tensor product Cl(8) x Cl(8)
where the two copies of Cl(8) can be denoted by Cl(8)G and Cl(8)SM

(in E8 Physics Cl(8)G gives Gravity with Dark Energy and Cl(8)SM gives the Standard Model)

Cl(8)G and Cl(8)SM each have 8-dim half-spinor spaces 8Gs+ 8Gs- and 8SMs+ 8SMs-

8Gs+ and 8SMs+ representing 8 Fermion Particles

8Gs- and 8SMs- representing 8 Fermion Antiparticles

so that

64s++ = 8Gs+ x 8SMs+ for First Generation Particles of E8 Physics

64s+- = 8Gs+ x 8SMs- for First Generation AntiParticles of E8 Physics

64s-+ = 8Gs- x 8SMs+ for AntiGeneration Particles (NOT in E8 Physics)

64s-- = 8Gs- x 8SMs- for AntiGeneration AntiParticles (NOT in E8 Physics)

where

+/- half-spinor of Cl(8)G determines +/- half-spinor of Cl(16)
and Generation or AntiGeneration (only +half-spinor Generation is in E8)

+/- half-spinor of Cl(8)SM determines Particle or AntiParticle

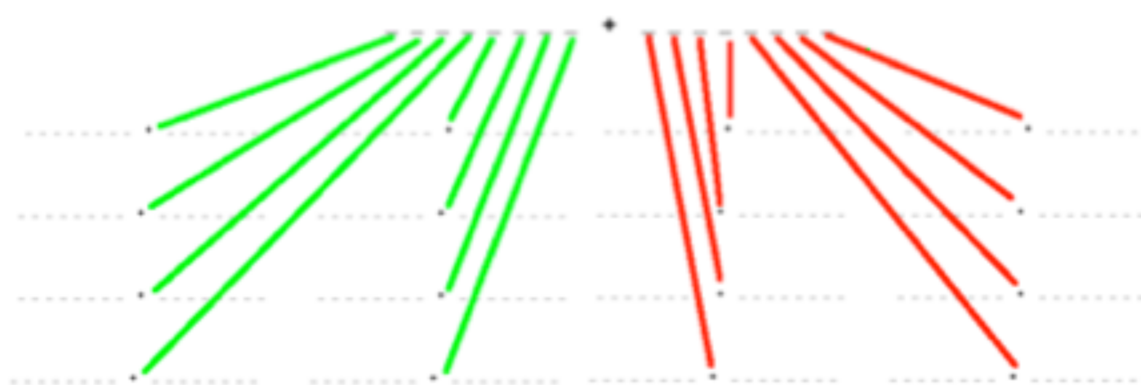
E8 Physics has Representation space for 8 Fermion Particles + 8 Fermion Antiparticles
on the original Cl(16) E8 Local Lagrangian Region that is 64s++ + 8 of 64s+- =

where a Fermion Representation slot _ of the 8+8 = 16 slots can be filled

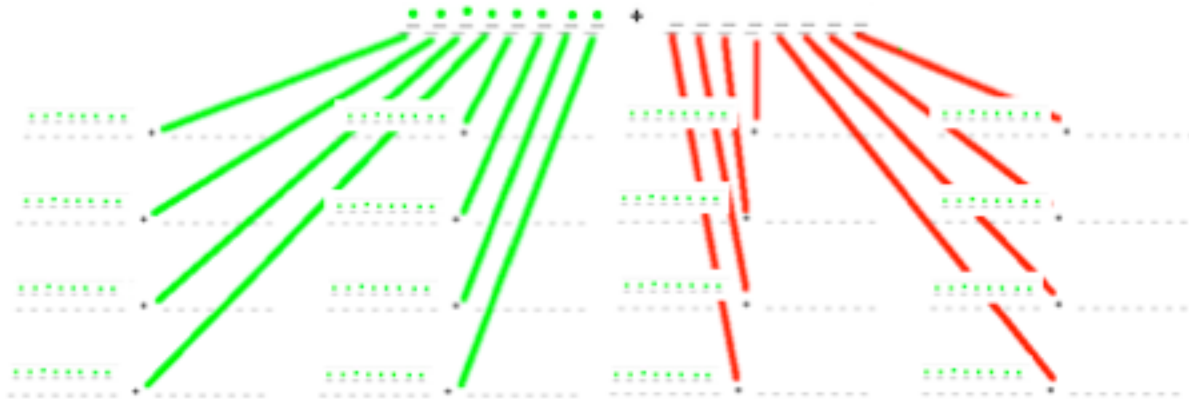
by Real Fermion Particles or Real Fermion Antiparticles
IF the Quantum Fluctuation(QF) has enough Energy to produce them as Real and
IF the Cl(16) E8 Local Lagrangian Region has an Effective Path from its QF Energy
to that Particular slot. (see Appendix III for Geoffrey Dixon's ideas and Effective Path of QF Energy)

Since E8 contains only the 128 +half-spinors and none of the 128 -half-spinors of Cl(16)
the only Effective Path of QF Energy to E8 Fermion Representation slots
goes to the only Fermion Particle slots that are also of type +
that is, to the 8 Fermion Particle Representation slots

Next, consider **the first Unfolding step of Octonionic Inflation**.It is based on
all 16 = 8 Fermion Particle slots + 8 Fermion Antiparticle Representation slots
whether or not they have been filled by QF Energy.
7 of the 8 Fermion Particle slots correspond to the 7 Imaginary Octonions and
therefore to the 7 Independent E8 Integral Domain Lattices and
therefore to 7 New Cl(16) E8 Local Lagrangian Regions.
The 8th Fermion Particle slot corresponds to the 1 Real Octonion and
therefore to the 8th E8 Integral Domain Lattice (not independent - see Kirmse's mistake) and
therefore to the 8th New Cl(16) E8 Local Lagrangian Region.
Similarly, the 8 Fermion Antiparticle slots Unfold into 8 more New New Cl(16) E8 Local
Lagrangian Regions, so that one Unfolding Step is a 16-fold multiplication
of Cl(16) E8 Local Lagrangian Regions:



If the QF Energy is sufficient, the Fermion Particle content after the first Unfolding is



so it is clear that **the Octonionic Inflation Unfolding Process creates Fermion Particles with no Antiparticles, thus explaining the dominance of Matter over AntiMatter in Our Universe**.

Each Unfolding has duration of the Planck Time Tplanck
and none of the components of the Unfolding Process Components are simultaneous,
so that **the total duration of N Unfoldings is 2^N Tplanck**.

Paola Zizzi in gr-qc/0007006 said: "... **during inflation, the universe can be described as a superposed state of quantum ... [qubits]**. the self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... **the decoherence time ... [Tdecoh = 10^9 Tplanck = 10^(-34) sec]** ... and **corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...".

Why decoherence at 64 Unfoldings = 2^64 qubits ?

2^64 qubits corresponds to the Clifford algebra Cl(64) = Cl(8x8).
By the periodicity-8 theorem of Real Clifford algebras, Cl(64) is the smallest Real Clifford algebra for which we can reflexively identify each component Cl(8) with a vector in the Cl(8) vector space. This reflexive identification/reduction causes our universe to decohere at N = 2^64 = 10^19
which is roughly the number of Quantum Consciousness Tubulins in the Human Brain.

The Real Clifford Algebra Cl(8) is the basic building block of Real Clifford Algebras due to 8-Periodicity whereby Cl(8N) = Cl(8) x ...(N times tensor product)... x Cl(8)

An Octonionic basis for the Cl(8) 8-dim vector space is {1,i,j,k,E,I,J,K}

NonAssociativity, NonUnitarity, and Reflexivity of Octonions is exemplified by the 1-1 correspondence between Octonion Basis Elements and E8 Integral Domains

1 <=> 0E8
i <=> 1E8
j <=> 2E8
k <=> 3E8
E <=> 4E8
I <=> 5E8
J <=> 6E8
K <=> 7E8

where 1E8,2E8,3E8,4E8,5E8,6E8,7E8 are 7 independent Integral Domain E8 Lattices and 0E8 is an 8th E8 Lattice (Kirmse's mistake) not closed as an Integral Domain.
Using that correspondence expands the basis {1,i,j,k,E,I,J,K} to
{0E8,1E8,2E8,3E8,4E8,5E8,6E8,7E8}

Each of the E8 Lattices has 240 nearest neighbor vectors so the total dimension of the Expanded Space is 240 x 240 x 240 x 240 x 240 x 240 x 240 x 240

Everything in the Expanded Space comes directly from the original Cl(8) 8-dim space so all Quantum States in the Expanded Space can be held in Coherent Superposition.
However,
if further expansion is attempted, there is no direct connection to original Cl(8) space and any Quantum Superposition undergoes Decoherence.

If each 240 is embedded reflexively into the 256 elements of Cl(8) the total dimension is
256 x 256 x 256 x 256 x 256 x 256 x 256 x 256 = 256^8 = 2^(8x8) = 2^64 =
= Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) = Cl(8x8) = Cl(64)
so the largest Clifford Algebra that can maintain Coherent Superposition is Cl(64) which is why Zizzi Quantum Inflation ends at the Cl(64) level.

At the end of 64 Unfoldings, Non-Unitary Octonionic Inflation ended having produced about (1/2) 16^64 = (1/2) (2^4)^64 = 2^255 = 6 x 10^76 Fermion Particles

The End of Inflation time was at about 10^(-34) sec = 2^64 Tplanck
and
the size of our Universe was then about 10^(-24) cm
which is **about the size of a Fermion Schwinger Source Kerr-Newman Cloud**.
(see viXra 1311.0088)

Calculation of CMB Fluctuation Q = 10^(-5)

Frank Dodd (Tony) Smith, Jr. - 2015 - viXra 1505.____

Frank Wilczek said, in his Physics Today (May 2006) Reference Frame:

"... The standard models contain ... numerical parameters ... of wide significance for present-day phenomena in the natural world ...
the amplitude of the primeval fluctuation spectrum. ...[is]... from cosmology ...".

Max Tegmark and Martin J. Rees said, in astro-ph/9709058 Why is the CMB Fluctuation Level 10^(-5) ?:

"... Q ... of order 10^(-5) ... is the amplitude that fluctuations in the gravitational potential have when they enter the horizon
... the baryon-to-photon ratio n = 10^(-9)
... the electromagnetic coupling constant a = e^2 = 1/137 ...".

Given Nbaryon the number of quarks is Nquark = 3 x Nbaryon.
Given Nphoton the number of geometric gravitions,
unsuppressed by factor of Planck mass squared,
is Ngeograv = 137 x Nphoton.

Nquark / Ngeograv = 3 x (Nbaryon / Nphoton) x 1/137 =
= 3 x 10^(-9) x 7 x 10^(-3) =
= 21 x 10^(-12) = 0.21 x 10^(-10)

David Layzer said, in his book Cosmogenesis (Oxford 1990):

"... random fluctuations obey well-understood statistical laws,
among them the "square-root-of-N law", which says that
if the average number ... in a sample is N , then
the random fluctuations of this number are likely to be near the square root of N ...".

Given Nquark / Ngeograv as the number of quarks per geograviton unit of spacetime
the amplitude of fluctuations of quark-matter in spacetime gravitational potential is

sqrt(Nquark / Ngeograv) = 0.45 x 10^(-5)

That is consistent with cosmological observation that Q is of the order 10^(-5).

Octonion Inflation produces Gravitational Waves that can now be observed in Polarization Patterns of the Cosmic Microwave Background.

BICEP2 in arXiv 1403.3985 said:
"... Inflation predicts ... a primordial background of ... gravitational waves ...[that]... would have imprinted a unique signature upon the CMB. **Gravitational waves induce local quadrupole anisotropies** in the radiation field within the last-scattering surface, inducing polarization in the scattered light ... **This polarization pattern will include a "curl" or ... inflationary gravitational wave (IGW) B-mode ... component**
at degree angular scales that cannot be generated primordially by density perturbations. The amplitude of this signal depends upon the **tensor-to-scalar ratio ... r = 0.20 +0.07 -0.05** ... which itself is a function of the energy scale of inflation. ...".

In the Cl(16)-E8 model,
Inflation is due to Non-Unitarity of Octonion Quantum Processes
that occur in 8-dim SpaceTime before freezing out of a preferred Quaternionic Frame ends Inflation and begins Ordinary Evolution in (4+4)-dim M4 x CP2 Kaluza-Klein.
The unit sphere in the Euclidean version of 8-dim SpaceTime (see viXra 1311.0088 for Schwinger's "unitary trick" to allow use of Euclidean SpaceTime) is the 7-sphere S7.

Curl-type B-modes (tensor) are Octonionic Quantum Processes on the surface of SpaceTime S7 which is a **7-dim NonAssociative Moufang Loop Malcev Algebra**.

(image below from Sky and Telescope)

B-modes look like   Spirals on the Surface of S7

Divergence-type E modes (scalar and tensor) are Octonionic Quantum Processes from SpaceTime S7
plus a spinor-type S7 representing Dirac Fermions living in SpaceTime
plus a 14-dim G2 Octonionic Derivation Algebra connecting the two S7 spheres
all of which is a **28-dim D4 Lie Algebra Spin(8)**.

(image below from Sky and Telescope)

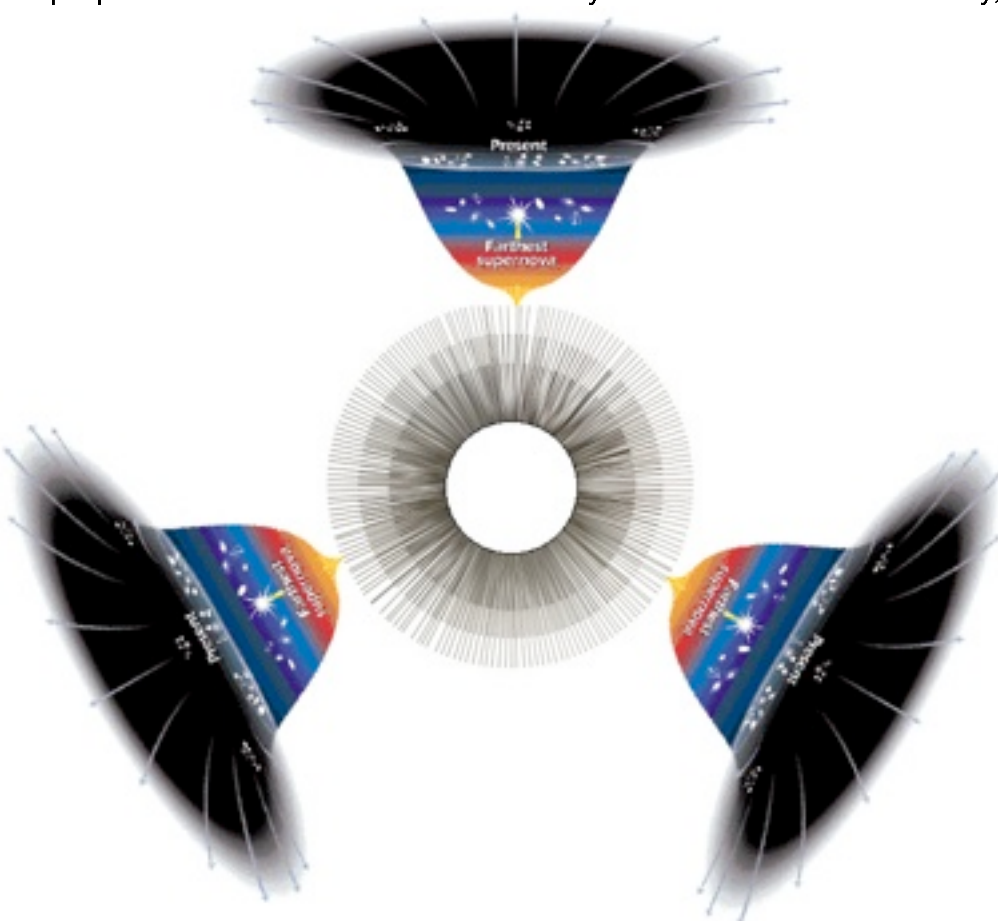
E-modes look like Fermion Pair Creation either

off (scalar)  or on (tensor)  the Surface of S7

Therefore: **for E8 Physics Octonionic Inflation the ratio r = 7 / 28 = 0.25**

End of Inflation and Low Initial Entropy in Our Universe:

Roger Penrose in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:
"... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... **the low-entropy states in the past are a puzzle**. ...".
The key to solving Penrose's Puzzle is given by Paola Zizzi in gr-qc/0007006:
"... **The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [Tdecoh = 10^9 Tplanck = 10(-34) sec] ... and corresponds to a superposed state of ... [10^19 = 2^64 qubits]**. ...
... **This is also the number of superposed tubulins-qubits in our brain ... leading to a conscious event**. ...".
The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of Quantum Theory,



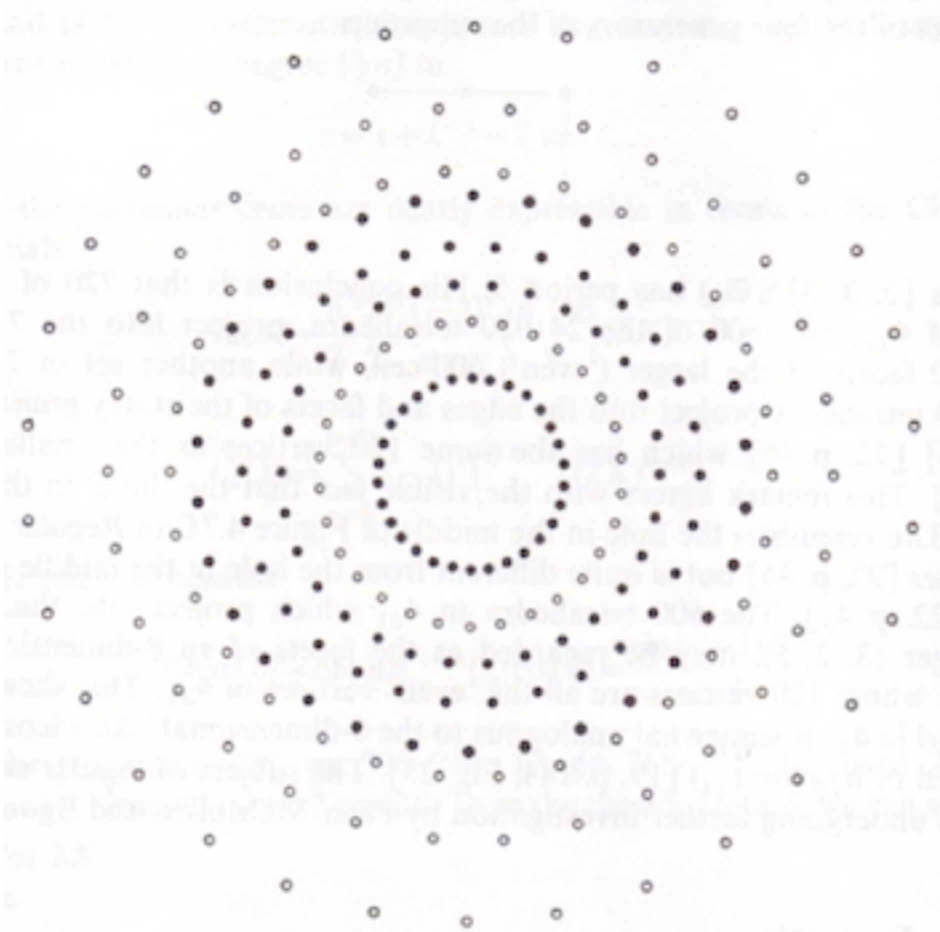
only one of which Worlds is our World. The central white circle is the Inflation Era in which everything is in Superposition; the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World.

Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

6. Quaternionic M4xCP2 Kaluza-Klein SpaceTime

At the end of Non-Unitary Octonionic Inflation Our Universe
had about (1/2) 16^64 = (1/2) (2^4)^64 = 2^255 = 6 x 10^76 Fermion Particles
The End of Inflation time was at about 10^(-34) sec = 2^64 Tplanck
and
the size of our Universe was then about 10^(-24) cm
which is about the size of a Fermion Schwinger Source Kerr-Newman Cloud
and
the Real Clifford Algebra of 8-dim SpaceTime was Cl(1,7) = Cl(0,8) = M(16,R)

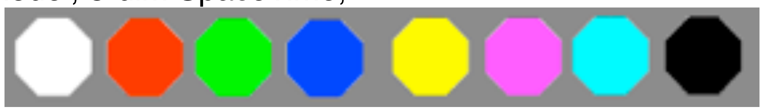
The Event that Ended Inflation was Decoherence of Zizzi Quantum Inflation that also produced decoherence of the D8 brane SpaceTime Planck-Scale Lattice superpositions of the 8 types of E8 Lattice 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8 which resulted in a decoherence choice of a particular E8 Lattice. The 240 origin-nearest-neighbor Root Vectors of such a chosen E8 Lattice can be represented as 8 circles of 30 vertices each

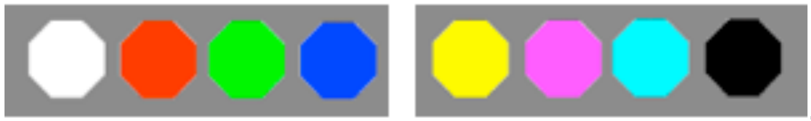


with 4x30 = 120 vertices (black dots) forming a 600-cell and
the other 4x30 = 120 vertices (white dots) forming another 600-cell at radii expanded from that of the black dots by a Golden Ratio factor. Since each 600-cell is 4-dim,
the Octonionic 8-dim E8 SpaceTime is decomposed into 2 Quaternionic 4-dim parts,

giving the Post-Inflation Cl(16)-E8 model a (4+4)-dim Kaluza-Klein SpaceTime
of the form M4 x CP2 where
M4 is 4-dim Physical Minkowski SpaceTime on which Gravity acts and
CP2 = SU(3) / U(2) is 4-dim Internal Symmetry Space for Standard Model Forces.

In the Cl(16)-E8 model, 8-dim SpaceTime,

both Octonionic 

and Quarternionic ,

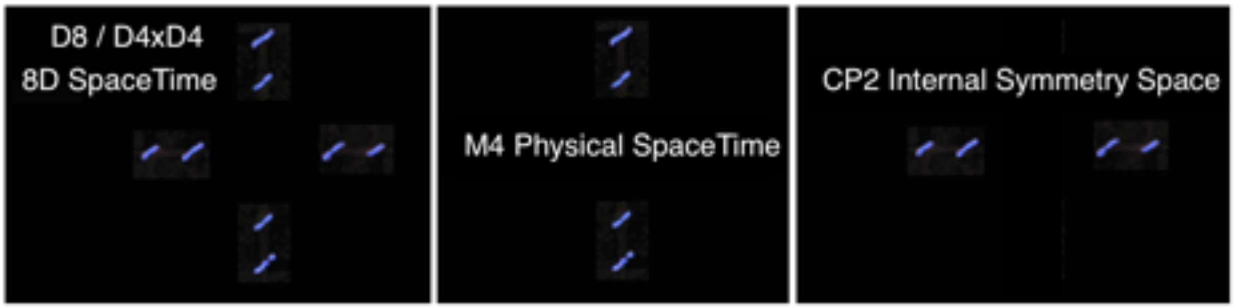
is represented by the 64-dim Adjoint D8 / D4xD4 part of E8
which is the A7 x R grade-0 part of the Maximal Contraction A7 x h92 with 5-grading
28 + 64 + (SL(8,R) + 1) + 64 + 28

In the Cl(16)-E8 model Gravity is most often written as in Chapter 18 of this paper
in terms of the MacDowell-Mansouri Conformal Group Spin(2,4) which is
the 15-dimensional Conformal BiVector Group of the 64-dim Cl(2,4) Clifford Algebra
but
it can also be written in terms of 64-dim grade-0 Maximal Contraction term SL(8,R) + 1
in which case it is known as Unimodular SL(8,R) Gravity which effectively describes
a generalized checkerboard of 8-dim SpaceTime HyperVolume Elements and,
with respect to Cl(16) = Cl(8)xCl(8), is the tensor product of the two 8v vector spaces
of the two Cl(8) factors of Cl(16). If those two Cl(8) factors are regarded
as Fourier Duals, then 8v x 8v describes Position x Momentum in 8-dim SpaceTime.

Conformal Spin(2,4) = SU(2,2) Gravity and Unimodular SL(4,R) = Spin(3,3) Gravity
seem to be effectively equivalent since, as Bradonjic and Stachel in arXiv 1110.2159
said: "... in ... Unimodular relativity ... the symmetry group of space-time is ...
the special linear group SL(4,R) ... the metric tensor ... break[s up] ... into
the conformal structure represented by a conformal metric ... with det = -1
and a four-volume element ... at each point of space-time ...[that]... may be
the remnant, in the ... continuum limit,
of a more fundamental discrete quantum structure of space-time itself ...".
Further,
Frampton, Ng, and Van Dam in J. Math. Phys. 33 (1992) 3881-3882 said:
"... Because of the existence of topologically nontrivial solutions, instantons, of the
classical field equations associated with quantum chromodynamics (QCD), the
quantized theory contains a dimensionless parameter ø (0 < ø < 2π) not explicit in the
classical lagrangian. Since ø multiplies an expression odd in CP, QCD predicts violation
of that symmetry unless the phase ø takes one of the special values ... 0 (mod π) ...
this fine tuning is the strong CP problem ... the quantum dynamics of ... unimodular
gravity ... may lead to the relaxation of ø to ø = 0 (mod π)
without the need ... for a new particle ... such as the axion ...".

End of Inflation and Quaternionic Structure

In Cl(16)-E8 Physics (vixra 1405.0030) Octonionic symmetry of 8-dim spacetime is broken at the End of Non-Unitary Octonionic Inflation to Quaternionic symmetry of (4+4)-dim Kaluza-Klein M4 x CP2 physical spacetime x internal symmetry space.



Here are some details about that transition:

The basic local entity of Cl(16)-E8 Physics is
Cl(0,16) = Cl(1,15) = Cl(4,12) = Cl(5,11) = Cl(8,8) = M(R,256) = 256x256 Real Matrices
which contains E8 with 8-dim Octonionic spacetime
and is the tensor product Cl(0,8) x Cl(0,8) = Cl(1,7) x Cl(1,7)
where Cl(0,8) = Cl(1,7) = M(R,16) is the Clifford Algebra of the 8-dim spacetime.

Non-Unitary Octonionic Inflation is based on Octonionic spacetime structure with superposition of E8 integral domain lattices. At the End of Inflation the superposition ends and Octonionic 8-dim structure is replaced by Quaternionic (4+4)-dim structure.

Since M(R,16) = M(Q,2) x M(Q,2) and M(Q,2) = Cl(1,3) = Cl(0,4)
Cl(0,8) = Cl(1,7) can be represented as Cl(1,3) x Cl(0,4)
where
Cl(1,3) is the Clifford Algebra for M4 physical spacetime
and
Cl(0,4) is the Clifford Algebra for CP2 = SU(3) / U(2) internal symmetry space
thus
making explicit the Quaternionic structure of (4+4)-dim M4 x CP2 Kaluza-Klein.

Cl(1,3) = Cl(0,4) = M(Q,2) has graded structure based on 1 2 1 grading of 2x2 matrices and 1 2 1 grading of the Quaternions, so that its total graded structure is

```
1  2  1
   2  4  2
      1  2  1
-------------
1  4  6  4  1
```

and its Spinor structure is 2x1 Quaternion matrices

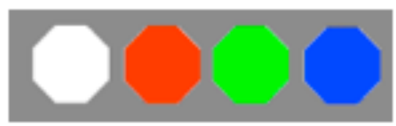
```
1 2 1
1 2 1
-----
2 4 2  =  1 2 1  +  1 2 1
```

1 2 1 = 4-dim Shilov Boundary for Lie Sphere Spin(6) / Spin(4)xU(1) =
= half-spinors for First Generation Lepton + 3 Quarks

4s+ for Electron + 3 Up Quarks
and

4s- for Neutrino + 3 Down Quarks

One copy of Cl(1,3) only has room for Particles, no AntiParticles



Cl(1,3) vectors can represent M4 physical spacetime
but

the CP2 part  of M4 x CP2 Kaluza-Klein is not directly represented by Cl(1,3).

Note that Cl(3,1) = Cl(2,2) = M(R,4) has the same Clifford Algebra dimension = 16 as does Cl(1,3) = Cl(0,4) = M(Q,2)
but
Cl(3,1) = Cl(2,2) = M(R,4) Spinors are 4x1 = 4-dimensional (Real Dirac Gammas) so physicists had to Complexify them in order to get realistic results
while
Cl(1,3) = Cl(0,4) = M(Q,2) Spinors are 2x4 = 8-dimensional and directly give the same realistic physical results of Complex Dirac Gammas.
Roughly,
Quaternification of the Clifford Algebra is like Complexification of Spinors.

Cl(0,8) = Cl(1,7) = M(R,16) = M(Q,2) x M(Q,2) has graded structure

1	4	6	4	1				
	4	16	24	16	4			
		6	24	36	24	6		
			4	16	24	16	4	
				1	4	6	4	1
1	8	28	56	70	56	28	8	1

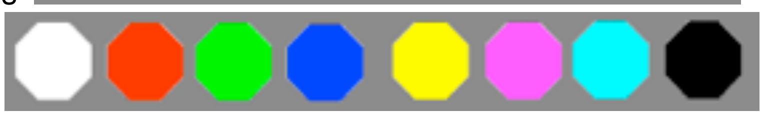
and its Spinor structure based on M(Q,2) = 16-dim is

Spinors = sqrt(16x16) = 16 = 8+8

Their Real / Octonionic M(R,16) structure is:

with 8-dim +half-spinors 

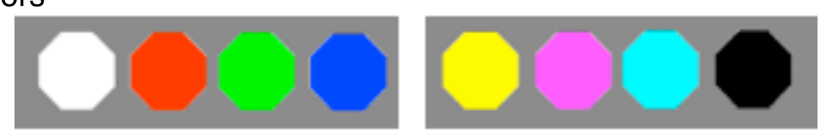
and 8-dim -half-spinors 

and 8-dim vectors 
related to each other by Triality

Their Quaternionic M(Q,2) x M(Q,2) structure is

with 2-Quaternionic +half-spinors 

and 2-Quaternionic -half-spinors 

and 2-Quaternionic vectors 
representing (4+4)-dim Kaluza-Klein M4 x CP2, related to half-spinors by Triality.

The 8-dim vectors of Cl(0,8) = Cl(1,7) correspond to B4 / D4 = OP1

Spinors = 8+8 = F4 / B4 = 52-36 = OP2

F4 = 8 + 28 + (8+8)

8 = Shilov Boundary for Lie Sphere Spin(10) / Spin(8)xU(1) =
= half-spinors for First Generation Fermion Particles / AntiParticles
8s+ for Particles and 8s- for AntiParticles
One copy of Cl(8) only has room for one Generation, no AntiGeneration
The AntiGeneration appears for Cl(16) = Cl(8) x Cl(8)
but is not in E8 which omits the AntiGeneration half-spinors of Cl(16)

Cl(0,16) = Cl(1,15) = M(R,256) = M(Q,2) x M(Q,2) x M(Q,2) x M(Q,2)
has graded structure

1	8	28	56	70	56	28	8	1
	8	64	...					
		28	...					
			...					...
1	16	120	...					

and its Spinor structure based on M(Q,2) = 16-dim is

Spinors = sqrt(16x16x16x16) = 16x16 = 256 = 128+128

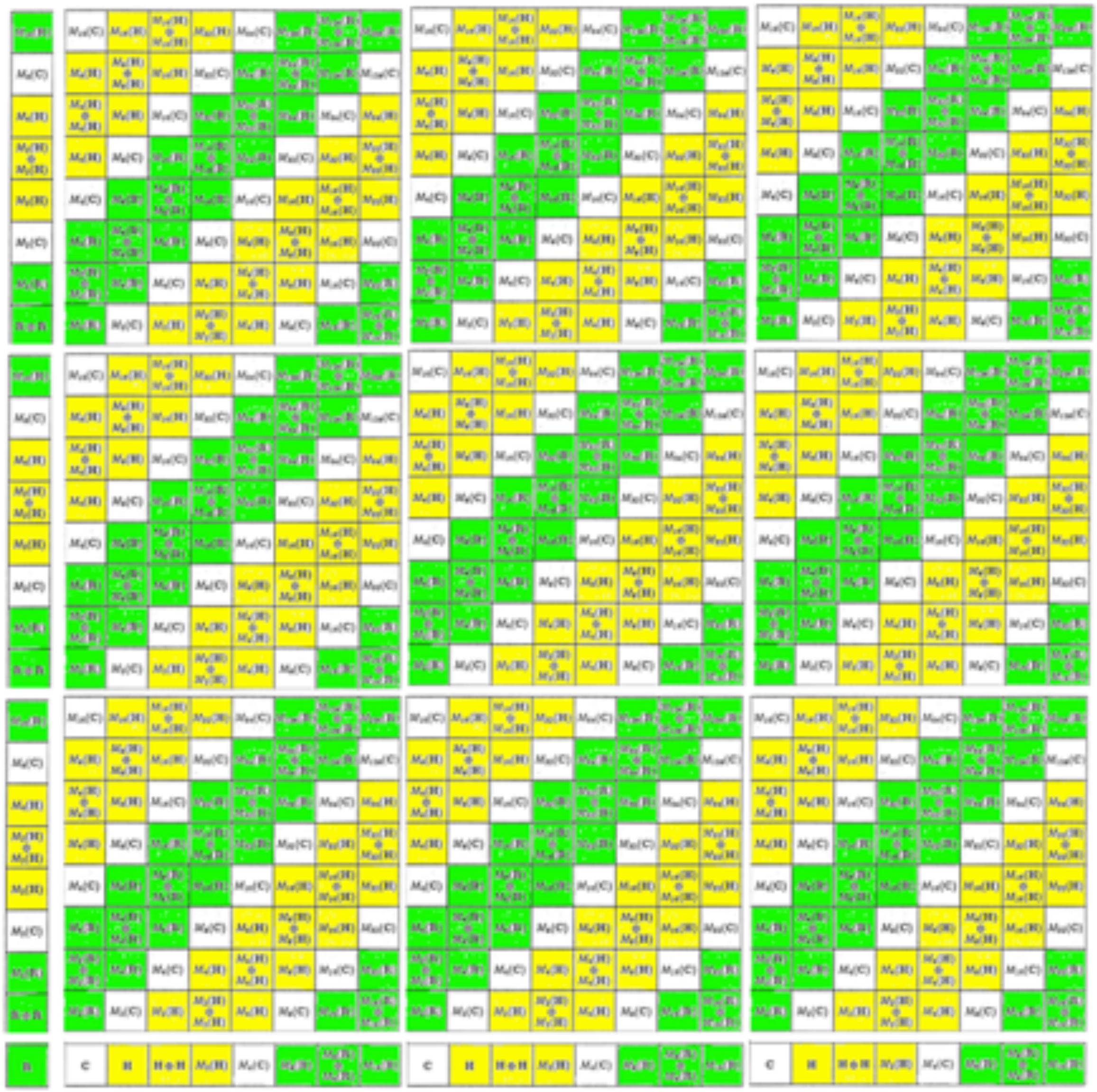
(equivalent to M(R,256) Spinors = 256x1 Real = 256 = 128 + 128)

Spinors = 128+128

E8 = 120 + 128

128 = Cl(16) half-spinors for One Generation Fermion Particles and AntiParticles

The other 128 is for One AntiGeneration that is not in E8



Quaternionic structure similar to that of Cl(1,3) = Cl(0,4) = M(Q,2) is seen in

Cl(2,4) = M(Q,4) = 4x4 Quaternion matrices with grading based on 4x4 = 1 4 6 4 1

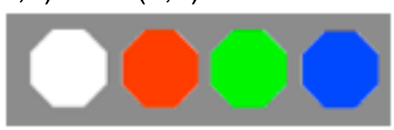
1	2	1				
	4	8	4			
		6	12	6		
			4	8	4	
				1	2	1
1	6	15	20	15	6	1

Conformal Gravity Spin(2,4) = SU(2,2) of Cl(2,4) = M(Q,4) 4x4 Quaternionic Matrices
have (4+4)x4 = 32-dim spinors
with

2-Quaternionic +half-spinors  
and

2-Quaternionic -half-spinors  

Cl(2,4) vectors are 6-dim but Spin(2,4) = SU(2,2) so the Twistor Correspondence

produces 1-Quaternionic Twistors 
that represent the M4 part of M4xCP2 Kaluza-Klein

with the CP2 part  not directly represented by Cl(2,4).

Spinors = 4x1 Quaternion 16 = 4 8 4 = 2 4 2 + 2 4 2

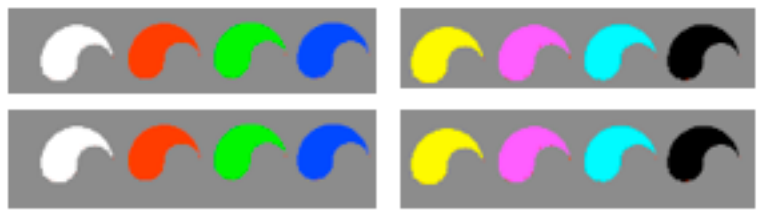
2 4 2 = 8 = Lie Sphere Spin(6) / Spin(4)xU(1) Complex Domain has Cl(1,3) half-spinor Shilov Boundary
Cl(2,4) is in some sense a (1,1) Complexification of Cl(1,3)

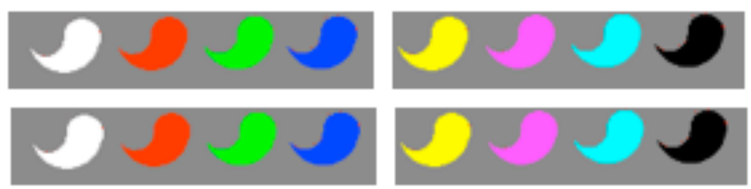
and in

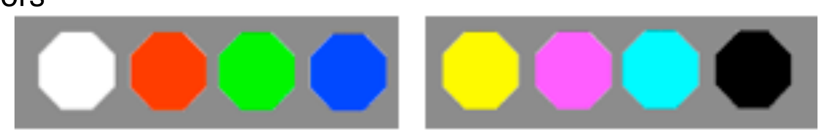
Cl(2,6) = Cl(3,5) = M(Q,8) = 8x8 Q-matrix grading based on 8x8 = 1 6 15 20 15 6 1

1	2	1						
	6	12	6					
		15	30	15				
			20	40	20			
				15	30	15		
					6	12	6	
						1	2	1
1	8	28	56	70	56	28	8	1

Quaternionic M(Q,8) 8x8 Quaternionic Matrices have (4+4)x4 = 32-dim spinors

with 4-Quaternionic +half-spinors 

and 4-Quaternionic -half-spinors 

and 2-Quaternionic vectors 
that represent the two 4-dim spaces of Kaluza-Klein M4 x CP2
The 8-dim vectors do not correspond to 16-dim D5 / D4xU(1) = (CxO)P1
If you were to expand the vectors to 16-dim you would go to Cl(16) = Cl(8)xCl(8)

Spinors = 8x1 Quaternion = E6 / D5xU(1) = 78-45-1 = 32

E6 = 15 + 30 + 32

32 = 8 16 8 = 4 8 4 + 4 8 4

4 8 4 = Lie Sphere Spin(10) / Spin(8)xU(1)

The Quaternionic half-spinors in Cl(2,6) correspond to Lie Sphere Complex Domains whereas
the half-spinors in Cl(1,7) = Cl(0,8) correspond to Shilov Boundaries
and
the E6 of Cl(2,6) is in some aspects a Complexification of the F4 of Cl(1,7) = Cl(0,8).

7. Batakis Standard Model Gauge Groups and Mayer-Trautman Higgs

The Mayer-Trautman Mechanism reduces
the Lagrangian integral over the 8-dim SpaceTime whose 8-Position x 8-Momentum is represented by 64-dim D8 / D4xD4 where D8 is the Adjoint part of E8.

$$\int_{\text{8-dim SpaceTime}} \text{Standard Model Gauge Gravity} + \text{Fermion Particle-AntiParticle}$$

to

a Lagrangian integral over the 4-dim M4 Minkowski Physical SpaceTime part of Kaluza-Klein M4 x CP2

$$\int_{\text{4-dim M4}} \text{SM GG} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

by integrating out the Lagrangian Density over the CP2 Internal Symmetry Space and so creating a new Higgs term in the Lagrangian Density integrated only over M4.

Since the D4 = U(2,2) of Gauge Gravity acts on the M4, there is no problem with it.

As to the D4 = U(4) of the Standard Model, U(4) contains as a subgroup color SU(3) which is also the global symmetry group of the CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space of M4 X CP2 Kaluza-Klein SpaceTime.

A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105 said:
"... In a standard Kaluza-Klein framework, M4 x CP2 allows
the classical unified description of an SU(3) gauge field with gravity ...
[and]
the possibility of an additional SU(2) x U(1) gauge field structure is uncovered. ...".

Since the CP2 = SU(3) / U(2) has global SU(3) action,
the SU(3) can be considered as a local gauge group acting on the M4,
so there is no problem with it.

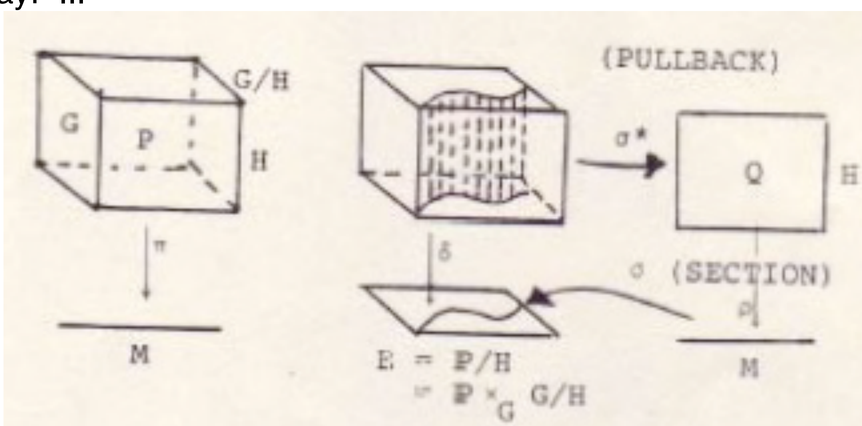
However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so **the local action of U(2) on CP2 must be integrated out to get the desired U(2) = SU(2)xU(1) local action directly on M4**.

Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4.

This is done by a mechanism due to Meinhard Mayer and A. Trautman in
“A Brief Introduction to the Geometry of Gauge Fields” and
“The Geometry of Symmetry Breaking in Gauge Theories”,
Acta Physica Austriaca, Suppl. XXIII (1981)

where they say: "...



... We start out from ... four-dimensional M [M4] ...[and]... R ...[that is]... obtained from ... G/H [CP2 = SU(3) / U(2)] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on M [M4] with values in H [SU(2)] ... the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ...
S_YM = Integral Tr (F /\ *F)
... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2] .
We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ...
Then the integrand ... becomes
Tr(F_!! F^!! + 2 F_!? F^!? + F_?? F^??)
...

The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory
...
the middle term .. becomes, symbolically,
Tr Sum D_! PHI(?) D^! PHI(?)
where PHI(?) is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect to the vector field ? , in the G/H [CP2] direction
...
the third term ... involves the contraction F_?? of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]
2 F_?? = [PHI(?) , PHI(?)] - PHI([?,?])
... Thus,
the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:
Tr F_?? F^?? = (1/4) Tr ([PHI , PHI] - PHI)^2
...
special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

See S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11: "...

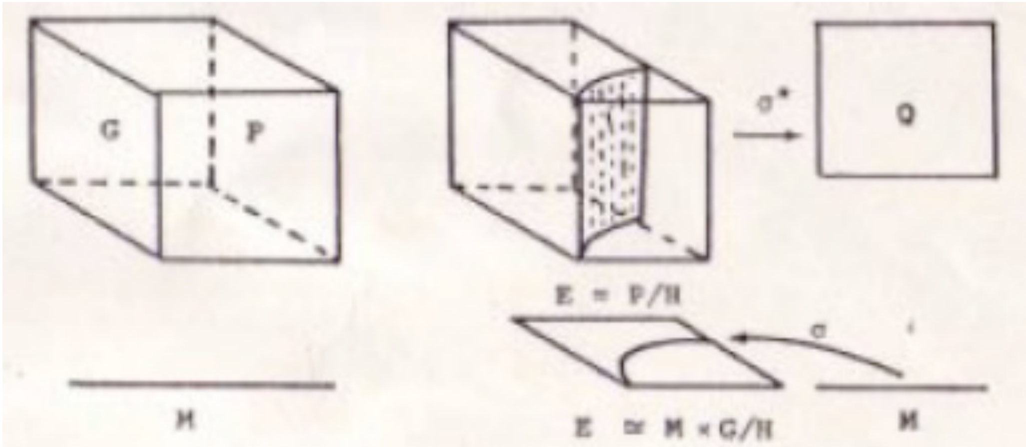
THEOREM 11.7. *Assume in Theorem* 11.5 *that* $\mathfrak{k}$ *admits a subspace* $\mathfrak{m}$ *such that* $\mathfrak{k} = \mathfrak{j} + \mathfrak{m}$ (*direct sum*) *and* $\mathrm{ad}\,(J)(\mathfrak{m}) = \mathfrak{m}$, *where* $\mathrm{ad}\,(J)$ *is the adjoint representation of* J *in* $\mathfrak{k}$. *Then* ...

The curvature form Ω *of the* K*-invariant connection defined by* $\Lambda_{\mathfrak{m}}$ *satisfies the following condition:*

$$2\Omega_{u_0}(\tilde{X}, \tilde{Y}) = [\Lambda_{\mathfrak{m}}(X), \Lambda_{\mathfrak{m}}(Y)] - \Lambda_{\mathfrak{m}}([X, Y]_{\mathfrak{m}}) - \lambda([X, Y]_{\mathfrak{j}}) \quad \textit{for } X, Y \in \mathfrak{m},$$

...".

Along the same lines, Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): "...



... each point of ... the ... fibre bundle ... E consists of a four- dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [SU(3) / U(2) = CP2] ... the components of the curvature lying in the homogeneous space G / H [= SU(3) / U(2)] could be reinterpreted as Higgs scalars (with respect to spacetime [M4]) ...
the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...".

8. 2nd and 3rd Generation Fermions

The 8 First Generation Fermion Particles
can each be represented by the 8 basis elements {1,i,j,k,E,I,J,K} of the Octonions O

1 <=> e-neutrino
i <=> red down quark
j <=> green down quark
k <=> blue down quark

E <=> electron
I <=> red up quark
J <=> green up quark
K <=> blue up quark with AntiParticles being represented similarly.

The Second and Third Generations can be represented by
Pairs of Octonions OxO and Triples of Octonions OxOxO
respectively.

When the non-unitary Octonionic 8-dim spacetime is reduced to the
Kaluza-Klein M4 x CP2 at the End of Inflation,
there are 3 possibilities for a fermion propagator from point A to point B:

1 - A and B are both in M4,
so its path can be represented by the single O;

2 - Either A or B, but not both, is in CP2,
so its path must be augmented by one projection from CP2 to M4,
which projection can be represented by a second O,
giving a second generation OxO;

3 - Both A and B are in CP2,
so its path must be augmented by two projections from CP2 to M4,
which projections can be represented by a second O and a third O,
giving a third generation OxOxO.

Combinatorics contributes to Fermion mass ratios. For example:

Blue Down Quark is 1 out of 8 and Blue Up Quark is 1 out of 8
so the Down Quark : Up Quark mass ratio is 1 : 1

Blue Strange Quark is 3 out of 8x8 = 64 and Blue Charm Quark is 17 out of 8x8 = 64
so the Strange Quark : Charm Quark mass ratio is 3 : 17

Blue Beauty Quark is 7 out of 8x8x8 = 512 and Blue Truth Quark is 161 out of 8x8x8 = 512
so the Beauty Quark : Truth Quark mass ratio is 7 : 161

9. Schwinger Sources with inherited Monster Group Symmetry have Kerr-Newman Black Hole structure size about 10^(-24) cm and Geometry of Bounded Complex Domains and Shilov boundaries

The Cl(16)-E8 model Lagrangian over 4-dim Minkowski SpaceTime M4 is

$$\int_{\text{4-dim M4}} \text{SM GG} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

Consider the Fermion Term.

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion.
The Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively themass term is, in the conventional picture, an ad hoc inclusion.

The Cl(16)-E8 model does not put in the mass m in an ad hoc way,
but constructs the Lagrangian integral such that
the mass m emerges naturally from the geometry of the spinor fermions by setting
the spinor fermion mass term as the volume of the Schwinger Source Fermions.

Effectively the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion
gives the volume of the Schwinger Source fermion and defines its mass,
which, since it is dressed with the particle/antiparticle pair cloud,
gives quark mass as constituent mass.

The Cl(16)-E8 model constructs the Lagrangian integral such that
the mass m emerges as the integral over the Schwinger Source spacetime region
of its Kerr-Newman cloud of virtual particle/antiparticle pairs plus the valence fermion
so that the volume of the Schwinger Source fermion defines its mass, which, being
dressed with the particle/antiparticle pair cloud, gives quark mass as constituent mass.

Fermion Schwinger Sources correspond to the Lie Sphere Symmetric space
Spin(10) / Spin(8)xU(1)
which has local symmetry of the Spin(8) gauge group
from which the first generation spinor fermions are formed as +half-spinor and -half-spinor spaces
and
Bounded Complex Domain D8 of type IV8 and Shilov Boundary Q8 = RP1 x S7

Consider the SM GG term from Gauge Gravity and Standard Model Gauge Bosons.
The process of breaking Octonionic 8-dim SpaceTime
down to Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein
creates differences in the way gauge bosons "see" 4-dim Physical SpaceTime

There 4 equivalence classes of 4-dimensional Riemannian Symmetric Spaces
with Quaternionic structure consistent with 4-dim Physical SpaceTime:

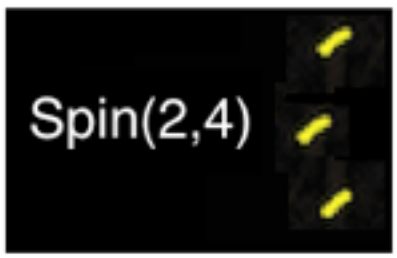
S4 = 4-sphere = Spin(5) / Spin(4) where Spin(5) = Schwinger-Euclidean version of the Anti-DeSitter subgroup of the Conformal Group that gives MacDowell-Mansouiri Gravity

CP2 = complex projective 2-space = SU(3) / U(2) with the SU(3) of the Color Force

S2 x S2 = SU(2)/U(1) x SU(2)/U(1) with two copies of the SU(2) of the Weak Force

S1 x S1 x S1 x S1 = U(1) x U(1) x U(1) x U(1) = 4 copies of the U(1) of the EM Photon
(1 copy for each of the 4 covariant components of the Photon)

The Gravity Gauge Bosons (Schwinger-Euclidean versions) live in
a Spin(5) subalgebra of the Spin(6) Conformal subalgebra of D4 = Spin(8).

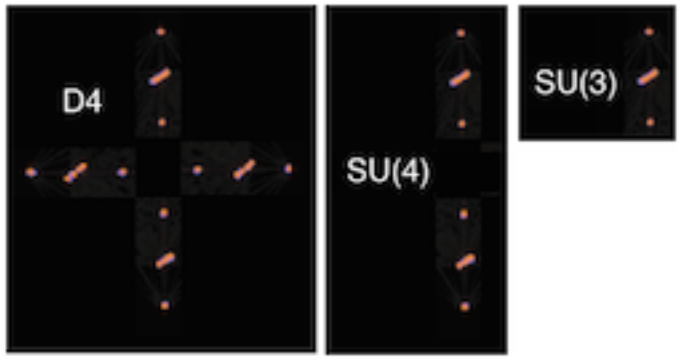


They "see" M4 Physical spacetime as the 4-sphere S4
so that their part of the Physical Lagrangian is

$$\int_{S4} \text{Gravity Gauge Boson Term} \quad .$$

an integral over SpaceTime S4.
The Schwinger Sources for GRb bosons are the Complex Bounded Domains and Shilov Boundaries for Spin(5) MacDowell-Mansouri Gravity bosons.
However, due to Stabilization of Condensate SpaceTime
by virtual Planck Mass Gravitational Black Holes,
for Gravity, the effective force strength that we see in our experiments
is not just composed of the S4 volume and the Spin(5) Schwinger Source volume,
but is suppressed by the square of the Planck Mass.
The unsuppressed Gravity force strength is the Geometric Part of the force strength.

The Standard Model SU(3) Color Force bosons live in
a SU(3) subalgebra of the SU(4) subalgebra of D4 = Spin(8).



They "see" M4 Physical spacetime as the complex projective plane CP2
so that their part of the Physical Lagrangian is

$$\int_{CP2} \text{SU(3) Color Force Gauge Boson Term} \quad .$$

an integral over SpaceTime CP2.
The Schwinger Sources for SU(3) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(3) Color Force bosons.
The Color Force Strength is given by
the SpaceTime CP2 volume and the SU(3) Schwinger Source volume.
Note that since the Schwinger Source volume is dressed with the particle/antiparticle pair cloud, the calculated force strength is
for the characteristic energy level of the Color Force (about 245 MeV).

The Standard Model SU(2) Weak Force bosons live in
a SU(2) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as two 2-spheres S2 x S2
so that their part of the Physical Lagrangian is

$$\int_{S2xS2} \text{SU(2) Weak Force Gauge Boson Term} \quad .$$

an integral over SpaceTime S2xS2.
The Schwinger Sources for SU(2) bosons are the Complex Bounded Domains and Shilov Boundaries for SU(2) Weak Force bosons.
However, due to the action of the Higgs mechanism,
for the Weak Force, the effective force strength that we see in our experiments
is not just composed of the S2xS2 volume and the SU(2) Schwinger Source volume,
but is suppressed by the square of the Weak Boson masses.
The unsuppressed Weak Force strength is the Geometric Part of the force strength.

The Standard Model U(1) Electromagnetic Force bosons (photons) live in
a U(1) subalgebra of the U(2) local group of CP2 = SU(3) / U(2)
They "see" M4 Physical spacetime as four 1-sphere circles S1xS1xS1xS1 = T4
(T4 = 4-torus) so that their part of the Physical Lagrangian is

$$\int_{T4} \text{(U(1) Electromagnetism Gauge Boson Term} \quad .$$

an integral over SpaceTime T4.
The Schwinger Sources for U(1) photons
are the Complex Bounded Domains and Shilov Boundaries for U(1) photons.
The Electromagnetic Force Strength is given by
the SpaceTime T4 volume and the U(1) Schwinger Source volume.

Schwinger Sources as described above are continuous manifold structures
of Bounded Complex Domains and their Shilov Boundaries
but
the Cl(16)-E8 model at the Planck Scale has spacetime condensing out of Clifford structures forming a Leech lattice underlying 26-dim String Theory of World-Lines
with 8 + 8 + 8 = 24-dim of fermion particles and antiparticles and of spacetime.

The automorphism group of a single 26-dim String Theory cell modulo the Leech lattice
is the Monster Group of order about 8 x 10^53.

When a fermion particle/antiparticle appears in E8 spacetime it does not remain a single
Planck-scale entity becauseTachyons create a cloud of particles/antiparticles.
The cloud is one Planck-scale Fundamental Fermion Valence Particle plus an effectively
neutral cloud of particle/antiparticle pairs forming a Kerr-Newman black hole.

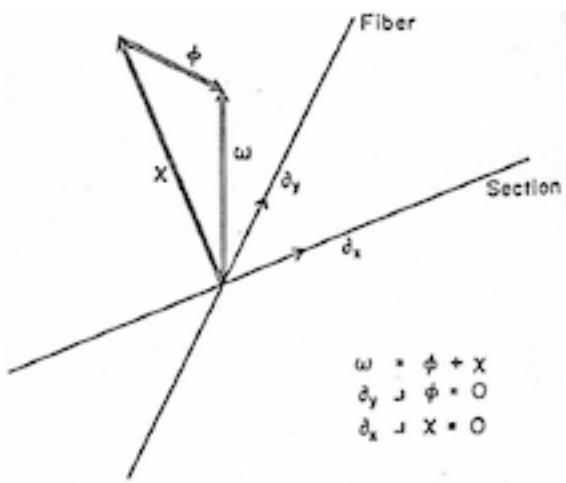
That cloud constitutes the Schwinger Source.
Its structure comes from the 24-dim Leech lattice part of the Monster Group which is
2^(1+24) times the double cover of Co1, for a total order of about 10^26.

(Since a Leech lattice is based on copies of an E8 lattice and since there are 7 distinct E8 integral domain
lattices there are 7 (or 8 if you include a non-integral domain E8 lattice)mdistinct Leech lattices.
The physical Leech lattice is a superposition of them, effectively adding a factor of 8 to the order.)

The volume of the Kerr-Newman Cloud is on the order of 10^27 x Planck scale,
so the Kerr-Newman Cloud should contain about 10^27 particle/antiparticle pairs
and its size should be about 10^(27/3) x 1.6 x 10^(-33) cm =
= roughly 10^(-24) cm.

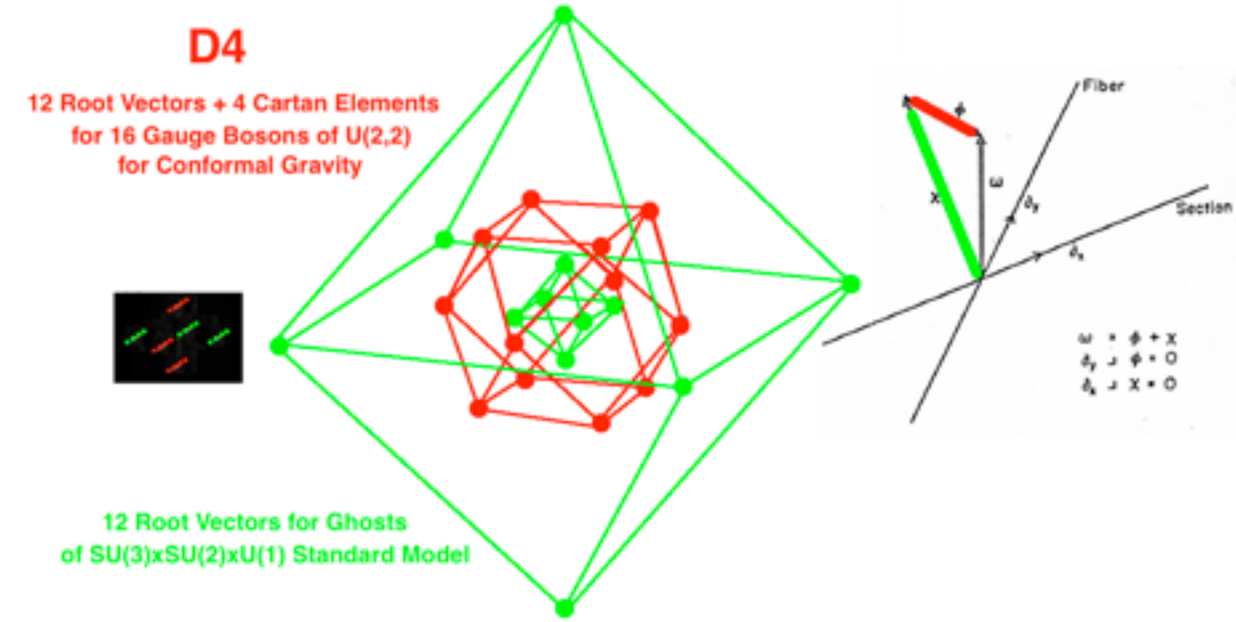
Ghosts

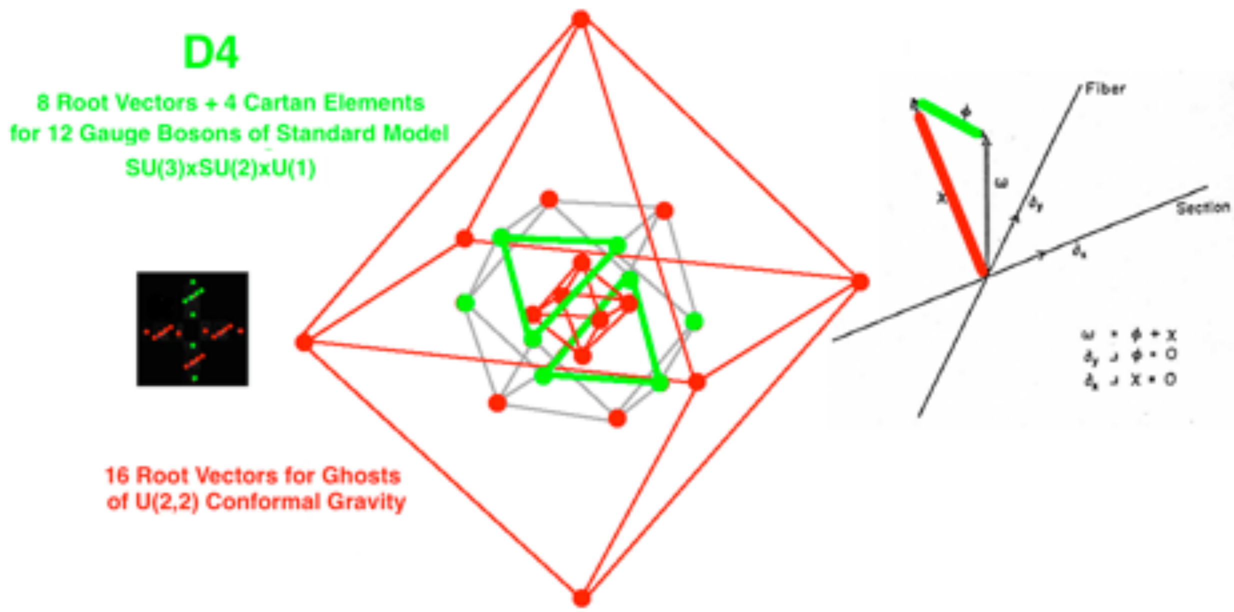
AQFT of Cl(16)-E8 Physics comes from the generalized von Neumann factor algebra constructed by completion of the union of all tensor products of Cl(16) Clifford Algebra where each Cl(16) contains E8 and a local Lagrangian constructed from E8.
The tensor product structure of Cl(16)-E8 AQFT is analogous to the sum-over-histories structure of Path Integral Quantization.
Jean Thierry-Mieg in J. Math. Phys. 21 (1980) 2834-2838 said: "... Because of gauge invariance, the classical Yang-Mills Lagrangian does not define a propagator for the gauge field. Using the path integral formulation of quantum field theory, Faddeev and Popov attributed this effect to the overcounting of gauge equivalent configurations. By fixing the gauge, Feynman diagrams are generated but unitarity is lost unless additional quantum fields are introduced: the ghost particles ...

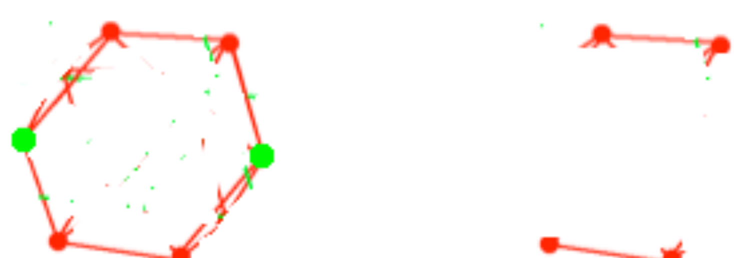


... FIG. 1. The ghost and the gauge field: The single lines represent a local coordinate system of a principal fiber bundle of base space-time. The double lines are 1 forms. The connection of the principle bundle w is assumed to be vertical. Its contravariant components PHI and X are recognized, respectively, as the Yang-Mills gauge field and the Faddeev-Popov ghost form ... By assumption, the ghost does not contribute to the description of motions tangent to the section. The exterior differential over ... the principal bundle ... of a function also splits, and its component normal to the section is recognized as the BRS operator ... the Cartan-Maurer structural theorem, which states the compatibility of the connection with the fibration, implies the BRS transformation rules of the gauge and ghost fields ... the ghost does not contribute to the curvature 2 form (field strength) and may thus be eliminated from the description of the classical theory. ... In ... the construction of the effective Lagrangian by using the generating functional ... No infinite constant has to be extracted, as the differential of the volume element of the group is actually lifted into the effective Lagrangian in the form of the ghost. The nongeometric transformation of the antighost, a Lagrange multiplier, is not recovered. However, the proof of renormalizability is not altered by the noninvariance of the effective Lagrangian, as one usually cancels the antighost variation via its equations of motion. On the contrary, the renormalized BRS operator is shown, as geometry suggests, not to act on the antighost ...".

There are two D4 in D8 in E8 in Cl(16): D4 Gravity and D4 Standard Model







SU(4) / SU(3) = 1 Cartan + CP3 = SU(2) + CP2

CP3 = Projective Twistors contains SU(2) and is Chiral **(Andrew Hodges "One to Nine")**

CP2 = SU(3) / SU(2)xU(1)

10. Fermion Mass Calculation

In the Cl(16)-E8 model, the first generation spinor fermions are seen as +half-spinor and -half-spinor spaces of Cl(1,7) = Cl(8).
Due to Triality,
Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime.

Take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:
the symmetric space Spin(10) / Spin(8)xU(1)
corresponding to a bounded domain of type IV8
whose Shilov boundary is RP^1 x S^7

Since all first generation fermions see the spacetime over which the integral is taken in the same way (unlike what happens for the force strength calculation), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.
Cl(16)-E8 model fermions correspond to Schwinger Source Kerr-Newman Black Holes, so the quark mass in the Cl(16)-E8 model is a constituent mass.

Fermion masses are calculated as a product of four factors:

V(Qfermion) x N(Graviton) x N(octonion) x Sym

V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold S^7 x RP^1 related to the fermion particle by photon, weak boson, or gluon interactions.

N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion.
The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2).
2 of them are in the Cartan subalgebra.
6 of them carry color charge, and therefore correspond to quarks.
The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons.
One graviton takes the electron into itself, and the other can only take the first-generation electron into the massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.

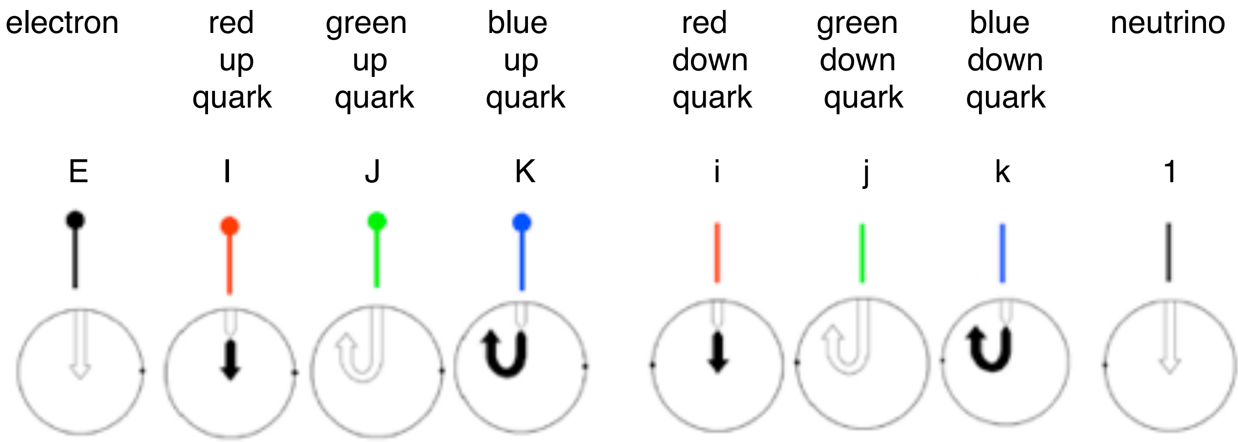
N(octonion) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.

Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

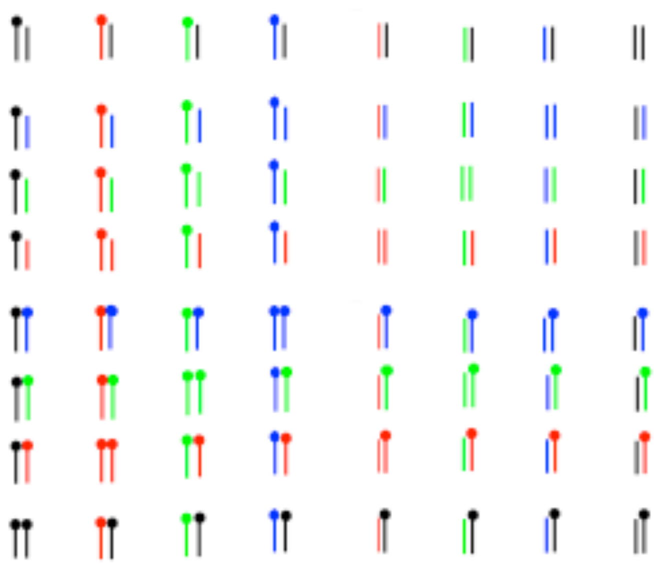
3 Generation Fermion Combinatorics

First Generation (8)

(geometric representation of Octonions is from arXiv 1010.2979)



Second Generation (64)

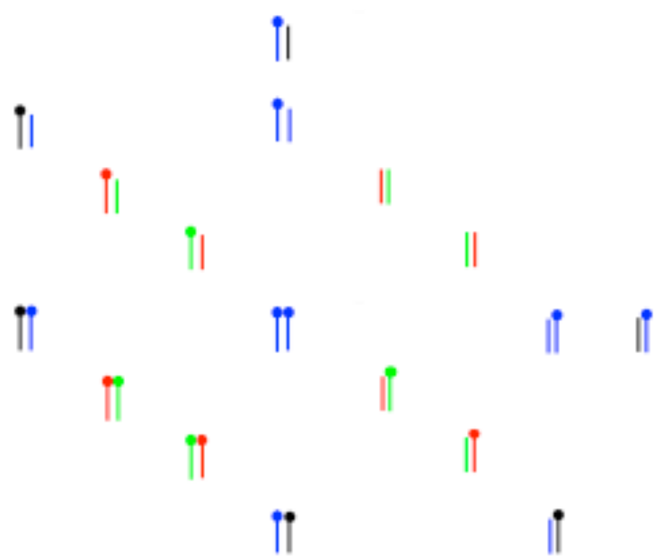


Mu Neutrino (1)
Rule: a Pair belongs to the Mu Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element) .

Muon (3)
Rule: a Pair belongs to the Muon if:
All elements are Colorless (black)
and at least one element is NonAssociative
(that is, is E which is the only Colorless NonAssociative element).

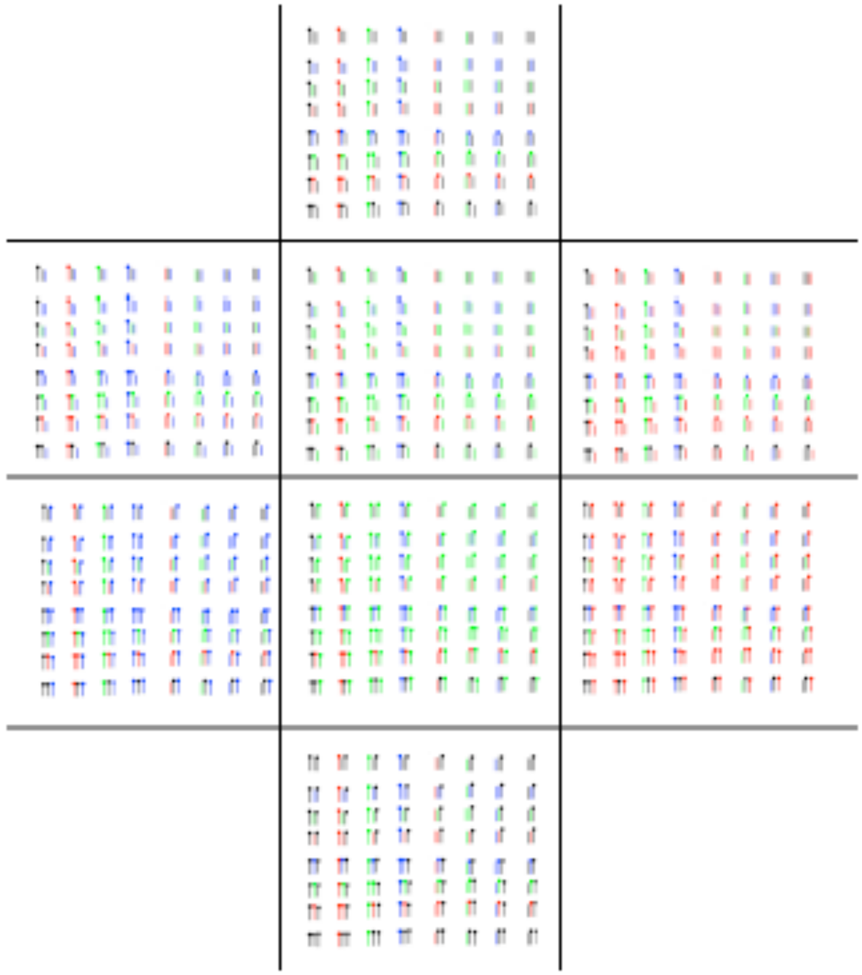
Blue Strange Quark **(3)**
Rule: a Pair belongs to the Blue Strange Quark if:
There is at least one Blue element and the other element is Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Charm Quark **(17)**
Rules: a Pair belongs to the Blue Charm Quark if:
1 - There is at least one Blue element and the other element is Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element (Red x Green = Blue).



(Red and Green Strange and Charm Quarks follow similar rules)

Third Generation (512)

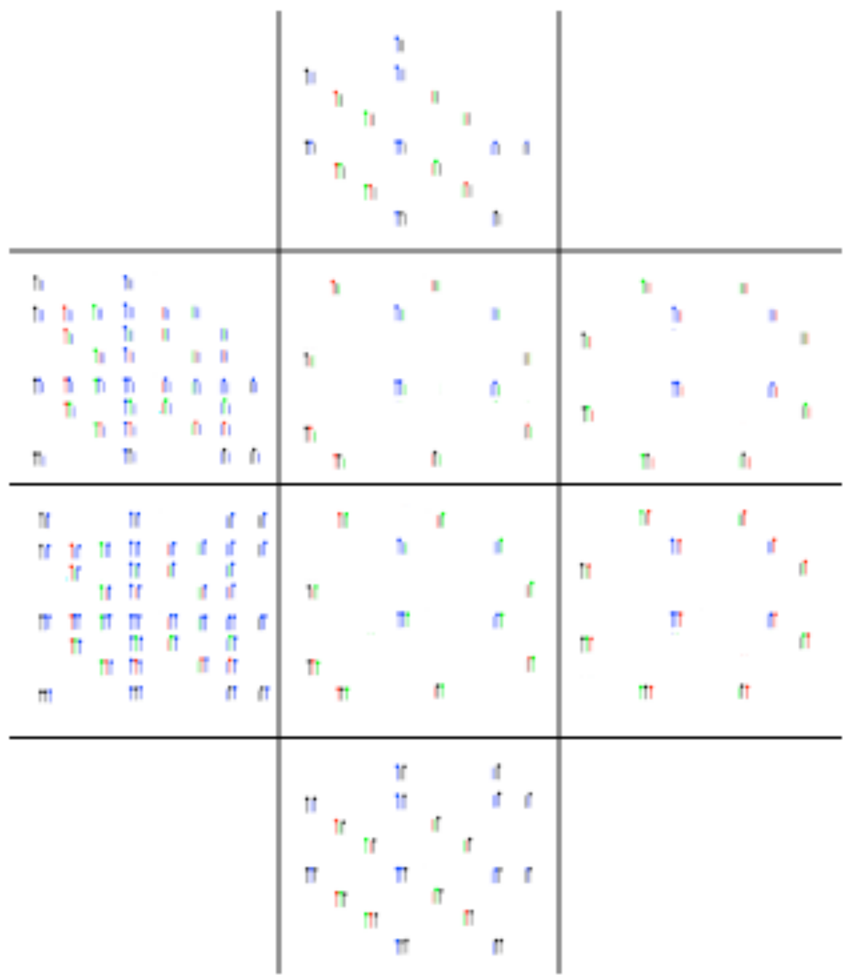


Tau Neutrino (1)
Rule: a Triple belongs to the Tau Neutrino if:
All elements are Colorless (black)
and all elements are Associative
(that is, is 1 which is the only Colorless Associative element)

Tauon (7)
Rule: a Triple belongs to the Tauon if:
All elements are Colorless (black)
and at least one element is NonAssociative (that is, is E which is the only Colorless NonAssociative element)

Blue Beauty Quark (7)
Rule: a Triple belongs to the Blue Beauty Quark if:
There is at least one Blue element and all other elements are Blue or Colorless (black)
and all elements are Associative (that is, is either 1 or i or j or k).

Blue Truth Quark (161)
Rules: a Triple belongs to the Blue Truth Quark if:
1 - There is at least one Blue element and all other elements are Blue or Colorless (black)
and at least one element is NonAssociative (that is, is either E or I or J or K)
2 - There is one Red element and one Green element and the other element is Colorless (Red x Green = Blue)
3 - The Triple has one element each that is Red, Green, or Blue,
in which case the color of the Third element (for Third Generation) is determinative
and must be Blue.



(Red and Green Beauty and Truth Quarks follow similar rules)

The first generation down quark constituent mass : electron mass ratio is:

The electron, E, can only be taken into the tree-level-massless neutrino, 1, by photon, weak boson, and gluon interactions.
The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks.
The neutrino, being massless at tree level, does not add anything to the mass formula for the electron.
Since the electron cannot be related to any other massive Dirac fermion, its volume V(Qelectron) is taken to be 1.

Next consider a red down quark i.
By gluon interactions, i can be taken into j and k, the blue and green down quarks.
By also using weak boson interactions,
it can also be taken into I, J, and K, the red, blue, and green up quarks.
Given the up and down quarks, pions can be formed from quark-antiquark pairs,
and the pions can decay to produce electrons and neutrinos.
Therefore the red down quark (similarly, any down quark)
is related to all parts of S^7 x RP^1,
the compact manifold corresponding to { 1, i, j, k, E, I, J, K }
and therefore
a down quark should have
a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.

The ratio of the down quark spinor manifold volume factor
to the electron spinor manifold volume factor is

V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md / me = 6 V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to I, J, and K, which are the octonion transforms under E of i, j, and k of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.
Since the model only gives ratios of masses,
the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,
and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:

Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

which is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions. There are 8^3 = 512 such triples.

The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and E correspond to the tauon:

{ E, E, E }
{ E, E, 1 }
{ E, 1, E }
{ 1, E, E }
{ 1, 1, E }
{ 1, E, 1 }
{ E, 1, 1 }

The symmetry of the 7 tauon triples is the same
as the symmetry of the first generation tree-level-massive fermions,
3 down, quarks, the 3 up quarks, and the electron,
so by the Sym factor the tauon mass should be the same as
the sum of the masses of the first generation massive fermion particles.

Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the tauon mass is about 2 GeV,
the effective tauon mass should be renormalized
from the energy level of 1 GeV at which the mass is 1.88 GeV
to the energy level of 2 GeV.
Such a renormalization should reduce the mass.

If the renormalization reduction were about 5 percent,
the effective tauon mass at 2 GeV would be about 1.78 GeV.
The 1996 Particle Data Group Review of Particle Physics gives a tauon mass of 1.777 GeV.

All triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.
They are triples of the same form as the 7 tauon triples involving 1 and E,
but for 1 and I, 1 and J, and 1 and K,
which correspond to the red, green, and blue beauty quarks,
respectively.

The seven red beauty quark triples correspond to the seven tauon triples,
except that
the beauty quark interacts with 6 Spin(0,5) gravitons
while the tauon interacts with only two.

The red beauty quark constituent mass should be the tauon mass times
the third generation graviton factor 6/2 = 3,
so the red beauty quark mass is mb = 5.63111 GeV.

The blue and green beauty quarks are similarly determined to also be 5.63111 GeV.

The calculated beauty quark mass of 5.63 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.
Therefore, the calculated beauty quark mass of 5.63 GeV
corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives
a lattice gauge theory beauty quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass
if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 5 GeV is about 0.22.

Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV
gives an MSbar 1-loop beauty quark mass of 4.6 GeV,
and
an MSbar 1,2-loop beauty quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV,
the MSbar mass decreases by about 1.3 GeV,
giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark
and found a 90 GeV MSbar beauty quark mass of about 2.67 GeV,
with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

The theoretical model calculated Beauty Quark mass of 5.63 GeV
corresponds to a pole mass of 5.32 GeV,
which is somewhat higher than the conventional value of 5.0 GeV.

However, the theoretical model calculated value
of the color force strength constant alpha_s at about 5 GeV is about 0.166,
while the conventional value
of the color force strength constant alpha_s at about 5 GeV is about 0.216,
and
the theoretical model calculated value
of the color force strength constant alpha_s at about 90 GeV is about 0.106,
while the conventional value
of the color force strength constant alpha_s at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that
is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV),
and a color force strength alpha_s at 5 GeV (0.166)
such that 1 + alpha_s = 1.166 is about 4 percent lower
than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Triples of the type { 1, I, J } , { I, J, K }, etc.,
do not correspond to the beauty quark, but to the truth quark.
The truth quark corresponds to those 512 - 1 - 7 - 21 = 483 triples,
so the constituent mass of the red truth quark
is 161 / 7 = 23 times the red beauty quark mass,
and the red T-quark mass is
mt = 129.5155 GeV

The blue and green truth quarks are similarly determined to also be 129.5155 GeV.

This is the value of the Low Mass State of the Truth calculated in the Cl(16)_E8 model.
The Middle Mass State of the Truth Quark has been observed by Fermilab since 1994.
The Low and High Mass States of the Truth Quark have, in my opinion, also been observed by Fermilab (see Chapter 17 of this paper) but the Fermilab and CERN establishments disagree.

All other masses than the electron mass
(which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value v = 252.514 GeV),
including the Higgs scalar mass and Truth quark mass,
are calculated (not assumed) masses in the Cl(16)-E8 model.
These results when added up give a total mass of third generation fermion particles:

Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.
There are 8^2 = 64 such pairs.

The pair { 1,1 } corresponds to the mu-neutrino.

The pairs { 1, E }, { E, 1 }, and { E, E } correspond to the muon.

For the Sym factor, compare the symmetries of the muon pairs
to the symmetries of the first generation fermion particles:
The pair { E, E } should correspond to the E electron.
The other two muon pairs have a symmetry group S2,
which is 1/3 the size of the color symmetry group S3
which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of
the { E, E } electron mass and
the { 1, E }, { E, 1 } symmetry mass, which is 1/3 of the up or down quark mass.
Therefore, mmu = 104.76 MeV .

According to the 1998 Review of Particle Physics of the Particle Data Group,
the experimental muon mass is about 105.66 MeV which may be consistent with
radiative corrections for the calculated tree-level mmu = 104.76 MeV as
Bailin and Love, in "Introduction to Gauge Field Theory", IOP (rev ed 1993), say:
"... considering the order alpha radiative corrections to muon decay ... Numerical details are contained in Sirlin ... 1980 Phys. Rev. D 22 971 ... who concludes that the order alpha corrections have the effect of increasing the decay rate about 7% compared with the tree graph prediction ...". Since the decay rate is proportional to mmu^5 the corresponding effective increase in muon mass would be about 1.36%, which would bring 104.8 MeV up to about 106.2 MeV.

All pairs corresponding to the muon and the mu-neutrino are colorless.

The red, blue and green strange quark each corresponds
to the 3 pairs involving 1 and i, j, or k.

The red strange quark is defined as the three pairs { 1, i }, { i, 1 }, { i, i }
because i is the red down quark.
Its mass should be the sum of two parts:
the { i, i } red down quark mass, 312.75 MeV, and
the product of the symmetry part of the muon mass, 104.25 MeV,
times the graviton factor.

Unlike the first generation situation,
massive second and third generation leptons can be taken,
by both of the colorless gravitons that may carry electric charge,
into massive particles.

Therefore the graviton factor for the second and third generations is 6/2 = 3.

So the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and
the red strange quark constituent mass is ms = 312.75 MeV + 312.75 MeV = 625.5 MeV

The blue strange quarks correspond to the three pairs involving j,
the green strange quarks correspond to the three pairs involving k,
and their masses are similarly determined to also be 625.5 MeV.
The charm quark corresponds to the remaining 64 - 1 - 3 - 9 = 51 pairs.

Therefore, the mass of the red charm quark should be the sum of two parts:
the { i, i }, red up quark mass, 312.75 MeV;
and
the product of the symmetry part of the strange quark mass, 312.75 MeV,
and the charm to strange octonion number factor 51 / 9,
which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is
mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are similarly determined to also be 2.085 GeV.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass,
that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a
conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the
Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known.
The conventional value of alpha_s at about 2 GeV is about 0.39, which is somewhat lower than the theoretical model value.
Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

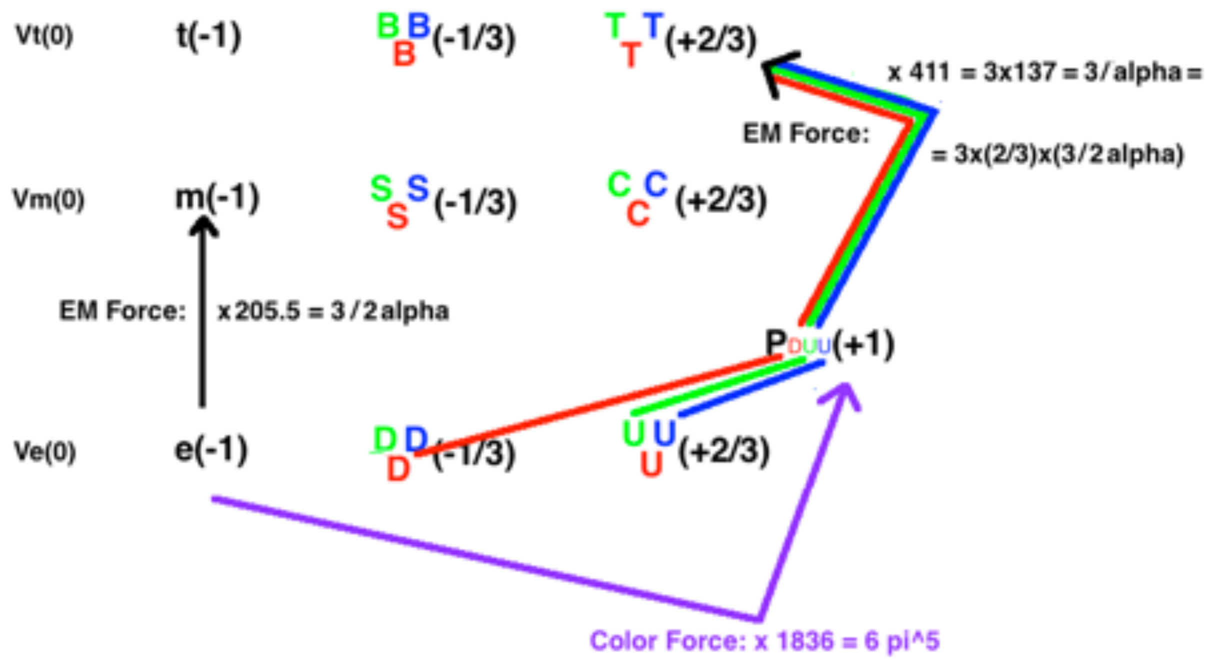
These results when added up give a total mass of second generation fermion particles:

Sigmaf2 = 32.9 GeV

Mendel Sachs and Particle Masses

Frank Dodd (Tony) Smith, Jr. - 2014

Mendel Sachs, in his books “General Relativity and Matter” (1982) and
“Quantum Mechanics from General Relativity” (1986) calculated
electron / muon and Proton / Tquark mass ratios substantially consistent with



Cl(16)-E8 Physics masses e = 0.511 MeV, m = 106 MeV, P = 938 MeV, T = 128.5 GeV
saying (my comments set off by brackets [[ ]]):
“... the inertial mass of an elementary (spinor) particle [i]s
determined by the curvature of space-time in its vicinity, representing
the coupling of this particle to its environment of particle-antiparticle pairs ...
[[In Cl(16)-E8 Physics the particle-antiparticle pairs form a Schwinger Source Kerr-Newman Black Hole]]
Because the coupling of the observed electron to the pairs ... is electromagnetic,
the electron's mass is proportional to the fine structure constant ...
[[In Cl(16)-E8 Physics the gauge symmetry of the force determines the geometry of the Schwinger Source and its Green’s Function.]]
The electron mass is one member of a mass doublet, predicted by this theory.
The other member, the muon, arises because occasionally the observed electron can
excite a pair of the background, which in turn changes the features of the geometry of
space-time in the vicinity of the electron. ...
[[In Cl(16)-E8 Physics the “excite” producing second and third generations is due to World-Lines traversing CP2 Internal Symmetry Space as well as M4 Physical Spacetime of M4xCP2 Kaluza-Klein]]
Because the excitation of the pair is due to an electromagnetic force,
the new mass ... is 3 / 2 alpha = 206 times greater than the old mass. ...
This theory also predicts that the proton should have a sister member of a doublet ...

To compute the inertial mass of the electron, consider first
the frame of reference whose spatial origin is at the site of the observed electron,
with the pairs of the background in motion relative to this point
...
Using the method of Green's functions ... we see that the quaternion metrical field ... in the linear approximation, reduce to an integral equation with ... solutions ...[that]... are the linear approximation ... to the spin-affine connection field ...
the solutions ... of the integral Equation ... lead directly to the (squared) mass eigenvalues ... the eigenvalues of the mass operator are the absolute values of the squares of the matrix elements above
...
The pairs interact with each other in a way that makes them appear to some 'observed' constituent electron as 'photons'. ... Nevertheless,the pairs do have 'inertia' by virtue of their bound electrons and positrons that are not , in fact, annihilated. ...
From a distance greater than a 'first Bohr orbit' of one of the particle components of a pair, it appears, as a unit , to be an electrically neutral object. But as the (observed) electron comes sufficiently close to the pair so as to interact with its separate components, energy is used up in exciting the pair, thereby decreasing the relative speed between the pair and the observed electron.
If the primary excitation of a pair (as 'seen' by the observed electron) is quadrupolar, and if the ground state of the pair corresponds to n = 1; then the
first excited Bohr orbital with a quadrupolar component is the state with n' = 3.

[[Quadrupolar implies 4+4 Kaluza-Klein of Cl(16)-E8 Physics]]

With these values ... it follows that the ratio of mass eigenvalues is ... 3 / 2 alpha = 206 ... The reason for this is that the curvature of space-time, in the vicinity of the observed electron, that gives rise to its inertia, is a consequence of the electromagnetic coupling between the matter components of the system. ...
Summing up, the inertial mass of an elementary (spinor) particle was determined by the curvature of space-time in its vicinity, representing the coupling of this particle to its environment of particle-antiparticle pairs.

[[Green’s functions for each force imply geometric structure of Schwinger Sources]]

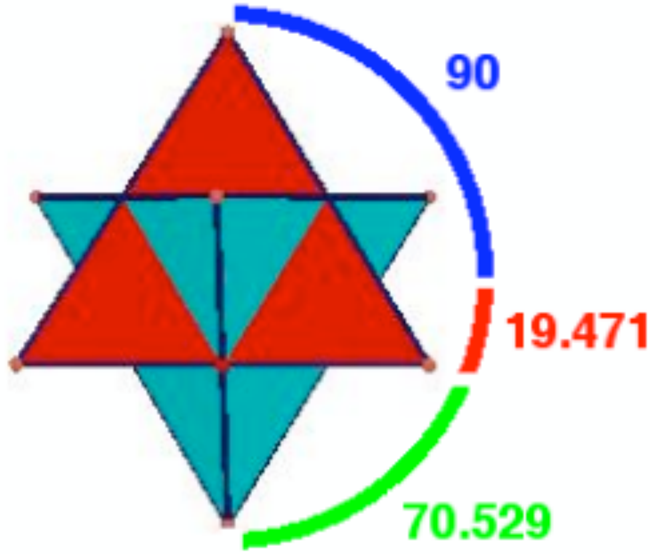
The significant domain of space populated by pairs that contributes to the electron mass is the order of 10^(-15) cm ,,,

[[Schwinger Source size in Cl(16)-E8 Physics is much smaller, about 10^(-24) cm]]

Because the coupling of the observed electron to the pairs - that gives it inertia - is electromagnetic, the electron's mass is proportional to the fine structure constant - which is a measure of the strength of this coupling. ...”.

11. Kobayashi-Maskawa Parameters

In E8 Physics the KM Unitarity Triangle angles can be seen on the Stella Octangula



The Kobayashi-Maskawa parameters are determined in terms of
the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by

Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted by

Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The resulting KM matrix is:

	d	s	b
u	0.975	0.222 0.00249	-0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Below the energy level of ElectroWeak Symmetry Breaking
the Higgs mechanism gives mass to particles.

According to a Review on the Kobayashi-Maskawa mixing matrix by Ceccucci, Ligeti, and Sakai in the 2010 Review of Particle Physics (note that I have changed their terminology of CKM matrix to the KM terminology that I prefer because I feel that it was Kobayashi and Maskawa, not Cabibbo, who saw that 3x3 was the proper matrix structure): "... the charged-current W± interactions couple to the ... quarks with couplings given by ...

Vud	Vus	Vub
Vcd	Vcs	Vcb
Vtd	Vts	Vtb

This Kobayashi-Maskawa (KM) matrix is a 3x3 unitary matrix.
It can be parameterized by three mixing angles and the CP-violating KM phase ...
The most commonly used unitarity triangle arises from
Vud Vub∗ + Vcd Vcb∗ + Vtd Vtb∗ = 0,
by dividing each side by the best-known one, Vcd Vcb∗
...
¯ρ + i¯η = −(Vud Vub∗)/(Vcd Vcb∗) is phase-convention- independent ...

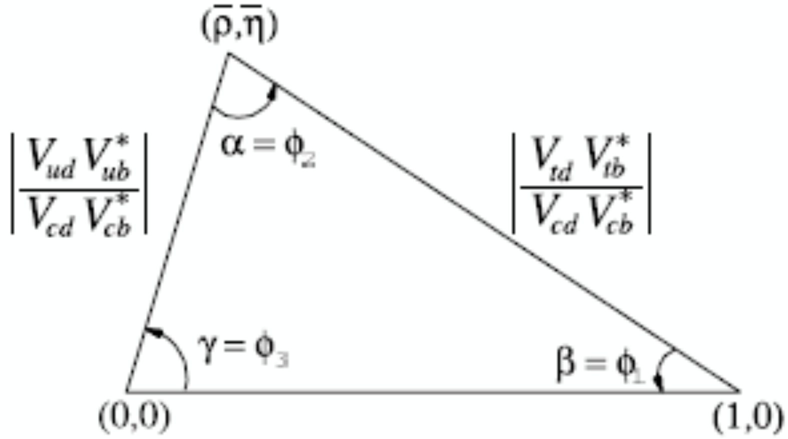


Figure 11.1: Sketch of the unitarity triangle.

... sin 2β = 0.673 ± 0.023 ... α = 89.0 +4.4 −4.2 degrees ... γ = 73 +22 −25 degrees ...
The sum of the three angles of the unitarity triangle, α + β + γ = (183 +22 −25) degrees, is ... consistent with the SM expectation. ...

The area... of ...[the]... triangle...[is]... half of the Jarlskog invariant, J, which is a phase-convention-independent measure of CP violation, defined by Im Vij Vkl Vil∗ Vkj∗ = J SUM(m,n) ε_ikm ε_jln

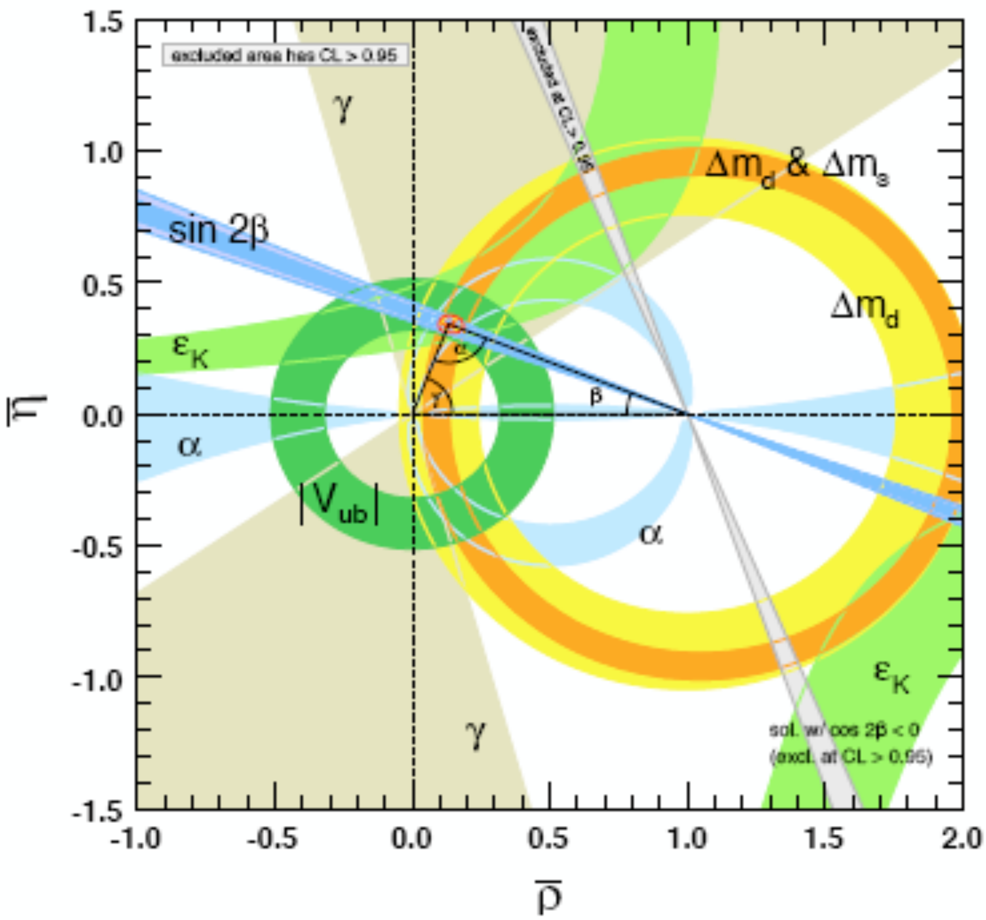


Figure 11.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

The fit results for the magnitudes of all nine KM elements are ...

0.97428 ± 0.00015	0.2253 ± 0.0007	0.00347 +0.00016 −0.00012
0.2252 ± 0.0007	0.97345 +0.00015 −0.00016	0.0410 +0.0011 −0.0007
0.00862 +0.00026 −0.00020	0.0403 +0.0011−0.0007	0.999152 +0.000030−0.000045

and the Jarlskog invariant is J = (2.91 +0.19-0.11)x10−5. ...".

Above the energy level of ElectroWeak Symmetry Breaking particles are massless.

Kea (Marni Sheppeard) proposed
that in the Massless Realm the mixing matrix might be democratic.
In Z. Phys. C - Particles and Fields 45, 39-41 (1989) Koide said: "...
the mass matrix ... MD ... of the type ... 1/3 x m x

1	1	1
1	1	1
1	1	1

... has name... "democratic" family mixing ...
the ... democratic ... mass matrix can be diagonalized by the transformation matrix A ...

1/sqrt(2)	-1/sqrt(2)	0
1/sqrt(6)	1/sqrt(6)	-2/sqrt(6)
1/sqrt(3)	1/sqrt(3)	1/sqrt(3)

as A MD At =

0	0	0
0	0	0
0	0	m

...".

Up in the Massless Realm you might just say that there is no mass matrix,
just a democratic mixing matrix of the form 1/3 x

1	1	1
1	1	1
1	1	1

with no complex stuff and no CP violation in the Massless Realm.

When go down to our Massive Realm by ElectroWeak Symmetry Breaking
then you might as a first approximation use m = 1
so that all the mass first goes to the third generation as

0	0	0
0	0	0
0	0	1

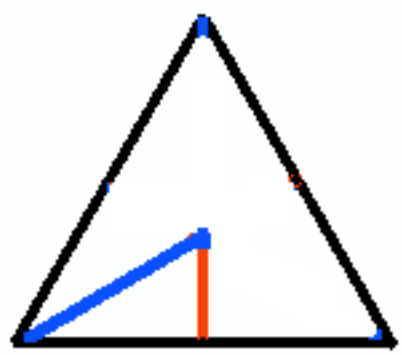
which is physically like the Higgs being a T-Tbar quark condensate.

Consider a 3-dim Euclidean space of generations:

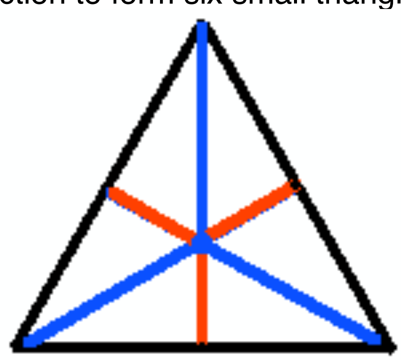
The case of mass only going to one generation
can be represented as a line or 1-dimensional simplex

in which the blue mass-line covers the entire black simplex line.

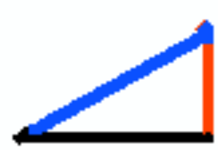
If mass only goes to one other generation
that can be represented by a red line extending to a second dimension
forming a small blue-red-black triangle



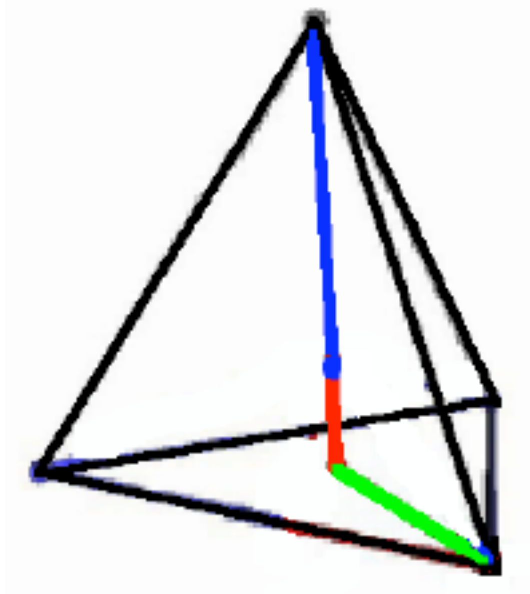
that can be extended by reflection to form six small triangles making up a large triangle



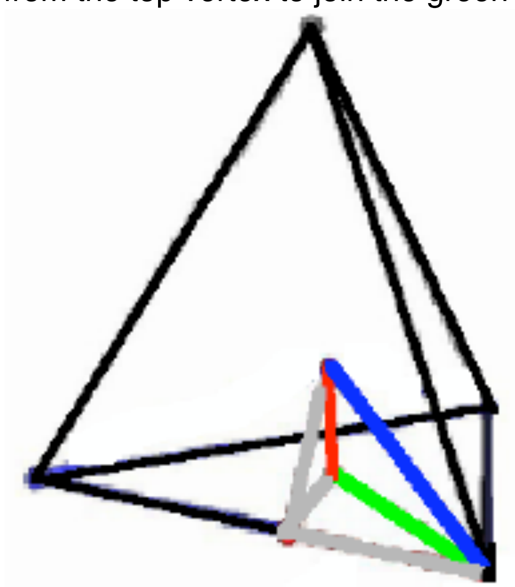
Each of the six component triangles has 30-60-90 angle structure:



If mass goes on further to all three generations
that can be represented by a green line extending to a third dimension



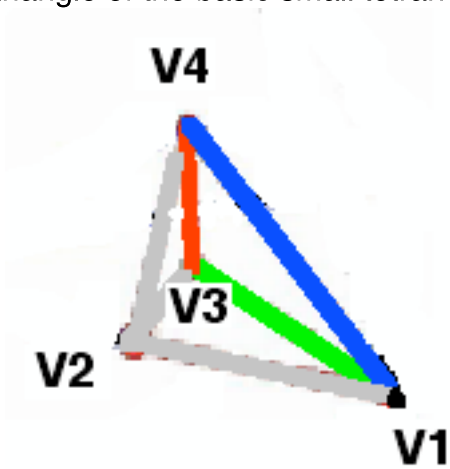
If you move the blue line from the top vertex to join the green vertex



you get a small blue-red-green-gray-gray-gray tetrahedron
that can be extended by reflection to form 24 small tetrahedra
making up a large tetrahedron.

Reflection among the 24 small tetrahedra corresponds
to the 12+12 = 24 elements of the Binary Tetrahedral Group.

The basic blue-red-green triangle of the basic small tetrahedron



has the angle structure of the K-M Unitary Triangle.

Using data from R. W. Gray's "Encyclopedia Polyhedra: A Quantum Module"
with lengths

V1.V2 = (1/2) EL ≡ Half of the regular Tetrahedron's edge length.
V1.V3 = (1 / sqrt(3)) EL ≅ 0.577 350 269 EL
V1.V4 = 3 / (2 sqrt(6)) EL ≅ 0.612 372 436 EL
V2.V3 = 1 / (2 sqrt(3)) EL ≅ 0.288 675 135 EL
V2.V4 = 1 / (2 sqrt(2)) EL ≅ 0.353 553 391 EL
V3.V4 = 1 / (2 sqrt(6)) EL ≅ 0.204 124 145 EL

the Unitarity Triangle angles are:

β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285

α = V1.V3.V4 = 90 degrees

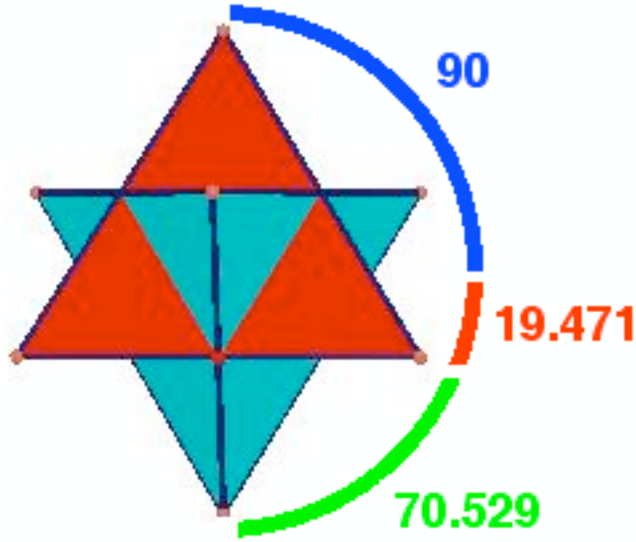
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

which is substantially consistent with the 2010 Review of Particle Properties

sin 2β = 0.673 ± 0.023 so β = 21.1495 degrees
α = 89.0 +4.4 –4.2 degrees
γ = 73 +22 –25 degrees

and so also consistent with the Standard Model expectation.

The constructed Unitarity Triangle angles can be seen on the Stella Octangula configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



In the Cl(16)-E8 model the Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by
Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted
by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8),
and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses
to calculate Kobayashi-Maskawa parameters:

phase angle d13 = gamma = 70.529 degrees

sin(theta12) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mmu^2+3ms^2+3mc^2]) = 0.222198

sin(theta13) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+
+ [mtau^2+3mb^2+3mt^2]) = 0.004608

sin(*theta23 = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+
+ [mmu^2+3ms^2+3mc^2])

sin(theta23) = s23 = sin(*theta23) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1
the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(theta23)	sin(theta23)
0	-sin(theta23)	cos(theta23)

cos(theta13)	0	sin(theta13)exp(-i d13)
0	1	0
-sin(theta13)exp(i d13)	0	cos(theta13)

cos(theta12)	sin(theta12)	0
-sin(theta12)	cos(theta12)	0
0	0	1

The resulting Kobayashi-Maskawa parameters
for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output,
or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level,
the lepton sector has no tree-level K-M mixing.

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model,
the only source of CP violation is the Kobayashi-Maskawa (KM) phase ...
The study of CP violation is, at last, experiment driven. ...
The CKM matrix provides a consistent picture
of all the measured flavor and CP violating processes. ...
There is no signal of new flavor physics. ...
Very likely,
the KM mechanism is the dominant source of CP violation in flavor changing processes.
... The result is consistent with the SM predictions. ...".

12. Proton-Neutron Mass Difference

An up valence quark, constituent mass 313 Mev,
does not often swap places with a 2.09 Gev charm sea quark,
but
a 313 Mev down valence quark
can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force
constituent mass of the down valence quark is heavier by about

(ms - md) (md/ms)^2 a(w) IVdsI = 312 x 0.25 x 0.253 x 0.22 Mev = 4.3 Mev,

(where a(w) = 0.253 is the geometric part of the weak force strength
and IVdsI = 0.22 is the magnitude
of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass Qmd of the down quark is

Qmd = 312.75 + 4.3 = 317.05 MeV.

Similarly, the up quark Quantum color force mass increase is about

(mc - mu) (mu/mc)^2 a(w) IV(uc)I = 1777 x 0.022 x 0.253 x 0.22 Mev = 2.2 Mev,

(where IVucI = 0.22 is the magnitude
of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass Qmu of the up quark is

Qmu = 312.75 + 2.2 = 314.95 MeV.

Therefore, the Quantum color force Neutron-Proton mass difference is

mN - mP = Qmd - Qmu = 317.05 Mev - 314.95 Mev = 2.1 Mev.

Since the electromagnetic Neutron-Proton mass difference is roughly

mN - mP = -1 MeV

the total theoretical Neutron-Proton mass difference is

mN - mP = 2.1 Mev - 1 Mev = 1.1 Mev,

an estimate that is comparable to the experimental value of 1.3 Mev.

13. Pion as Sine-Gordon Breather

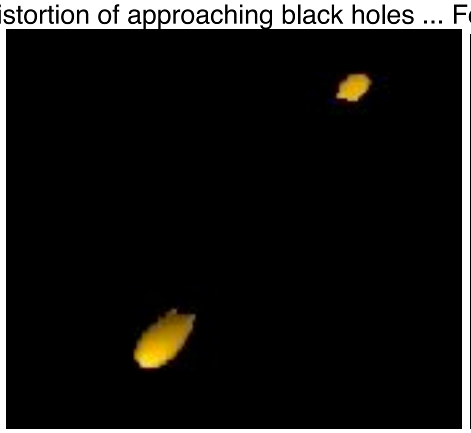
The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Schwinger Source Kerr-Newman Black Hole
with constituent mass M 312 MeV.

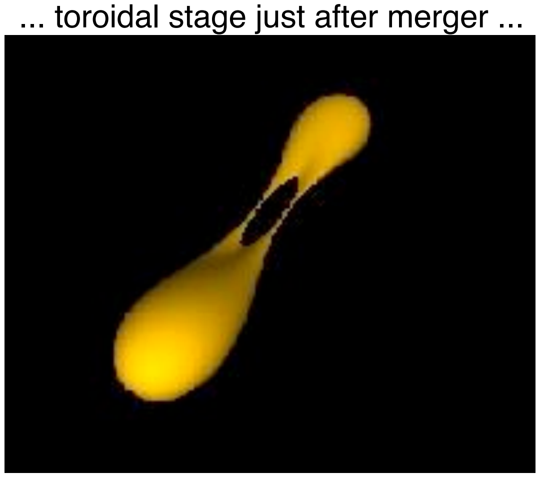
The antiquark is also a Schwinger Source Kerr-Newman Black Hole,
with constituent mass M 312 MeV.

According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):
"... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger. ...

Tidal distortion of approaching black holes ... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion,
the toroidal black hole remains a torus.
The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):
"... The surface r = r+ is ... the event horizon ... and is a null surface ...

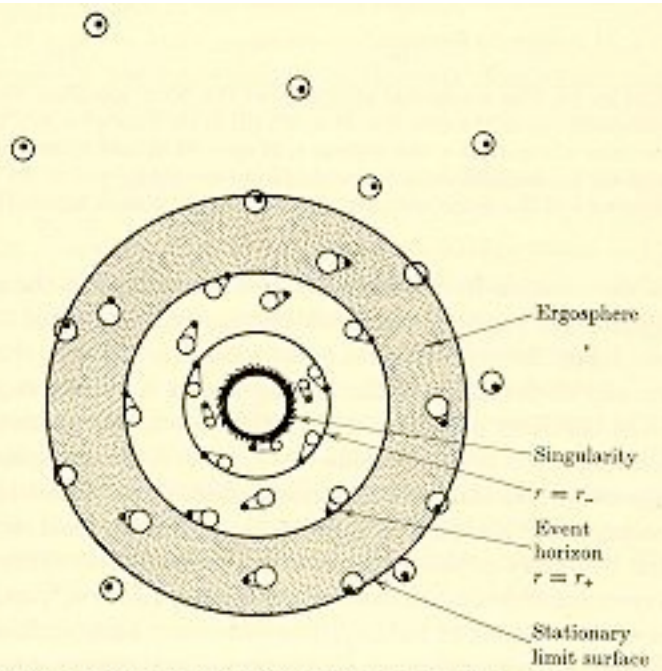


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather.
The soliton and antisoliton of a Sine-Gordon Breather correspond
to the quark and antiquark that make up the pion,
analagous to the Massive Thirring Model.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper
Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985),
where he writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

L = (1 / B^2) ((1/2) (df)^2 + A (cos(f) - 1))

Coleman says: “... We see that, in classical physics, B is an irrelevant parameter:
if we can solve the sine-Gordon equation for any non-zero B,
we can solve it for any other B.
The only effect of changing B is the trivial one of changing the energy and momentum assigned to a given solution of the equation. This is not true in quantum physics, because the relevant object for quantum physics is not L but [eq. 4.4]

L / hbar = (1 / (B^2 hbar)) ((1/2) (df)^2 + A (cos(f) - 1))

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ...
the classical limit, vanishing hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...
... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

f(x, t) = (4 / B) arctan((n sin(w t) / cosh(n w x))

where [eq. 4.60] n = sqrt(A - w^2) / w and w ranges from 0 to A.
This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As sin(w t) increases, the soliton and antisoliton move farther apart from each other. When sin(w t) passes through one,
they turn around and begin to approach one another. As sin(w t) comes down to zero ... the soliton and antisoliton are on top of each other ...
when sin(w t) becomes negative .. the soliton and antisoliton have passed each other.

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

E = 2 M sqrt(1 - (w^2 / A))

where [eq. 4.65] M = 8 sqrt(A) / B^2 is the soliton mass.

Note that the mass of the doublet is always less than twice the soliton mass,
as we would expect from a soliton-antisoliton pair. ...

Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974).
...[found that]... there is only a single series of bound states, labeled by the integer N ...
The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]
B'^2 = B^2 / (1 - (B^2 / 8 pi)) and M is the soliton mass.
M is not given by Eq. (4.65), but is the soliton mass corrected by the DHN formula,
or, equivalently, by the first-order weak coupling expansion. ...
I have written the equation in this form .. to eliminate A,
and thus avoid worries about renormalization conventions.
Note that the DHN formula is identical to the Bohr-Sommerfeld formula,
except that B is replaced by B'. ...
Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter
family of periodic motions, labeled by the period, T,
then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula,
but it is much more general;
it is always the leading approximation for any dynamical system ...
Dashen et al speculate that Eq. (4.82) is exact. ...
the sine-Gordon equation is equivalent ... to the massive Thirring model.
This is surprising,
because the massive Thirring model is a canonical field theory
whose Hamiltonian is expressed in terms of fundamental Fermi fields only.
Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent
to a free massive Dirac theory, in one spatial dimension. ...
Furthermore, we can identify the mass term in the Thirring model
with the sine-Gordon interaction, [eq. 5.13]

M = - (A / B^2) N_m cos(B f)

.. to do this consistently ... we must say [eq. 5.14]

B^2 / (4 pi) = 1 / (1 + g / pi)

....[where]... g is a free parameter, the coupling constant [for the Thirring model]...
Note that if B^2 = 4 pi , g = 0 ,
and the sine-Gordon equation is the theory of a free massive Dirac field. ...
It is a bit surprising to see a fermion appearing as a coherent state of a Bose field.
Certainly this could not happen in three dimensions,
where it would be forbidden by the spin-statistics theorem.
However, there is no spin-statistics theorem in one dimension,
for the excellent reason that there is no spin. ...
the lowest fermion-antifermion bound state of the massive Thirring model
is an obvious candidate for the fundamental meson of sine-Gordon theory. ...
equation (4.82) predicts that
all the doublet bound states disappear when B^2 exceeds 4 pi .

This is precisely the point where
the Thirring model interaction switches from attractive to repulsive. ...
these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ...
define identical physics. ...
I have computed the predictions of ...[various]... approximation methods
for the ration of the soliton mass to the meson mass for three values of B^2 :
4 pi (where the qualitative picture of the soliton as a lump totally breaks down),
2 pi, and pi . At 4 pi we know the exact answer ...
I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b]

E = 8 sqrt(A) / B^2

...[is the]... energy of the lump ... of sine-Gordon theory ...
frequently called 'soliton...' in the literature ...
[Zeroth-order is the classical case, or classical limit.] ...
... Coherent-state variation always gives
the same result as the ... Zeroth-order weak coupling expansion ... .
The ... First-order weak-coupling expansion ...
explicit formula ... is (8 / B^2) - (1 / pi). ...".

Using the Cl(16)-E8 model constituent mass of the Up and Down quarks and
antiquarks, about 312.75 MeV, as the soliton and antisoliton masses,
and setting B^2 = pi and using the DHN formula,
the mass of the charged pion is calculated to be (312.75 / 2.25) MeV = 139 MeV
which is close to the experimental value of about 139.57 MeV.

Why is the value B^2 = pi the special value that gives the pion mass ?

(or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi)

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula. In other words,

The physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

14. Neutrino Masses Beyond Tree Level

Consider the three generations of neutrinos:
nu_e (electron neutrino); nu_m (muon neutrino); nu_t
and three neutrino mass states: nu_1 ; nu_2 : nu_3
and
the division of 8-dimensional spacetime into
4-dimensional physical Minkowski spacetime
plus
4-dimensional CP2 internal symmetry space.

The heaviest mass state nu_3 corresponds to a neutrino
whose propagation begins and ends in CP2 internal symmetry
space,lying entirely therein. According to the Cl(16)-E8 model
the mass of nu_3 is zero at tree-level
but it picks up a first-order correction
propagating entirely through internal symmetry space by merging
with an electron through the weak and electromagnetic forces,
effectively acting not merely as a point
but
as a point plus an electron loop at beginning and ending points
so
the first-order corrected mass of nu_3 is given by
M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E
where the factor (1/sqrt(2)) comes from the Ut3 component
of the neutrino mixing matrix
so that

M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =
= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =
= 7.35 / 137 = 5.4 x 10^(-2) eV.

The neutrino-plus-electron loop can be anchored by weak force
action through any of the 6 first-generation quarks
at each of the beginning and ending points, and that the
anchor quark at the beginning point can be different from
the anchor quark at the ending point,
so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino
whose propagation begins or ends in CP2 internal symmetry space
and ends or begins in M4 physical Minkowski spacetime,
thus having only one point (either beginning or ending) lying
in CP2 internal symmetry space where it can act not merely
as a point but as a point plus an electron loop.

According to the Cl(16)-E8 model the mass of nu_2 is zero at
tree-level but it picks up a first-order correction at only one
(but not both) of the beginning or ending points
so that so that there are 6 different possible anchorings
for nu_2 first-order corrections, as opposed to the 36 different
possible anchorings for nu_3 first-order corrections,
so that
the first-order corrected mass of nu_2 is less than
the first-order corrected mass of nu_3 by a factor of 6,
so

the first-order corrected mass of nu_2 is
M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6
= 9 x 10^(-3)eV.

The low mass state nu_1 corresponds to a neutrino
whose propagation begins and ends in physical Minkowski
spacetime.
thus having only one anchoring to CP2 interna symmetry space.

According to the Cl(16)-E8 model the mass of nu_1 is zero at
tree-level but it has only 1 possible anchoring to CP2
as opposed to the 36 different possible anchorings for nu_3
first-order corrections
or the 6 different possible anchorings for nu_2 first-order
corrections
so that
the first-order corrected mass of nu_1 is less than
the first-order corrected mass of nu_2 by a factor of 6,
so

the first-order corrected mass of nu_1 is
M_nu_1 = M_nu_2 / Vol(CP2) = 9 x 10^(-3) / 6
= 1.5 x 10^(-3)eV.

Therefore:

the mass-squared difference D(M23^2) = M_nu_3^2 - M_nu_2^2 =
= (2916 - 81) x 10^(-6) eV^2 =
= 2.8 x 10^(-3) eV^2

and

the mass-squared difference D(M12^2) = M_nu_2^2 - M_nu_1^2 =
= (81 - 2) x 10^(-6) eV^2 =
= 7.9 x 10^(-5) eV^2

The 3x3 unitary neutrino mixing matrix neutrino mixing matrix U

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

can be parameterized (based on the 2010 Particle Data Book)
by 3 angles and 1 Dirac CP violation phase

```
      c12 c13                         s12 c13                         s13 e-id

U = - s12 c23 - c12 s23 s13 eid     c12 c23 - s12 s23 s13 eid     s23 c13

      s12 s23 - c12 c23 s13 eid   - c12 s23 - s12 c23 s13 eid     c23 c13
```

where cij = cos(theta_ij) , sij = sin(theta_ij)

The angles are

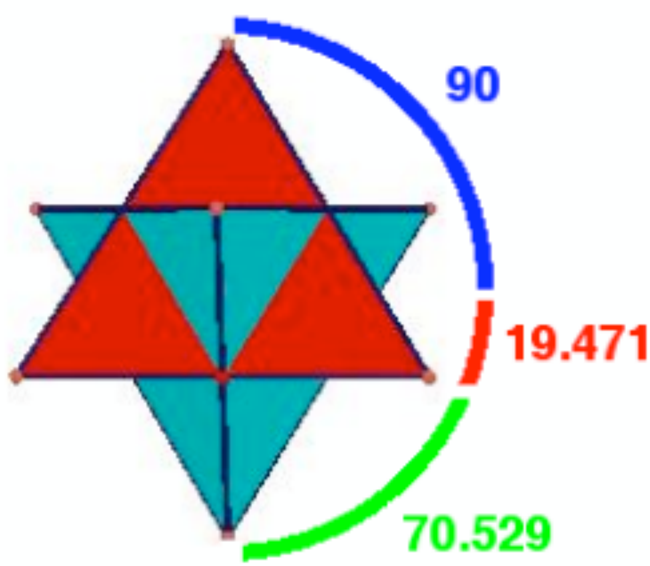
theta_23 = pi/4 = 45 degrees
because
nu_3 has equal components of nu_m and nu_t so
that Um3 = Ut3 = 1/sqrt(2) or, in conventional
notation, mixing angle theta_23 = pi/4
so that cos(theta_23) = 0.707 = sqrt(2)/2 = sin(theta_23)

theta_13 = 9.594 degrees = asin(1/6)
and cos(theta_13) = 0.986
because sin(theta_13) = 1/6 = 0.167 = |Ue3| = fraction of nu_3 that is nu_e

theta_12 = pi/6 = 30 degrees
because
sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points
that are in the physical spacetime where massless nu_e lives
so that cos(theta_12) = 0.866 = sqrt(3)/2

d = 70.529 degrees is the Dirac CP violation phase
ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
This is because the neutrino mixing matrix has 3-generation structure
and so has the same phase structure as the KM quark mixing matrix
in which the Unitarity Triangle angles are:
β = V3.V1.V4 = arccos(2 sqrt(2) / 3) ≅ 19.471 220 634 degrees so sin 2β = 0.6285
α = V1.V3.V4 = 90 degrees
γ = V1.V4.V3 = arcsin(2 sqrt(2) / 3) ≅ 70.528 779 366 degrees

The constructed Unitarity Triangle angles can be seen on the Stella Octangula
configuration of two dual tetrahedra (image from gauss.math.nthu.edu.tw):



Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.866 x 0.986	0.50 x 0.986	0.167 x e-id
nu_m	-0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986
nu_t	0.5 x 0.707 -0.866 x 0.707 x 0.167 x eid	-0.866 x 0.707 -0.5 x 0.707 x 0.167 x eid	0.707 x 0.986

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.167 e-id
nu_m	-0.354 -0.102 eid	0.612 -0.059 eid	0.697
nu_t	0.354 -0.102 eid	-0.612 -0.059 eid	0.697

Since ei(70.529) = cos(70.529) + i sin(70.529) = 0.333 + 0.943 i
and .333e-i(70.529) = cos(70.529) - i sin(70.529) = 0.333 - 0.943 i

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.354 -0.034 - 0.096 i	0.612 -0.020 - 0.056 i	0.697
nu_t	0.354 -0.034 - 0.096 i	-0.612 -0.020 - 0.056 i	0.697

for a result of

	nu_1	nu_2	nu_3
nu_e	0.853	0.493	0.056 - 0.157 i
nu_m	-0.388 - 0.096 i	0.592 - 0.056 i	0.697
nu_t	0.320 - 0.096 i	0.632 - 0.056 i	0.697

which is consistent with the approximate experimental values of mixing angles shown in the Michaelmas Term 2010 Particle Physics handout of Prof Mark Thomson if the matrix is modified by taking into account the March 2012 results from Daya Bay observing non-zero theta_13 = 9.54 degrees.

15. Planck Mass as Superposition Fermion Condensate

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum superposition of all possible virtual first-generation particle-antiparticle fermion pairs allowed by the Pauli exclusion principle to live on that vertex. (The second generation fermions live on two vertices and the third-generation fermions live on three vertices which pairs or triples of vertices act at our energy levels very much like one vertex.)

Once a Planck-mass black hole is formed, it is stable in the E8 model.
Less mass would not be gravitationally bound at the vertex.
More mass at the vertex would decay by Hawking radiation.

There are 8 fermion particles and 8 fermion antiparticles whose average mass is about
(0 + 0.0005 + 6 x 0.312) / 8 = 0.234 GeV.

There are 8x8 = 64 particle-antiparticle pairs
and 2^64 = 1.8 x 10^19 combinations of pairs, ranging in size from 1 to 64 pairs.

The 64-pair mass is about 64 x 2 x 0.234 = 29.952 GeV
and the 32-pair mass is about 14.976 GeV.

If the 32-pair mass is taken to be typical, then the total mass of all 2^64 combinations would be about 14.976 x 1.8 x 10^19 = 26.957 x 10^19 GeV.

However, the Pauli exclusion principle would prevent participation of pairs of fermions unless the pairs formed a bosonic pion-type state.
Of the 64 pairs, only 12 are bosonic pion-type states,
and
a pion-type state has mass about 139.57 / 625.5 times the mass of its two fermions,
so
the realistic total mass should be about (139.57 / 625.5) (12 / 64) x 26.957 x 10^19 =
= 1.128 x 10^19 GeV.

The value for the Planck mass given by Particle Data Group (2013)
is 1.221 x 10^19 GeV.

16. Force Strength and Boson Mass Calculation

Cl(8) bivector Spin(8) is the D4 Lie algebra two copies of which are in the Cl(16)-E8 model Lagrangian (as the D4xD4 subalgebra of the D8 subalgebra of E8)

$$\int_{\text{4-dim M4}} \text{GG SM} + \text{Fermion Particle-AntiParticle} + \text{Higgs}$$

.

with the Higgs term coming from integrating over the CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein by the Mayer-Trautman Mechanism

This shows that the **Force Strength is made up of two parts**:

the relevant spacetime manifold of gauge group global action
and
the relevant symmetric space manifold of gauge group local action.

The 4-dim spacetime Lagrangian GG SM gauge boson term is:
the integral over spacetime as seen by gauge boson acting globally
of the gauge force term of the gauge boson acting locally
for the gauge bosons of each of the four forces:

U(1) for electromagnetism
SU(2) for weak force
SU(3) for color force
Spin(5) - compact version of antiDeSitter Spin(2,3) subgroup of Conformal Spin(2,4) for gravity by the MacDowell-Mansouri mechanism.

In the conventional picture,
for each gauge force the gauge boson force term contains the force strength,
which in Feynman's picture is the amplitude to emit a gauge boson,
and can also be thought of as the probability = square of amplitude,
in an explicit (like g IFI^2) or an implicit (incorporated into the IFI^2) form.
Either way, the conventional picture is that the force strength g is an ad hoc inclusion.

The Cl(16)-E8 model does not put in force strength g ad hoc,
but constructs the integral such that
the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime as it is seen by that gauge boson, that is, in terms of the symmetric space with global symmetry of the gauge boson:

the U(1) photon sees 4-dim spacetime as T^4 = S1 x S1 X S1 x S1
the SU(2) weak boson sees 4-dim spacetime as S2 x S2
the SU(3) weak boson sees 4-dim spacetime as CP2
the Spin(5) of gravity sees 4-dim spacetime as S4

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with local symmetry of the gauge boson.
The nontrivial Shilov boundaries are:

for SU(2) Shilov = RP^1xS^2
for SU(3) Shilov = S^5
for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strengths.

Each gauge group is the global symmetry of a symmetric space

S1 for U(1)
S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
CP2 = SU(3)/SU(2)xU(1) for SU(3)
S4 = Spin(5)/Spin(4) for Spin(5)

Each gauge group is the local symmetry of a symmetric space

U(1) for itself
SU(2) for Spin(5) / SU(2)xU(1)
SU(3) for SU(4) / SU(3)xU(1)
Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces
correspond to bounded complex domains

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries

SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

Very roughly, think of the force strength as
integral over global symmetry space of physical (ie Shilov Boundary) volume =
= strength of the force.

That is:
the geometric strength of the force is given by the product of
the volume of a 4-dim thing with global symmetry of the force and
the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some tricky normalization stuff),
you see that roughly:

Volume product for gravity is the largest volume
so since (as Feynman says) force strength = probability to emit a gauge boson means
that the highest force strength or probability should be 1
the gravity Volume product is normalized to be 1, and so (approximately):

Volume product for gravity = 1
Volume product for color = 2/3
Volume product for weak = 1/4
Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

1 - for massive gauge bosons, a suppression by a factor of 1 / M^2
2 - renormalization running (important for color force)

Consider Massive Gauge Bosons:

Gravity as curvature deformation of SpaceTime, with SpaceTime as a condensate of
Planck-Mass Black Holes, must be carried by virtual Planck-mass black holes,
so that the geometric strength of gravity should be reduced by 1/Mp^2

The weak force is carried by weak bosons,
so that the geometric strength of the weak force should be reduced by 1/MW^2

That gives the result (approximate):

gravity strength = G (Newton's G)
color strength = 2/3
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

Consider Renormalization Running for the Color Force:: That gives the result:

gravity strength = G (Newton's G)
color strength = 1/10 at weak boson mass scale
weak strength = G_F (Fermi's weak force G)
electromagnetism = 1/137

he use of compact volumes is itself a calculational device,
because it would be more nearly correct,
instead of the integral over the compact global symmetry space of
the compact physical (ie Shilov Boundary) volume=strength of the force
to use
the integral over the hyperbolic spacetime global symmetry space
of the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is ratios,
and the compact volumes (finite and easy to look up in the book by Hua)
have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices,
and
that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Here are less approximate more detailed force strength calculations:

The force strength of a given force is

alphaforce = (1 / Mforce^2) (Vol(MISforce)) (Vol(Qforce) / Vol(Dforce)^(1 / mforce))

where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the relevant part of the target Internal Symmetry Space;

Vol(MISforce) stands for volume of MISforce and is sometimes also denoted by Vol(M);

Qforce represents the link from the origin to the relevant target for the gauge boson;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain
of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce, which is
4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to have two copies of QW for SpaceTime),
1 for Electromagnetism (which therefore is considered to have four copies of QE for SpaceTime)

Vol(Dforce)^(1 / mforce) stands for a dimensional normalization factor
(to reconcile the dimensionality of the Internal Symmetry Space of the target vertex
with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space, and Dforce manifolds for the four forces are:

Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [unit radius scale].

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk

4-dim T^4 = S^1 x S^1 x S^1 x S^1 = topological boundary of 8-dim 4-polydisk
Shilov Boundary of 8-dim 4-polydisk = S^1 + S^1 + S^1 + S^1 =
= 1-dim wire frame of 4-dim T^4

Note (thanks to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.
Note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
Weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Note (thanks to Carlos Castro for noticing this) that the volume listed for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Using the above numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.
The effect is particularly pronounced with the color force.
The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give a Color Force Strength of about 0.125 at about 91 GeV

Higgs:

As with forces strengths, the calculations produce ratios of masses,
so that only one mass need be chosen to set the mass scale.

In the Cl(16)-E8 model,
the value of the fundamental mass scale vacuum expectation value v = <PHI>
of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons,
W+, W-, and Z0, whose tree-level masses will then be shown by ratio calculations to be
80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively,
and therefore the electron mass will be 0.5110 MeV.

The relationship between the Higgs mass and v is given
by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, i
n the notation of quant-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says:
"... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:

lambda = 3 (M_H / PHI)^2

which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming that lambda = (cos(pi / 6))^2 = 0.866^2
(a value consistent with the Higgs-Tquark condensate model of Michio Hashimoto,
Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165)
we have

M_H^2 / v^2 = (cos(pi / 6))^2 / 3

In the Cl(16)-E8 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and v is set to be 252.514 GeV
so that

M_H = v cos(pi / 6) / sqrt(1 / 3) = 126.257 GeV

This is the value of the Low Mass State of the Higgs observed by the LHC.
MIddle and High Mass States come from a Higgs-Tquark Condensate System.
The Middle and High Mass States may have been observed by the LHC at 20% of the Low Mass State cross section, and that may be confirmed by the LHC 2015-1016 run.

A Non-Condensate Higgs is represented by a Higgs at a point in M4
that is connected to a Higgs representation in CP2 ISS by a line whose length
represents the Higgs mass

```
Higgs     Higgs in CP2 Internal Symmetry Space
  |            |
  |            |
  |            |
  | mass = 145 | Non-Condensate Higgs Mass = 145
  |            |
  |            |
  |            |
Higgs     Higgs  in M4  spacetime
```

and the value of lambda is 1 = 1^2
so that the Higgs mass would be M_H = v /sqrt(3) = 145.789 GeV

However, in the Cl(16)-E8 model, the Higgs has structure of a Tquark condensate

```
mass = 145
T ---------Tbar     Effective  Higgs  in CP2 Internal Symmetry Space
 \    |    /                 |
  \   |   /                  |
   \  |  /mass = 145         | Higgs Effective Mass =
    \ | /                    | = 145 x cos(pi/6)= 145 x 0.866 = 126
     \|/                     |
   Higgs               Higgs in M4 spacetime
```

in which the Higgs at a point in M4 is connected to a T and Tbar in CP2 ISS
so that the vertices of the Higgs-T-Tbar system are connected
by lines forming an equilateral triangle composed of 2 right triangles
(one from the CP2 origin to the T and to the M4 Higgs and
another from the CP2 origin to the Tbar and to the M4 Higgs).
In the T-quark condensate picture
lambda = 1^2 = lambda(T) + lambda(H) = (sin(pi / 6))^2 + (cos(pi / 6))^2
and
lambda(H) = (cos(pi / 6))^2

Therefore the Effective Higgs mass observed by LHC is:

Higgs Mass = 145.789 x cos(pi/6) = 126.257 GeV.

To get W-boson masses,
denote the 3 SU(2) high-energy weak bosons
(massless at energies higher than the electroweak unification)
by W+, W-, and W0,
corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair,
so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification
is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar,
which carries the Higgs mechanism by which the W0 becomes the physical Z0,
so that the total mass of the triplet { W+, W-, Z0 }
is equal to the vacuum expectation value v of the Higgs scalar field, v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 } which can be represented by the 3-sphere S^3.
The Hopf fibration of S^3 as

S^1 --> S^3 --> S^2

gives a decomposition of the W bosons into the neutral W0 corresponding to S^1
and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0
should be the volume ratio of the S^2 in S^3 to the S^1 in S3.
The unit sphere S^3 in R^4 is normalized by 1 / 2.
The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).
The ratio of the sum of the W+ and W- masses to the W0 mass should then be
(2 / sqrt3) V(S^2) / (2 / sqrt2) V(S^1) = 1.632993

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV,
the total mass of a T - Tbar pair, and the charged weak bosons have equal mass,
we have

M_W+ = M_W- = 80.326 GeV and M_W0 = 98.379 GeV.

The charged W+/- neutrino-electron interchange must be symmetric
with the electron-neutrino interchange, so that the tree-level absence
of right-handed neutrino particles requires that

the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector,
so that the
charged W+/- SU(2) weak bosons act only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral W0 must be equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-,
there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction is proportional to
(M_W+/-^2 / M_W0^2) acting on left-handed fermions
and
(1 - (M_W+/-^2 / M_W0^2)) acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2 and denoted by K,
and if the strength of the W+/- charged weak force
(and of the custodial SU(2) symmetry) is denoted by T,
then the W0 neutral weak interaction can be written as W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force
and as W0LR with respect to the parity conserving U(1) electromagnetic force,
the W0 mass mW0 has two components:
the parity violating SU(2) part mW0L that is equal to M_W+/-
and the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR,
and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using the volume V(S^2) of an S^2 \subset R^3,
normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres,
while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2,
each of which contains one 1-sphere of the *alphaE force.

Therefore

*alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),

*e = e / (4th root of 6) = e / 1.565 ,

and
the mass mW0LR must be reduced to an effective value

M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV

for the *alphaE force to act like an electromagnetic force in the E8 model:

*e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct Cl(16)-E8 model values for weak boson masses and the Weinberg angle theta_w are:

M_W+ = M_W- = 80.326 GeV;

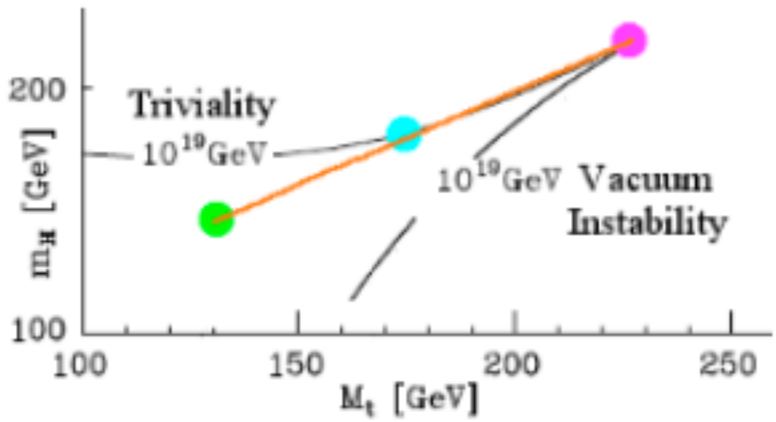
M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree- level values somewhat.

17. Higgs - Truth Quark Condensate System with 3 Mass States

The Cl(16)-E8 model identifies the Higgs with Primitive Idempotents
of the Cl(8) real Clifford algebra, whereby the Higgs is not seen as a simple-minded single fundamental scalar particle, but rather the Higgs is seen as a quantum process that creates a fermionic condensate and effectively a 3-state Higgs-Tquark System.



(from hep-ph/0307138)

The Magenta Dot is the high-mass state of a 220 GeV Truth Quark and a 240 GeV Higgs. It is at the critical point of the Higgs-Tquark System with respect to Vacuum Instability and Triviality. It corresponds to the description in hep-ph/9603293 by Koichi Yamawaki of the Bardeen-Hill-Lindner model. That high-mass Higgs is around 250 GeV in the range of the Higgs Vacuum Instability Boundary
which range includes the Higgs VEV.

The Gold Line leading down from the Critical Point roughly along the Triviality Boundary line is based on Renormalization Group calculations with the result that MH / MT = 1.1 as described by Koichi Yamawaki in hep-ph/9603293 .

The Cyan Dot where the Gold Line leaves the Triviality Boundary to go into our Ordinary Phase is the middle-mass state of a 174 GeV Truth Quark and Higgs around 200 GeV. It corresponds to the Higgs mass calculated by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they show that
for 8-dimensional Kaluza-Klein spacetime with the Higgs as a Truth Quark condensate $172 < MT < 175$ GeV and $178 < MH < 188$ GeV.
That mid-mass Higgs is around the 200 GeV range of the Higgs Triviality Boundary at which the composite nature of the Higgs as T-Tbar condensate in (4+4)-dim Kaluza-Klein becomes manifest.

The Green Dot where the Gold Line terminates in our Ordinary Phase
is the low-mass state of a 130 GeV Truth Quark and a 126 GeV Higgs.

The conventional Standard Model has structure:

spacetime is a base manifold
particles are representations of gauge groups
gauge bosons are in the adjoint representation
fermions are in other representations (analagous to spinor)
Higgs boson is in scalar representation

The Cl(16)-E8 model has structure
(from 248-dim E8 = 120-dim adjoint D8 + 128-dim half-spinor D8):

spacetime is in the adjoint D8 part of E8 (64 of 120 D8 adjoints)
gauge bosons are in the adjoint D8 part of E8 (28+28 = 56 of the 120 D8 adjoints)
fermions are in the half-spinor D8 part of E8 (64+64 of the 128 D8 half-spinors.

There is no room for a fundamental Higgs directly appearing in the E8,
rather, it emerges from the Mayer-Trautman Mechanism
with formation of Quaternionic (4+4)-dim M4 x CP2 Kaluza-Klein SpaceTime.
To see how that Higgs works in terms of the Cl(16) = Cl(8)xCl(8) Clifford Algebra,
embed 248-dim E8 into the 256-dim real Clifford algebra Cl(8):

Cl(8)	256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1
Primitive Idempotent	16 = 1 + 6 + 8 + 1
E8 Root Vectors	240 = 8 + 28 + 56 + 56 + 56 + 28 + 8
E8	248 = 8 + 28 + 56 + 64 + 56 + 28 + 8

The Cl(8) Primitive Idempotent is 16-dimensional and can be decomposed
into two 8-dimensional half-spinor parts each of which is related by Triality
to 8-dimensional spacetime and has Octonionic structure.

In that decomposition: the 1+6+1 = (1+3)+(3+1) is related to two copies of
a 4-dimensional Associative Quaternionic subspace of the Octonionic structure
and
the 8 = 4+4 is related to two copies of
a 4-dimensional Co-Associative subspace of the Octonionic structure
(see the book “Spinors and Calibrations” by F. Reese Harvey)

The 8 = 4+4 Co-Associative part of the Cl(8) Primitive Idempotent
when combined with the 240 E8 Root Vectors forms the full 248-dimensional E8.
It represents a Cartan subalgebra of the E8 Lie algebra.

**The (1+3)+(3+1) Associative part of the Cl(8) Primitive Idempotent
corresponds to the Higgs of the Cl(16)-E8 model.**

The half-spinors generated by the Higgs part of the Cl(8) Primitive Idempotent represent

neutrino; red, green, blue down quarks; red, green, blue up quarks; electron

so the E8 Higgs effectively creates/annihilates the fundamental fermions
and

the E8 Higgs is effectively a condensate of fundamental fermions.

In the Cl(16)-E8 model the high-energy 8-dimensional Octonionic spacetime reduces,
by freezing out a preferred 4-dim Associative Quaternionic subspace,
to a 4+4 -dimensional Batakis Kaluza-Klein of the form M4 x CP2
with 4-dim M4 physical spacetime.

The (1+3)+(3+1) part of the Cl(8) Primitive Idempotent includes

the 1of Cl(8) grade-0 scalar (that determines M4 transformation properties)
and 3+3 = 6 of the Cl(8) grade-4
and the 1 of Cl(8) grade-8

so the Cl(16)-E8 model Higgs transforms as a scalar
with respect to 4-dim M4 Physical SpaceTime
and is consistent with LHC observations (see arXiv 1307.1432).

Not only does the Cl(16)-E8 model Higgs fermion condensate transform
with respect to 4-dim physical spacetime like the Standard Model Higgs
but
the geometry of the reduction from 8-dim Octonionic spacetime
to (4+4)-dimensional Batakis Kaluza-Klein,
by the Mayer-Trautman Mechanism,
gives the Cl(16)-E8 Higgs ElectroWeak Symmetry-Breaking Ginzburg-Landau structure.

Since the second and third fermion generations emerge dynamically
from the reduction from 8-dim to 4+4 -dim Kaluza-Klein,
they are also created/annihilated by the Primitive Idempotent Cl(16)-E8 Higgs
and are present in the fermion condensate.

Since the Truth Quark is so much more massive that the other fermions,
the Cl(16)-E8 model Higgs is effectively a Truth Quark condensate.

When Triviality and Vacuum Stability are taken into account,
the Cl(16)-E8 model Higgs and Truth Quark system has 3 mass states.

As to composite Higgs and the Triviality boundary, Pierre Ramond says in his book Journeys Beyond the Standard Model (Perseus Books 1999) at pages 175-176:

"... The Higgs quartic coupling has a complicated scale dependence. It evolves according to d lambda / d t = (1 / 16 pi^2) beta_lambda
where the one loop contribution is given by
beta_lambda = 12 lambda^2 - ... - 4 H ...
The value of lambda at low energies is related [to] the physical value of the Higgs mass according to the tree level formula
m_H = v sqrt(2 lambda)
while the vacuum value is determined by the Fermi constant
...
for a fixed vacuum value v, let us assume that the Higgs mass and therefore lambda is large. In that case, beta_lambda is dominated by the lambda^2 term, which drives the coupling towards its Landau pole at higher energies.
Hence the higher the Higgs mass, the higher lambda is and the close[r] the Landau pole to experimentally accessible regions.
This means that for a given (large) Higgs mass,
we expect the standard model to enter a strong coupling regime
at relatively low energies, losing in the process our ability to calculate.
This does not necessarily mean that the theory is incomplete,
only that we can no longer handle it ...
it is natural to think that this effect is caused by new strong interactions,
and that the Higgs actually is a composite ...
The resulting bound on lambda is sometimes called the triviality bound.
The reason for this unfortunate name (the theory is anything but trivial)
stems from lattice studies where the coupling is assumed to be finite everywhere;
in that case the coupling is driven to zero, yielding in fact a trivial theory.
In the standard model lambda is certainly not zero. ...".

Composite Higgs as Tquark condensate studies by Yamawaki et al have produced realistic models that are consistent with the Cl(16)-E8 model with a 3-State System:

1 - The basic Cl(16)-E8 model state
with **Tquark mass = 130 GeV and Higgs mass = 126 GeV**

2 - Triviality boundary 8-dim Kaluza-Klein state described by Hashimoto, Tanabashi, and Yamawaki in hep-ph/0311165 where they say:
“... "... We perform the most attractive channel (MAC) analysis in the top mode standard model with TeV-scale extra dimensions, where the standard model gauge bosons and the third generation of quarks and leptons are put in D(=6,8,10,...) dimensions. In such a model, bulk gauge couplings rapidly grow in the ultraviolet region. In order to make the scenario viable, only the attractive force of the top condensate should exceed the critical coupling, while other channels such as the bottom and tau condensates should not. We then find that the top condensate can be the MAC for D=8 ... We predict masses of the top (m_t) and the Higgs (m_H) ...

based on the renormalization group for the top Yukawa and Higgs quartic couplings with the compositeness conditions at the scale where the bulk top condenses ... for ...[Kaluza-Klein type]... dimension... D=8 ...
m_t = 172-175 GeV and m_H=176-188 GeV ...".

3 - Critical point BHL state
with **Tquark mass = 218 +/- 3 GeV and Higgs mass = 239 +/- 3 GeV**
As Yamawaki said in hep-ph/9603293: "... the BHL formulation of the top quark condensate ... is based on the RG equation combined with the compositeness condition ... start[s] with the SM Lagrangian which includes explicit Higgs field at the Lagrangian level ... BHL is crucially based on the perturbative picture ...[which]... breaks down at high energy near the compositeness scale /\ ...[10^19 GeV]... there must be a certain matching scale /_Matching such that the perturbative picture (BHL) is valid for mu < /_Matching, while only the nonperturbative picture (MTY) becomes consistent for mu > /_Matching ...
However, thanks to the presence of a quasi-infrared fixed point,
BHL prediction is numerically quite stable against ambiguity at high energy region, namely, rather independent of whether this high energy region is replaced by MTY or something else. ... Then we expect mt = mt(BHL) = ... = 1/(sqrt(2)) ybart v within 1-2%, where ybart is the quasi-infrared fixed point given by Beta(ybart) = 0 in ... the one-loop RG equation ...
The composite Higgs loop changes ybart^2 by roughly the factor Nc/(Nc +3/2) = 2/3 compared with the MTY value, i.e., 250 GeV -> 250 x sqrt(2/3) = 204 GeV, while the electroweak gauge boson loop with opposite sign pulls it back a little bit to a higher value. The BHL value is then given by mt = 218 +/- 3 GeV, at /\ = 10^19 GeV.
The Higgs boson was predicted as a tbar-t bound state
with a mass MH = 2mt based on the pure NJL model calculation.
Its mass was also calculated by BHL through the full RG equation ...
the result being ... MH / mt = 1.1) at /.\ = 10^19 GeV ...
... the top quark condensate proposed by Miransky, Tanabashi and Yamawaki (MTY) and by Nambu independently ... entirely replaces the standard Higgs doublet by a composite one formed by a strongly coupled short range dynamics (four-fermion interaction) which triggers the top quark condensate.
The Higgs boson emerges as a tbar-t bound state and hence is deeply connected with the top quark itself. ... MTY introduced explicit four-fermion interactions responsible for the top quark condensate in addition to the standard gauge couplings. Based on the explicit solution of the ladder SD equation, MTY found that even if all the dimensionless four-fermion couplings are of O(1), only the coupling larger than the critical coupling yields non-zero (large) mass ... The model was further formulated in an elegant fashion by Bardeen, Hill and Lindner (BHL) in the SM language, based on the RG equation and the compositenes condition.
BHL essentially incorporates 1/Nc sub-leading effects such as those of the composite Higgs loops and ... gauge boson loops which were disregarded by the MTY formulation. We can explicitly see that BHL is in fact equivalent to MTY at 1/Nc-leading order. Such effects turned out to reduce the above MTY value 250 GeV down to 220 GeV ...".

8-dim Kaluza-Klein spacetime physics as required by Hashimoto, Tanabashi, and Yamawaki for the Middle State of the 3-State System was described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-LI05 in terms a M4xCP2 structure similar to that of the Cl(16)-E8 model. Although spacetime and Standard Model gauge bosons worked well for Batakis, he became discouraged by difficulties with fermions, perhaps because he did not use Clifford Algebras with natural spinor structures for fermions.

Calculations of the Low-Mass State of Higgs and Truth Quark have been given in Chapters 10 and 16 of this paper. Here are similar details for Middle and High Mass:

Middle Mass State:

In the Cl(16)-E8 model, the Middle-Mass Higgs has structure that is not restricted to Effective M4 Spacetime as is the case with the Low-Mass Higgs Ground State

but extends to the full 4+4 = 8-dim structure of M4xCP2 Kaluza-Klein.

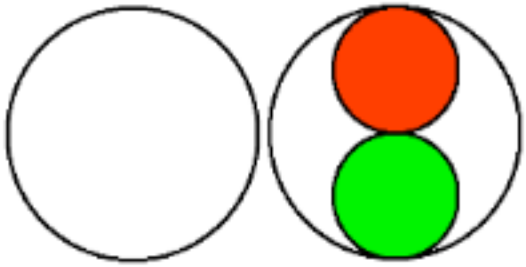
```
T ----------- Tbar        in CP2 Internal Symmetry Space
 \           /
  \         /
   \       /
    \     /
     \   /
    Higgs                 in M4 Physical SpaceTime
```

Therefore the Mid-Mass Higgs looks like a 3-particle system of Higgs + T + Tbar.

The T and Tbar form a Pion-like state.
Since Tquark Mid-Mass State is 174 GeV
the Middle-Mass T-Tbar that lives in the CP2 part of (4+4)-dim Kaluza-Klein
has mass (174+174) x (135 / (312+312) = 75 GeV.

The Higgs that lives in the M4 part of (4+4)-dim Kaluza-Klein
has, by itself, its Low-Mass Ground State Effective Mass of 125 GeV.
So, the total Mid-Mass Higgs lives in full 8-dim Kaluza-Klein
with mass 75+125 = 200 GeV.
This is consistent with the Mid-Mass States of the Higgs and Tquark
being on the Triviality Boundary of the Higgs - Tquark System
and with the 8-dim Kaluza-Klein model in hep-ph/0311165 by Hashimoto, Tanabashi, and Yamawaki.

As to the cross-section of the Middle-Mass Higgs



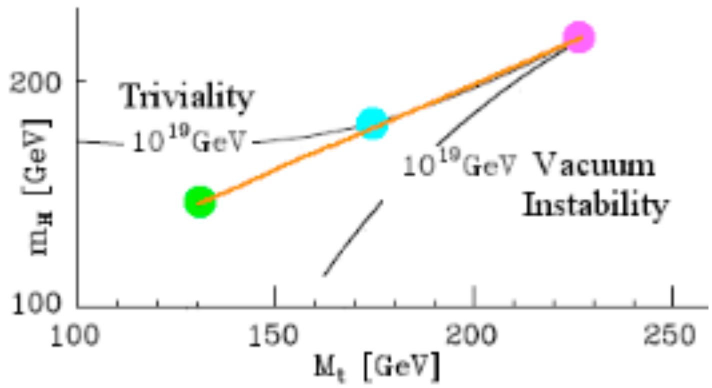
consider that the entire Ground State cross-section lives only in 4-dim M4 spacetime
(left white circle)
while the Middle-Mass Higgs cross-section lives in full 4+4 = 8-dim Kaluza-Klein
(right circle with red area only in CP2 ISS and white area partly in CP2 ISS
with only green area effectively living in 4-dim M4 spacetime)
so that
our 4-dim M4 Physical Spacetime experiments only see for the Middle-Mass Higgs
a cross-section that is 25% of the full Ground State cross-section.
The 25% may also be visualized in terms of 8-dim coordinates {1,i,j,k,E,I,J,K}

	1	i	j	k	E	I	J	K
1	11	1i	1j	1k	1E	1I	1J	1K
i	i1	ii	ij	ik	iE	iI	iJ	iK
j	j1	ji	jj	jk	jE	jI	jJ	jK
k	k1	ki	kj	kk	kE	kI	kJ	kK
E	E1	Ei	Ej	Ek	EE	EI	EJ	EK
I	I1	Ii	Ij	Ik	IE	II	IJ	IK
J	J1	Ji	Jj	Jk	JE	JI	JJ	JK
K	K1	Ki	Kj	Kk	KE	KI	KJ	KK

in which {1,i,j,k} represent M4 and {E,I,J,K} represent CP2.

High Mass State:

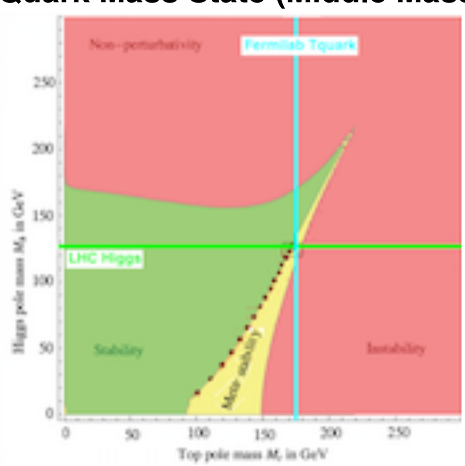
In the Cl(16)-E8 model, the the High-Mass Higgs State is at the Critical Point of the Higgs-Tquark System



(from hep-ph/0307138)

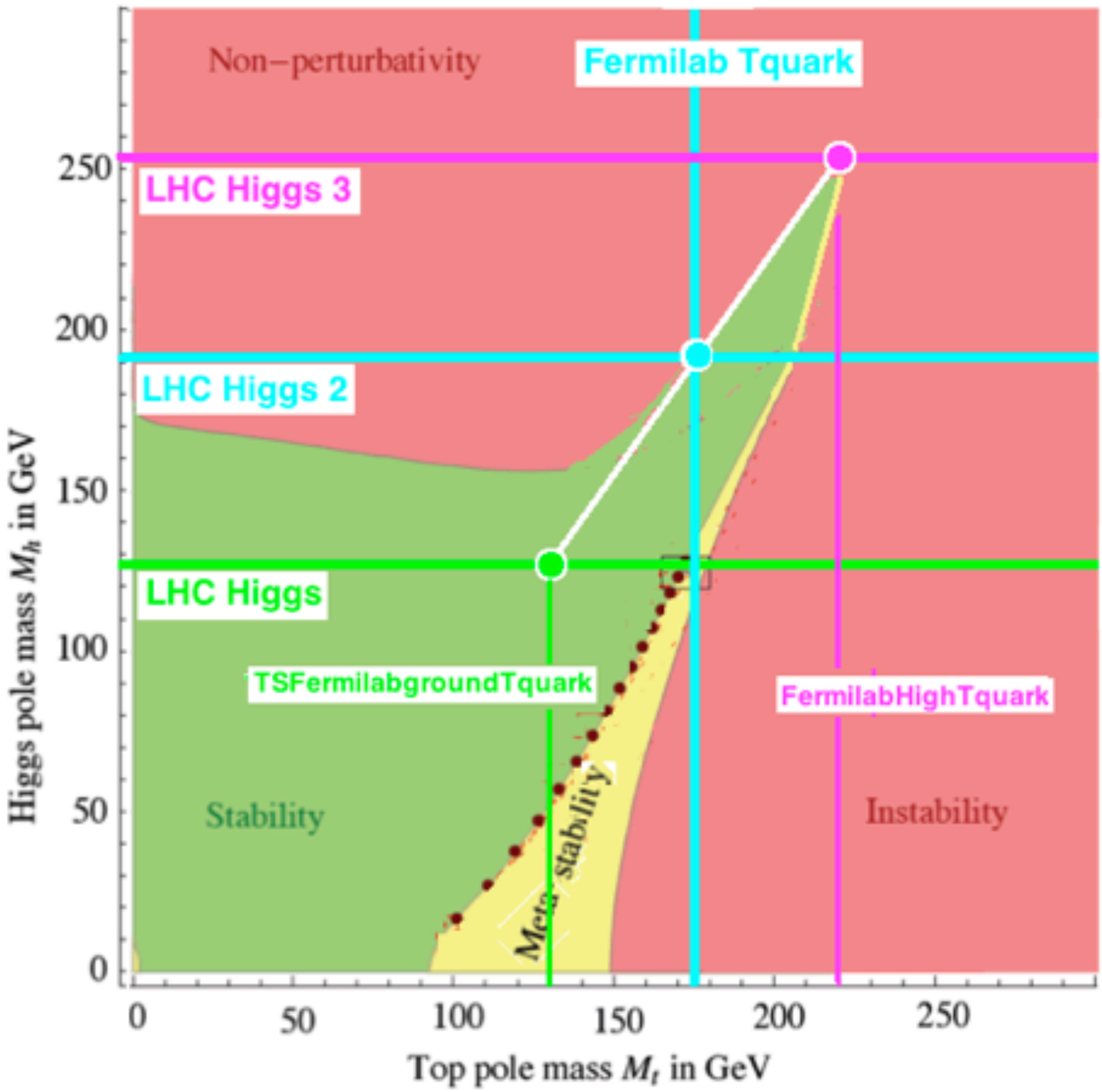
where the Triviality Boundary intersects the Vacuum Instability Boundary which is also at the Higgs Vacuum Expectation Value VEV around 250 GeV.
As with the Middle-Mass Higgs,
the High-Mass Higgs lives in all 4+4 = 8 Kaluza-Klein dimensions
and so has a cross-section that is about 25% of the Higgs Ground State cross-section.

The Cl(16)-E8 model view is 3 Mass States for Higgs and Truth Quark.
Opposed to the Cl(16)-E8 view is the Fermilab / CERN / Establishment view that there is only one Higgs Mass State (Low Mass around 126 GeV) and only one Truth Quark Mass State (Middle Mass around 174 GeV).



Their view is represented in the above Mh-Mt diagram adapted from arXiv 1307.3536 by Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, and Strumia who say "... from data ... of the Higgs ... and the ... [Tquark] Yukawa coupling ... we extrapolate ... SM parameters up to large energies ... Then we study the phase diagram of the Standard Model in term of high-energy parameters, finding that
the measured Higgs mass roughly corresponds to ... vacuum metastability ...
the SM Higgs vacuum is not the true vacuum ... our universe is potentially unstable ...".

The Cl(16)-E8 model has no vacuum metastability problem
because it has 3 sets of Higgs-Tquark mass states which modify the phase diagram



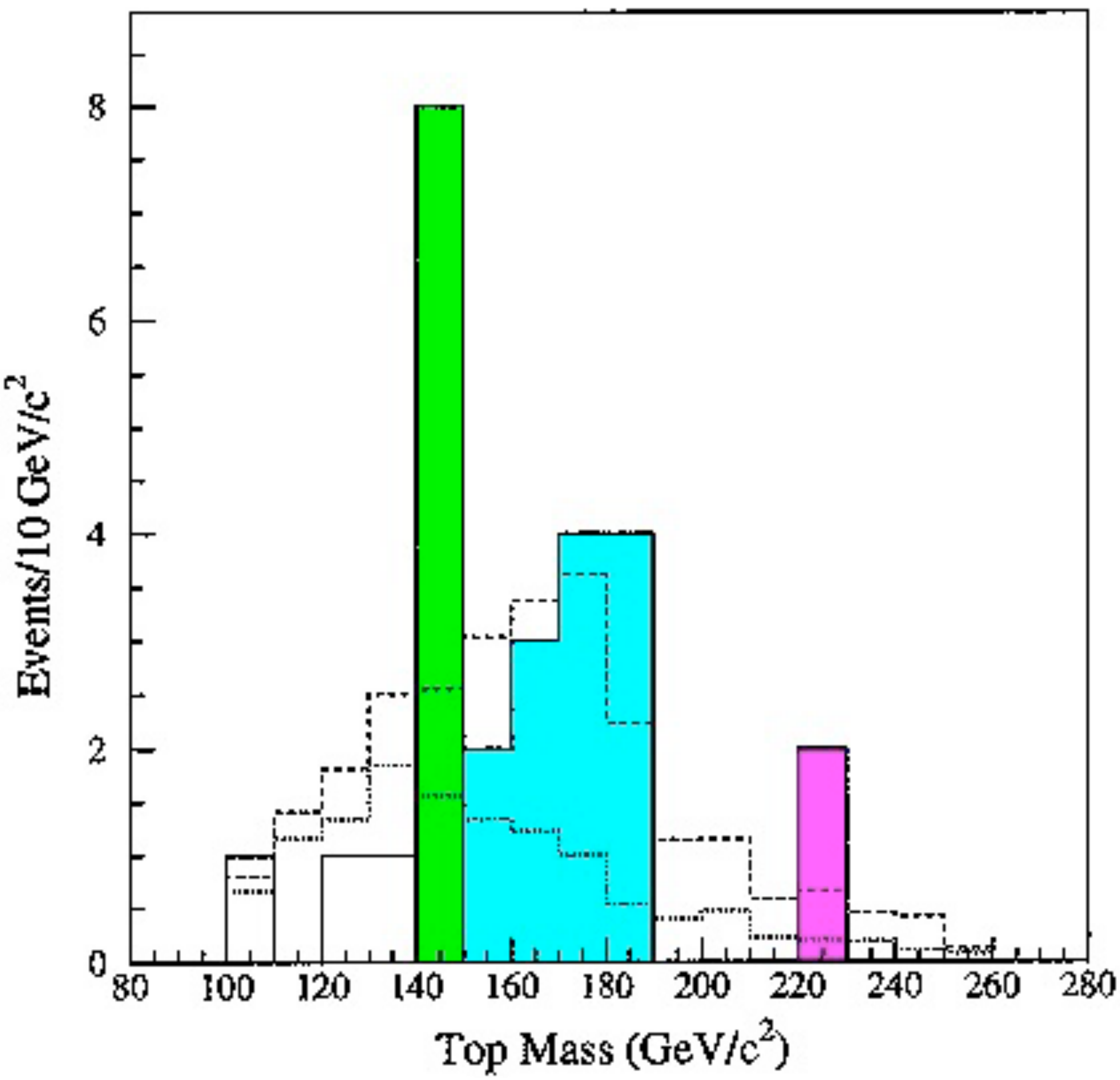
so that the Low-Mass Ground State is in the region of Stability
and the Middle-Mass State is at the boundary of Non-Perturbativity
and the High-Mass State at the Critical Point has Higgs mass = Higgs VEV.

The two additional Tquark mass states and Higgs mass states are
not recognized by the Fermilab/CERN/Establishment.

The two Tquark states (TSFermilabgroundTquark and FermilabHighTquark) have been seen at Fermilab and
the LHC has seen indications of the Two Higgs states (LHC Higgs 2 and LHC Higgs 3) whose status should be clarified by the 2015-2016 LHC Run.

Here are details of those additional Fermilab and LHC states:

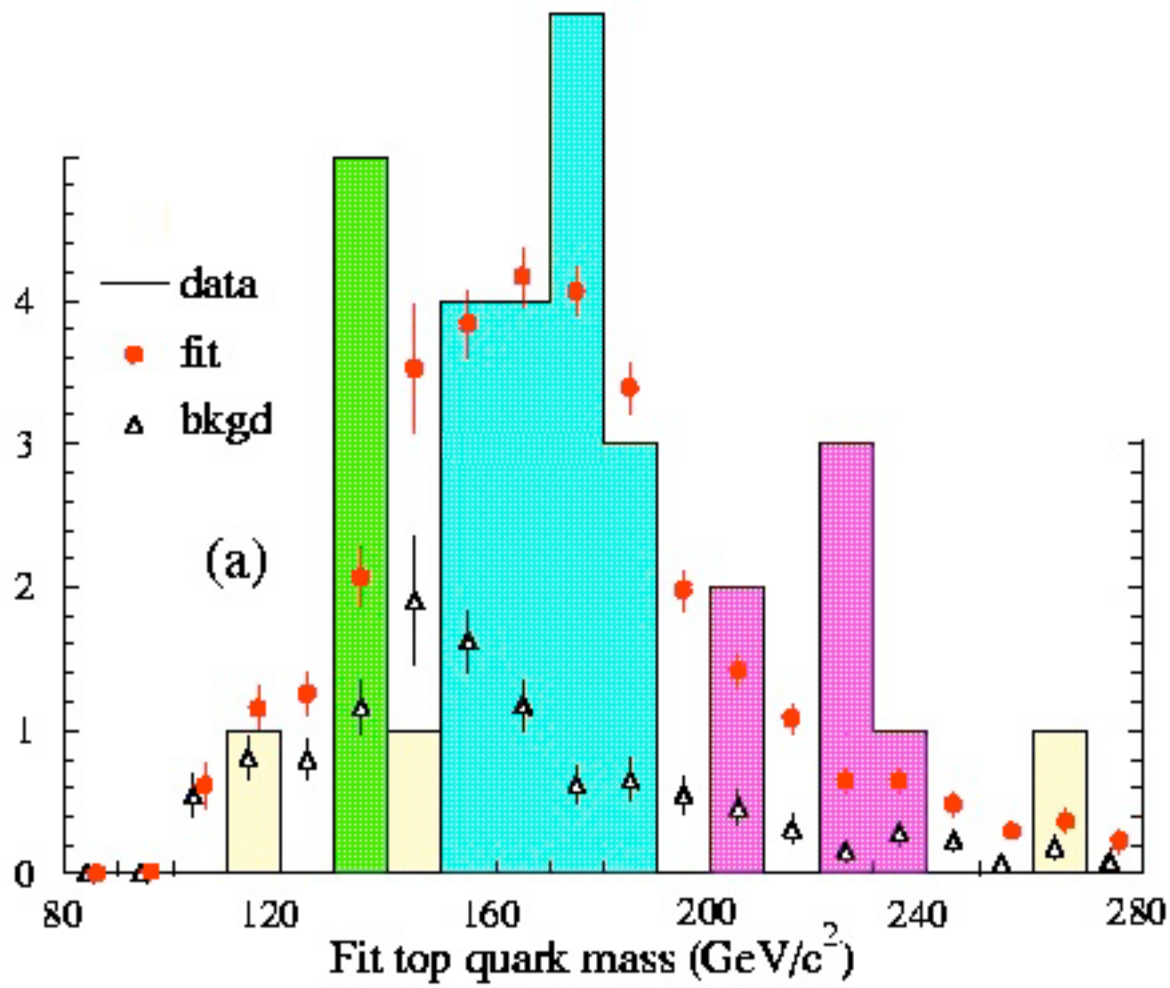
In 1994 a semileptonic histogram from CDF



(from FERMILAB-PUB-94/097-E)

seems to me to show all three states of the T-quark.

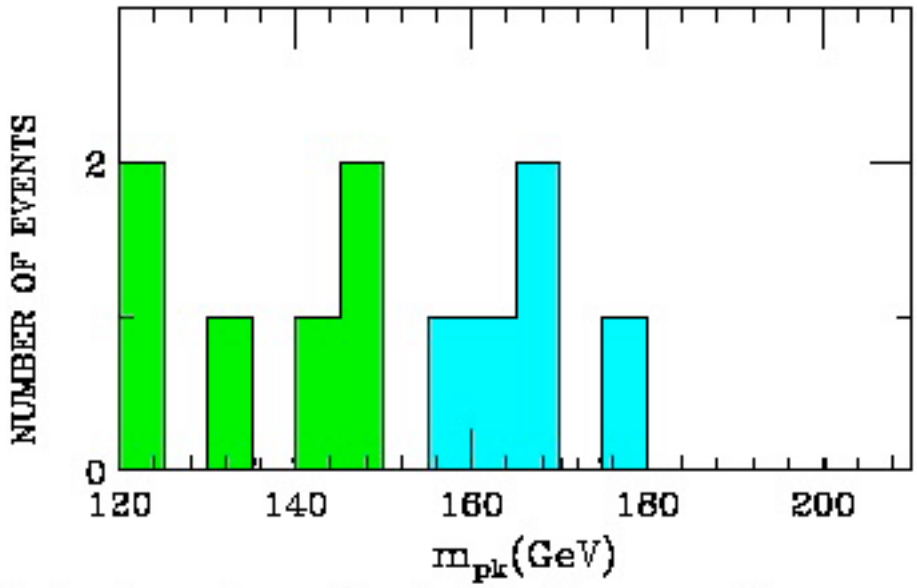
In 1997 a semileptonic histogram from D0



(from hep-ex/9703008)

also seems to me to show all three states of the T-quark.
The fact that the low (green) state showed up in both independent detectors indicates
a significance of 4 sigma.
Some object that the low (green) state peak should be as wide as the peak for the middle (cyan) state,
but
my opinion is that the middle (cyan) state should be wide because it is on the Triviality boundary where the composite nature of the Higgs as T-Tbar condensate becomes manifest and
the low (cyan) state should be narrow because it is in the usual non-trivial region where the T-quark acts more nearly as a single individual particle.

In 1998 a dilepton histogram from CDF

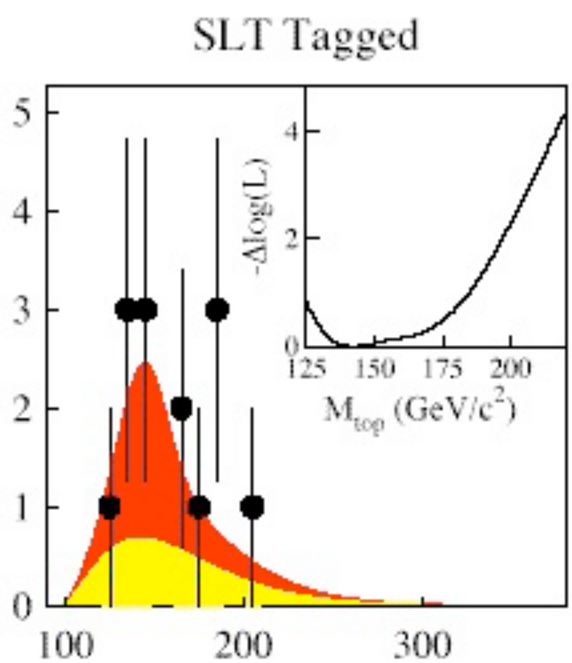


The distribution of m_{pk} values determined from 11 CDF dilepton events available empirically.

(from hep-ex/9802017)

seems to me to show both the low (green) state and the middle (cyan) T-quark state.

In 1998 an analysis of 14 SLT tagged lepton + 4 jet events by CDF

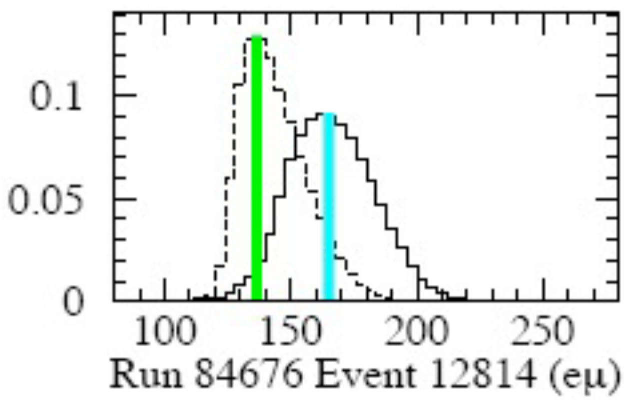


(from hep-ex/9801014)

showed a T-quark mass of 142 GeV (+33,-14) that seems to me to be consistent with the low (green) state of the T-quark.

In 1997 the Ph.D. thesis of Erich Ward Varnes (Varnes-fermilab-thesis-1997-28) at page 159 said:

"... distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest ET jets in the reconstruction ...



...” (colored bars added by me)

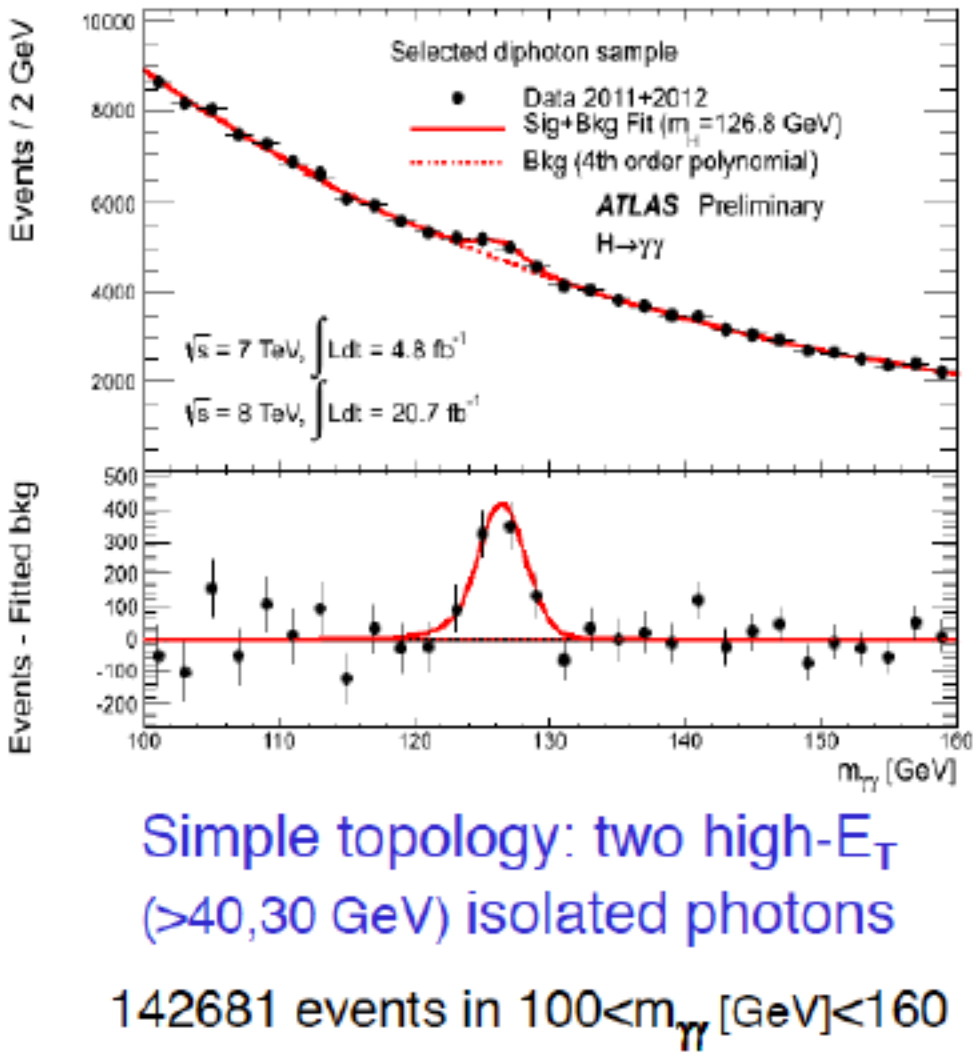
The event for all 3 jets (solid curve) seens to me to correspond to
decay of a middle (cyan) T-quark state
with one of the 3 jets corresponding to
decay from the Triviality boundary down to the low (green) T-quark state,
whose immediately subsequent decay is corresponds to the 2-jet (dashed curve) event at the low (green) energy level.

After 1998 Fermilab and CERN have focussed attention on detailed analysis
of the middle (cyan) T-quark state, getting much valuable detailed information
about it but not producing much information about the low or high Tquark states.

In the 25/fb of data collected through the run ending with the long shutdown at the end of 2012,
the LHC has observed a 126 GeV state of the Standard Model Higgs boson.

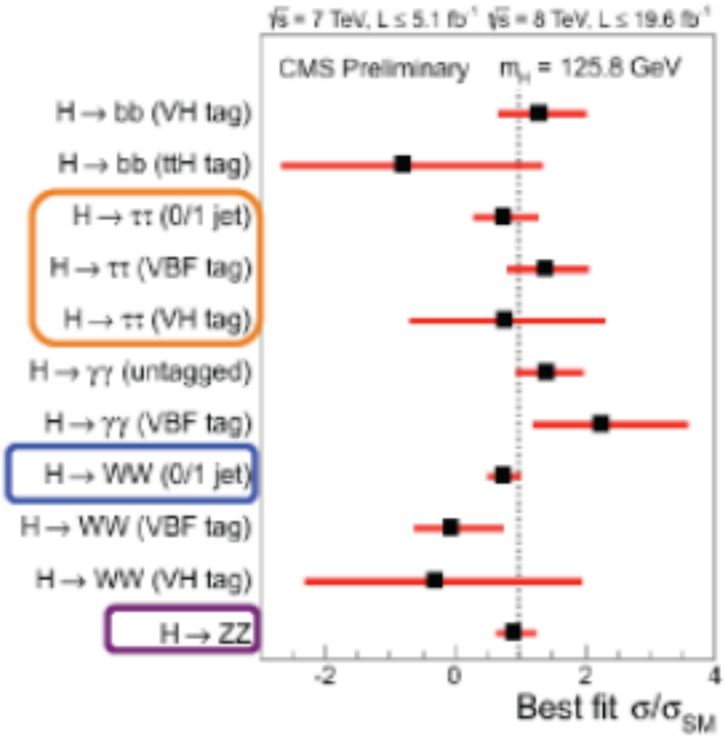
Here are some details about the LHC observation at 126 GeV and related results shown at Moriond 2013:

The digamma histogram for ATLAS



shows only one peak below 160 GeV and it is around 126 GeV.

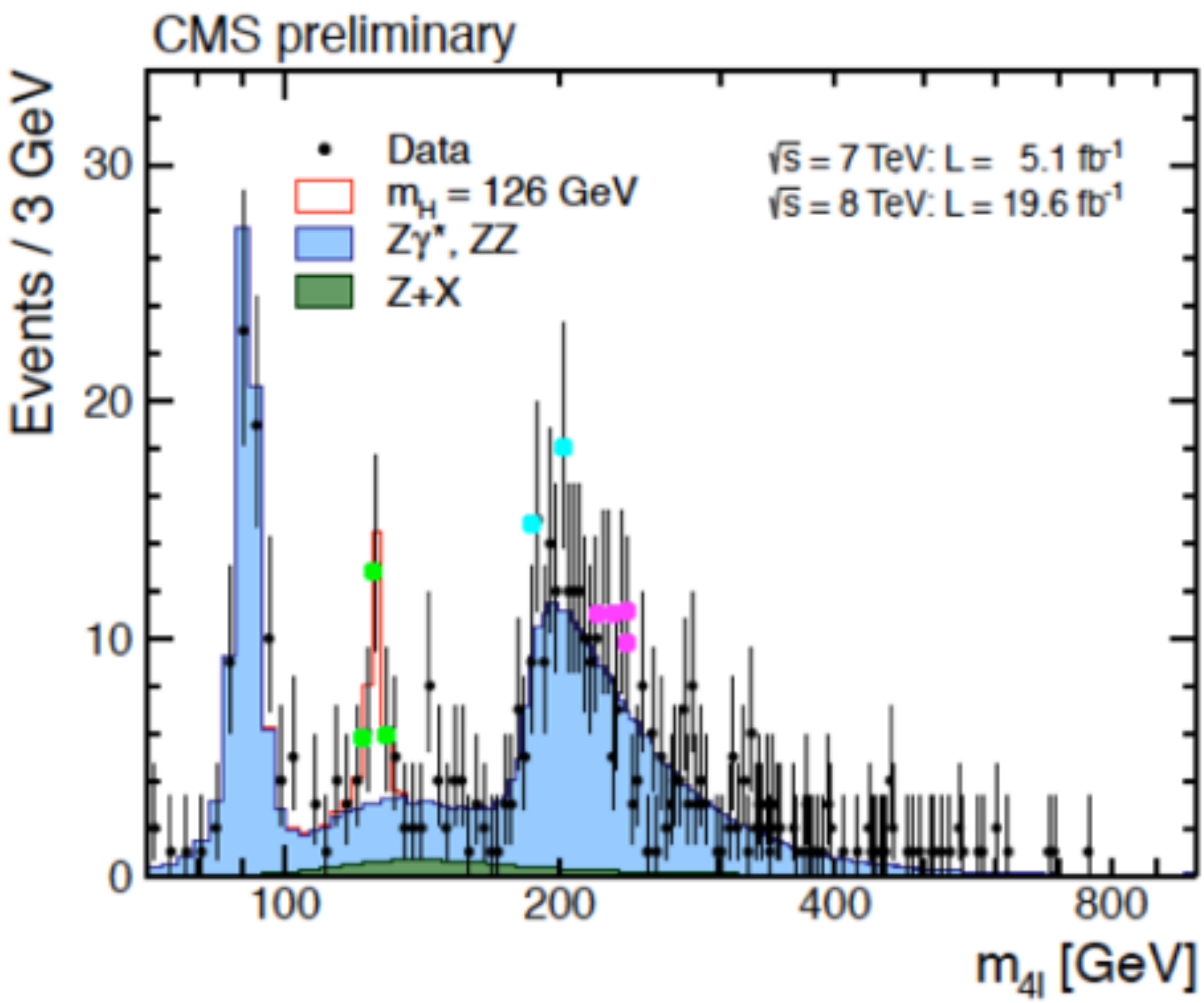
CMS shows the cross sections for Higgs at 125.8 GeV

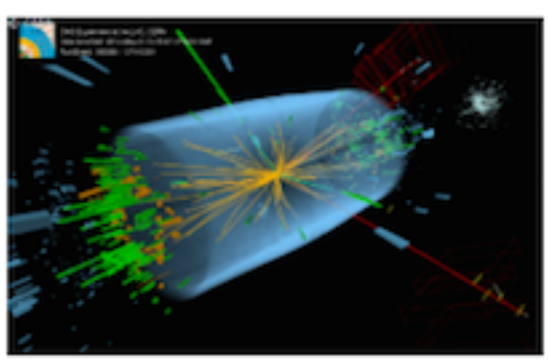


$H \rightarrow ZZ$(0/1 jet) : $0.84^{+0.32}_{-0.26}$
$H \rightarrow ZZ$(dijet tag): $1.22^{+0.84}_{-0.57}$

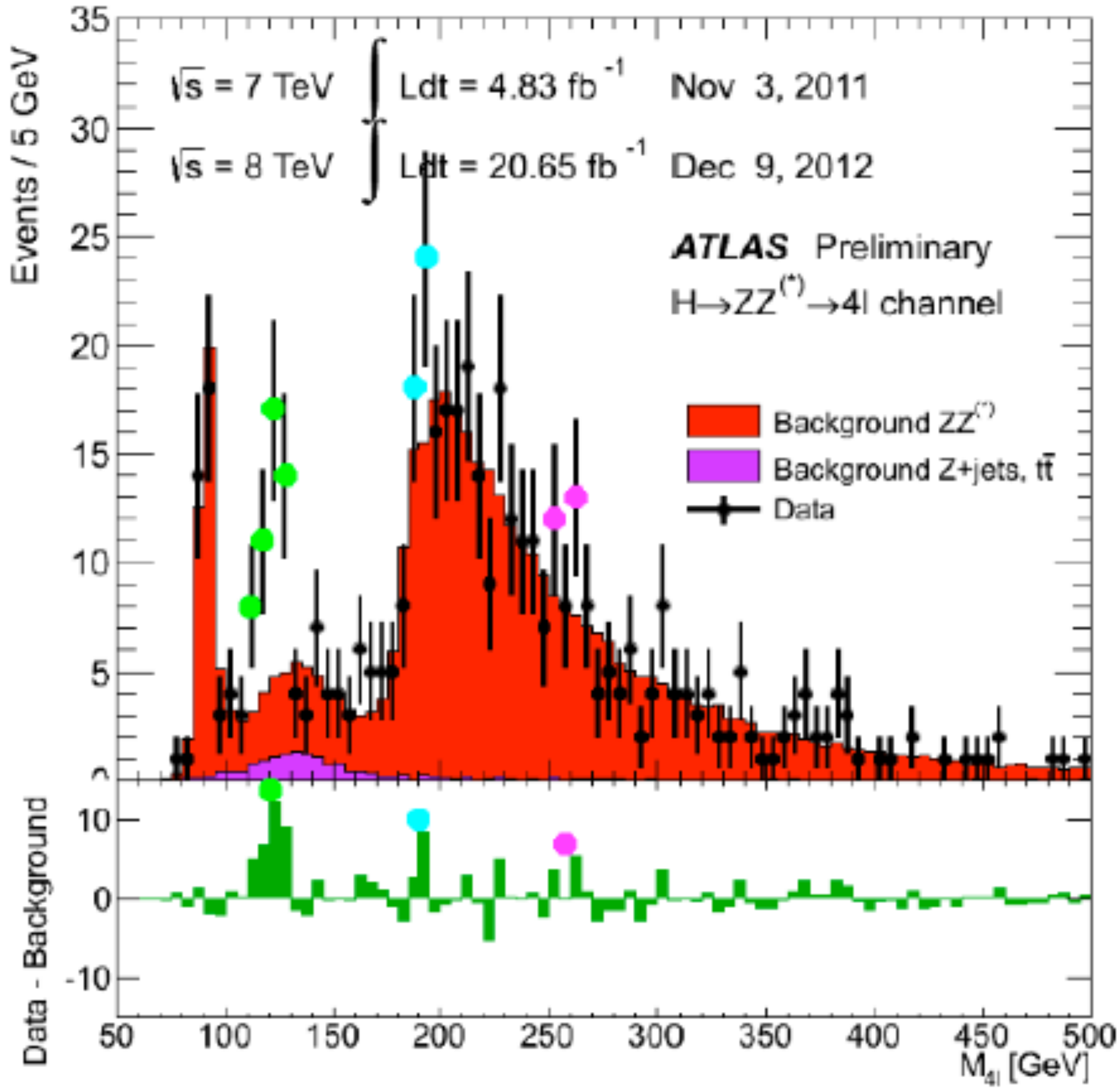
to be substantially consistent with the Standard Model for the WW and ZZ channels, a bit low for tau-tau and bb channels (but that is likely due to very low statistics there), and a bit high for the digamma channel (but that may be due to phenomena related to the Higgs as a Tquark condensate).

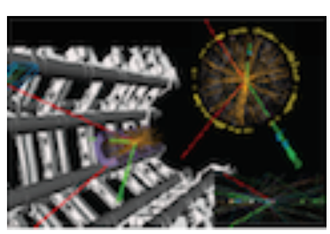
A CMS histogram (some colors added by me) for the Golden Channel Higgs to ZZ to 4l shows the peak around 126 GeV (green dots - lowHiggs mass state).
The CMS histogram also indicates other excesses
around 200 GeV (cyan dots - midHiggs mass state)
and around 250 GeV (magenta dots - highHiggs mass state).
An image of one of the events is shown below the histogram.



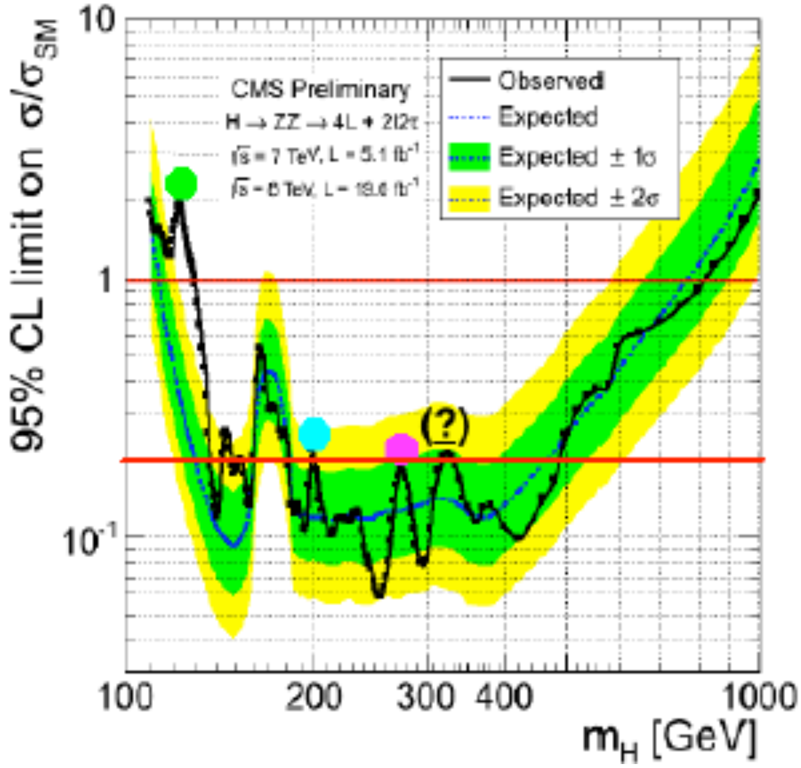


An ATLAS ZZ to 4l histogram (some colors added by me)
show the peak around 126 GeV (green dots - low Higgs mass state.
The ATLAS histogram also indicates other excesses
around 200 GeV (cyan dots - middle Higgs mass state)
and around 250 GeV (magenta dots - high Higgs mass state) .
An image of one of the events is shown below the histogram.





CMS showed a Brazil Band Plot for the High Mass Higgs to ZZ to 4l/2l2tau channel where
the top red line represents the expected cross section of a single Standard Model Higgs and the lower red line represents about 20% of the expected Higgs SM cross section.



The green dot peak is at the 126 GeV Low Mass Higgs state
with expected Standard Model cross section.
The cyan dot peak is around the 200 GeV Mid Mass Higgs state
expected to have about 25% of the SM cross section.
The magenta dot peak is around the 250 (+/- 20 or so) GeV High Mass Higgs state
expected to have about 25% of the SM cross section.

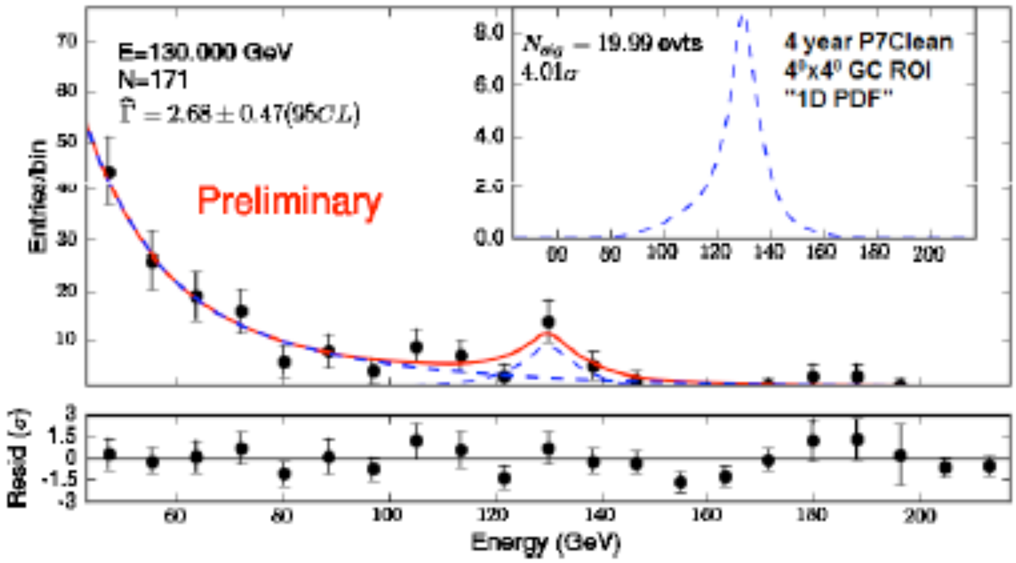
The (?) peak is around 320 GeV where I would not expect a Higgs Mass state and I note that in fact it seems to have gone away in the full ATLAS ZZ to 4l histogram shown above because between 300 and 350 GeV the two sort-of-high excess bins are adjacent to deficient bins .
It will probably be no earlier than 2015 (after the long shutdown) that the LHC will produce substantially more data than the 25/fb available at Moriond 2013 and therefore no earlier than 2016 for the green and yellow Brazil Bands to be pushed down (throughout the 170 GeV to 500 GeV region) below 10 per cent (the 10^(-1) line) of the SM cross section as is needed to show whether or not the cyan dot, magenta dot, and/or (?) peaks are real or statistical fluctuations.

My guess (based on the Cl(16)-E8 model) is that the cyan dot and magenta dot peaks will prove to be real and that the (?) peak will go away as a statistical fluctuation.

Sgr A* and Higgs = Tquark-Tantiquark Condensate

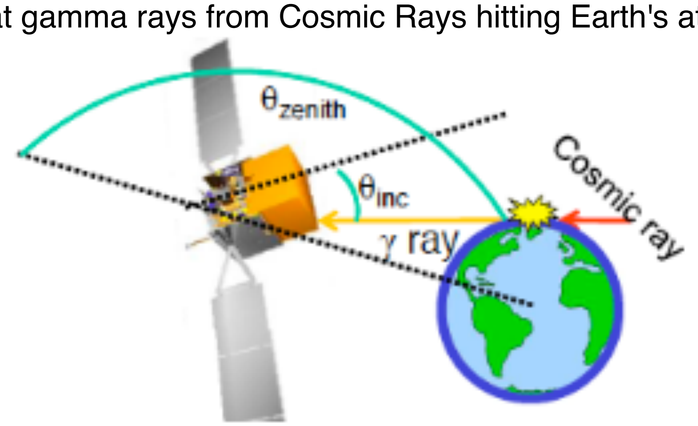
Sagittarius A* (Sgr A*) is a very massive black hole in the center of our Galaxy into which large amounts of Hydrogen fall. As the Hydrogen approaches Sgr A* it increases in energy, ionizing into protons and electrons, and eventually producing a fairly dense cloud of infalling energetic protons whose collisions with ambient protons are at energies similar to the proton-proton collisions at the LHC.

Andrea Albert at The Fermi Symposium 11/2/2012 said: "... gamma rays detectable by the Fermi Large Area Telescope [FLAT] ...



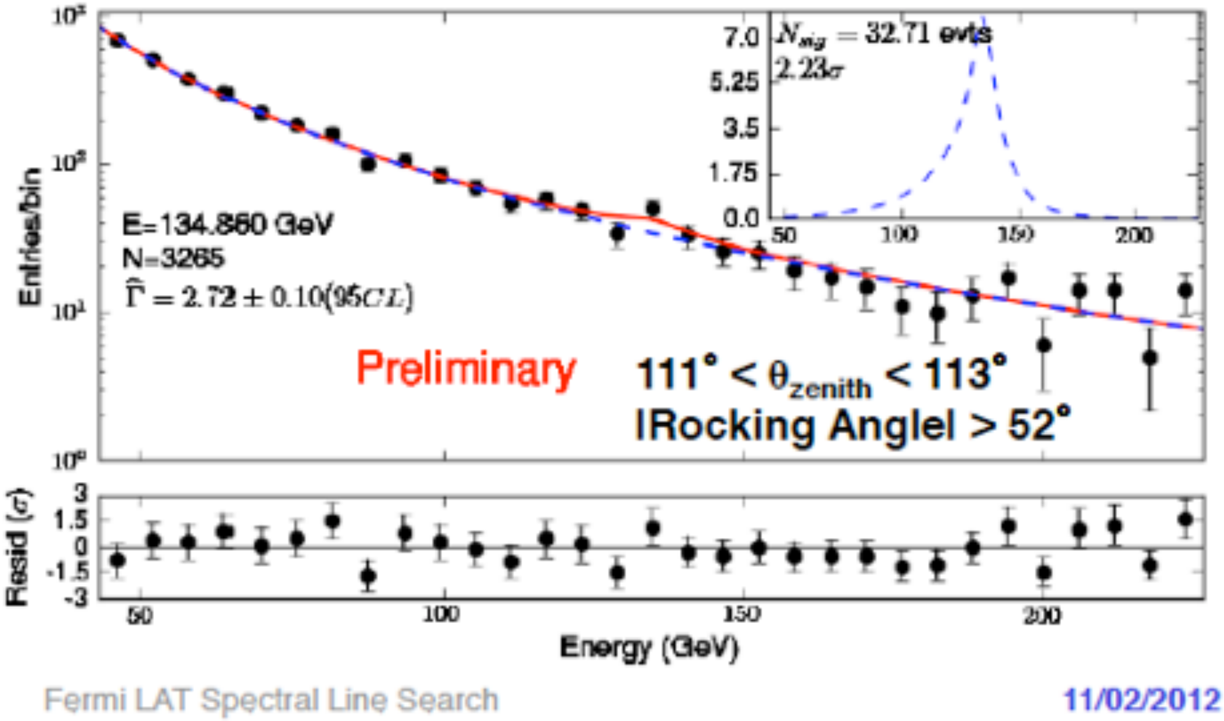
... Line-like Feature near 135 GeV ... localized in the galactic center ...".

In addition to the Galactic Center observations,
Fermi LAT looked at gamma rays from Cosmic Rays hitting Earth's atmosphere



by looking at the Earth Limb.

Andrea Albert at The Fermi Symposium 11/2/2012 also said: "... Earth Limb is a bright gamma-ray source ... From cosmic-ray interactions in the atmosphere ...



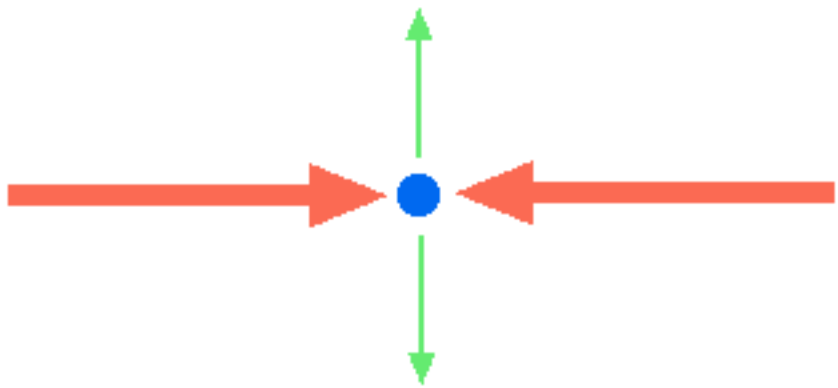
... Line-like feature ... at 135 GeV .. Appears when LAT is pointing at the Limb ...".

Since 90% of high-energy Cosmic Rays are Protons and since their collisions with Protons and other nuclei in Earth's atmosphere produce gamma rays,
the 135 GeV Earth Limb Line seen by Fermi LAT is also likely to be the Higgs produced by collisions analagous to those at the LHC.

Olivier K. in a comment in Jester's blog on 10 November 2012 said:
"... Could the 135 GeV bump be related ... to current Higgs ... properties ? ...
The coincidence between GeV figures ...[for LHC] Higgs mass and
this [Fermi LAT] bump is thrilling for an amateur like me...".

Jester in his resonaances blog on 17 April 2012 said, about Fermi LAT:
"... the plot shows the energy of *single* photons as measured by Fermi,
not the invariant mass of photon pairs ...".
Since the LHC 125 GeV peak is for "invariant mass of photon pairs"
and the Fermi LAT 135 GeV peak is for ""single" photons"
how could both correspond to a Higgs mass state around 130 GeV ?

The LHC sees collisions of high-energy protons (red arrows) forming Higgs (blue dot)



with the Higgs at rest decaying into a photon pair (green arrows)
giving the observed Higgs peak (around 130 GeV) as invariant mass of photon pairs.

Fermi LAT at Galactic Center and Earth Limb sees collisions of one high-energy proton
with a low-energy (relatively at rest) proton forming Higgs

with Higgs moving fast from momentum inherited from the high-enrgy proton decaying
into two photons: one with low energy not observed by Fermi LAT
and the other being observed by Fermi LAT as a high-energy gamma ray carrying
almost all of the Higgs decay energy (around 130 GeV) as a "single" photon.

Therefore, the coincidence noted by Olivier K. is probably a realistic phenomenon.

18. Segal-type Conformal gravity with conformal generator structure giving Dark Energy, Dark Matter, and Ordinary Matter ratio

MacDowell-Mansouri Gravity is described by Rabindra Mohapatra in section 14.6 of his book “Unification and Supersymmetry”:

§14.6. Local Conformal Symmetry and Gravity

Before we study supergravity, with the new algebraic approach developed, we would like to discuss how gravitational theory can emerge from the gauging of conformal symmetry. For this purpose we briefly present the general notation for constructing gauge covariant fields. The general procedure is to start with the Lie algebra of generators X_A of a group

$$[X_A, X_B] = f_{AB}^{C} X_C, \tag{14.6.1}$$

where f_{AB}^{C} are structure constants of the group. We can then introduce a gauge field connection h_μ^A as follows:

$$h_\mu = h_\mu^A X_A. \tag{14.6.2}$$

Let us denote the parameter associated with X_A by ε^A. The gauge transformations on the fields h_μ^A are given as follows:

$$\delta h_\mu^A = \partial_\mu \varepsilon^A + h_\mu^B \varepsilon^C f_{CB}^{A} \equiv (D_\mu \varepsilon)^A. \tag{14.6.3}$$

We can then define a covariant curvature

$$R_{\mu\nu}^A = \partial_\nu h_\mu^A - \partial_\mu h_\nu^A + h_\nu^B h_\mu^C f_{CB}^{A}. \tag{14.6.4}$$

Under a gauge transformation

$$\delta_{\text{gauge}} R_{\mu\nu}^A = R_{\mu\nu}^B \varepsilon^C f_{CB}^{A}. \tag{14.6.5}$$

We can then write the general gauge invariant action as follows:

$$I = \int d^4x\, Q_{AB}^{\mu\nu\rho\sigma} R_{\mu\nu}^A R_{\rho\sigma}^B. \tag{14.6.6}$$

Let us now apply this formalism to conformal gravity. In this case

$$h_\mu = P_m e_\mu^m + M_{mn}\omega_\mu^{mn} + K_m f_\mu^m + D b_\mu. \tag{14.6.7}$$

The various $R_{\mu\nu}$ are

$$R_{\mu\nu}(P) = \partial_\nu e_\mu^m - \partial_\mu e_\nu^m + \omega_\mu^{mn} e_\nu^n - \omega_\nu^{mn} e_\mu^n - b_\mu e_\nu^m + b_\nu e_\mu^m, \tag{14.6.8}$$

$$R_{\mu\nu}(M) = \partial_\nu \omega_\mu^{mn} - \partial_\mu \omega_\nu^{mn} - \omega_\nu^{mp}\omega_{\mu,p}^{n} - \omega_\mu^{mp}\omega_{\nu,p}^{n} - 4(e_\mu^m f_\nu^n - e_\nu^m f_\mu^n), \tag{14.6.9}$$

$$R_{\mu\nu}(K) = \partial_\nu f_\mu^m - \partial_\mu f_\nu^m - b_\mu f_\nu^m + b_\nu f_\mu^m + \omega_\mu^{mn} f_\nu^n - \omega_\nu^{mn} f_\mu^n, \tag{14.6.10}$$

$$R_{\mu\nu}(D) = \partial_\nu b_\mu - \partial_\mu b_\nu + 2e_\mu^m f_\nu^m - 2e_\nu^m f_\mu^m. \tag{14.6.11}$$

The gauge invariant Lagrangian for the gravitational field can now be written down, using eqn. (14.6.6), as

$$S = \int d^4x\, \varepsilon_{mnrs}\varepsilon^{\mu\nu\rho\sigma} R_{\mu\nu}^{mn}(M) R_{\rho\sigma}^{rs}(M). \tag{14.6.12}$$

We also impose the constraint that

$$R_{\mu\nu}(P) = 0, \tag{14.6.13}$$

which expresses ω_μ^{mn} as a function of (e, b). The reason for imposing this constraint has to do with the fact that P_m transformations must be eventually identified with coordinate transformation. To see this point more explicitly let us consider the vierbein e_μ^m. Under coordinate transformations

$$\delta_{GC}(\xi^\nu)e_\mu^m = \partial_\mu \xi^\lambda e_\lambda^m + \xi^\lambda \partial_\lambda e_\mu^m. \tag{14.6.14}$$

Using eqn. (14.6.8) we can rewrite

$$\delta_{GC}(\xi^\nu)e_\mu^m = \delta_P(\xi e^n)e_\mu^m + \delta_M(\xi\omega^{mn})e_\mu^m + \delta_D(\xi b)e_\mu^m + \xi^\nu R_{\mu\nu}^m(P),$$

where

$$\delta_P(\xi^n)e_\mu^m = \partial_\mu \xi^m + \xi^n \omega_\mu^{mn} + \xi^m b_\mu. \tag{14.6.15}$$

If $R^{\mu\nu}(P) = 0$, the general coordinate transformation becomes related to a set of gauge transformations via eqn. (14.6.15).

At this point we also wish to point out how we can define the covariant derivative. In the case of internal symmetries $D_\mu = \partial_\mu - iX_A h_\mu^A$; now since momentum is treated as an internal symmetry we have to give a rule. This follows from eqn. (14.6.15) by writing a redefined translation generator $\bar{P}$ such that

$$\delta_{\bar{P}}(\xi) = \delta_{GC}(\xi^\nu) - \sum_{A'} \delta_{A'}(\xi^m h_m^A), \tag{14.6.16}$$

where A' goes over all gauge transformations excluding translation. The rule is

$$\delta_{\bar{P}}(\xi^m)\phi = \xi^m D_m^C \phi. \tag{14.6.17}$$

We also wish to point out that for fields which carry spin or conformal charge, only the intrinsic parts contribute to D_m^C and the orbital parts do not play any rule.

Coming back to the constraints we can then vary the action with respect to f_μ^m to get an expression for it, i.e.,

$$e_\nu^m f_{\mu m} = -\tfrac{1}{4}[e_m^\lambda e_{n\nu} R_{\mu\lambda}^{mn} - \tfrac{1}{6} g_{\mu\nu} R], \tag{14.6.18}$$

where f_μ^m has been set to zero in R written in the right-hand side.

This eliminates (from the theory the degrees of freedom) ω_μ^{mn} and f_μ^m and we are left with e_μ^m and b_μ. Furthermore, these constraints will change the transformation laws for the dependent fields so that the constraints do not change.

Let us now look at the matter coupling to see how the familiar gravity theory emerges from this version. Consider a scalar field ϕ. It has conformal weight $\lambda = 1$. So we can write a convariant derivative for it, eqn. (14.6.17)

$$D_\mu^C \phi = \partial_\mu \phi - \phi b_\mu. \tag{14.6.19}$$

We note that the conformal charge of ϕ can be assumed to be zero since $K_m = x^2 \partial$ and is the dimension of inverse mass. In order to calculate $\Box^c \phi$ we start with the expression for d'Alambertian in general relativity

$$\frac{1}{e}\partial_\nu(g^{\mu\nu}eD^C_\mu\phi). \tag{14.6.20}$$

The only transformations we have to compensate for are the conformal transformations and the scale transformations. Since

$$\delta b_\mu = -2\xi^m_k e_{m\mu}, \qquad \delta(\phi b_\mu) = \phi\delta b_\mu = -2\phi f^m_\mu e^\mu_m = +\tfrac{2}{12}\phi R, \tag{14.6.21}$$

where, in the last step, we have used the constraint equation (14.6.18). Putting all these together we find

$$\Box^C\phi = \frac{1}{e}\partial_\nu(g^{\mu\nu}eD^c_\mu\phi) + b_\mu D^c_\mu\phi + \tfrac{2}{12}\phi R. \tag{14.6.22}$$

Thus, the Lagrangian for conformal gravity coupled to matter fields can be written as

$$S = \int e\, d^4x\, \tfrac{1}{2}\phi\Box^C\phi. \tag{14.6.23}$$

Now we can use conformal transformation to gauge $b_\mu = 0$ and local scale transformation to set $\phi = \kappa^{-1}$ leading to the usual Hilbert action for gravity. To summarize, we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. We will adopt the same procedure for supergravity. An important technical point to remember is that, $\Box^C$, the conformal d'Alambertian contains R, which for constant ϕ, leads to gravity. We may call ϕ the auxiliary field.

After the scale and conformal gauges have been fixed,
the conformal Lagrangian becomes a de Sitter Lagrangian.

Einstein-Hilbert gravity can be derived from the de Sitter Lagrangian, as was first shown by MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739).
(Frank Wilczek, in hep-th/9801184 says that the MacDowell-Mansouri "... approach to casting gravity as a gauge theory was initiated by MacDowell and Mansouri ... S. MacDowell and F. Mansouri, Phys. Rev. Lett. 38 739 (1977) ... , and independently Chamseddine and West ... A. Chamseddine and P. West Nucl. Phys. B 129, 39 (1977); also quite relevant is A. Chamseddine, Ann. Phys. 113, 219 (1978). ...".)

The minimal group required to produce Gravity,
and therefore **the group that is used in calculating Force Strengths**,
is the [anti] de Sitter group, as is described by
Freund in chapter 21 of his book Supersymmetry (Cambridge 1986) (chapter 21 is a Non-Supersymmetry chapter leading up to a Supergravity description in the following chapter 22):
"... Einstein gravity as a gauge theory ... we expect a set of gauge fields w^ab_u for the Lorentz group and a further set e^a_u for the translations, ...
Everybody knows though, that Einstein's theory contains but one spin two field, originally chosen by Einstein as g_uv = e^a_u e^b_v n_ab
(n_ab = Minkowski metric).
What happened to the w^ab_u ?
The field equations obtained from the Hilbert-Einstein action by varying the w^ab_u are algebraic in the w^ab_u ... permitting us to express the w^ab_u in terms of the e^a_u ... The w do not propagate ...
We start from the four-dimensional de-Sitter algebra ... so(3,2).
Technically this is the anti-de-Sitter algebra ...
We envision space-time as a four-dimensional manifold M.
At each point of M we have a copy of SO(3,2) (a fibre ...) ...
and we introduce the gauge potentials (the connection) h^A_mu(x)
A = 1,..., 10 , mu = 1,...,4. Here x are local coordinates on M.
From these potentials h^A_mu we calculate the field-strengths
(curvature components) [let @ denote partial derivative]
R^A_munu = @_mu h^A_nu - @_nu h^A_mu + f^A_BC h^B_mu h^C_nu
...[where]... the structure constants f^C_AB ...[are for]... the anti-de-Sitter algebra ....
We now wish to write down the action S as an integral over
the four-manifold M ... S(Q) = INTEGRAL_M R^A /\ R^B Q_AB
where Q_AB are constants ... to be chosen ... we require
... the invariance of S(Q) under local Lorentz transformations
... the invariance of S(Q) under space inversions ...
...[AFTER A LOT OF ALGEBRA NOT SHOWN IN THIS QUOTE]...
we shall see ...[that]... the action becomes invariant
under all local [anti]de-Sitter transformations ...[and]... we recognize ... t
he familiar Hilbert-Einstein action with cosmological term in vierbein notation ...
Variation of the vierbein leads to the Einstein equations with cosmological term.
Variation of the spin-connection ... in turn ... yield the torsionless Christoffel connection ... the torsion components ... now vanish.
So at this level full sp(4) invariance has been checked.
... Were it not for the assumed space-inversion invariance ...
we could have had a parity violating gravity. ...
Unlike Einstein's theory ...[MacDowell-Mansouri].... does not require Riemannian invertibility of the metric. ... the solution has torsion ... produced by an interference between parity violating and parity conserving amplitudes.
Parity violation and torsion go hand-in-hand.
Independently of any more realistic parity violating solution of the gravity equations this raises the cosmological question whether
the universe as a whole is in a space-inversion symmetric configuration. ...".

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:
"... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological constant limit can be considered as limiting cases of the fundamental symmetry. ...
... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations. Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...
... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.
The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the conespace N. The points at infinity of M are concentrated in the vertex of the conespace N, and those on the light-cone of M becomes the infinity of N. It is concepts of space isotropy and equivalence between inertial frames in the conespace N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group,
which is made up of:

3 Rotations
3 Boosts
4 Translations
4 Special Conformal transformations
1 Dilatation

The **Cosmological Constant / Dark Energy** comes from
the **10 Rotation, Boost, and Special Conformal generators**
of the Conformal Group Spin(2,4) = SU(2,2),
so the fractional part of our Universe of the Cosmological Constant
should be **about 10 / 15 = 67% for tree level**.

Black Holes, including **Dark Matter Primordial Black Holes**, are curvature singularities in our 4-dimensional physical spacetime,
and since Einstein-Hilbert curvature comes from the **4 Translations**
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2)
through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate),
the fractional part of our Universe of Dark Matter Primordial Black Holes
should be **about 4 / 15 = 27% at tree level**.

Since **Ordinary Matter** gets mass from the Higgs mechanism
which is related to the **1 Scale Dilatation**
of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2),
the fractional part of our universe of Ordinary Matter
should be **about 1 / 15 = 6% at tree level**.

However,
as Our Universe evolves the Dark Energy, Dark Matter, and Ordinary Matter densities evolve at different rates,
so that the differences in evolution must be taken into account
from the initial End of Inflation to the Present Time.

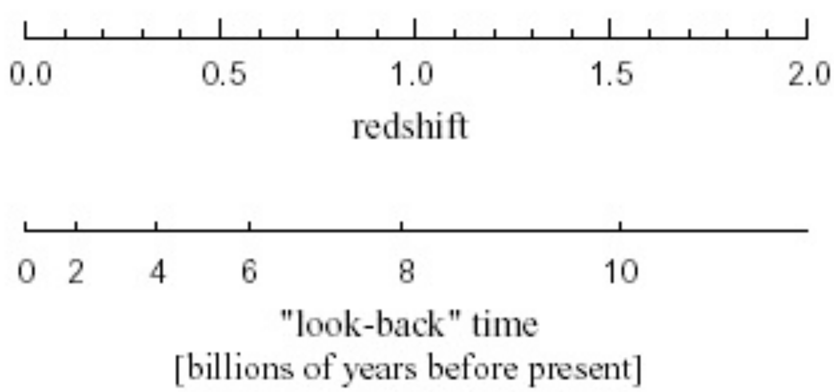
Without taking into account any evolutionary changes with time,
our Flat Expanding Universe should have roughly:

67% Cosmological Constant
27% Dark Matter - possilbly primordial stable Planck mass black holes
6% Ordinary Matter

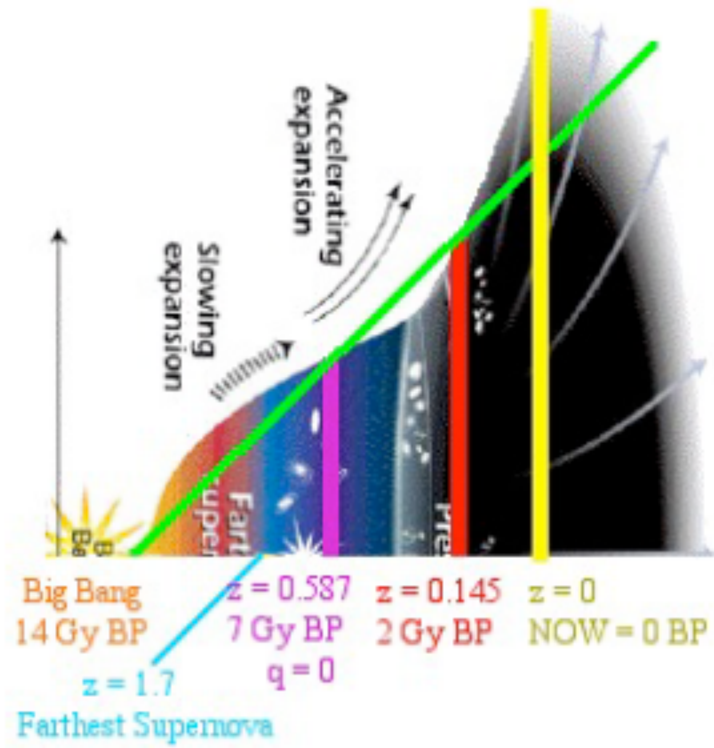
As Dennnis Marks pointed out to me,
since density rho is proportional to (1+z)^3(1+w) for red-shift factor z
and a constant equation of state w:
w = -1 for Λ and the average overall density of Λ Dark Energy remains constant with time and the expansion of our Universe;
and
w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands;
and
w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;
so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.
Therefore,
the above calculated ratio 0.67 : 0.27 : 0.06 is valid
only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio now ?

Since WMAP observes Ordinary Matter at 4% NOW,
the time when Ordinary Matter was 6% would be
at redshift z such that
1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145.
To translate redshift into time,
in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf to see that
the time when Ordinary Matter was 6%
would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:
the Big Bang Beginning of Inflation (about 13.7 Gy BP);

1 - the End of Inflation = Beginning of Decelerating Expansion
(beginning of green line also about 13.7 Gy BP);

2 - the End of Deceleration (q=0) = Inflection Point =
= Beginning of Accelerating Expansion
(purple vertical line at about z = 0.587 and about 7 Gy BP).
According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shallow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...".
According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...".
According to astro-ph/0106051 by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at z > 0.5 ... SN 1997ff at z = 1.7 provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

3 - the Last Intersection of the Accelerating Expansion of our Universe
of Linear Expansion (green line) with the Third Intersection
(at red vertical line at $z = 0.145$ and about 2 Gy BP),
which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

4 - Now.

Those four Special Times define four Special Epochs:

The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).

The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.

The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.

The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant Λ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.

Now happens to be about 2 billion years into the Late Accelerating Expansion Epoch.

What about Dark Energy : Dark Matter : Ordinary Matter now ?

As to how the Dark Energy Λ and Cold Dark Matter terms have evolved
during the past 2 Gy, a rough estimate analysis would be:

Λ and CDM would be effectively created during expansion in their natural ratio
67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;
CDM Black Hole decay would be ignored; and
pre-existing CDM Black Hole density would decline by the same 1 / R^3 factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the Λ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21
for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for now of

0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

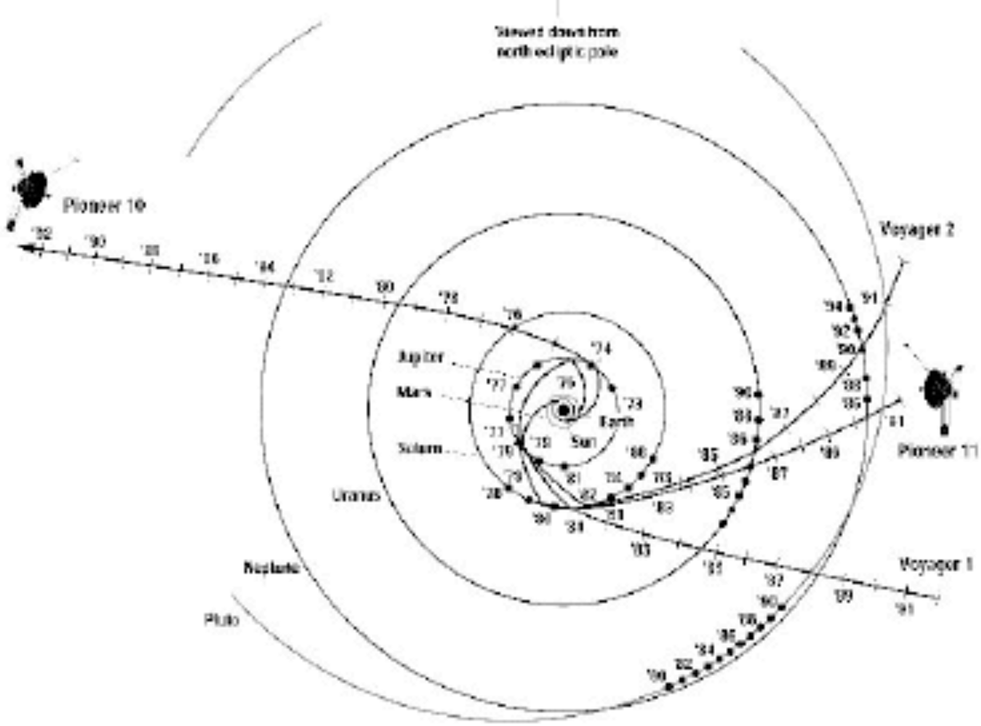
2013 Planck Data (arxiv 1303.5062) showed "... anomalies ... previously observed in the WMAP data ... alignment between the quadrupole and octopole moments ... asymmetry of power between two ... hemispheres ... Cold Spot ... are now confirmed at ... 3 sigma ... but a higher level of confidence ...".

Now the Cl(16)-E8 model rough evolution calculation is: DE : DM : OM = 75 : 20 : 05
WMAP: DE : DM : OM = 73 : 23 : 04
Planck: DE : DM : OM = 69 : 26 : 05
basic E8 Conformal calculation: DE : DM : OM = 67 : 27 : 06

Since uncertainties are substantial, I think that there is reasonable consistency.

19. Dark Energy explanations for Pioneer Anomaly and Uranus spin-axis tilt

After the Inflation Era and our Universe began its current phase of expansion, some regions of our Universe become Gravitationally Bound Domains (such as, for example, Galaxies) in which the 4 Conformal GraviPhoton generators are frozen out, forming domains within our Universe like IceBergs in an Ocean of Water.
On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.



Pioneer spacecraft are not bound to our Solar System and are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.
In their Study of the anomalous acceleration of Pioneer 10 and 11 gr-qc/0104064 John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about 68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....
[the above image is] Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager

trajectories. Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.
... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ...
The last communication from Pioneer 11 was received in November 1995, when the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should pass close to the nearest star in the constellation Aquila in about 4 million years ...
... Calculations of the motion of a spacecraft are made on the basis of the range time-delay and/or the Doppler shift in the signals. This type of data was used to determine the positions, the velocities, and the magnitudes of the orientation maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ... analyses of radio Doppler ... data ... indicated that an apparent anomalous acceleration is acting on Pioneer 10 and 11 ... The data implied an anomalous, constant acceleration with a magnitude a_P = 8 x 10^(-8) cm/cm/s^2, directed towards the Sun ...
... the size of the anomalous acceleration is of the order c H, where H is the Hubble constant ...
... Without using the apparent acceleration, CHASMP shows a steady frequency drift of about -6 x 10^(-9) Hz / s, or 1.5 Hz over 8 years (one-way only). ... This equates to a clock acceleration, -a_t, of -2.8 x 10^(-18) s / s^2 . The identity with the apparent Pioneer acceleration is a_P = a_t c. ...
... Having noted the relationships
a_P = c a_t
and that of ...
a_H = c H -> 8 x 10^(-8) cm / s^2
if H = 82 km / s / Mpc ...
we were motivated to try to think of any ... "time" distortions that might ... fit the CHASMP Pioneer results ... In other words ...
Is there any evidence that some kind of "time acceleration" is being seen?
... In particular we considered ... Quadratic Time Augmentation. This model adds a quadratic-in-time augmentation to the TAI-ET (International Atomic Time - Ephemeris Time) time transformation, as follows
ET -> ET + (1/2) a_ET ET^2
The model fits Doppler fairly well
...
There was one [other] model of the ...[time acceleration]... type that was especially fascinating. This model adds a quadratic in time term to the light time as seen by the DSN station:
delta_TAI = TAI_received - TAI_sent ->
-> delta_TAI + (1/2) a_quad (TAI_received^2 - TAI_sent^2)
It mimics a line of sight acceleration of the spacecraft, and could be thought of as an expanding space model.
Note that a_quad affects only the data. This is in contrast to the a_t ... that affects both the data and the trajectory. ... This model fit both Doppler and range very well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant and a_P ... remains an interesting conjecture. ...".

In his book “Mathematical Cosmology and Extragalactic Astronomy” (Academic Press 1976) (pages 61-62 and 72), Irving Ezra Segal says:
"... Temporal evolution in ... Minkowski space ... is
H -> H + s I
... unispace temporal evolution ... is ...
H -> (H + 2 tan(a/2)) / (1 - (1/2) H tan(a/2)) = H + a I + (1/4) a H^2 + O(s^2)
...".

Therefore,
the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085:
"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates;
its constant value is ... H c ...
This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".

The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.
The Rosales and Sanchez-Gomez paper very nicely unites:

the physical 2-phase (bounded and unbounded orbits) view;
the Foucault pendulum idea; and the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez,
can be summarized as a 2-phase model based on Segal's work
which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands;
and
a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.

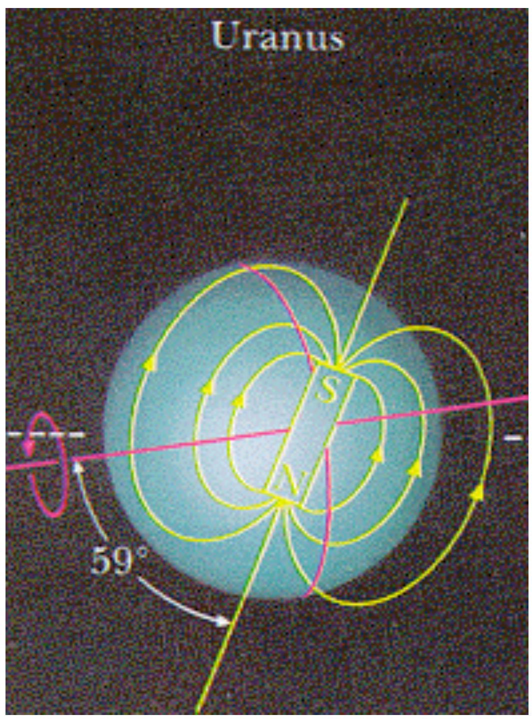
If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064.

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where c^2 /\ dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU) whereby ordinary matter dominates inside that distance from the sun
and exotic dark energy / dark matter appears at greater distances,
then Jack's model could also explain the Pioneer anomaly
and it may be that Jack's model with ordinary and exotic phases
and my model with deSitter/Poincare and Conformal phases
may be two ways of looking at the same thing.
As to what might be the physical mechanism of the phase transition, Jack says "... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".
In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe).
That physical interpretation is consistent with my view.

Transition at Orbit of Uranus:
It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than nuclear energy.

In gr-qc/0104064 Anderson et al say:
"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",

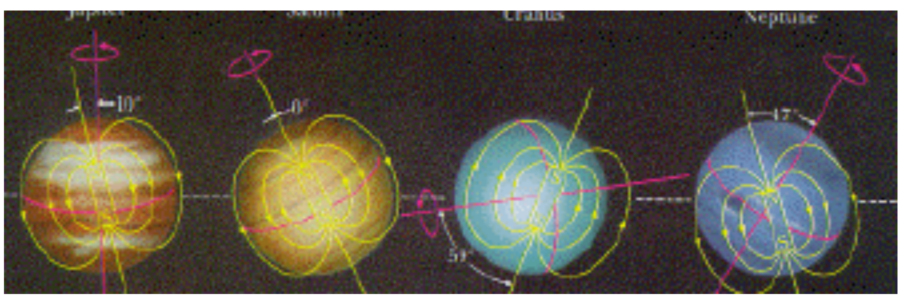
so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.
The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.

Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now.
However, such an effect may have been been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.
Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from “Universe”, 4th ed, by William Kaufmann, Freeman 1994).



According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:
"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [or 2 x 10^11 cm] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... . [C]omets ... suffer ... from outgassing ... [and their nuclei are hard to locate precisely] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

20. Dark Energy experiment by BSCCO Josephson Junctions and geometry of 600-cell

I. E. Segal proposed a MInkowski-Conformal 2-phase Universe
and
Beck and Mackey proposed 2 Photon-GraviPhoton phases:
Minkowski/Photon phase locally Minkowski with ordinary Photons and
Gravity weakened by 1 / (M_Planck)^2 = 5 x 10^(-39).
so that we see Dark Energy as only 3.9 GeV/m^3
Conformal/GraviPhoton phase with GraviPhotons and Conformal symmetry
(like the massless phase of energies above Higgs EW symmetry breaking)
With massless Planck the 1 / M_Planck^2 Gravity weakening goes away
and the Gravity Force Strength becomes the strongest possible = 1
so Conformal Gravity Dark Energy should be enhanced by M_Planck^2
from the Minkowski/Photon phase value of 3.9 GeV/m^3.

The Energy Gap of our Universe as superconductor condensate spacetime
is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length).
Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.
Neutrino masses are involved because their mass is zero at tree level
and their masses that we observe come from virtual graviphotons becoming virtual neutrino-antineutrino pairs.

BSCCO superconducting crystals are by their structure natural Josephson Junctions. Dark Energy accumulates (through graviphotons) in the superconducting layers of BSCCO.
Josephson Junction control voltage acts as a valve for access to the BSCCO Dark Energy, an idea due to Jack Sarfatti.

Christian Beck and Michael C. Mackey in astro-ph/0703364 said: "... Electromagnetic dark energy .... is based on a Ginzburg-Landau ... phase transition for the gravitational activity of virtual photons ... in two different phases:
gravitationally active [GraviPhotons] ...
and gravitationally inactive [Photons]
...
Let IPI^2 be the number density of gravitationally active photons ...
start from a Ginzburg-Landau free energy density ...

F = a IPI^2 + (1/2) b IPI^4

... The equilibrium state Peq is ... a minimum of F ... for T > Tc ...

Peq = 0 [and] Feq = 0

... for T< Tc

IPeqI^2 = - a / b [and] Fdeq = -(1/2) a^2 / b

... temperature T [of] virtual photons underlying dark energy ... is ..

h v = ln3 k T

... dark energy density ...[is]...

rho_dark = (1/2) (pi h / c^3) (v_c)^4

... The currently observed dark energy density in the universe of about 3.9 GeV/m^3 implies that the critical frequency v_c is ...

v_c = 2.01 THz

... BCS Theory yields ... for Fermi energy ... in copper ... 7.0 eV and the critical temperature of ... YBCO ... around 90 K ...

h v_c = 8 x 10^(-3) eV

... Solar neutrino measurements provide evidence for a neutrino mass of about m_v c^2 = 9 x 10^-3 eV ...

[the Cl(16)-E8 model has first-order masses for the 3 generations of neutrinos as
1 x 10^(-3) and 9 x 10^(-3) and 5.4 x 10(-2) eV]

... in solid state physics the critical temperature is essentially determined by the energy gap of the superconductor ... (i.e. the energy obtained when a Cooper pair forms out of two electrons) ...
for [graviphotons] ... at low temperatures (frequencies) Cooper-pair like states [of neutrino-antineutrino pairs] can form in the vacuum ... the ... energy gap would be of the order of typical neutrino mass differences ...".

Clovis Jacinto de Matos and Christian Beck in arXiv 0707.1797 said: "...
Tajmar's experiments ... at Austrian Research Centers Gmbh-ARC ...
with ... rotating superconducting rings ... demonstrated ...
a clear azimuthal acceleration ... directly proportional to the superconductive ring angular acceleration, and
an angular velocity orthogonal to the ring's equatorial plane ...
In 1989 Cabrera and Tate, through the measurement of the London moment magnetic trapped flux, rekported an anomalous Cooper pair mass excess in thin rotating Niobium supeconductive rings ...
A non-vanishing cosmological constant (CC) /\ can be interpreted in terms of a non-vanishing vacuum energy density

rho_vac = (c^4 / 8 pi G) /\

which corresponds to dark energy with equation of state w = -1.
The ... astronomically observed value [is]... /\ = 1.29 x 10^(-52) [1/m^2] ...
Graviphotons can form weakly bounded states with Cooper pairs, increasing their mass slightly from m to m' .
The binding energy is Ec = u c^2 :

m' = m + my - u

... Since the graviphotons are bounded to the Cooper pairs, their zeropoint energies form a condensate capable of the gravitoelectrodynamic properties of superconductive cavities. ...
Beck and Mackey's Ginzburg-Landau-like theory leads to a finite dark energy density dependent on the frequency cutoff v_c of vacuum fluctuations:

rho* = (1/2) (pi h / c^3) (v_c)^4

in vacuum one may put rho* = rho_vac from which the cosmological cutoff frequency v_cc is estimated as

v_cc = 2.01 THz

The corresponding "cosmological" quantum of energy is:

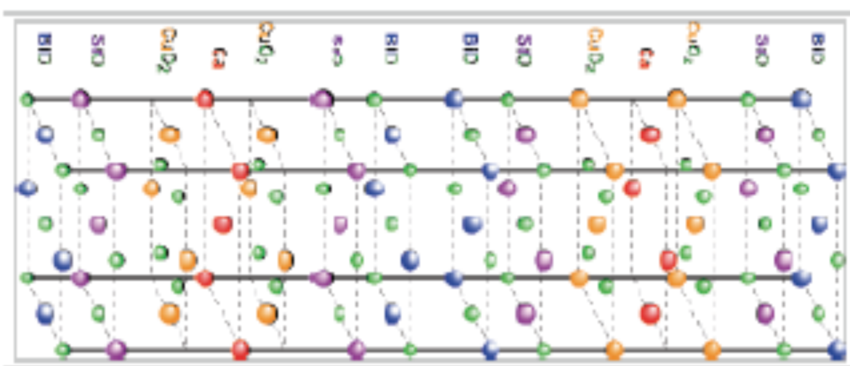
Ecc = h v_cc = 8.32 MeV

... In the interior of superconductors ... the effective cutoff frequency can be different ... h v = ln 3 k T ... we find the cosmological critical temprature Tcc

Tcc = 87.49 K

This temperature is characteristic of the BSCCO High-Tc superconductor. ...".

Xiao Hu and Shi-Zeng Lin in arXiv 0911.5371 said: "... The Josephson effect is a phenomenon of current flow across two weakly linked superconductors separated by a thin barrier, i.e. Josephson junction, associated with coherent quantum tunneling of Cooper pairs. ... The Josephson effect also provides a unique way to generate high-frequency electromagnetic (EM) radiation by dc bias voltage ... The discovery of cuprate high-Tc superconductors accelerated the effort to develop novel source of EM waves based on a stack of atomically dense-packed intrinsic Josephson junctions (IJJs), since the large superconductivity gap covers the whole terahertz (THz) frequency band. Very recently, strong and coherent THz radiations have been successfully generated from a mesa structure of Bi2Sr2CaCu2O8+d single crystal ...[BSCCO image from Wikipedia



]

which works both as the source of energy gain and as the cavity for resonance. This experimental breakthrough posed a challenge to theoretical study on the phase dynamics of stacked IJJs, since the phenomenon cannot be explained by the known solutions of the sine-Gordon equation so far. It is then found theoretically that, due to huge inductive coupling of IJJs produced by the nanometer junction separation and the large London penetration depth ... of the material,
a novel dynamic state is stabilized in the coupled sine-Gordon system, in which +/- pi kinks in phase differences are developed responding to the standing wave of Josephson plasma and are

stacked alternately in the c-axis. This novel solution of the inductively coupled sine-Gordon equations captures the important features of experimental observations.
The theory predicts an optimal radiation power larger than the one observed in recent experiments by orders of magnitude ...".

What are some interesting BSCCO JJ Array configurations ?

Christian Beck and Michael C. Mackey in astro-ph/0605418 describe
"... the AC Josephson effect ...
a Josephson junction consists of two superconductors with an insulator sandwiched in between. In the Ginzburg-Landau theory each superconductor is described by a complex wave function whose absolute value squared yields the density of superconducting electrons. Denote the phase difference between the two wave functions ... by P(t).
...
at zero external voltage a superconductive current given by Is = Ic sin(P) flows between the two superconducting electrodes ... Ic is the maximum superconducting current the junction can support.
...
if a voltage difference V is maintained across the junction, then the phase difference P evolves according to

d P / dt = 2 e V / hbar

i.e. the current ... becomes an oscillating curent
with amplitude Ic and frequency v = 2 e V / h
This frequency is the ... Josephson frequency ... The quantum energy h v ... can be interpreted as the energy change of a Cooper pair that is transferred across the junction ...".

Xiao Hu and Shi-Zeng Lin in arXiv 1206.516 said:
"... to enhance the radiation power in teraherz band based on the intrinsic Josephson Junctions of Bi2Sr2CaCu2O8+d single crystal ...
we focus on the case that the Josephson plasma is uniform along a long crystal as established by the cavity formed by the dielectric material. ...
A ... pi kink state ... is characterized by static +/- pi phase kinks in the lateral directions of the mesa, which align themselves alternatingly along the c -axis. The pi phase kinks provide a strong coupling between the uniform dc current and the cavity modes, which permits large supercurrent flow into the system at the cavity resonances, thus enhances the plasma oscillation and radiates strong EM wave ...
The maximal radiation power ... is achieved when the length of BSCCO single crystal at c-axis equals the EM wave length. ...".

Each long BSCCO single crystal looks geometrically like a line so configure the JJ Array using BSCCO crystals as edges.

The simplest polytope, the **Tetrahedron, is made of 6 edges**:
Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 said:
"... Superconducting tetrahedral quantum bits ...

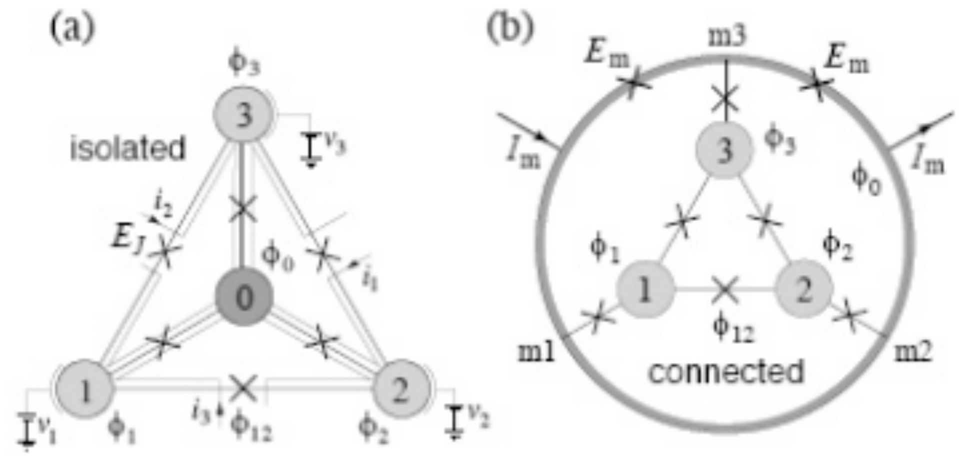
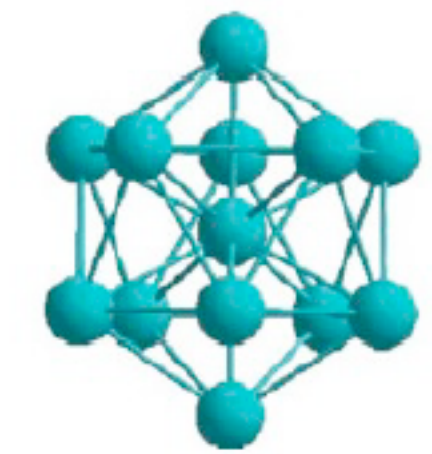


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling E_J and charging energy E_C); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages v_i and bias currents i_i. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_m \gg E_J$ on the outer ring which are driven by external currents I_m (schematic, see Fig. 6 for details); the large coupling E_m effectively binds the ring segments into one island.

... tetrahedral qubit design ... emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit.
Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin;
the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space. ...".

42 edges make an Icosahedron plus its center
(image from Physical Review B 72 (2005) 115421 by Rogan et al)

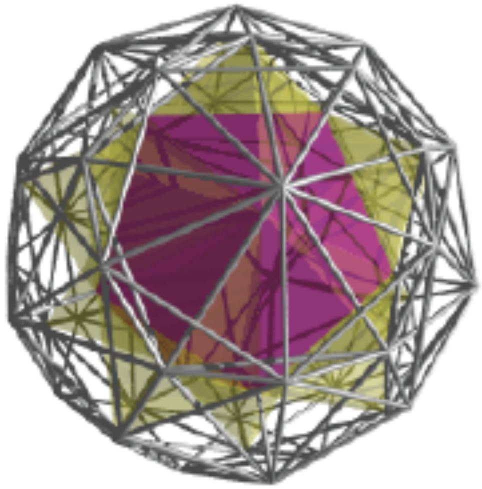


with 30 exterior edges and 12 edges from center to vertices.
It has 20 cells which are approximate Tetrahedra in flat 3-space but become exact regular Tetrahedra in curved 3-space.

Could an approximate-20Tetrahedra-Icosahedron configuration of 42 BSCCO JJ tap into Dark Energy so that the Dark Energy might regularize the configuration to exact Tetrahedra and so curve/warp spacetime from flat 3-space to curved 3-space ?

720 edges make a 4-dimensional 600-cell

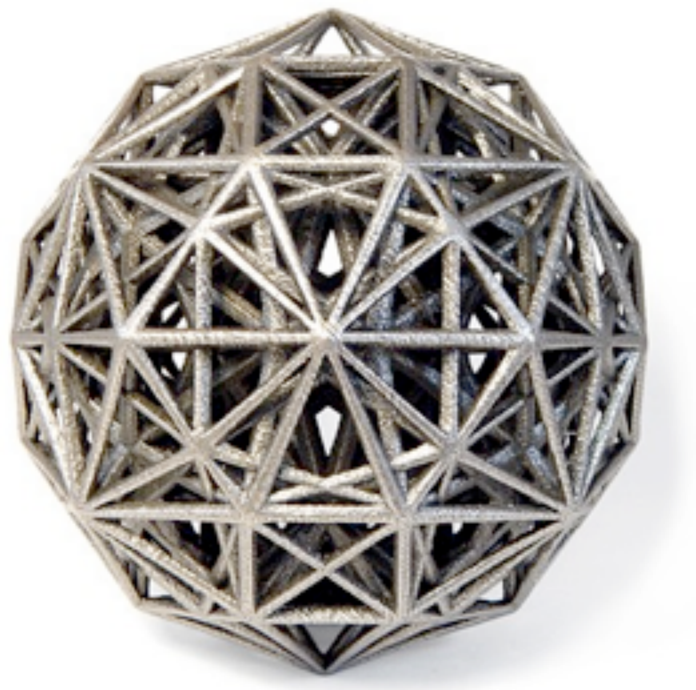
(image from Wikipedia)



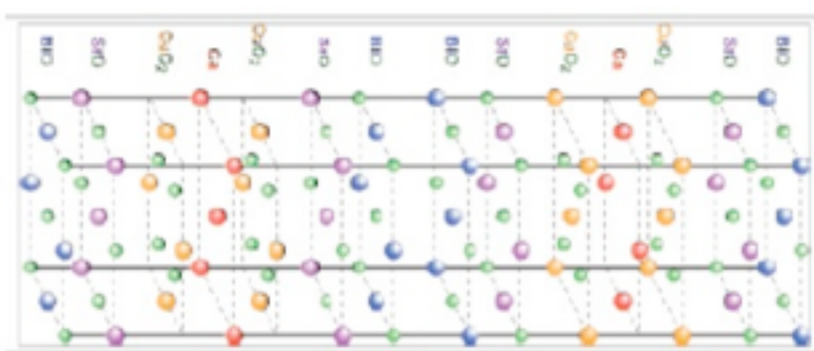
At each vertex 20 Tetrahedral faces meet forming an Icosahedron
which is exact because the 600-cell lives on a curved 3-shere in 4-space.
It has 600 Tetrahedral 3-dim faces and 120 vertices

Could a 600 approximate-Tetrahedra configuration of 720 BSCCO JJ
approximating projection of a 600-cell into 3-space
tap into Dark Energy so that the Dark Energy might regularize
the configuration to exact Tetrahedra and an exact 600-cell
and so curve/warp spacetime from flat 3-space to curved 3-space ?

The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell as follows: Consider 3-dim models of 600-cell such as metal sculpture from Bathsheba Grossman who says:
"… for it I used an orthogonal projection rather than the Schlegel diagrams of the other polytopes I build.
… In this projection all cells are identical, as there is no perspective distortion. …".



For the Dark Energy experiment each of the 720 lines would be made of a single BSCCO crystal



whose layers act naturally to make the BSCCO crystal an intrinsic Josephson Junction. (see Wikipedia and arXiv 0911.5371)

Each of the 600 tetrahedral cells of the 600-cell has 6 BSCCO crystal JJ edges.

Since the 600-cell is in flat 3D space the tetrahedra are distorted.

According to the ideas of Beck and Mackey (astro--ph/0703364) and of Clovis Jacinto de Matos (arXiv 0707.1797) the superconducting Josephson Junction layers of the 720 BSCCO crystals will bond with Dark Energy GraviPhotons that are pushing our Universe to expand.

My idea is that the Dark Energy GraviPhotons will not like being configured as edges of tetrahedra that are distorted in our flat 3D space and

they will use their Dark Energy to make all 600 tetrahedra to be exact and regular by curving our flat space (and space-time).

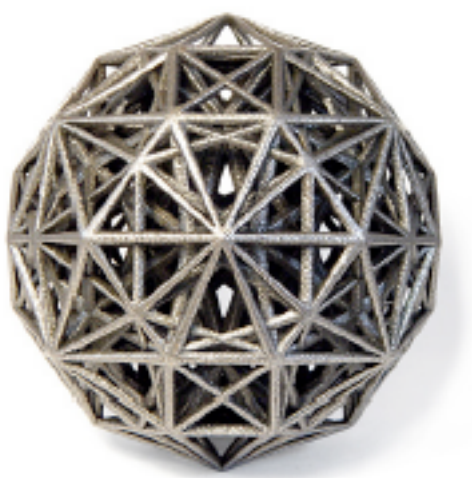
My view is that the Dark Energy Graviphotons will have enough strength to do that because their strength will NOT be weakened by the (1 / M_Planck)^2 factor that makes ordinary gravity so weak.

It seems to me to be a clearly designed experiment that will either

1- not work and show my ideas to be wrong or

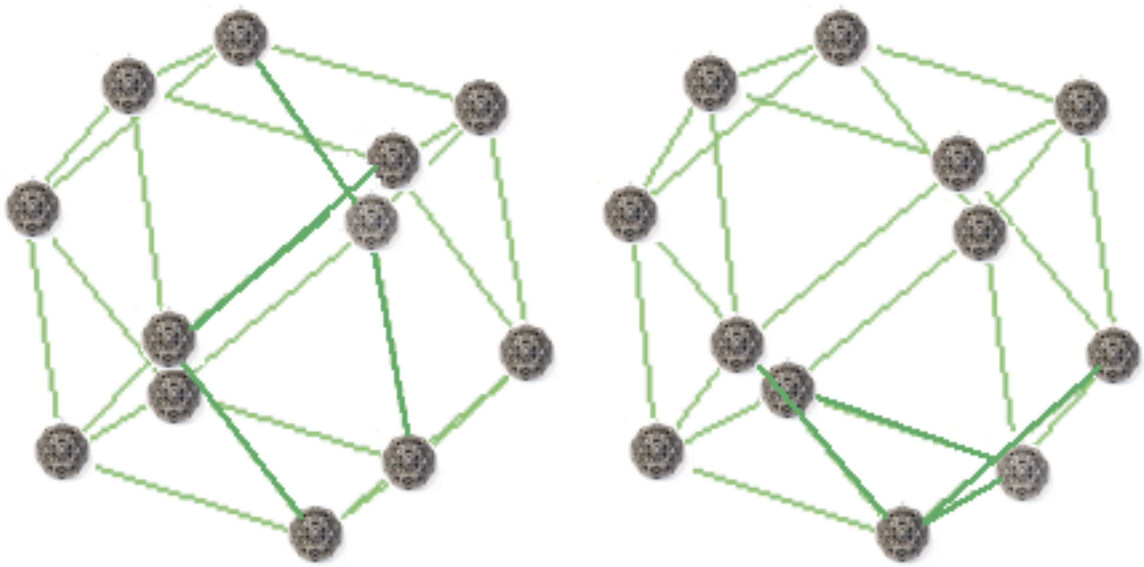
2 - work and open the door for humans to work with Dark Energy.

Consider BSCCO JJ 600-cells

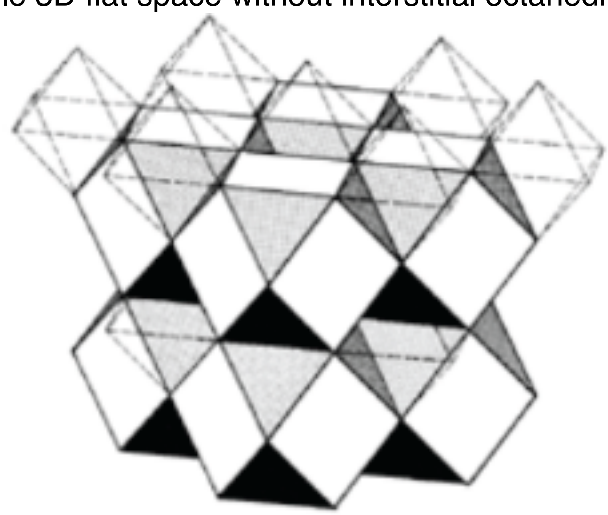


in this configuration:

First put 12 of the BSCCO JJ 600-cells at the vertices of a cuboctahedron shown here as a 3D stereo pair:

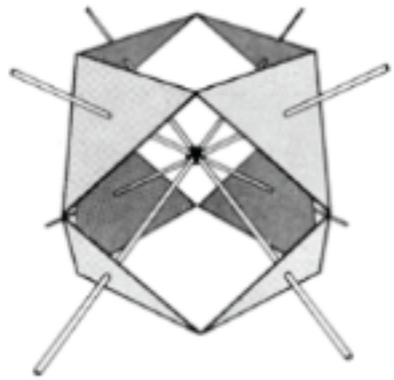
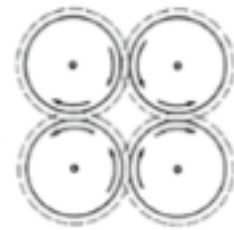


Cuboctahedra do not tile 3D flat space without interstitial octahedra

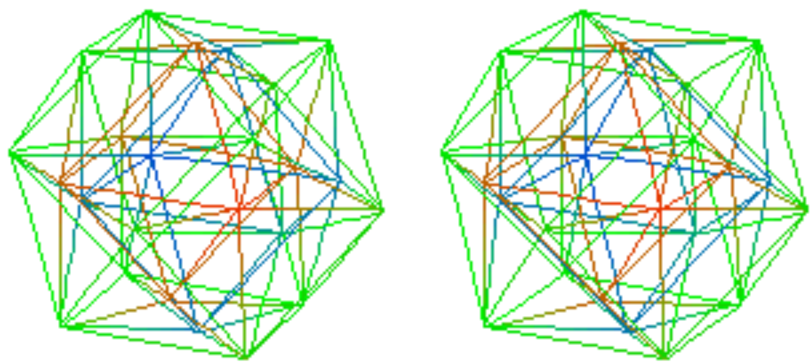


but BSCCO JJ 600-cell cuboctahedra can be put together square-face-to-square-face in flat 3D configurations including flat sheets.

As Buckminster Fuller described, the 8 triangle faces of a cuboctahedron

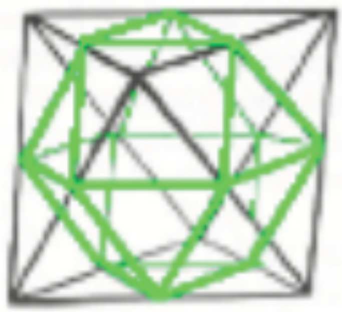
give it an inherently 4D structure consistent with the green cuboctahedron

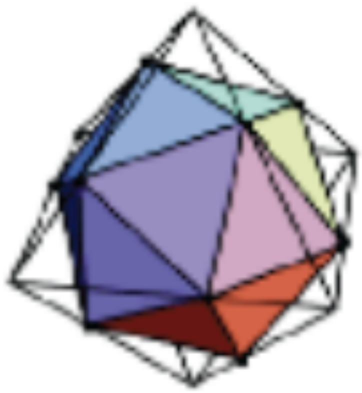


central figure of a 24-cell (3D stereo 4thD blue-green-red color)
that tiles flat Euclidean 4D space.

So, cuboctahedral BSCCO JJ 600-cell structure likes flat 3D and 4D space
but
if BSCCO JJ Dark Energy act to transform flat space into curved space
like a 720-edge 600-cell with 600 regular tetrahedra
then
Dark Energy should transform cuboctahedral BSCCO JJ 600-cell structure
into
a 720-edge BSCCO JJ 600-cell structure that likes curved space.

There is a direct JItterbug transformation of
the 12-vertex cuboctahedron to the 12-vertex icosahedron

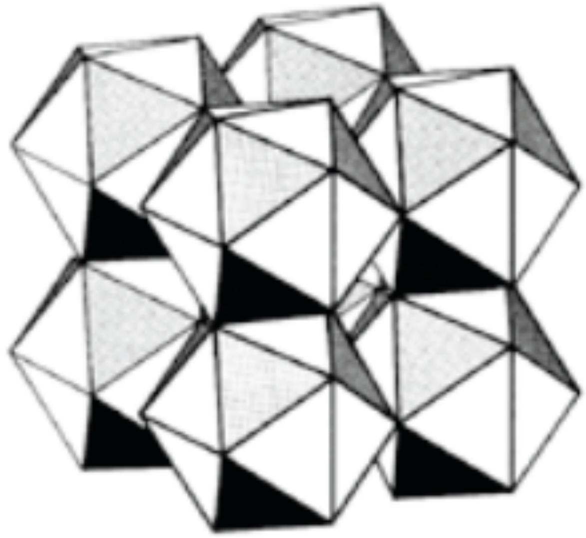




whereby the 12 cuboctahedron vertices as midpoints of octahedral edges
are mapped to
12 icosahedron vertices as Golden Ratio points of octahedral edges.
There are two ways to map a midpoint to a Golden Ratio point.
For the Dark Energy experiment the same choice of mapping
should be made consistently throughout the BSCCO JJ 600-cell structure.

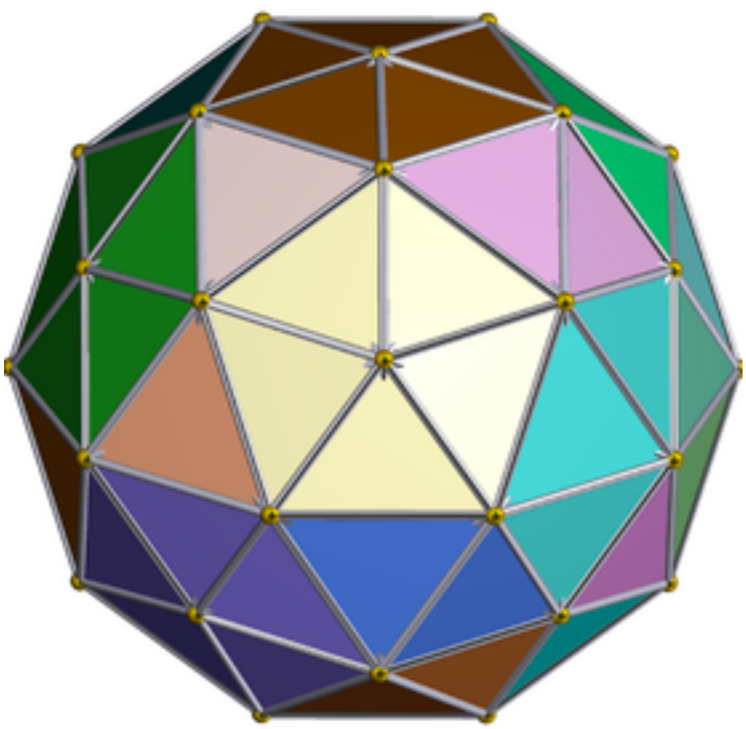
The result of the Jitterbug mapping is that
each cuboctahedron in the BSCCO JJ 600-cell structure
with its 12 little BSCCO JJ 600-cells at its 12 vertices
is mapped to
an icosahedron with 12 little BSCCO JJ 600-cells at its 2 vertices

and the overall cuboctahedral BSCCO JJ 600-cell structure
is transformed into
an overall icosahedral BSCCO JJ 600-cell structure



does not fit in flat 3D space in a naturally characteristic way
(This is why icosahedral QuasiCrystal structures do not extend as simply throughout flat 3D space as do cuboctahedral structures).

However, the BSCCO JJ 600-cell structure JItterbug icosahedra
do live happily in 3-sphere curved space within the icosahedral 120-cell



which has the same 720-edge arrangement as the 600-cell (see Wikipedia).
The icosahedral 120-cell is constructed by 5 icosahedra around each edge.
It has:

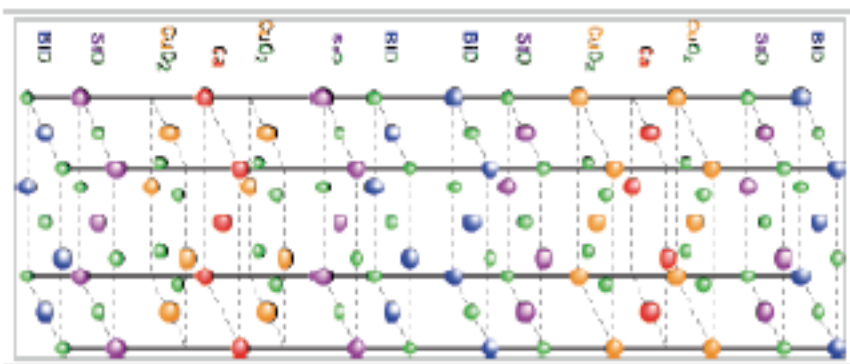
cells - 120 {3,5}
faces - 1200 {3}
edges - 720
vertices - 120
vertex figure - {5,5/2}
symmetry group H4,[3,3,5]
dual - small stellated 120-cell

In summary,

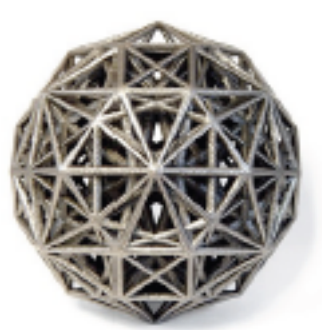
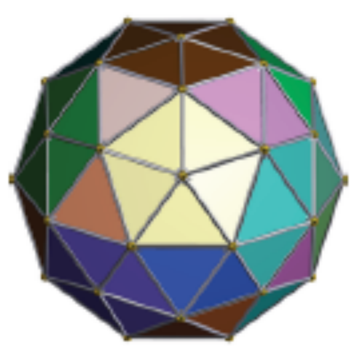
Jitterbug transformations and BSCCO Josephson Junctions
may be the Geometric Key to controlling Dark Energy

(as were Chain Reactions for Nuclear Fission and Ellipsoidal Focussing for H-Bombs)

The Energy Gap of our Universe as superconductor condensate spacetime is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length).
Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses =
= critical temperature Tc of BSCCO superconducting crystals.



BSCCO superconducting crystals are natural Josephson Junctions.
Dark Energy accumulates in the superconducting layers of BSCCO.
The basic idea of Dark Energy from BSCCO Josephson Junctions is based on the 600-cell each of whose 720 edge-lines would be made of a single BSCCO crystal. It may be useful to use a Jitterbug-type transformation between a 600-cell configuration and a configuration based on icosahedral 120-cells which also have 720 edge-lines:

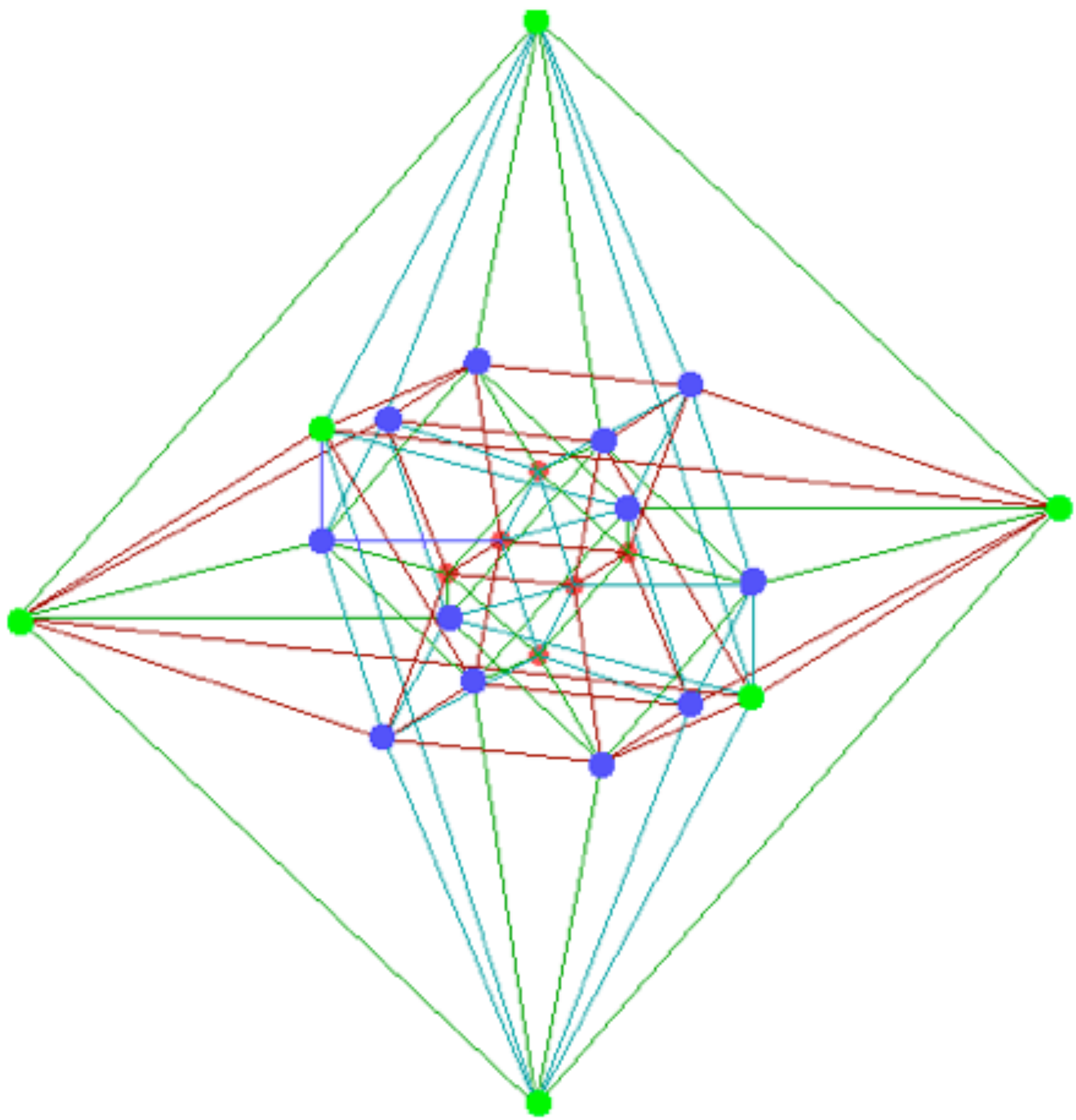



21. 600-cell Geometry of Cl(16)-E8 Physics

Start by building a 600-cell from a 24-cell.

24-cell diagrams here are adapted from those of Frans Marcelis at
http://members.home.nl/fg.marcelis/24-cell.htm#stereographic%20projection

The 24-cell is made up of an Outer Octahedron (green),
a Central Cuboctahedron (blue), and an Inner Octahedron (red).
Physically, it corresponds to the 24 Root Vectors of a D4 Gauge Group
that can represent either Gravity + Dark Energy or the Standard Model.

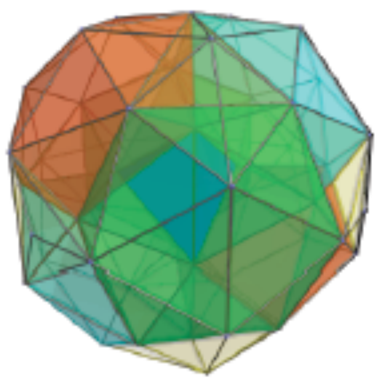


To build a 600-cell, first surround each of the 24 vertices with 5 Tetrahedra
which gives you 120 of its 600 Tetrahedra.

Next, look at the 24 Octahedra that fill up the volume of the 24-cell.
Each Octahedron contains an Icosahedron

(image from wolfram mathworld)

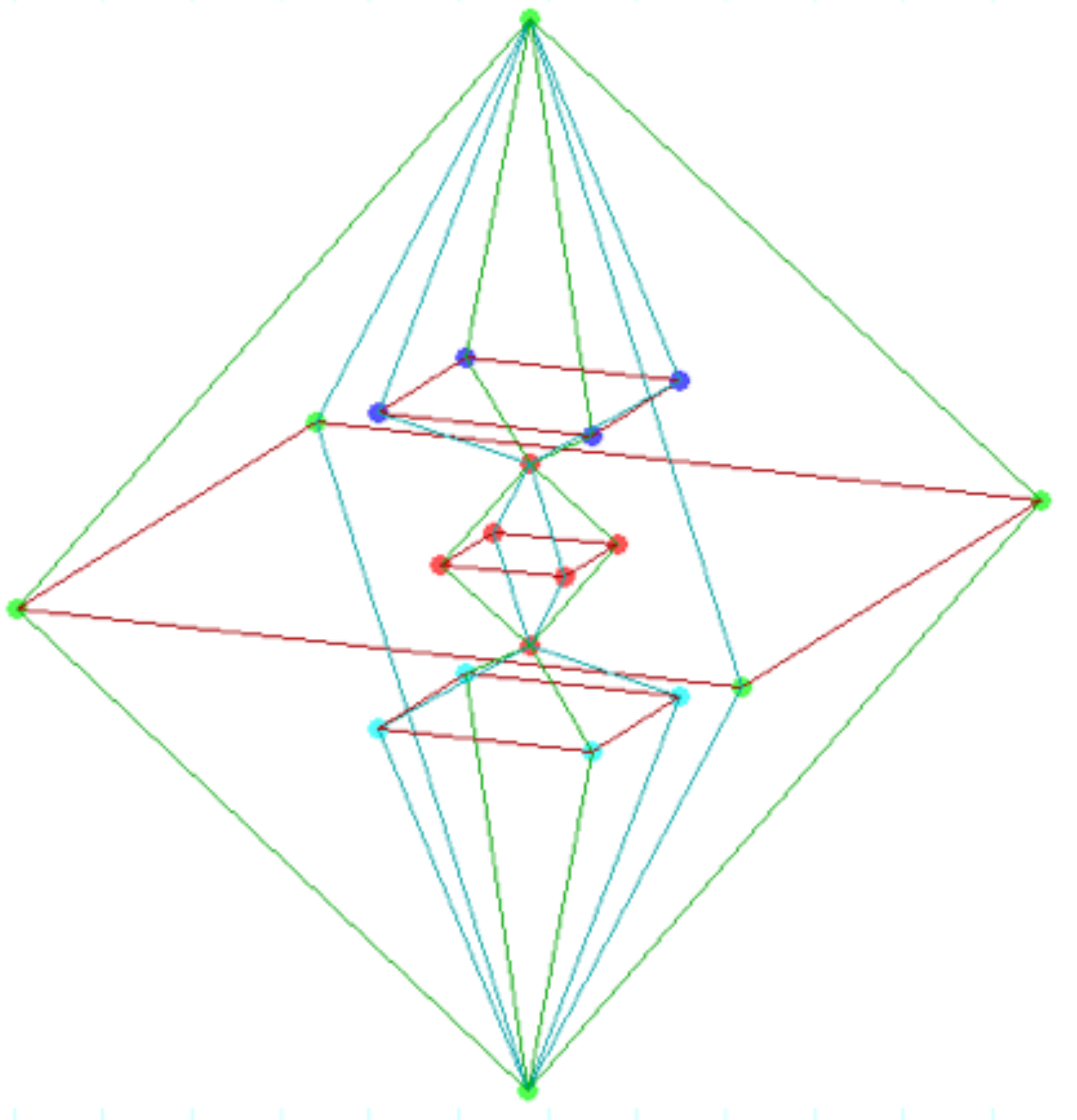
plus some extra volume in each Octahedron.
The extra volume can be divided into 24 vertex Tetrahedra + 96 edge Tetrahedra
so the 24-cell becomes a Snub 24-cell with 24 Icosahedral and 120 Tetrahedral cells

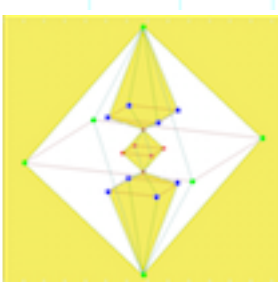


(image from eusebia.dyndns.org)

Each of the 24 Icosahedra contains 20 Tetrahedra for a total of 480 Tetrahedra
which when added to the 24+96 = 120 Tetrahedra outside the Icosahedra
(Tetrahedra are only approximately regular in 3D space but become regular in 4D)
give you the 480+120 = 600 Tetrahedra of the 600-cell.

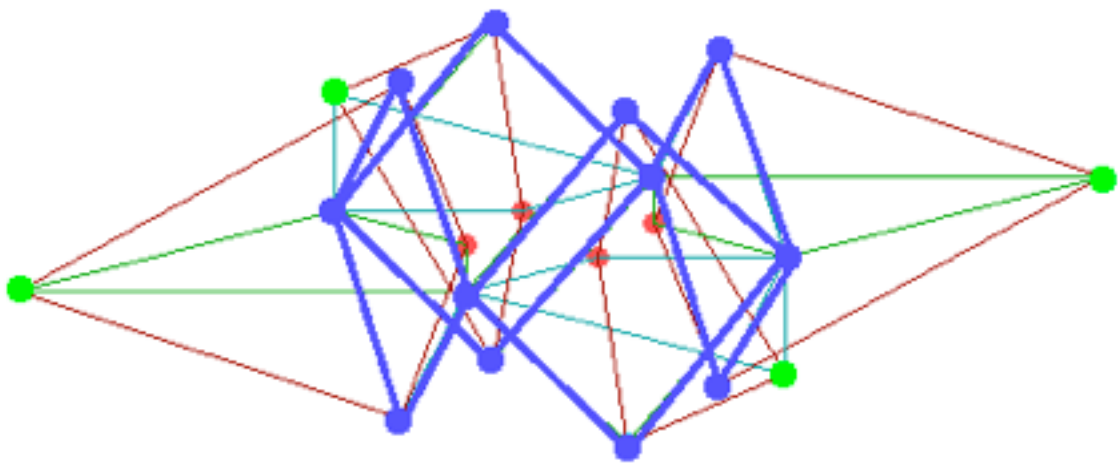
These are 4 of the Octahedra corresponding to 4-dim M4 physical spacetime
within (4+4)-dim M4 x CP2 Kaluza-Klein of B4 / D4
They account for 4x20 = 80 of the 600-cell Tetrahedra.

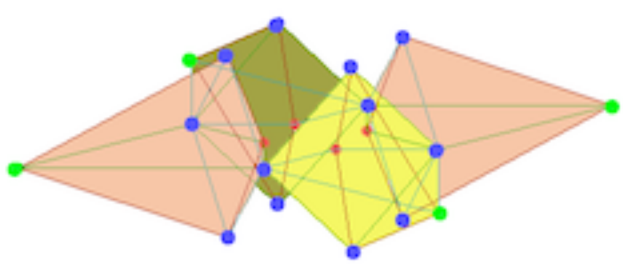




These are 4 of the Octahedra corresponding to 4-dim CP2 internal symmetry space within the (4+4)-dim M4 x CP2 Kaluza-Klein of B4 / D4

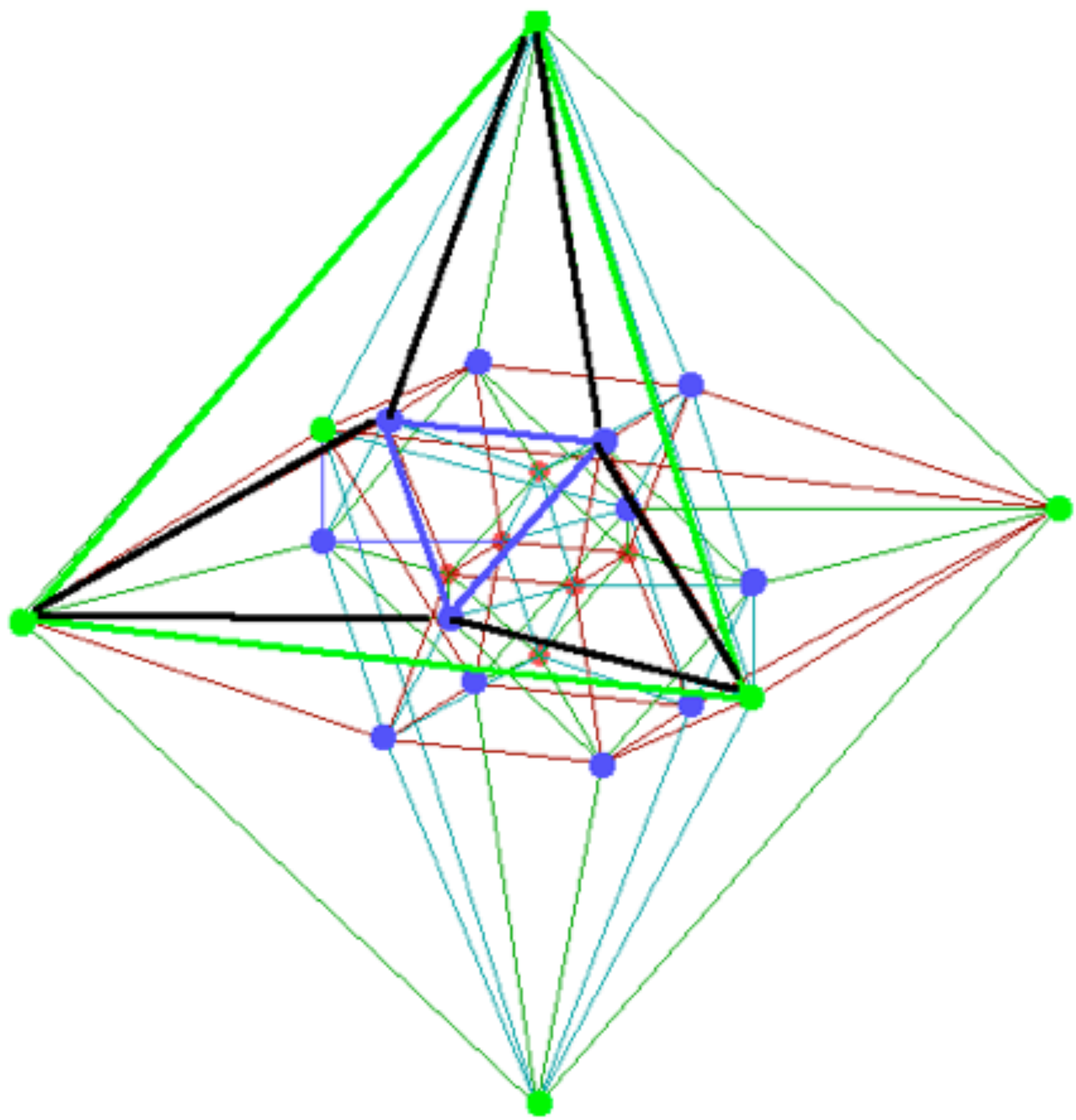
They account for 4x20 = 80 of the 600-cell Tetrahedra.





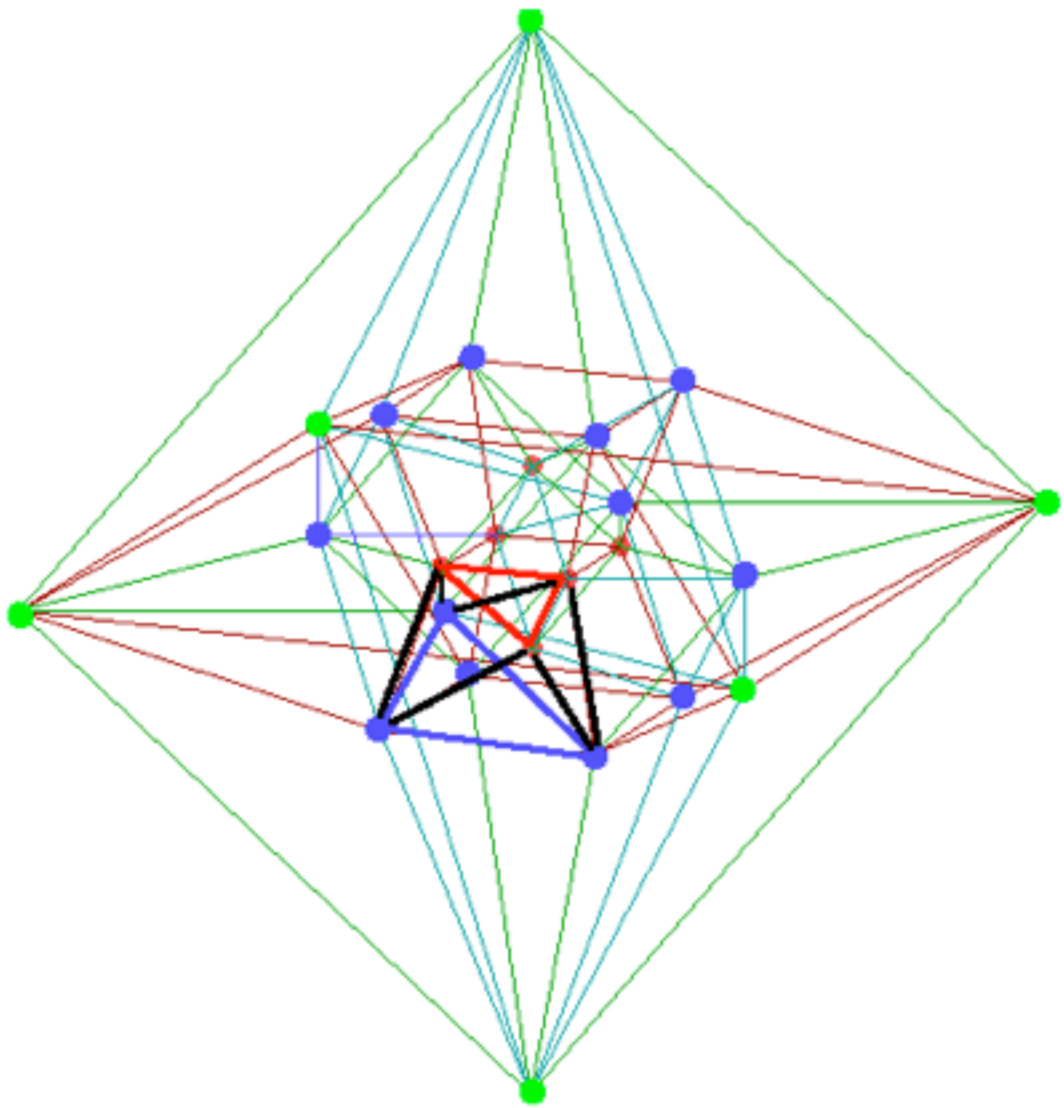
This is one of the 8 Octahedra corresponding to 8 fundamental Fermion Particles
within the (8+8)-dim spinor-type Octonionic Projective Plane of F4 / B4.
The other 7 are similarly configured
on each of the other 7 faces of the outer (green) Octahedron.

These 8 account for 8x20 = 160 of the 600-cell Tetrahedra.



This is one of the 8 Octahedra corresponding to 8 fundamental Fermion AntiParticles within the (8+8)-dim spinor-type Octonionic Projective Plane of F4 / B4.
The other 7 are similarly configured
on each of the other 7 faces of the inner (red) Octahedron.

These 8 account for 8x20 = 160 of the 600-cell Tetrahedra.

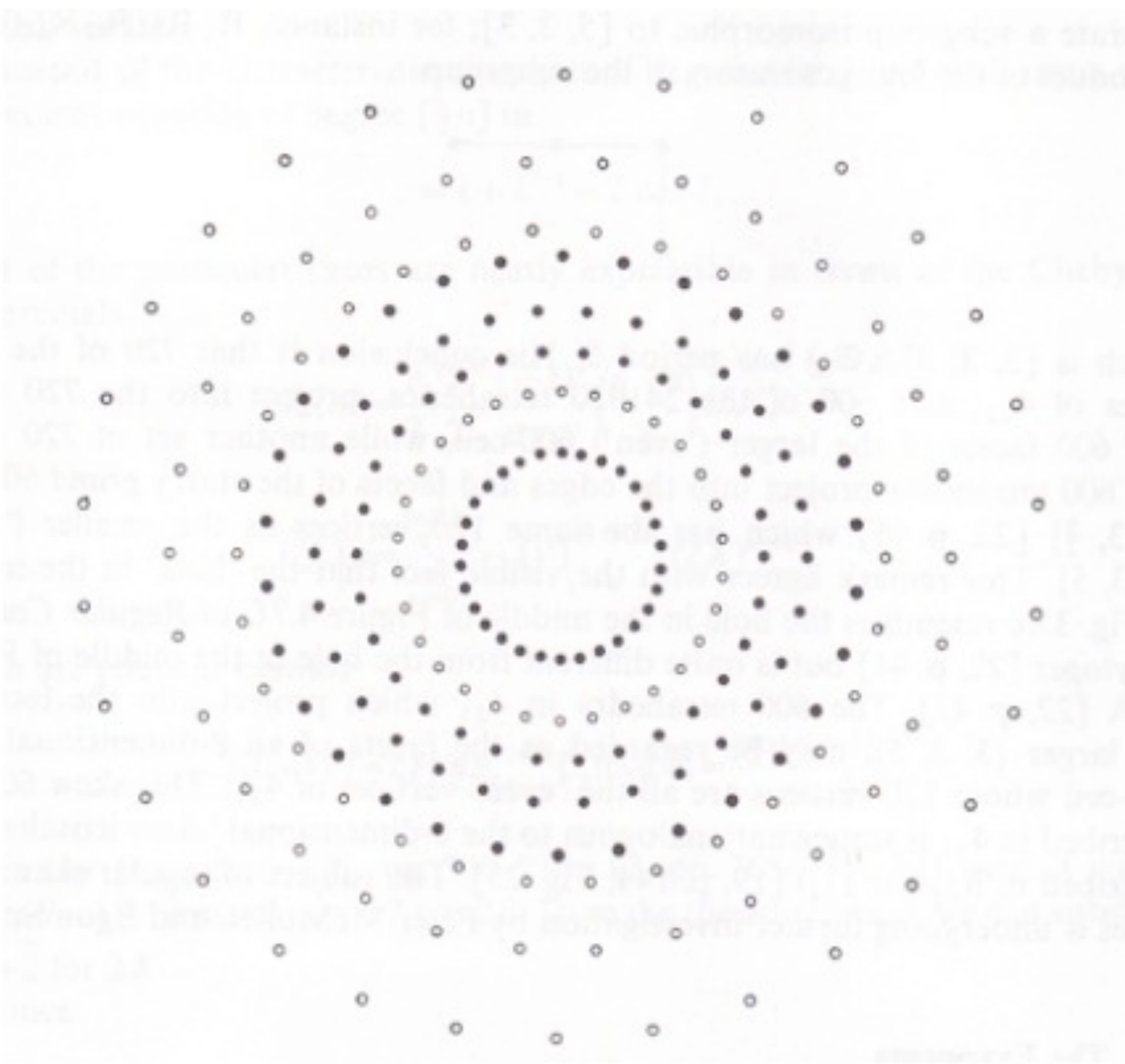


That is all 600 = 120 + 80 + 80 + 160 + 160 = 280 + 320 Tetrahedra of the 600-cell corresponding to Cl(16)-E8 Physics through the structure of F4 as follows:

120 to 28-dim D4 for SU(2,2) Gauge Gravity or 28-dim D4 for Standard Model SU(3)
80 + 80 to B4 / D4 for (4+4)-dim Kaluza-Klein M4 x CP2
160 +160 to F4 / B4 for (8+8)-dim OP2 spinor fermions

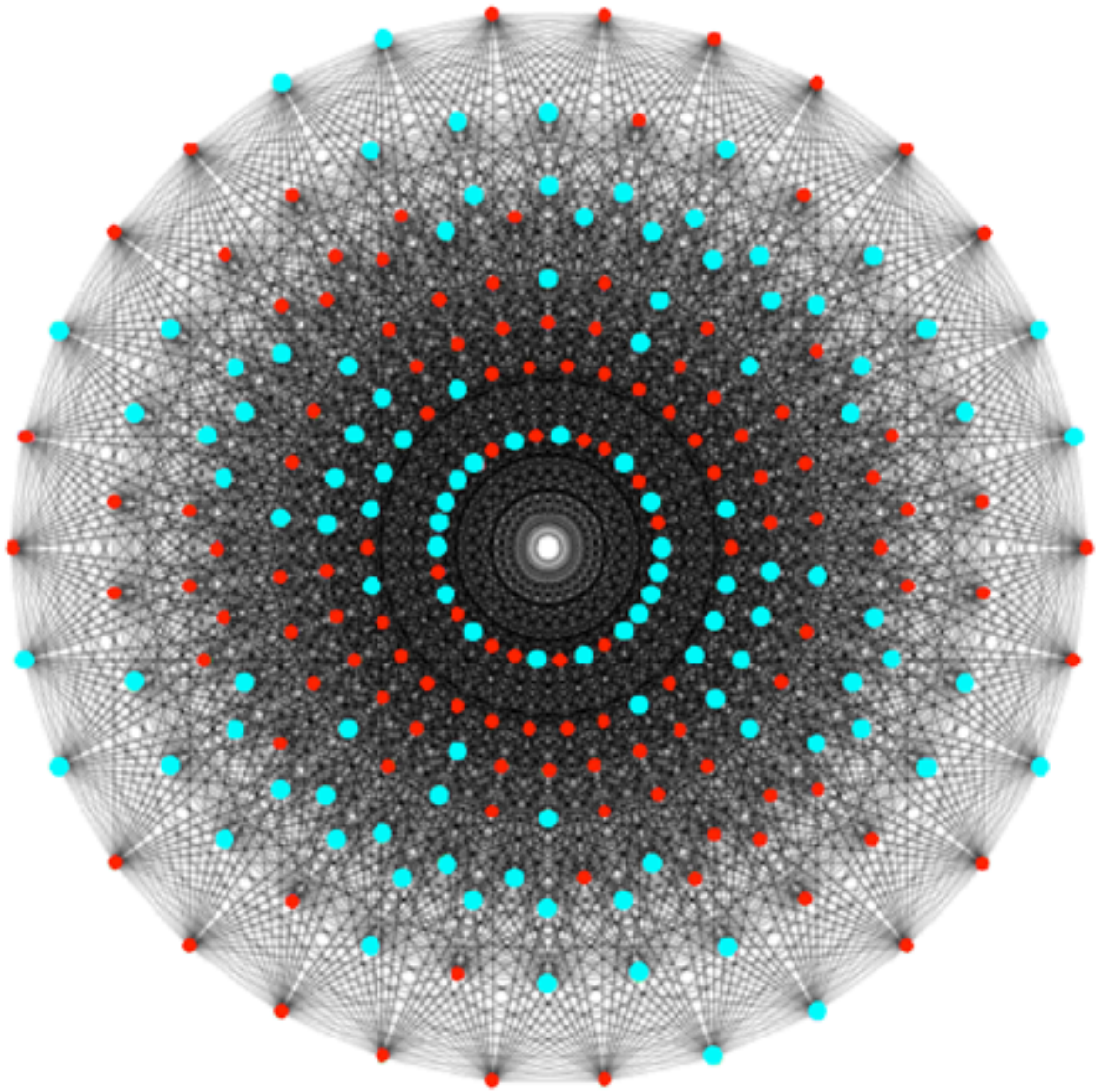
Here is another way to look at 600-cells with respect to Cl(16)-E8 Physics:
The 240 Root Vectors of E8 can be represented by two 120-vertex 600-cells.
One 600 cell corresponds to 4-dim Minkowski SpaceTime and to Gravity + Dark Energy and to 4 of the 8 coordinates of each Fermion Particle and AntiParticle
while
the other 600 cell corresponds to CP2 Internal Symmetry Space and to Standard Model Gauge Bosons
and to the other 4 of the 8 coordinates of each Fermion Particle and AntiParticle.

Here is a more detailed discussion:
The 240 Root Vectors of E8 can be projected onto 2D as 8 circles of 30 vertices each as shown in this diagram from Regular and Semi-Regular Polytopes III by Coxeter



in which there are 4 circles of white dots and 4 circles of black dots
with the 120 white-dots being like the 120 black dots expanded by the Golden Ratio.

The 120 + 120 division of the 240 is not the division into spacetime + particles.
That division is shown by
the 240 Root Vectors of E8 being projected onto 2D as 8 circles of 30 vertices each
in which there are 112 large dots (colored cyan) and 128 small dots (colored red)



as shown in this diagram adapted from http://www.madore.org/~david/math/e8w.html
where Madore says: "... E8 roots can be described, in the coordinate system we have chosen, as the (112) points having coordinates (±1,±1,0,0,0,0,0,0) (where both signs can be chosen independently and the two non-zero coordinates can be anywhere) together with those (128) having coordinates (±½,±½,±½,±½,±½,±½,±½,±½) (where all signs can be chosen independently except that there must be an even number of minuses) ...

the E_8 root system ... can be described as a remarkable polytope in 8 dimensions (also known as the Gosset 4_{21} polytope) having 240 vertices (known, in this context, as "*roots*"), and 6720 edges ...

all vertices are on a sphere with the origin as center; this is specific to E_8 ...
the opposite of each root is again a root, and each one is orthogonal to 126 others, while forming an angle of $\pi/3$ with 56 others (those that are connected to it by an edge): the only possible angles between two roots are 0, $\pi/3$, $\pi/2$, $2\pi/3$ and π.
The group of symmetries of this object is the group, known as the Weyl group of E_8, generated by the (orthogonal) reflections about the hyperplane orthogonal to each root: this is a group of order 696729600 which can also be described as $O_8^+(2)$.
It is also the group of automorphism of the adjacency graph of the polytope.

Those 112 roots which have coordinates of the form (±1,±1,0,0,0,0,0,0) are shown as larger dots, and constitute a so-called D_8 root system inside the E_8 root system, which, as a polytope, is a rectified octacross; the reflections determined by those vertices generate a subgroup of order 5160960 (the Weyl group of D_8, a subgroup of index 2 in $\{\pm 1\} \wr S8$) of the full Weyl group of E_8.

The 128 remaining vertices (forming a demiocteract) are shown as smaller dots; alone, they are not a root system because the reflection determined by one of them does not fix that subset.
Note that this division of the 240 vertices as 112+128 is particular to the chosen coordinate system and is not preserved by symmetries of the whole (except, precisely, by those living in the smaller Weyl group of D_8;
so there are 135 ways of making this decomposition).

One can further divide the roots in two by calling half of them "*positive*" in such a way that the sum of two positive roots, if it is a root, is always positive, and that for every root either it or its opposite is positive; there are many ways to do this (in fact, precisely as many as there are elements in the Weyl group), and we have chosen the division given by a lexicographic order on the coordinates: we call positive those roots such that the leftmost nonzero coordinate is positive (or, by numbering the roots lexicographically from 0 to 239, the positive ones are those numbered 120 through 239). A choice of positive roots is equivalent to a choice of *fundamental* (or *simple*) roots: these are the positive roots which cannot be written as a sum of two positive roots, and it then turns out that these form a basis of the ambient 8-space and, remarkably, that every positive root can be written as a linear combination of fundamental roots with nonnegative integer coefficients (equivalently, the fundamental roots form a non-orthogonal basis in which the coordinates of every root are either all nonnegative or all nonpositive; there is a uniquely defined *greatest root*, whose coordinates in terms of fundamental roots dominates that of every other root, and which happens to be one half the sum of *all* positive roots, fundamental or not: for E_8, it is $\langle 4,3,6,5,4,3,2,2 \rangle$ and, for our choices, it is root number 239, or (1,1,0,0,0,0,0,0)). Any choice of positive/fundamental roots can be brought to any other choice by a unique element of the Weyl group.

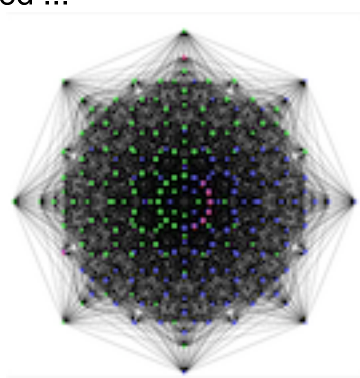
If we represent the eight fundamental roots and connect two by a line whenever they form an angle of $2\pi/3$ (the only other possibility being that they are orthogonal: in the case of E_8, the angles of $3\pi/4$ and $5\pi/6$ do not occur), we obtain the so-called *Dynkin diagram*, which in the case of E_8 has seven nodes in a simple chain and an eighth branching from the third. Here, we number the fundamental roots in the same total order as chosen to define the positive roots (i.e., lexicographic order on the coordinates; then the fundamental roots 1 through 8 are the roots numbered 120, 121, 122, 126, 132, 140, 150 and 162), and the Dynkin diagram has fundamental roots 8–1–3–4–5–6–7 in a chain and fundamental root number 2 branching off from 3.

The fundamental roots are important because the reflection with respect to them suffice to generate the Weyl group. Furthermore, the minimal length of an expression of a given element of the Weyl group as such a product of fundamental reflections (the *length* relative to the given element for the chosen system of fundamental roots) is equal to the number of positive roots whose image is a negative root; and composing by a fundamental reflection will always increase or decrease by 1 the length of the Weyl group element. ...

reflection with respect ...[a]... root ... permutes that root with its opposite,
fixes 126 others, and exchanges the 112 remaining roots as 56 pairs ...
The 696729600 elements of the Weyl group are generated by such reflections ...

Each element of the Weyl group can be written as a product (of a uniquely defined length) of reflections by eight fundamental roots ...

The default ... projection ... in which positive roots (for the particular order chosen) are represented in blue and negative roots in green, and the eight fundamental roots (relative to that order) are labeled ...



... is related to the chosen coordinate system in that it can be described by linearly combining the coordinates with coefficients given by eight consecutive complex sixteenth roots of unity. ...”.

The 112 large cyan dots correspond to the D8 subalgebra of E8
which represents SpaceTime and Gauge Bosons

The 128 small red dots correspond to Fermion Particles and AntiParticles

The circles break down like this:
inner - black dots - 18 SpaceTime Gauge Boson and 12 Fermion
second from center - white dots - 10 SpaceTime Gauge Boson and 20 Fermion
third from center - black dots - 10 SpaceTime Gauge Boson and 20 Fermion
fourth from center - black dots - 10 SpaceTime Gauge Boson and 20 Fermion
fifth from center - black dots - 18 SpaceTime Gauge Boson and 12 Fermion
sixth from center - white dots - 18 SpaceTime Gauge Boson and 12 Fermion
seventh from center - white dots - 18 SpaceTime Gauge Boson and 12 Fermion
eighth from center (outer) - white dots - 10 SpaceTime Gauge Boson and 20 Fermion

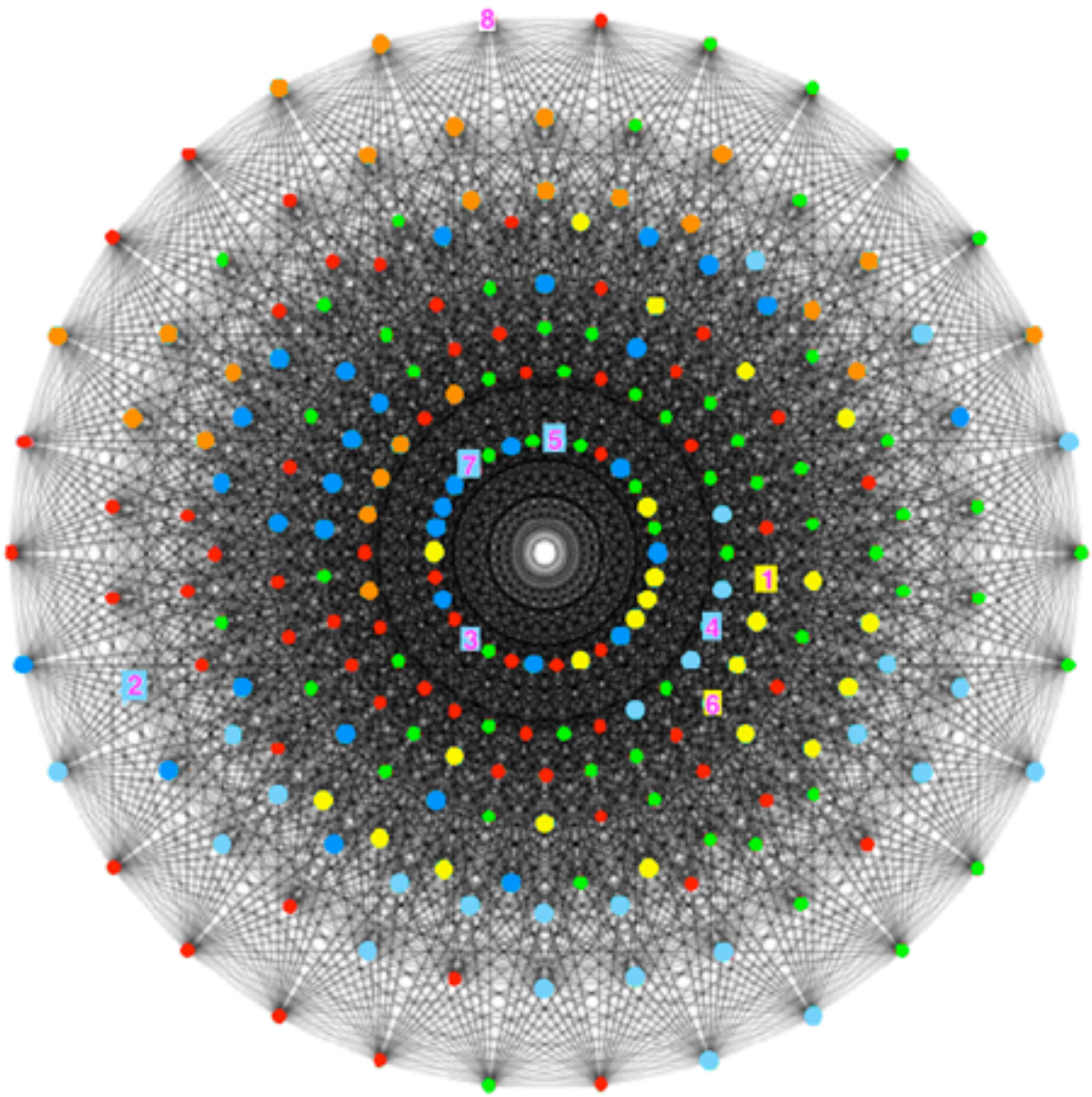
There are 4x12 + 4x20 = 128 Fermion small red dots
and
4x18 + 4x10 = 112 SpaceTime Gauge Boson large cyan dots

The black-dot 4 circles of the small 600-cell contain
56 SpaceTime Gauge Boson cyan dots and 64 Fermion red dots.
You can take the small 600-cell to correspond to M4 4D physical spacetime
so that
24 of the 56 give Gauge Bosons for Gravity + Dark Energy
and
32 of the 56 give 4 M4 spacetime components of 8-dim Momentum
and
32 of the 64 give 4 M4 spacetime components of 8 fundamental Fermion Particles
and
32 of the 64 give 4 M4 spacetime components of 8 fundamental Fermion AntiParticles

The white-dot 4 circles of the large (by Golden ratio) 600-cell also contain
56 SpaceTime Gauge Boson cyan dots and 64 Fermion red dots.
You can take the small 600-cell to correspond to CP2 4D Internal Symmetry Space
so that
24 of the 56 give Gauge Bosons for Standard Model SU(3)
and
32 of the 56 give 4 CP2 internal symmetry space components of 8-dim Momentum
and
32 of the 64 give 4 CP2 internal symmetry components of 8 Fermion Particles
and
32 of the 64 give 4 CP2 internal symmetry components of Fermion AntiParticles

In terms of the Madore 8 circles of 30 version of the 240 E8 Root Vectors:

8 Fundamental Root Vectors 1-8 of which 5 are in the 64 representing SpaceTime
and 2 are in the 24 representing Conformal Gravity
and the 8th is in the 64 representing Fermion Particles.



E8 / D8 = 128 = 64 + 64:
63 green representing 63 of the 64 representing Fermion Particles
64 red representing Fermion AntiParticles
59 blue (light and dark) representing 59 of the 64 representing D8 / D4xD4 SpaceTime
24 orange representing D4 containing the Standard Model SU(3)
22 yellow of the 24 representing D4 containing Conformal SU(2,2) = Spin(2,4) Gravity

There are:
22 yellow dots + 2 Fundamental Root Vector (nos. 1,6 of 8) = 24
(+ 4 Cartan Elements) for Gravity + Dark Energy:
5 in black circle 1 (inner)
5 in black circle 3
5 in black circle 4
9 in black circle 5
3 + 4+2 + 3 = 12 Conformal SU(2,2)=Spin(2,4) Root Vectors

24 orange dots (+ 4 Cartan Elements) for the Standard Model:
5 in white circle 2
8 in white circle 6
7 in white circle 7
4 in white circle 8 (outer)
1+2+2+1 = 6 Standard Model SU(3) Root Vectors

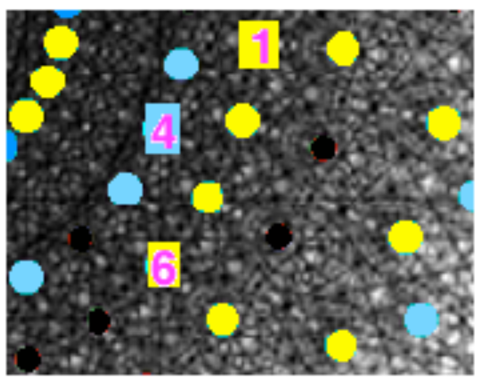
63 green dots + 1 Fundamental Root Vector (no. 8 of 8) = 64 for Fermion Particles:
6 in black circle 1 (inner)
10 in white circle 2
10 in black circle 3
10 in black circle 4
6 in black circle 5
6 in white circle 6
6 in white circle 7
10 in white circle 8 (outer)

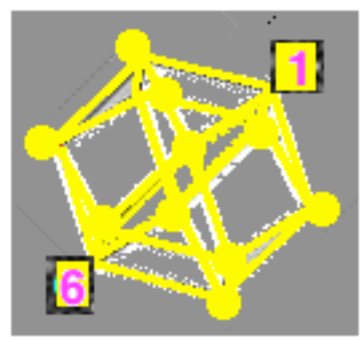
64 red dots for Fermion AntiParticles:
6 in black circle 1 (inner)
10 in white circle 2
10 in black circle 3
10 in black circle 4
6 in black circle 5
6 in white circle 6
6 in white circle 7
10 in white circle 8 (outer)

59 blue dots + 5 Fundamental Root Vector (nos. 2,3,4,5,7 of 8) =
= (28+4)+32 (positive+negative) = 64 for SpaceTime:
4+9 = 13 in black circle 1 (inner)
5+0 = 5 in white circle 2
0+5 = 5 in black circle 3
0+5 = 5 in black circle 4
0+9 = 9 in black circle 5
9+1 = 10 in white circle 6
9+2 = 11 in white circle 7
5+1 = 6 in white circle 8 (outer)

E8 / D8 Fermion Particles and AntiParticles are distributed through all 8 circles
D8 / D4xD4 SpaceTime is distributed through all 8 circles
32 dark blue Negative Root Vectors (28 of them in the 4 circles of the inner 600-cell)
correspond to CP2 internal symmetry space of M4xCP2 (4+4)-dim Kaluza-Klein
that is directly related to
the D4 (orange) of Standard Model in 24 Negative Root Vectors in the outer 600-cell
and
27 light blue Positive Root Vectors in the outer Golden Ratio 600-cell 4 circles
plus 2 Fundamental Root Vectors 4 and 2 in the outer Ratio 600-cell
plus 3 Fundamental Root Vectors 3-5-7 in the inner 600-cell
correspond (27+2+3 = 32) to M4 physical spacetime of M4xCP2 Kaluza-Klein

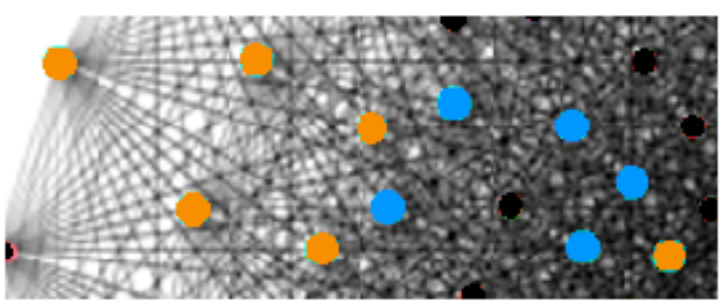
D4 (yellow) of Conformal Gravity is in 22 Positive Root Vectors in the inner 600-cell
and 2 Fundamental Root Vectors 1 and 6 in the inner 600-cell for 22+2 = 24
These 12 D4 Conformal Gravity Root Vectors = cuboctahedron polytope

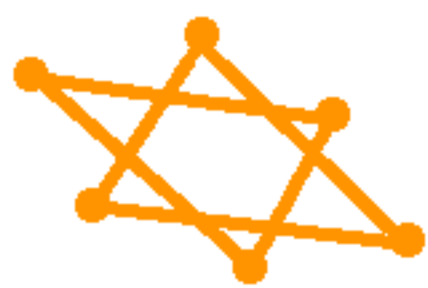




represent the Conformal D3 = SU(2,2) = Spin(2,4) subgroup of that D4

D4 (orange) of Standard Model SU(3) is in 24 Negative Root Vectors in the outer 600-cell
These 6 D4 Standard Model Root Vectors = Star of David polytope

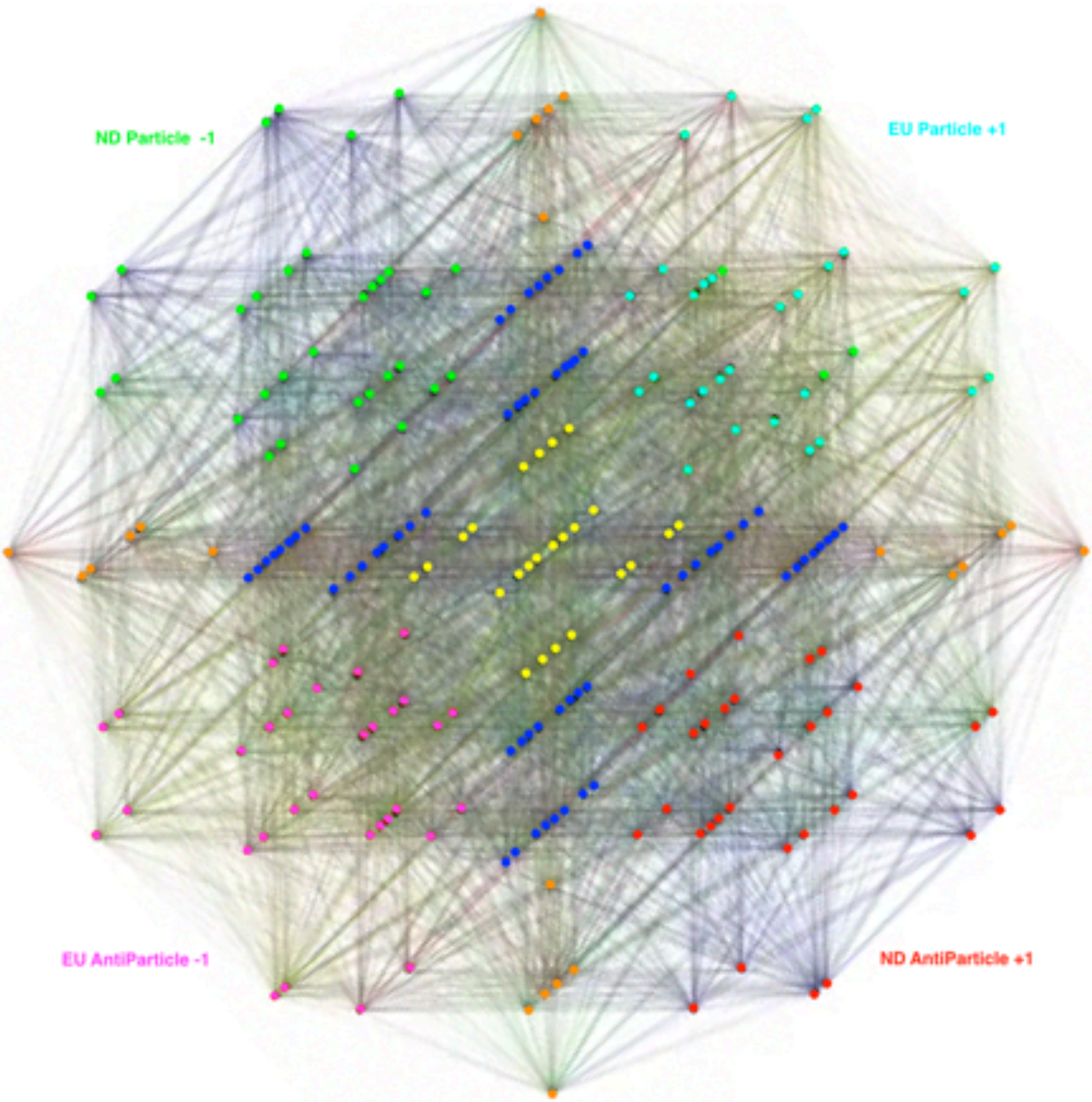




represent the SU(3) subgroup of that D4

The Madore 8 circles of 30 version gives realistic physics
but the physics interpretation of the vertices is not clear and obvious to me.

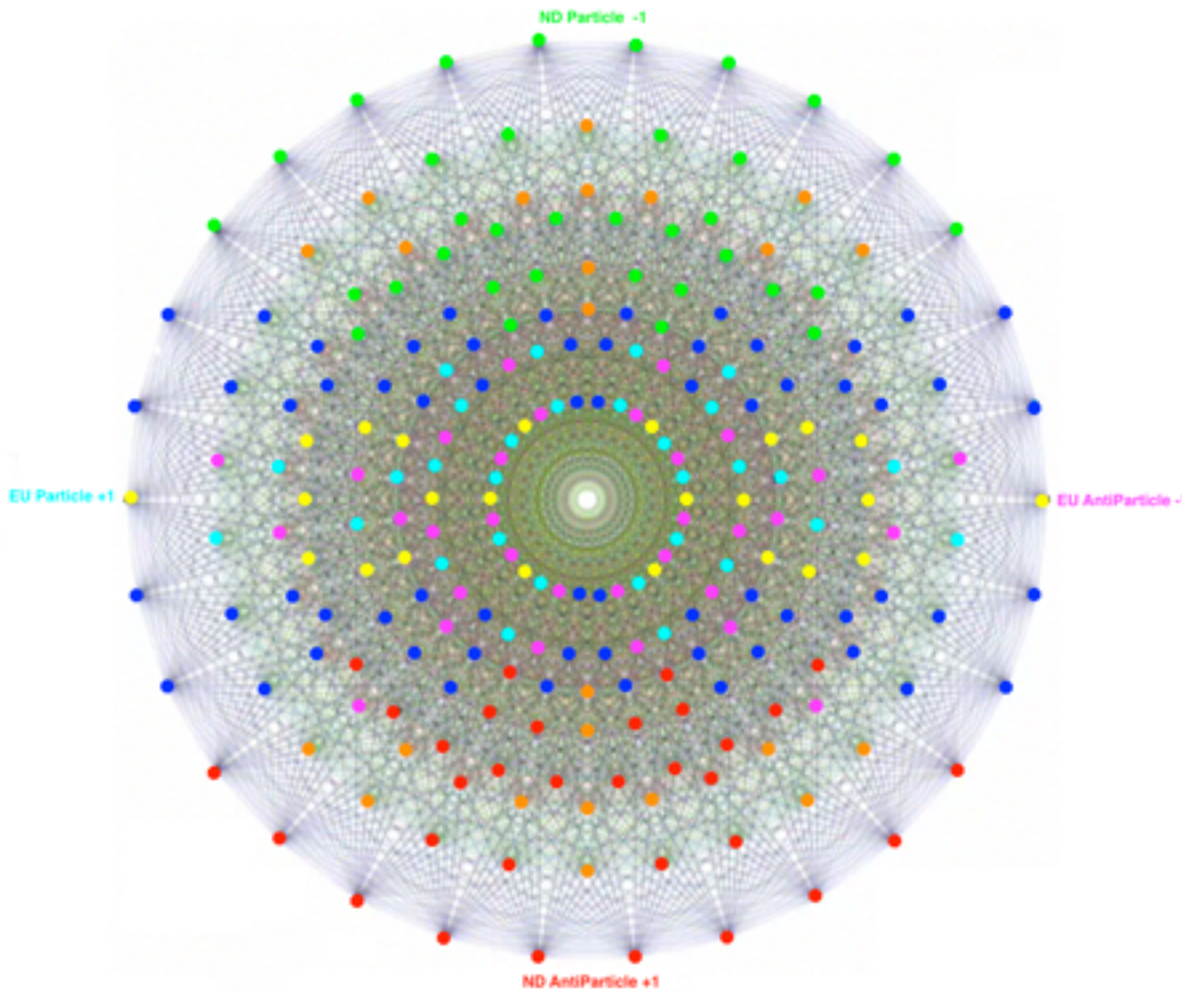
As Madore says there are many versions of 8 circles of 30 and
as to clearly visualizing how to build a realistic Lagrangian
I prefer a version of 8 circles of 30 that is derived from the square/cube type projection



that I use in my paper at http://vixra.org/pdf/1405.0030v9.pdf
in which it the physical interpretations of the root vectors are clear:

green / cyan and red / magenta for fermion particles and antiparticles (E8 / D8)
blue for M4 x CP2 Kaluza-Klein SpaceTime (D8 / D4xD4)
yellow for Gravity + Dark Energy (one of the D4)
orange for Standard Model SU(3) (the other D4)

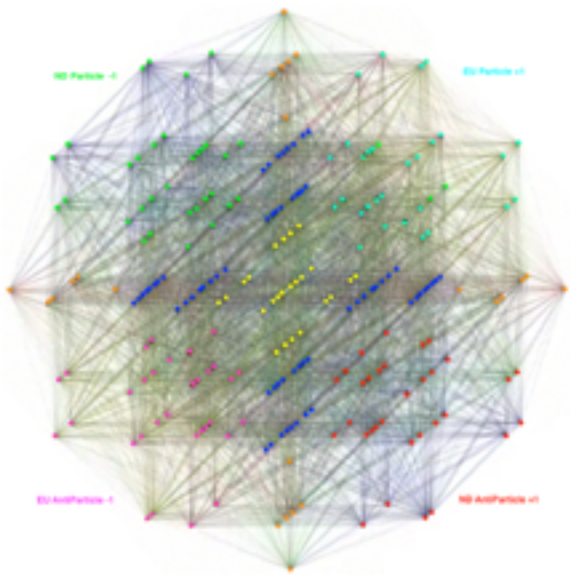
The square/cube version transforms into this 8x30 version

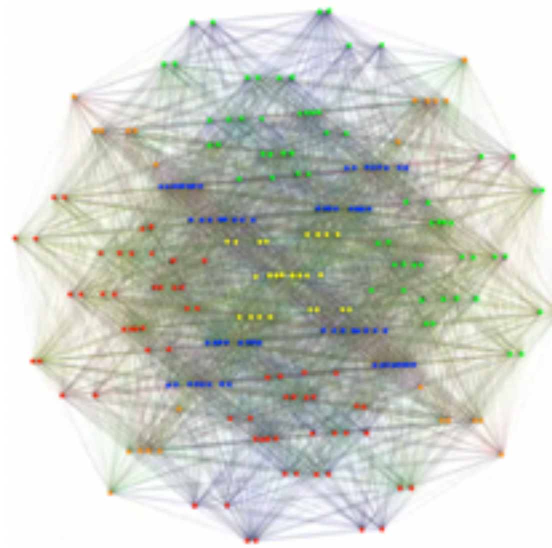


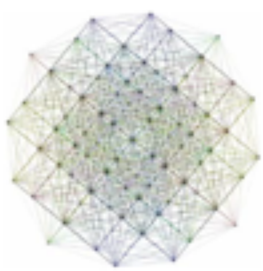
by means a video from mathematica code by Garrett Lisi ca 2007.

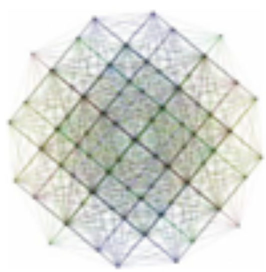
I have converted the video into a pdf slide sequence
and added vertex-colored square images at the beginning
and vertex-colored circle images at the end which pdf file is at
http://tony5m17h.net/E8squarecirclepdf.pdf

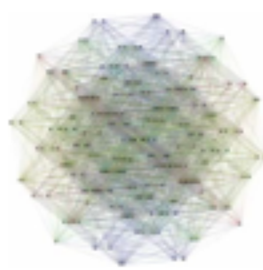
Here are some small images from that pdf file:

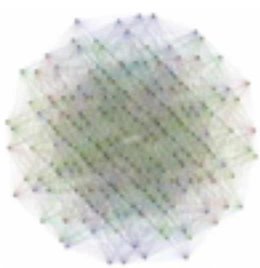


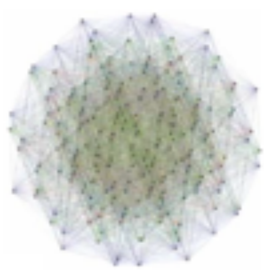


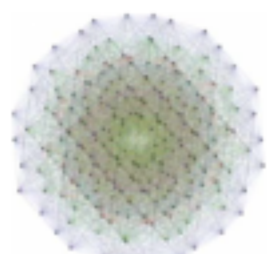


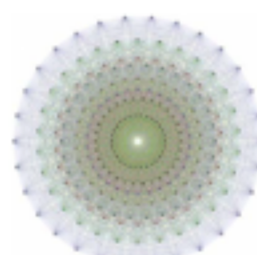


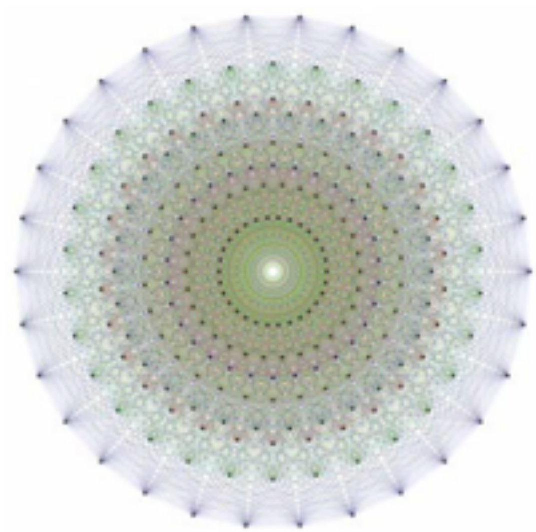


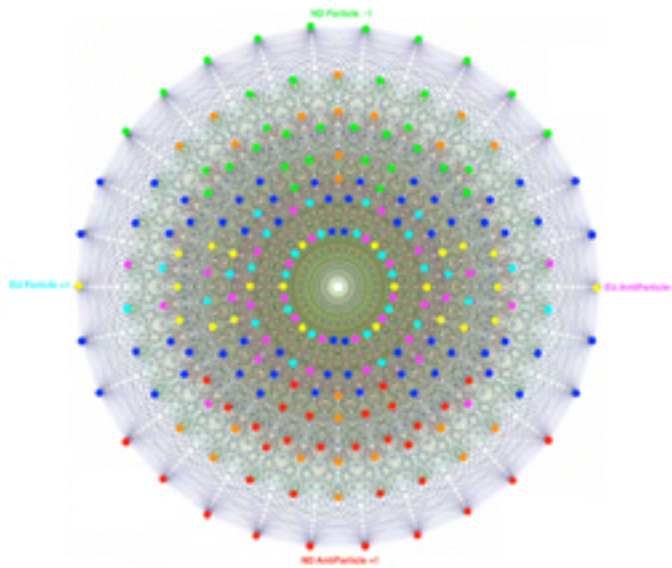




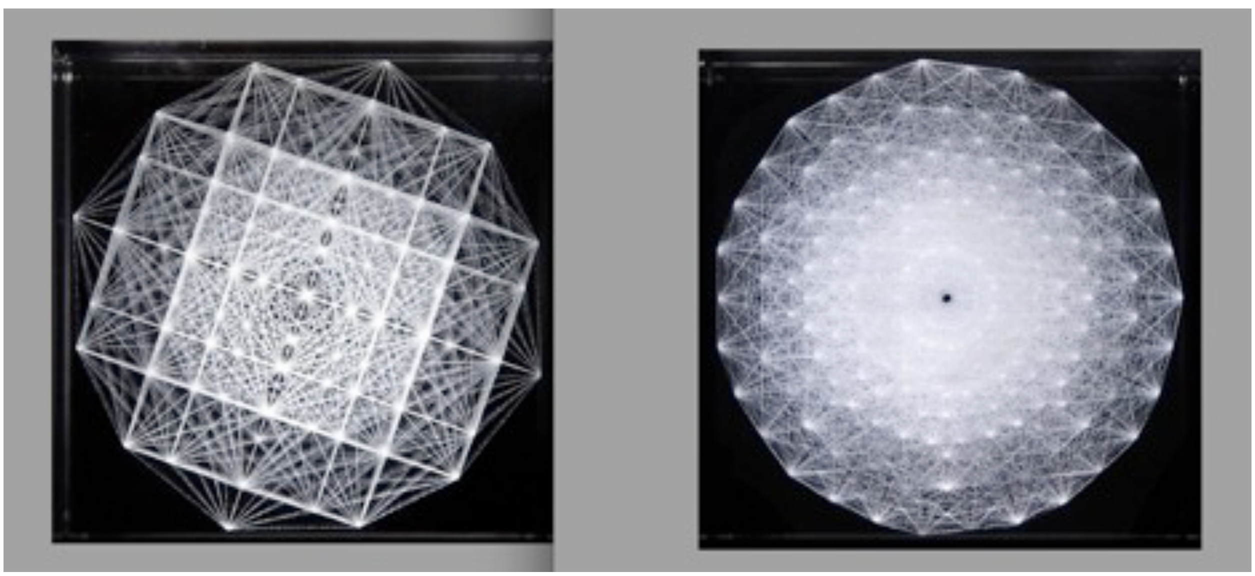








Similar code was used by Bathsheba Grossman in making her E8 cystal cube
two of whose faces



show that the square and circle projections are of the same E8.

22. From SU(2) to E8 for Cl(16)-E8 Physics

Frank Dodd (Tony) Smith, Jr. - 2014

Cl(16)-E8 Physics is described in viXra 1405.0030 from a top-down point of view of fundamental Clifford Algebra structure containing E8 leading to Lagrangian based on starting with E8 (the top Lie algebra) and then looking down at its substructures:

E8 / D8 = half-spinor Fermions (8 components of 8 Particles and 8 AntiParticles)
D8 / D4sm x D4g = (4+4)-dim M4 x CP2 Kaluza-Klein position x momentum
D4g = Conformal Gravity + Dark Energy and ghosts for Standard Model
D4sm = Standard Model Gauge Groups and ghosts for Gravity + Dark Energy

This paper takes a complementary bottom-up point of view to show that you can
start with the simplest Non-Abelian Gauge Group SU(2)
and
add the next step SU(3)
and
then go to SU(4) with cuboctahedron root vector polytope
and
then go to the D4sm Lie algebra with 24-cell root vector polytope
and
then go to a 600-cell whose 120 vertices give half of the 240 of E8

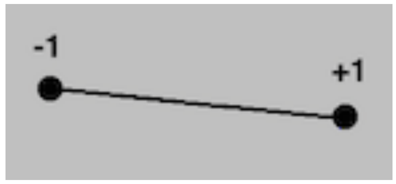
The you can get the other half of E8 by
starting with another cuboctahedron root vector polytope,
for U(2,2) of Conformal Gravity + Dark Energy
and
then go to the D4g Lie algebra with 24-cell root vector polytope
and
then go to a second 600-cell whose 120 vertices give the other half of E8

The two approaches, top-down of viXra 1405.0030 and bottom-up here,
give the same physics results
but
I think that you can get a deeper intuitive understanding of the physics
by looking at Cl(16)-E8 Physics from both points of view.

With that in mind, I have written this paper with heavy emphasis on intuitive graphics bearing in mind that technical issues have already been covered in viXra 1405.0030 .

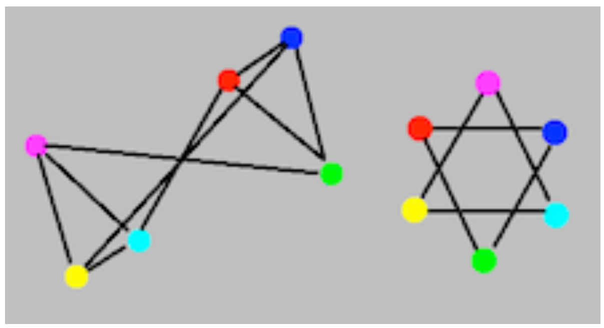
Standard Model

dipole



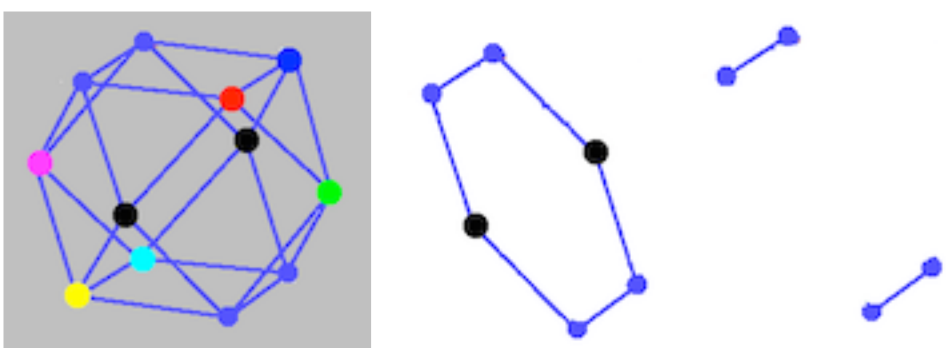
The 2 vertices of the dipole correspond
to
the 2 electric-charged gauge bosons W+ and W-
of the SU(2) Weak Force Gauge Group.

color

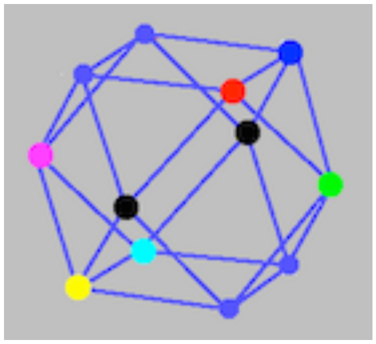


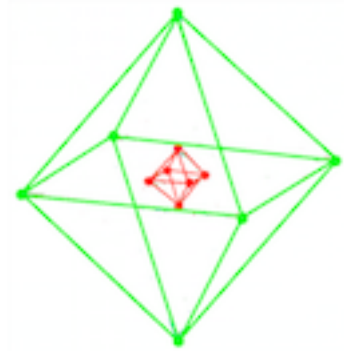
The 6 vertices of the Star of David correspond
to
the 6 color-charged gauge bosons Gluons (RY, RM, BM, BC, GC, GY)
of the SU(3) Color Force Gauge Group.

cuboctahedron



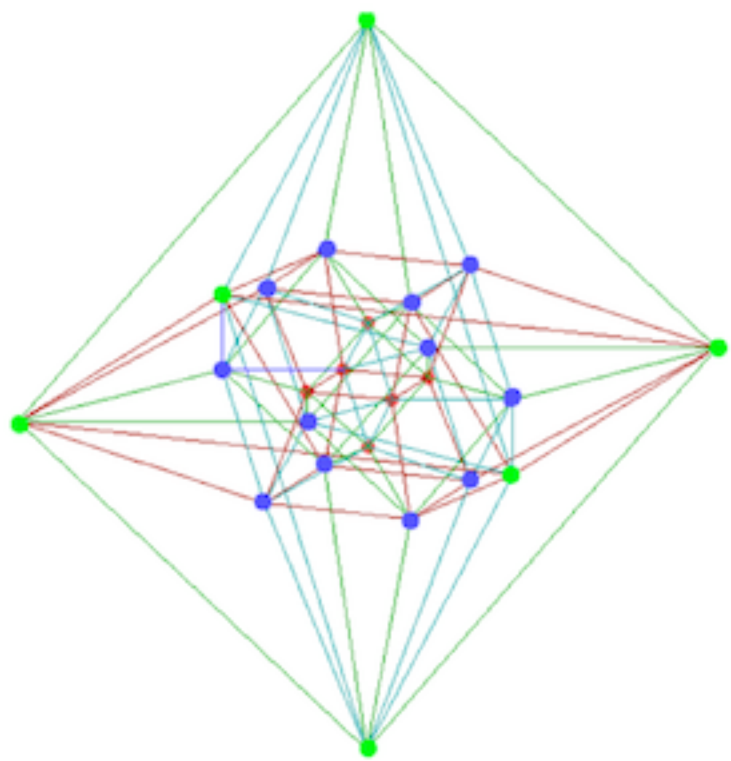
The 12 vertices of the cuboctahedron correspond to
the 6 charged gauge boson Gluons of the SU(3) Color Force Gauge Group
and 6-dim CP3 = SU(4) / U(3) Projective Twistors which include
2 charged gauge bosons W+ and W- of the Chiral SU(2) Weak Force Gauge Group
and
4-dim CP2 subspace of CP3 as ghosts for Special Conformal Transformations.





The 12 vertices of the cuboctahedron plus the 6+6 = 12 vertices of two octahedra make the 24 vertices of the 24-cell, the root vectors of the D4 Lie Algebra

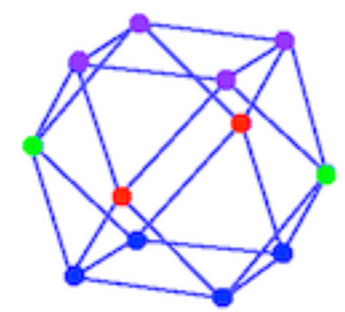
D4sm for the Standard Model



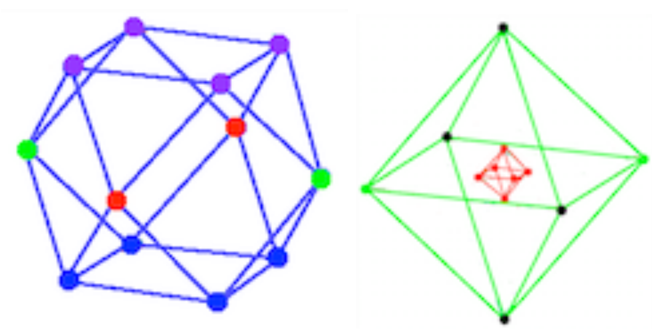
The 6+6 = 12 vertices of the two octahedra (red+green) represent ghosts
for 4 SpaceTime Translations
and 6 Lorentz Group Generators (including 2 Cartan elements)
and the other 2 Cartan elements of Conformal Gravity + Dark Energy U(2,2).

Conformal Gravity + Dark Energy

cuboctahedron



The 12 vertices of the cuboctahedron correspond to
the 2 generators of 3-dim space rotations (red) represented by Quaternions {i,j} (ij = k)
the 2 generators of space-time boosts (green) also represented by {i,j}
the 4 spacetime translations (blue)
the 4 spacetime Special Conformal Transformations (purple)

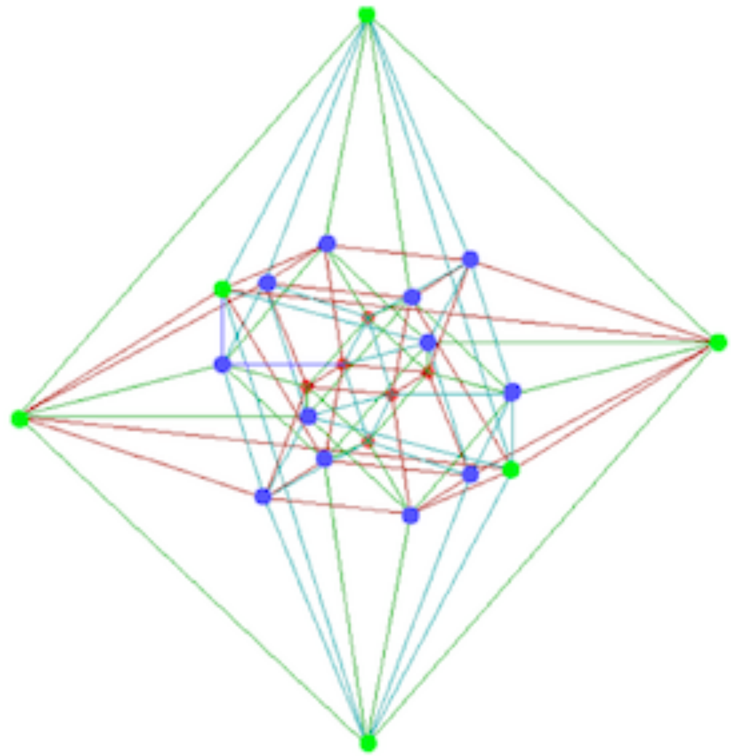


The 12 vertices of the cuboctahedron plus the 6+6 = 12 vertices of two octahedra make the 24 vertices of the 24-cell, the root vectors of the D4 Lie Algebra

D4g for Conformal Gravity + Dark Energy

The 6+6 = 12 vertices of the two octahedra represent ghosts
for the 12 generators of the Standard Model Gauge Groups:
SU(3) (red 6 charged Gluons) x SU(2) (green 2 charged W-bosons x U(1) (black)
plus 3 Cartan elements (2 for SU(3) and 1 for SU(2)) (black)

D4sm and D4g are each represented by the 24-cell



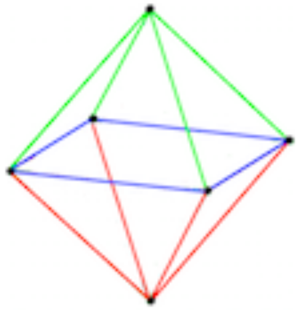
Each of the 24 octahedral cells of the 24-cell contains an icosahedron.

Each of the 24 icosahedra contains 20 tetrahedra for a total of 480 tetrahedra.
Each of the 24 vertices of the 24-cell is surrounded
by 5 tetrahedra that fill up the space of the 24 octahedra not in the 24 icosahedra,
for a total of 24x5 = 120 more tetrahedra so that
the 24-cell has been mapped into a 480+120 = 600-cell with 600 tetrahedral cells.
However,
it is not the tetrahedral cells that correspond to fundamental physics entities,
but rather it is the vertices. The 600-cell has 120 vertices:

24 from the 24-cell corresponding to gauge bosons and ghosts
and
96 from icosahedral vertices on each of the 96 edges of the 24-cell.

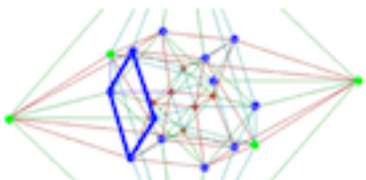
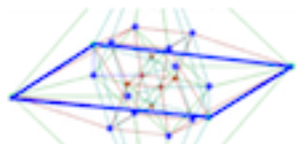
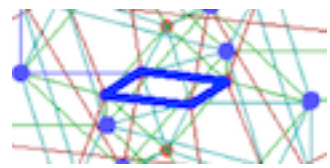
What fundamental physics entities correspond to the 96 vertices ?

Each of the 96 vertices lives on one of the 96 edges of the 24-cell which are edges of the octahedral cells of the 24-cell, so look at their relative positions in the octahedra.



4 square edges (blue) correspond to spacetime

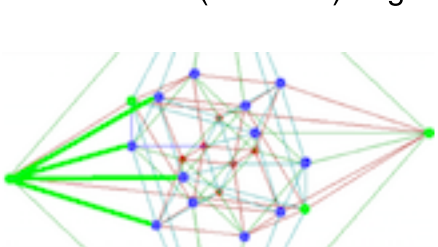
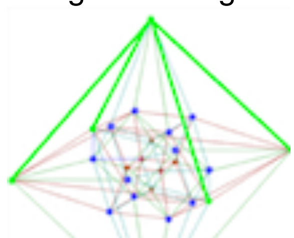
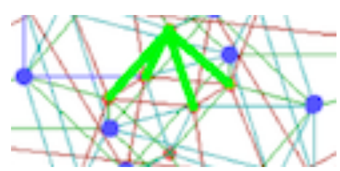
There are (6x4 = 24) edges + 4 edges + 4 edges = 32 edges of this type:

The D4sm 600-cell has 32 spacetime vertex-entities
and the D4g 600-cell also has 32 spacetime vertex-entities.
The total 32+32 = 64 = 8x8 position x momentum for (4+4)-dim M4 x CP2 Kaluza-Klein
The D4sm entities act on CP2 = SU(3) / U(2) Internal Symmetry Space.
The D4g entitites act on M4 Minkowski Physical Spacetime.

4 edges (green) going down from a common vertex correspond to Fermion Particles

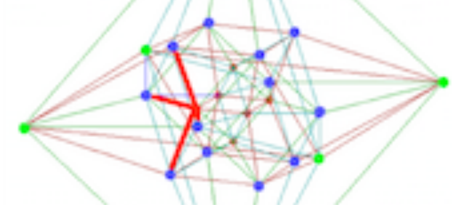
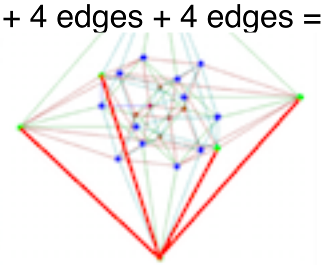
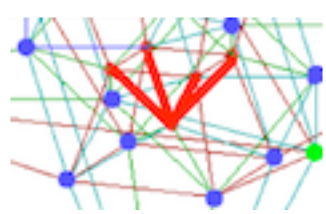
There are (6x4 = 24) edges + 4 edges + 4 edges = 32 edges of this type:

The D4sm particles are 32 = 4x8 = 4 CP2 coordinates of 8 Fundamental Particles.
The D4g particles are 32 = 4x8 = 4 M4 coordinates of 8 Fundamental Particles.

4 edges (red) going up from a common vertex correspond to Fermion AntiParticles

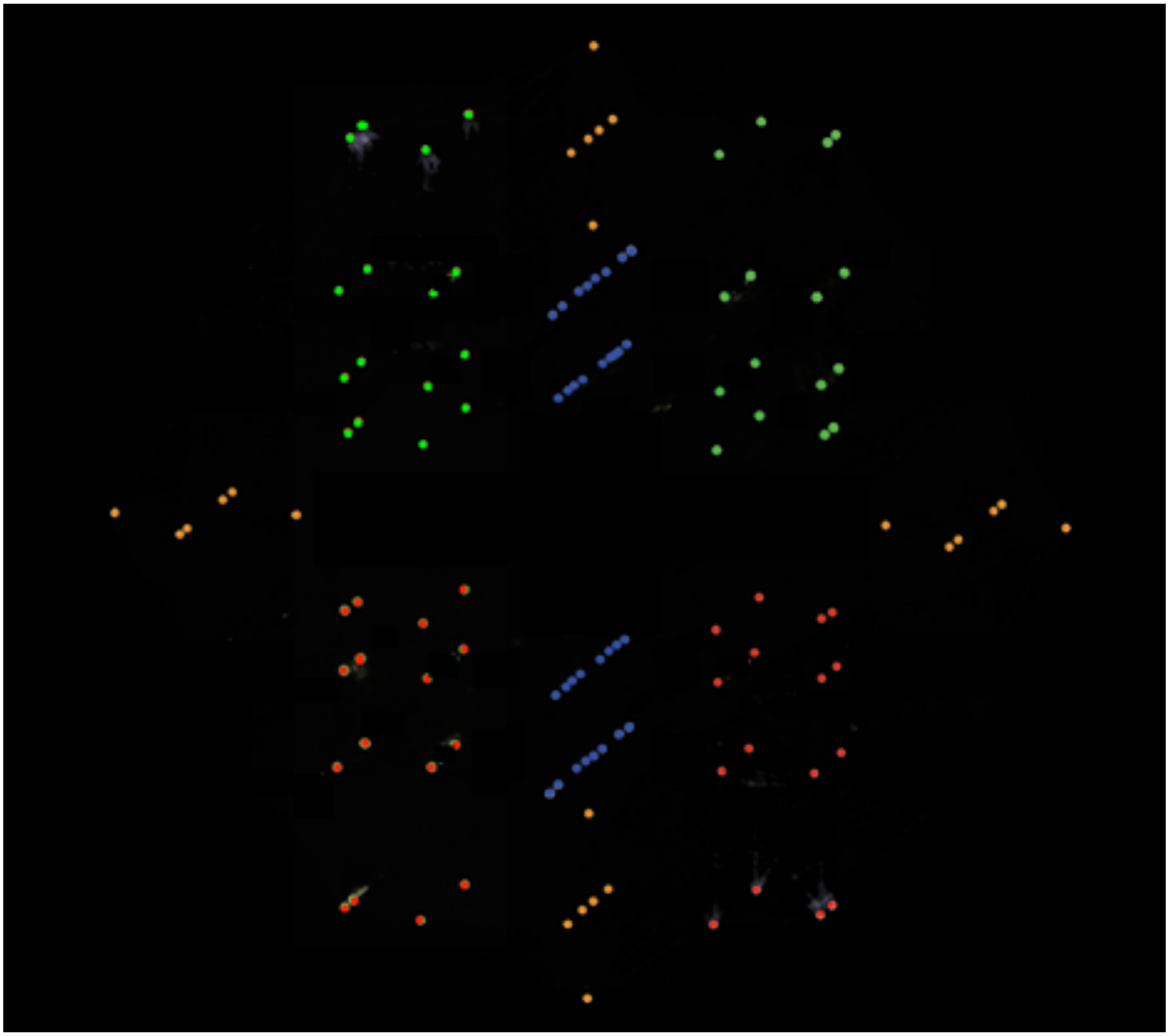
There are (6x4 = 24) edges + 4 edges + 4 edges = 32 edges of this type:

The D4sm particles are 32 = 4x8 = 4 CP2 coordinates of 8 Fundamental AntiParticles.
The D4g particles are 32 = 4x8 = 4 M4 coordinates of 8 Fundamental AntiParticles.

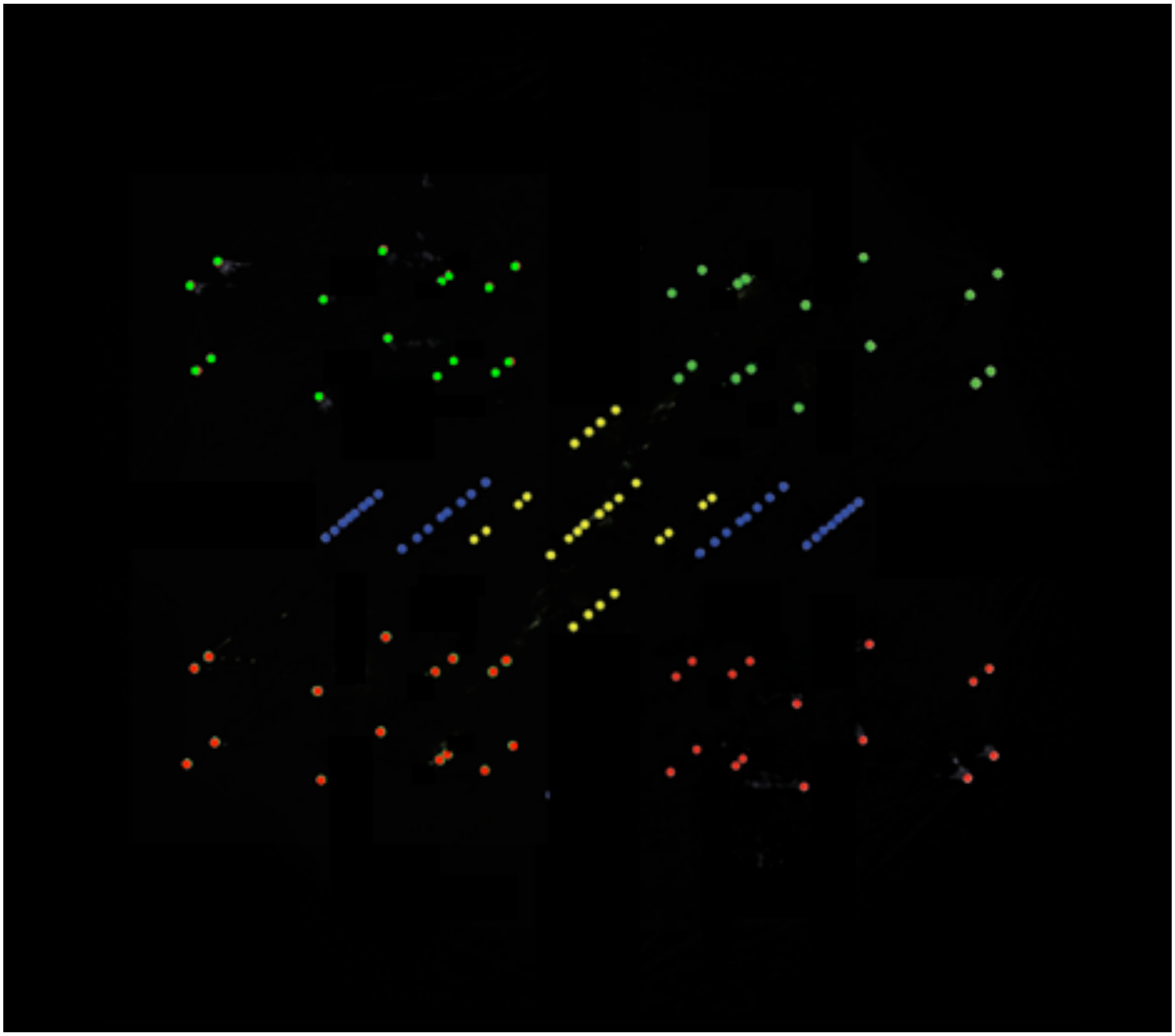
The 120 + 120 vertices of

the Standard Model D4sm 600-cell

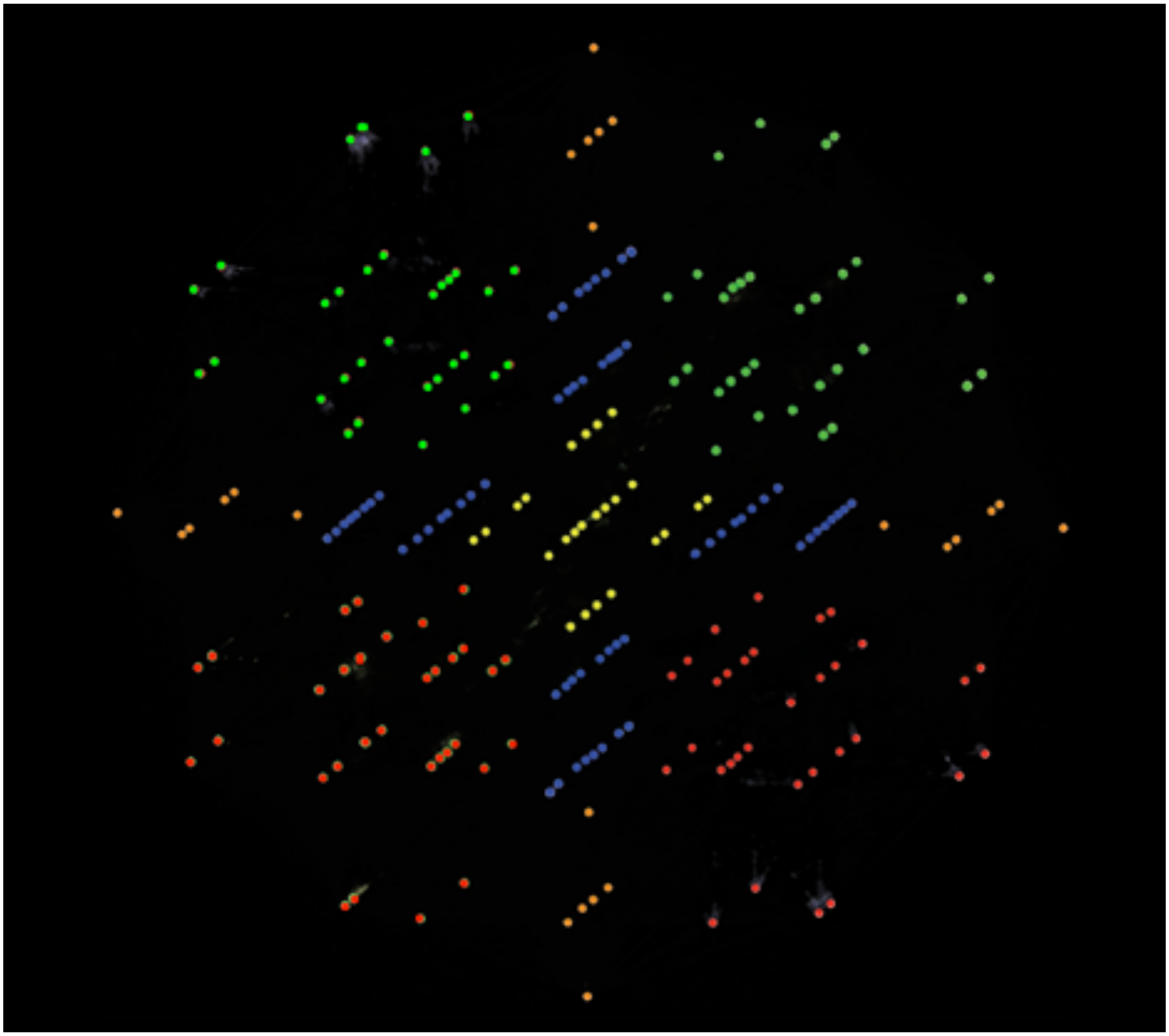


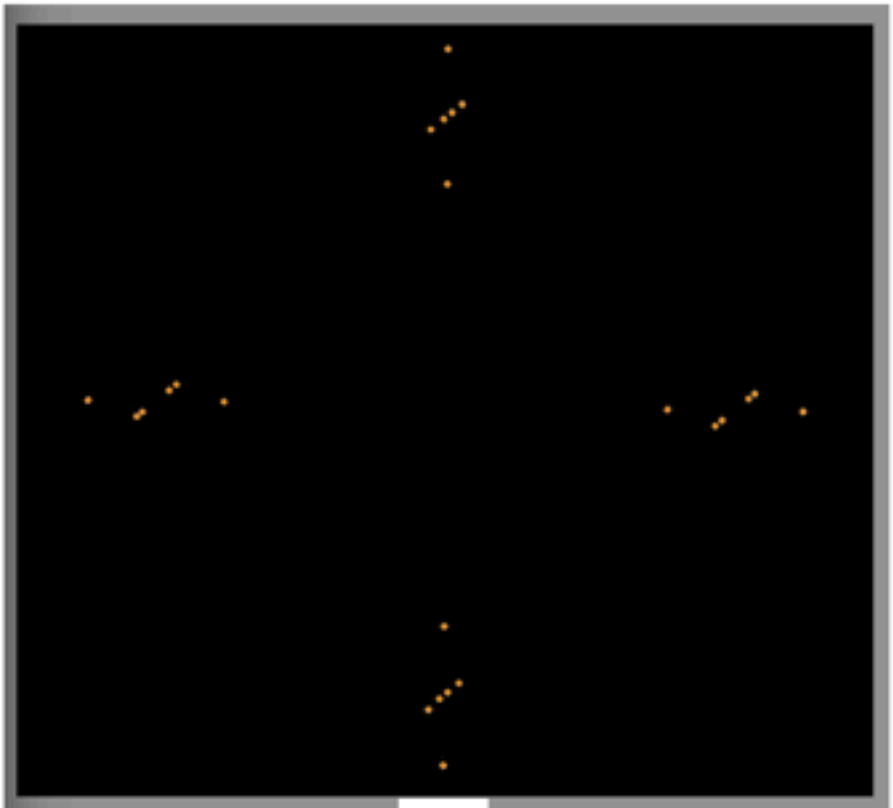
and

the Gravity + Dark Energy D4g 600-cell



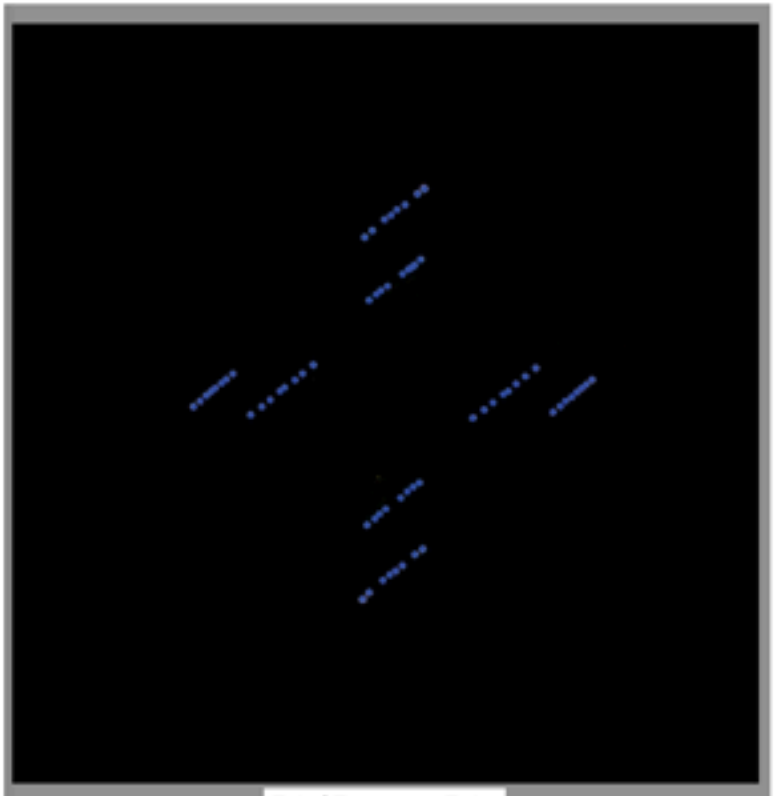
combine
to form the 240 root vector vertices of E8



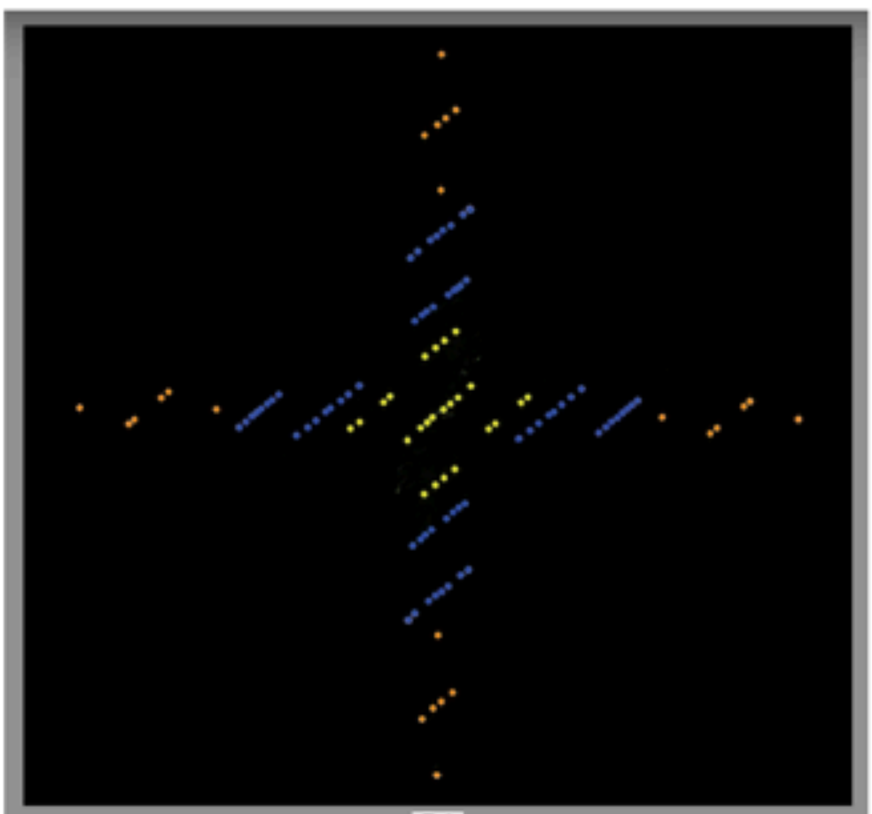


D4sm

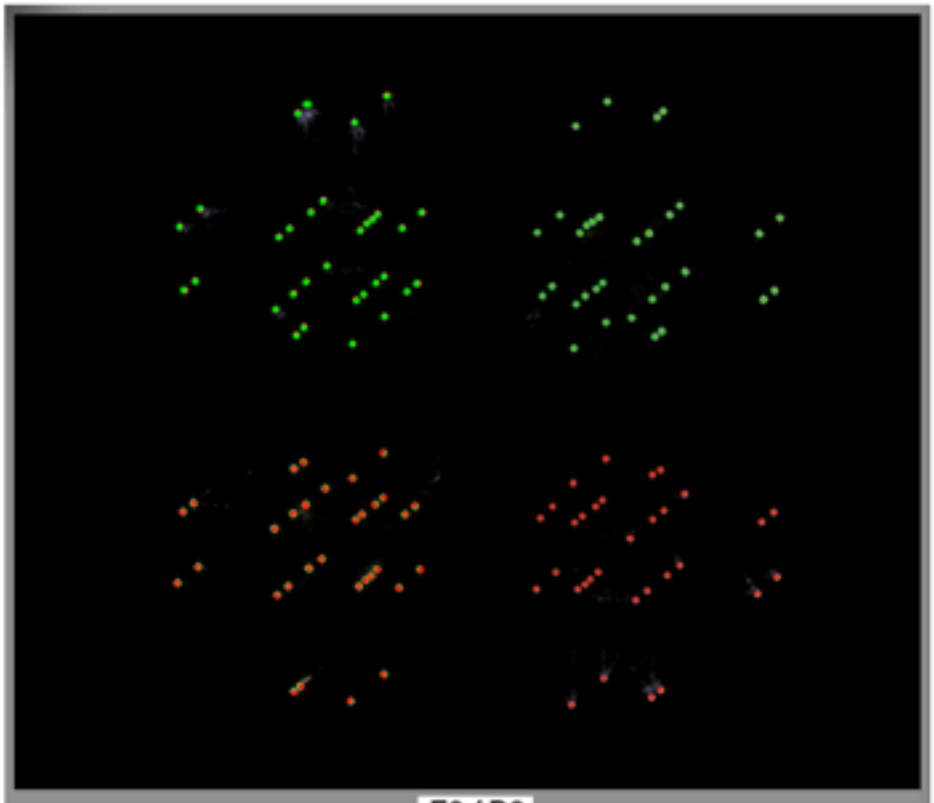
D4g



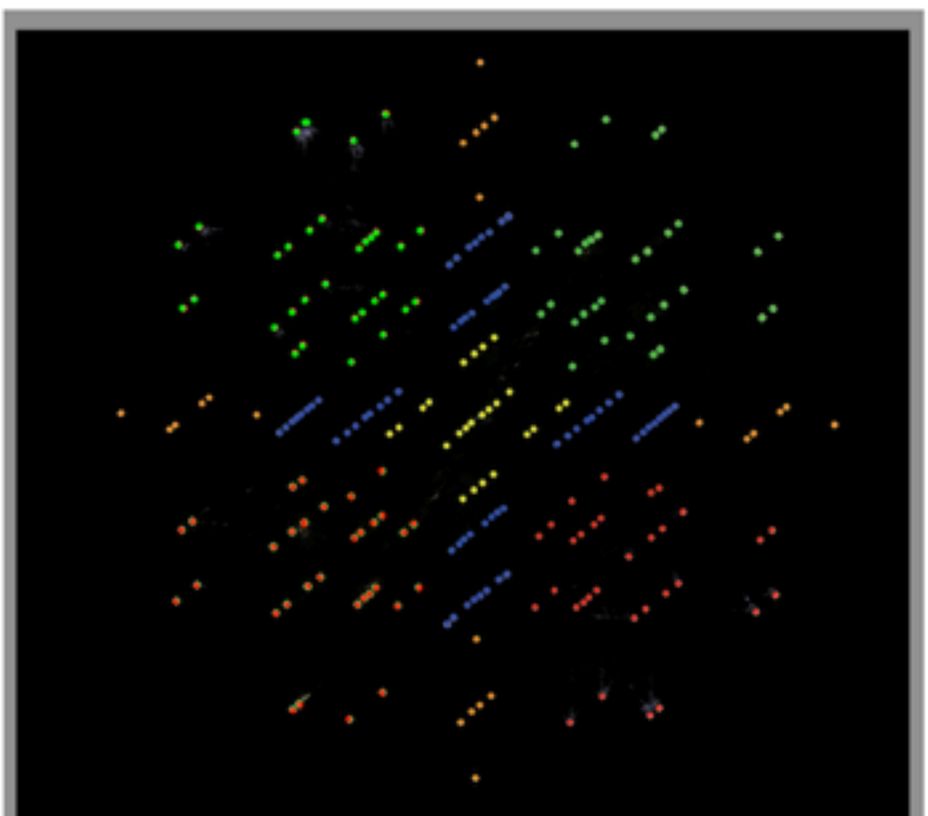
D8 / D4sm x D4g



D8



E8 / D8



E8

23. How Garrett Lisi's E8 differs from my Clifford Algebra based Cl(16)-E8 Physics model:

Frank Dodd (Tony) Smith< Jr. - 2014 msg to Ben Goertzel

Garrett Lisi uses E8 as a gauge group over a separate 4-dim spacetime
while
my E8 is NOT just a gauge group
but includes (4+4)-dim Kaluza-Klein spacetime as part of E8

I see E8 not as merely a gauge group but
as an algebraic structure that tells you how build a Lagrangian
with (in terms of Lie algebras E8, D8, D4g, D4sm as described below)

E8 / D8 gives Densities for Fermions
and
D4g and D4sm give Densities for Gravity and the Standard Model
Densities are integrated over
Kaluza-Klein spacetime represented by D8 / D4g x D4sm

===

The bottom line is:

Garrett's E8 does not contain his external 4-dim spacetime
but
does contain 166 unobserved things

My Cl(16)-E8 model contains all things that have been observed
and nothing that is not observed
and (4+4)-dim Kaluza-Klein spacetime is included in my E8.

Here are details:

Most recently Lisi's E8 has been described by Douglas and Repka in
http://arxiv.org/pdf/1305.6946v3.pdf
and Garrett himself tweeted on 25 Sep 2014 that the paper was "Nice."
so I will use it as well as Garrett's paper
http://arxiv.org/pdf/1006.4908v1.pdf
as a basis for comparison.

First I will describe Garrett's 2010 paper at arXiv 1006.4908

248-dim E8 = 120-dim D8 + 128-dim half-spinor of D8
where D8 represents spin(12,4)
so that

Lisi breaks down E8 into:

120 = 91-dim spin(11,3) + 29-dim D8 / spin(11,3)
+
128 = 64-dim positive chiral half-spinor of spin(11,3) + 64-dim negative half-spinor of spin(11,3)

In Lisi's scheme
91-dim spin(11,3) breaks down into:

6 for Gravity spin(1,3) = Lorentz + boosts
40 for a frame-Higgs + 45 for spin(10) GUT gauge bosons which 45 break down into:
1 photon
8 gluons
3 weak bosons
3 new weak bosons
30 X-bosons

29-dim D8 / spin(11,3) breaks down into:

20 more X-bosons
1 Peccei-Quinn w-boson
8 more frame Higgs including two axions

64-dim positive chiral spin(3,11) half-spinor = one generation of fermions
based on
2 chirality staes x 2 charge states of 8 fermion particles (e; R,G,B up quarks : Neu; R,G,B down quarks
andt
2 chirality states x 2 charge states of 8 fermion antiparticles

64-dim negative chiral spin(3,11) half-spinor = one generation of mirror fermions
These are not observed now.

==

Now I will describe the more recent arXiv 1305.6946v3
that uses a complex so(14)C instead of Spin(11,3). Both are 91-dimensional
and
instead of adding just a 64-dim fermion thing to Spin(11,3)
there is added to 91-dim so(14)C a 78-dim thing that breaks down into 14-dim + 64-dim

The 64-dim thing is a half-spinor of so(14) and has the same physics interpretation
as in Lisi's 2010 model.

In Lisi's new expanded scheme scheme
91-dim so(14)C lives in 120-dim D8 and breaks down into:

6 for Gravity spin(1,3) = Lorentz + boosts
40 for a frame-Higgs + 45 for spin(10) GUT gauge bosons which 45 break down into:
1 photon
8 gluons
3 weak bosons
3 new weak bosons
30 X-bosons

14-dim thing is translations in 14-dim vector space.

The 120 - 91 - 14 = 15 things are

6 more X-bosons = 20 - 14
1 Peccei-Quinn w-boson
8 more frame Higgs including two axions

64-dim positive chiral so(14)C half-spinor = one generation of fermions
based on
2 chirality staes x 2 charge states of 8 fermion particles (e; R,G,B up quarks : Neu; R,G,B down quarks
andt
2 chirality states x 2 charge states of 8 fermion antiparticles

64-dim negative chiral so(14)C half-spinor = one generation of mirror fermions
These are not observed now.

==

Compare the Garrett Lisi breakdown with what I do:

I also break down E8 into:
120 = D8
+
128 = chiral half-spinor of D8 =
= 64-dim positive chiral half-spinor of spin(14) + 64-dim negative half-spinor of spin(14)

but in my scheme
E8 / D8 = 64 + 64 =
8 Kaluza-Klein 8-dim components of 8 fermion particles
and
8 Kaluza-Klein 8-dim components of 8 fermion antiparticles

D8 contains two copies of 28-dim D4,
one D4g for Gravity + Dark Energy and the other D4sm for the Standard Model

D4g = 16 generators for Gravity + 12 ghosts for the Standard Model

D4sm = (1+8+3) = 12 generators for the Standard Model +16 ghosts for Gravity

D8 / D4g x D4sm = 8x8 = 64-dim representation of 8-dim Kaluza-Klein 8-position x 8-momentum

==

Both Garrett and I have as fundamental only the first generation of fermions.
In my model, the second and third generations come from the geometry of (4+4) -dim Kaluza-Klein which geometry also produces the Higgs.

Garrett's E8 does not contain his external 4-dim spacetime.

It has 166 unobserved things

40 frame-Higgs
3 new weak bosons
30 X-bosons
14 translations in so(14)C vector pace
6 more X-bosons
1 Peccei-Quinn w-boson
8 more frame Higgs including two axions
64 half-spinors for one generation of mirror fermions

and 82 observed things

6 for Gravity spin(1,3) = Lorentz + boosts
1 photon
8 gluons
3 weak bosons
64 half-spinors for one generation of mirror fermions

My Cl(16)-E8 model contains all things that have been observed
and nothing that is not observed
and (4+4)-dim Kaluza-Klein spacetime is included in my E8.

SuperNova Precession Period of Peace

Frank Dodd (Tony) Smith, Jr. - 2014

viXra 1404.0057

Abstract:

The 24,925 year period
beginning with the Shock Wave of the Geminga SuperNova 36,525 years ago
and
ending with the Vela Pulsar SuperNova 11,600 years ago
corresponds roughly to one 26,000 year Earth Precession Period.

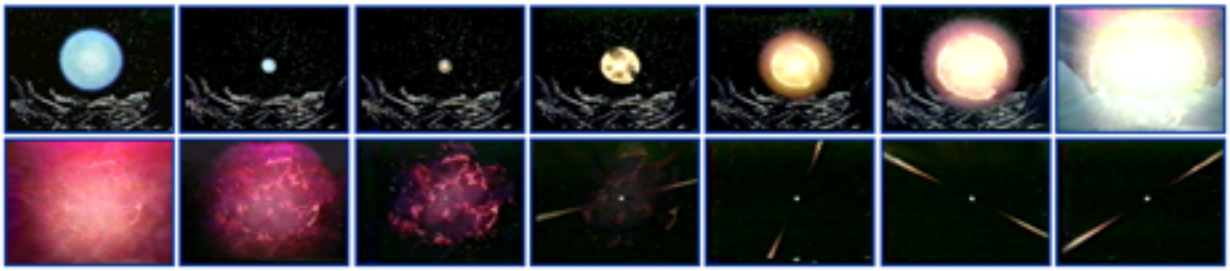
No written historical record has been found for that SuperNova Precession Period during which Humans had migrated out of Africa to populate much of the Earth continuing the African IFA divination system, producing Cave Art, developing Sanskrit and the Rig Veda and the Meru Prastara, and building the Great Pyramid complex at Giza.

Human Civilization during much of that SuperNova Precession Period seems to have been Peaceful, living in Harmony with Nature and with Each Other, so Manetho characterized it as the time of the Rule of Gods, Demigods and Spirits and I would call it the SuperNova Precession Period of Peace.

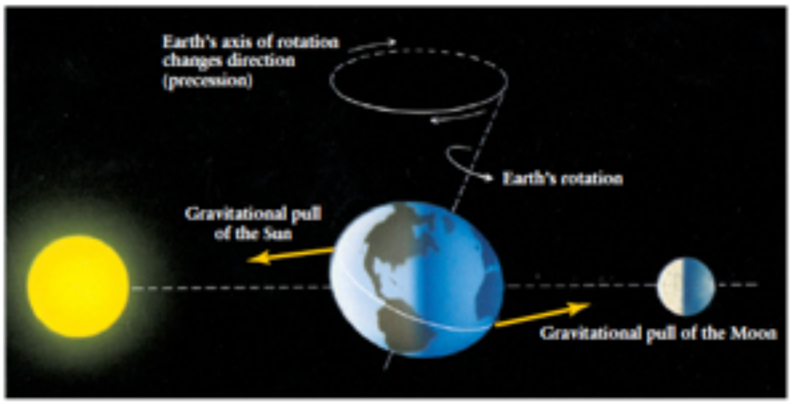
However, after the Vela Pulsar SuperNova 11,600 years ago Human Civilization entered the Present Precession Period, Manetho's time of the Rule of Mortal Humans characterized by (to paraphrase Terence McKenna) scarcity, preservation of privilege, and ever-more-sophisticated use of technology and ideology to control people.

This paper is an attempt to describe the SuperNova Precession Period of Peace and to explore possible future paths of the Present Precession Period, including possible Global Nuclear War and possible Global Reconciliation and Peace.

Manetho (Egyptian historian ca. 343 BC) wrote about **the SuperNova Precession Period**



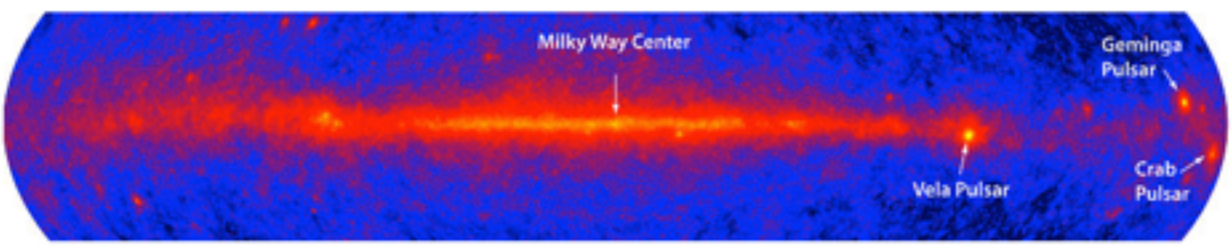
(SuperNova image sequence from NASA Goddard)



Precession Because the Earth's rotation axis is tilted, the gravitational pull of the Moon and the Sun on the Earth's equatorial bulge together cause the Earth to precess. As the Earth precesses, its axis of rotation slowly traces out a circle in the sky

(image from Universe by Kaufmann et al 8th ed)

from about 36,525 years ago - **Geminga SuperNova Shock Wave** hit Earth (815 lyr)



(image from Wikipedia)

to about 11,600 years ago - **Vela Pulsar SuperNova** seen at Earth (959 lyr) as bright as a Quarter Moon (magnitude -10)

Manetho characterizes the **SuperNova Precession Period** as
the **Rule of Gods, Demigods and Spirits of the Dead.**

Developments then include **IFA**,
Rig Veda,
Cave Art,
Meru Prastara,
and the **Great Pyramid**.

As would befit Gods, Demigods, and Spirits, **ALL THESE WERE PEACEFUL**.

After the SuperNova Precession Period Rule of Gods, Demigods and Spirits
that is
under what Manetho calls the Rule of Mortal Humans,

we have seen a transition described by Terence McKenna (1993 OMNI interview) as

"... From 75,000 to about 15,000 years ago,
there was a kind of human paradise on Earth.
...[Then around 11,600 years ago, when the Vela Pulsar supernova was seen in Earth,
a very sudden (50 years or so) warming event ended the Ice Age and marked the start
of the Holocene Age of warm climate and glacial retreat.
Rising sea levels flooded much productive land creating shortages of food
and consequently competition for available food,
and what Terence McKenna called "tooth-and-claw dominance"]...
For 10,000 years ... we've pursued an agenda of beasts and demons ...
If history goes off endlessly into the future,
it will be about scarcity, preservation of privilege, forced control of populations,
the ever-more-sophisticated use of ideology to enchain and delude people.
We are at the breakpoint.

It's like when a woman comes to term ...
if the child is not severed from the mother and launched into its own separate existence,
toxemia will set in and create a huge medical crisis.
...
[But if we launch ourselves into the Birth of a New Period of Peace]...
What lies ahead is a dimension of such freedom and transcendence,
that once in place, the idea of returning to the womb will be preposterous.
We will ... expand infinitely into pleasure, caring, attention, and connectedness. ...".

Hoping that understanding the SuperNova Precession Period of Peace
might be exemplary of what Mortal Humans need to do to better their condition,
here on the following pages are some details about
IFA, **Rig Veda**, **Cave Art**, **Meru Prastara**, and the **Great Pyramid**.
(more details about that Period are in Appendix II)

After that, I describe some events of the Rule of Mortal Humans so far,
and speculate about possible future events.

IFA

70,000 years ago Mount Toba in Sumatra Erupted 800 cubic kilometers of ash into the air producing a bottleneck minimum of human population -
a few thousand people huddled together in Africa.

According to the National Geographic Genographic Project (the source of the following 3 map images)
Y-chromosome DNA shows that humans were still living only in Africa
as late as about **55,000 years ago**.

The wisdom of Ancient Africa was encoded in **the 256-element IFA divination system that is equivalent to the 256-dimensional Cl(8) Real Clifford Algebra** that,
since Cl(8N) = Cl(8) x...(N times tensor product)...x Cl(8) by Real Clifford 8-Periodicity,
includes Cl(16) from which emerges E8 Physics Algebraic Quantum Field Theory.
(see viXra 1312.0036 and 1310.0182 and related papers for details of E8 Physics)
Since all humans lived in the same neighborhood
knowledge was transmitted from generation to generation by Oral tradition
with no need for Written texts or Art paintings for accurate preservation.

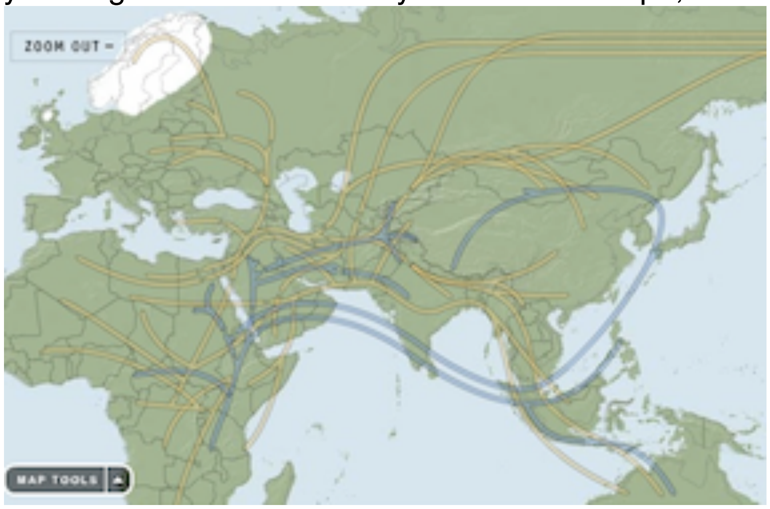
About **50,000 years ago** Y-chromosome DNA indicates that basic human physiology

had emerged from Africa to India then to Japan and on to Tibet.

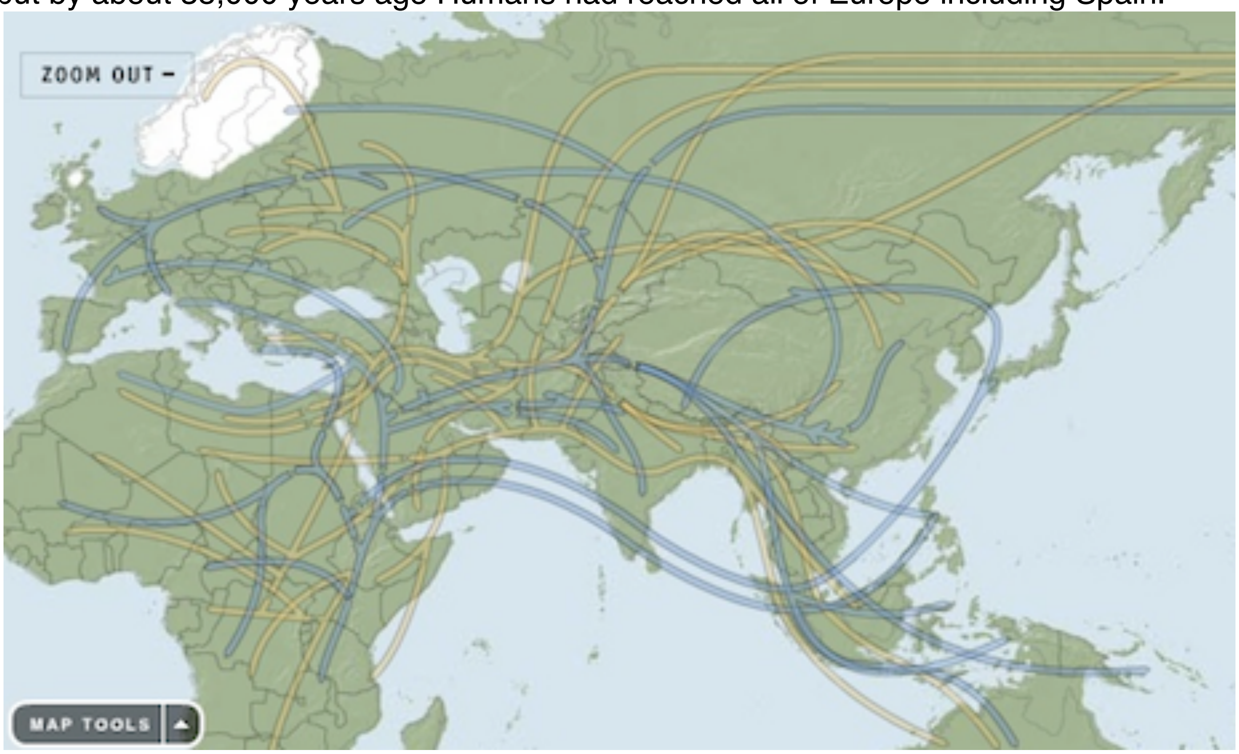
Rig Veda

Indians were separated from their African Homeland by the Arabian Sea
which was close enough to Africa to maintain regular contact
but far enough that they felt isolated from the very close contact needed
to maintain the details of the oral traditions of IFA, so the Indian priests of IFA chose
to put the IFA Information System into writing and to do so developed Sanskrit
and wrote the Rig Veda.
Japanese were even further isolated from Africa so they retained only 1/2 of the 256 IFA elements in their 128-element Shinto Futomani Divination system.
Tibetans were the most isolated of the first 3 groups migrating from Africa.
They retained only 1/4 of the 256 IFA elements in their 64-element I Ching.

By about 40,000 years ago Humans had not yet reached Europe,



but by about 35,000 years ago Humans had reached all of Europe including Spain.

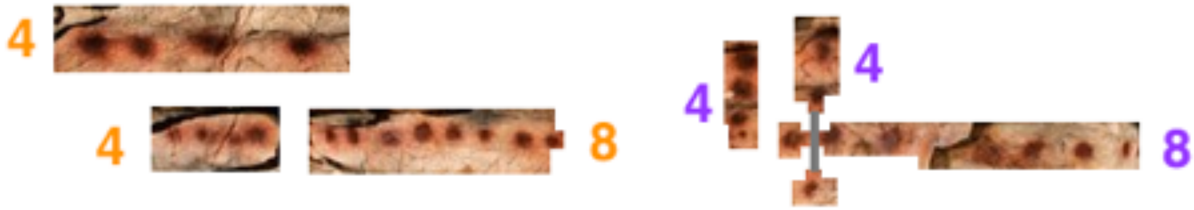


(Note that 35,000 years ago the Black and Mediterranean Seas were much smaller and there was a lot more land around Indonesia, New Guinea, and China. The present-day large Black and Mediterranean Seas and high Sea levels near Indonesia, New Guinea, China, and the rest of the Earth have only existed in the past 11,600 years or so.)

Cave Art

Jonathan Amos, in BBCnews 15 June 2012, said:
"... Red dot becomes 'oldest cave art' ... The symbols on the walls at ... Spanish ... El Castillo ...[include]... a red disk ... older than 40,800 years ... The oldest dates ...

... coincide with the first known immigration into Europe of modern humans. ...".
If you look closely at the red dots you see that they fall into two groups



8+4+4 = 16 dots and 8+4+4 = 16 dots

Since 16x16 = 256 is the fundamental number of elements of IFA divination it seems that the first thing done by the newly arrived migrants into Europe may have been to encode IFA divination on the Walls of their Caves.

(Note that Spain is close to Africa so that the New Europeans could verify directly with their African neighbors that 16x16 = 256 does indeed encode IFA.)

That time period roughly coincides with the Geminga SuperNova Shock Wave arriving at Earth and with what Manetho Egyptian historian ca. 343 BC) described as **the beginning about 36,525 years ago of the Rule of Gods, Demigods and Spirits of the Dead that ended around 11,600 years ago** when the Vela Pulsar SuperNova was seen on Earth and the Younger Dryas Cold Snap was immediately followed by a very sudden (50 year or so) beginning of

the **Warm Interval in the Ice Age in which we Mortal Humans now live**.

(It has high sea levels and large Black and Mediterranean Seas. Since we are in a Warm Interval of an Ice Age it is not clear what will come next: a Cold End of the Warm Interval or a Hot End of the Ice Age, and it is also not clear to what degree Humans can influence the outcome by controlling Industrial Activity - which should be done anyway for other reasons including but not limited to pollution control etc.)

Meru Prastara

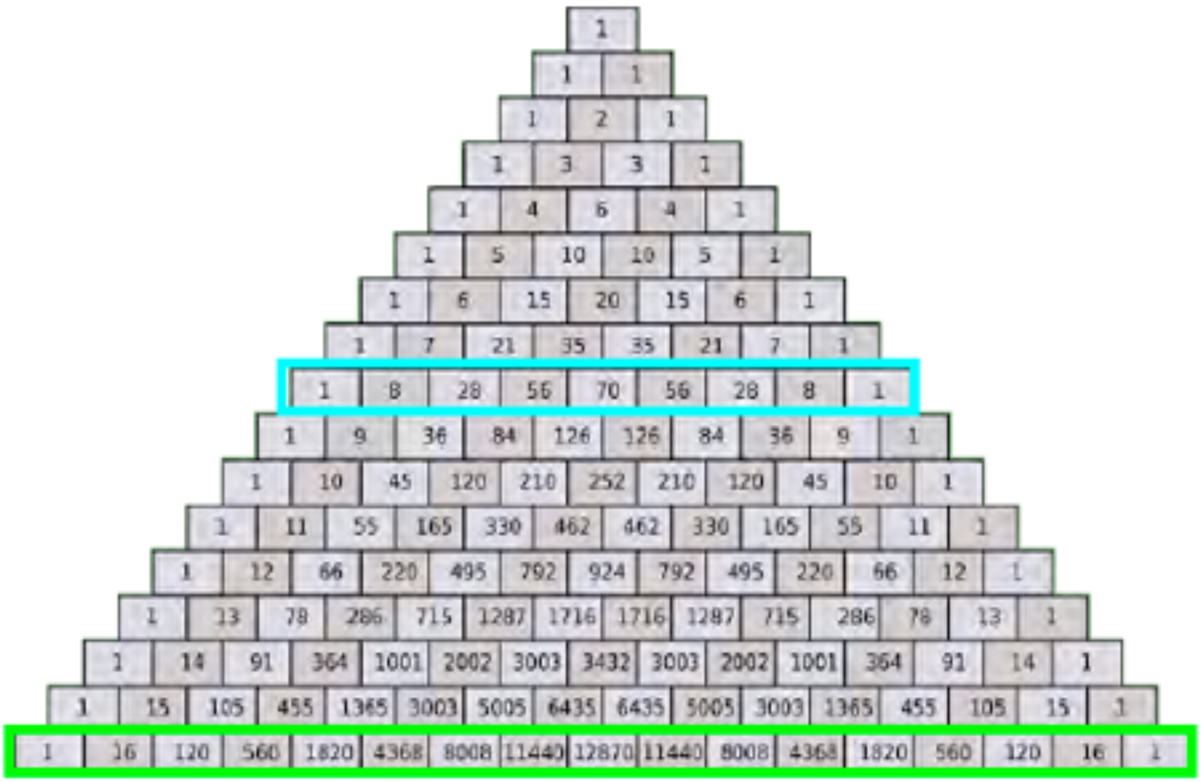
However,

during the period from 36,525 years ago to 11,600 years ago,

a period of 24,925 years roughly similar to Earth's 26,000 year Precession Period,

(according to the Indian National Science Academy web site insaindia.org)

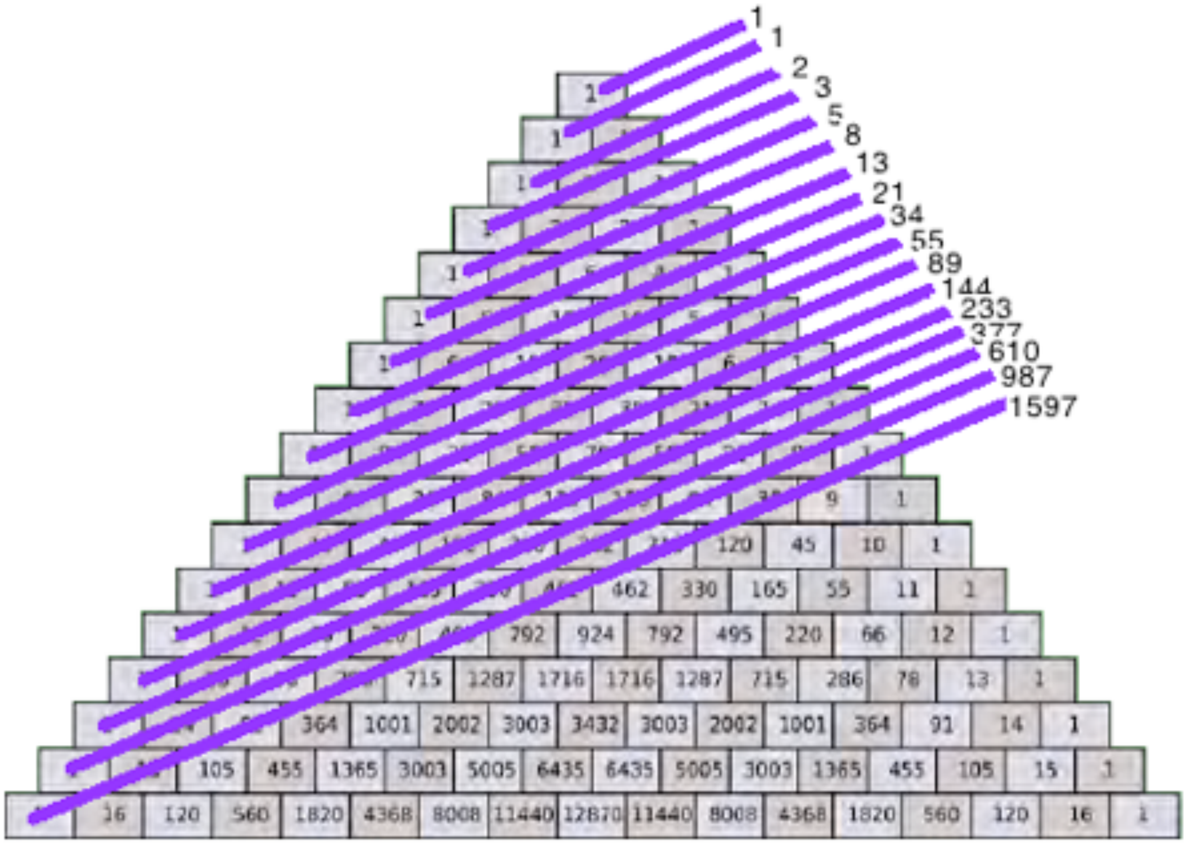
"... The Vedic Civilization ... evolved around ... the Vedas ... Vedic meters ... permutations and combinations of long and short sounds ... led ... to discover[y of] the Meru Prastara ...



now known as Pascal's Triangle ...".

The row I have outlined in cyan contains the 1+8+28+56+70+56+28+8+1 = 256 = 16x16 elements of the Cl(8) Real Clifford Algebra of African IFA divination.
The other rows contain the 2^N elements of Cl(N) where N is the second number from the left in each row, so that the Meru Prastara describes all Real Clifford Algebras Cl(N), with the figure above showing Cl(0) through Cl(16) which I have outlined in green.

The Meru Prastara also encodes Fibonacci numbers and therefore related processes:

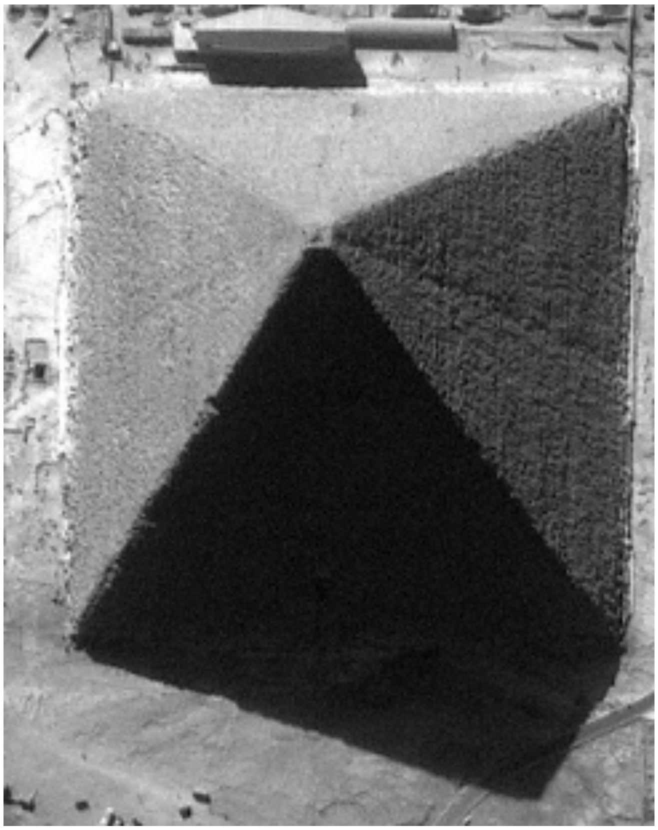


According to Wikipedia: The Fibonacci numbers occur as the ratio of successive convergents of the continued fraction for the Golden Ratio ϕ

$$\varphi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ddots}}}$$

the **Great Pyramid**

The Golden Ratio structure and pyramidal shape show that the representation of Ancient African IFA by the Meru Prastara of African Migrants to India 50,000 years ago corresponds to its representation by the Great Pyramid of Giza of African Nile Migrants In the same Earth Precession Period (36,525 years ago to 11,600 years ago).



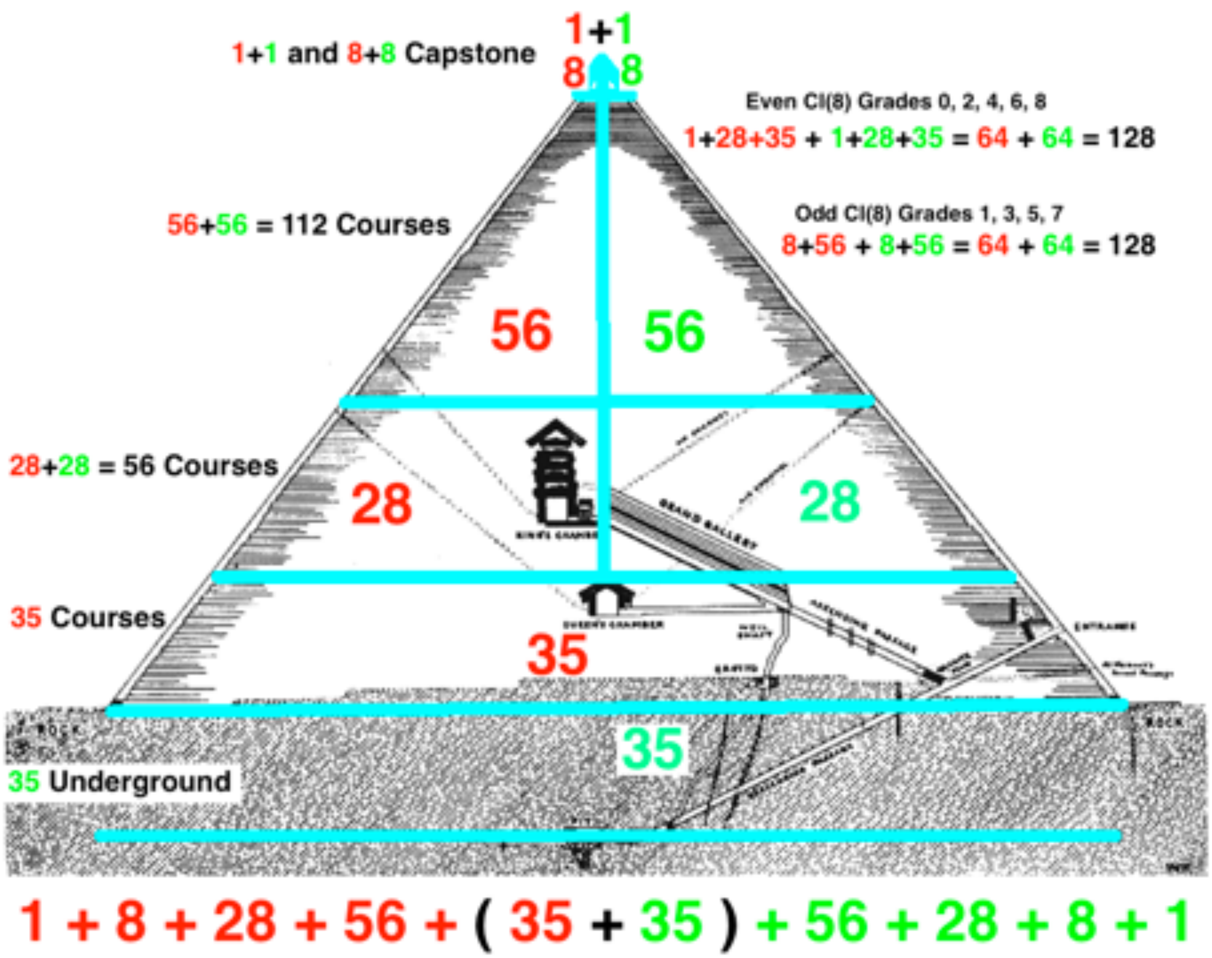
The Builders of the Great Pyramid who had migrated throughout the length of the Nile along which substantially contiguous settlements enabled them to maintain enough contact to maintain the details of the oral traditions of IFA so that when they built the earliest of the pyramids, the Great Pyramid, they did not deface it with any writing but instead encoded the IFA Clifford Algebra in the structure of the Pyramid itself:

The Great Pyramid is built of 203 layers (courses) plus a now-missing capstone for a total number of layers = 204 = 64 + 49 + 36 + 25 + 16 + 9 + 4 + 1 = SPN(8) the Square Pyramidal Number of order 8.

The Subterranean Pit is as deep below ground level as Queen's Chamber is above it so

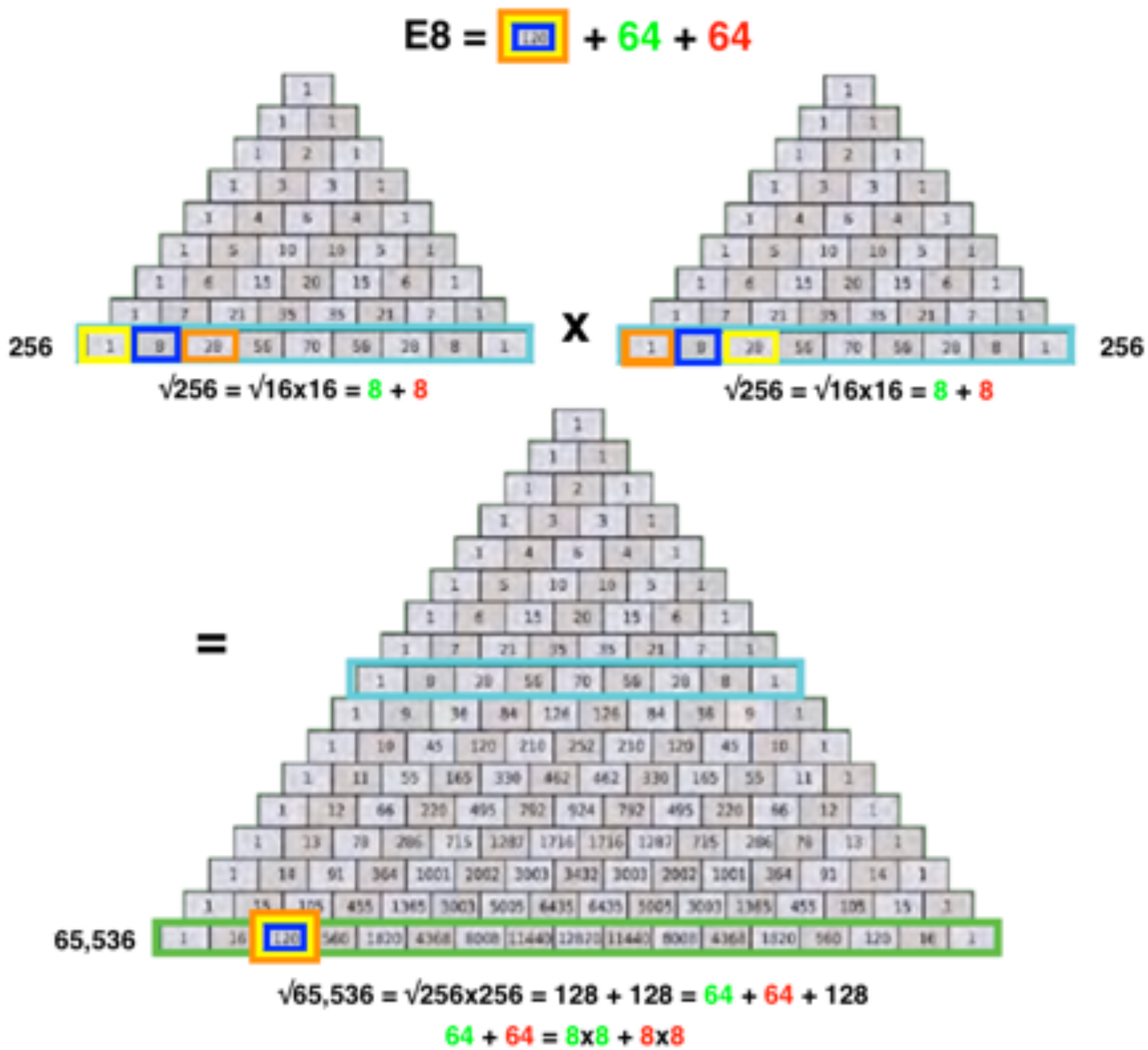
the Subterranean Pit depth equivalent to 35 courses is dual to the Queen's Chamber height of 35 courses just as the 70 mid-grade grade 4 elements of the Cl(8) Clifford Algebra are (35+35) 35 elements plus 35 elements, dual to each other.
(3+3) of (35+35) are the middle components of the 1+3+3+1 Higgs Primitive Idempotent whose 1+1 scalar+pseudoscalar compontents are the 1+1 of the Capstone so that

Cl(8) = 1 + 8 + 28 + 56 + (35 + 35) + 56 + 28 + 8 + 1



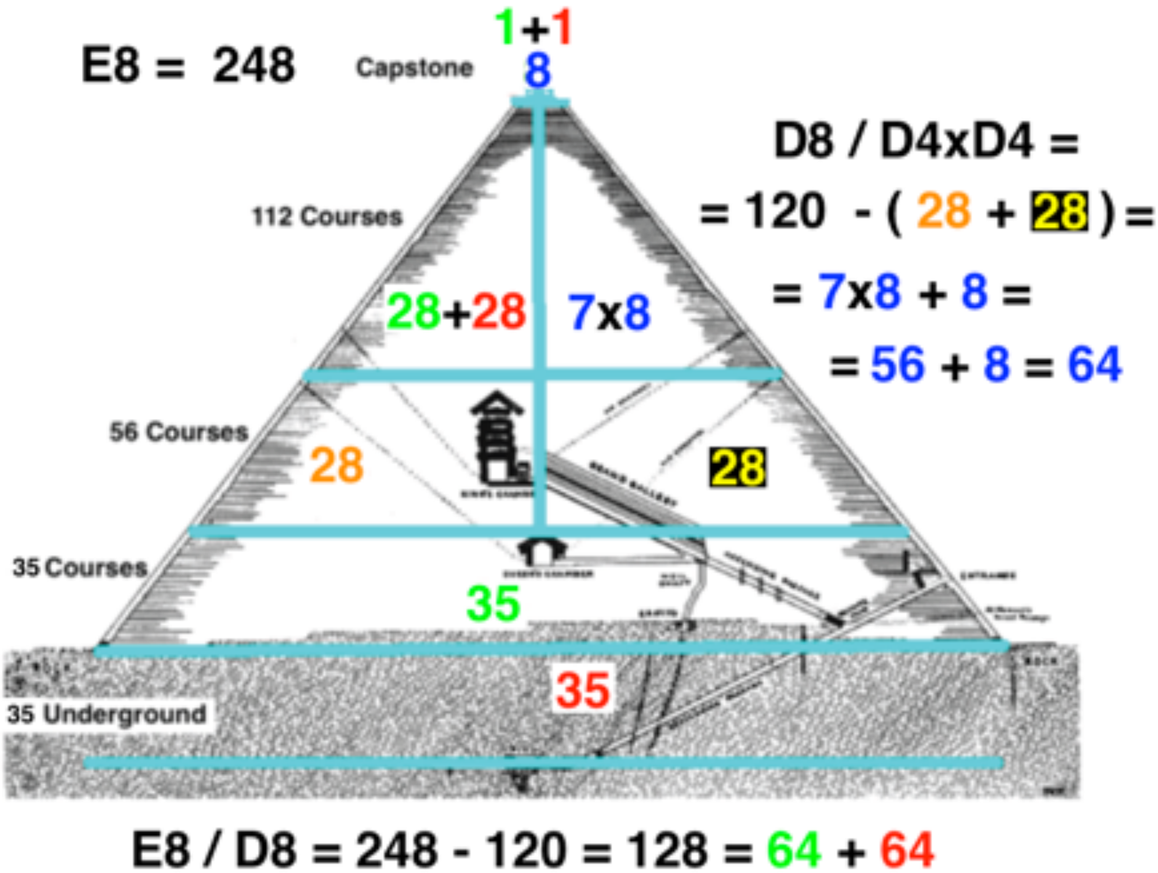
(image adapted from David Davidson image -
for a larger version of this image go to tony5m17h.net/GreatPyrCl8.png
or valdostamuseum.com/hamsmith/GreatPyrCl8.png)
(for more details about the Great Pyramid Geometry correspondences see viXra 1305.0060)

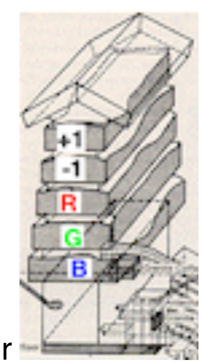
By 8-periodicity of Real Clifford Algebras the tensor product Cl(8)xCl(8) = Cl(16)
and Cl(16) contains E8 = bivectors of Cl(16) + half-spinor of Cl(16)



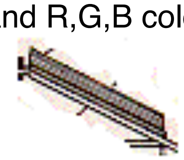
(for a larger version of the above image and the following one
go to tony5m17h.net/E8Cl8xCl8Cl16.png and tony5m17h.net/E8GreatPyr.png
or
valdostamuseum.com/hamsmith/E8Cl8xCl8Cl16.png
and
valdostamuseum.com/hamsmith/E8GreatPyr.png)

248-dim E8 (like 256-dim Cl(8)) can also be seen in terms of the Great Pyramid
(the 8-dim difference is related to the Cl(8) Primitive Idempotent and the Higgs)



The 28 is in the area of the Upper Chamber 

which has 5 slabs that represent the 5 charges (+1,-1 electric and R,G,B color) of the Standard Model.

The 28 is in the area of the Grand Gallery  which rises at a slope of about 26 degrees, or about half of the Golden Ratio slope of the Great Pyramid which is arccosine(1 / ((1 + sqrt(5))/2)) = 51.8 degrees.
The Grand Gallery could represent a segment of a space-time path (World-Line) in the context of Conformal Gravity.

Present Precession Period

The Present Precesssion Period after the Vela Pulsar SuperNova of 11,600 years ago is characterized by Manetho as a time of the Rule of Mortal Humans, during which emerged Human Organizations not directly related to ancient IFA and Vedism.
Some examples, along with individuals associated with them, are:

Zoroastrianism - Zarathrustra
Confucianism - Confucius
HuangLao Daoism - Lao Tze
Buddhism - Gautama Sakyamuni
Judaism - Moses
Christianity - Jesus
Islam - Muhammad
Industrial Capitalism - Socialist Materialism - Karl Marx
Finance Capitalism - Rudolf Hilferding

Although there are common threads among all those Human Organizations they have evolved to be Competitive rather than Cooperative, thus Fueling the Flames of Strife and War.

Some historical examples:

3,000+ years ago Moses led the Exodus from African Egypt;
2,200 years ago HuangLao Daoism was displaced by Confucianism in Han China;
1,700 years ago Rome selectively adopted only parts of Jesus's Christianity;
1,400 years ago Islam displaced Vedism in much of the Middle East;
1,000 years ago Crusades and Wars were fought between Christianity and Islam;
700 years ago Christianity expelled Judaism and Islam from Spain.
Also about 700 years ago Ramon Llull's Art showed how Geometry of IFA
encodes a realistic E8 Physics Model (see Appendix I) and
indicates a Unifying Harmony of Judaism, Christianity, and Islam.
It was not well received by the Paris Establishment, the Vatican, or Islamists.
600 years ago Confucian China terminated Ocean Voyages by Daoist Zheng He.
500 years ago Pope Alexander VI divided the New World between Spain and Portugal.
400 years ago Protestant England defeated Catholic Spain.
200 years ago USA became independent of Britain.
100 years ago USA and UK won World War I and USSR was born.
70 years ago USA (Finance Capitalism) and USSR (Socialist Materialism) won WWII.
30 years ago China (Industrial Capitalism + Confucianism) becomes dominant Factory.
20 years ago USSR dissolved
leaving Russia (Moscow Christianity + Nuclear Military)

Now we have:
USA (Finance Capitalism) unstable $$Hundreds-Trillions Ponzi with Fed Money Printing
China (Industrial Capitalism + Confucianism) dominating Manufacturing.
Russia (Moscow Christianity + Nuclear Military) with First-Use-Nuke Doctrine.

With the USA trying to maintain Hegemony opposed by Russia+China, how will this play out ?

USA Finance Capitalism cannot use USA Military Dominant Conventional Forces
because
Russia Military has First-Use-Nuke Doctrine if attacked by Conventional Forces.

USA Finance Capitalism cannot set prices of Manufactured Goods
because
China (Industrial Capitalism + Confucianism) controls the Factories.

Through Mercantile Trade of Manufactured Goods China's Finance Banks
have Real Assets comparable to USA Finance Banks

USA Big Five Banks have $ 303 Trillion of Real Assets + Ponzi Scheme Claims
but
$ 295 Trillion of their Total Wealth is in really worthless Ponzi Scheme Claims
leaving
the Big Five USA Banks with only their Real Asset Wealth of $ 8 Trillion

The Top 5 Banks of China have Real Asset Wealth of $ 10 Trillion

so

China's Finance Capitalism Sector is a Realistic Alternative
to USA Finance Capitalism Hegemony.

Two Possible Scenarios: Nuclear War or Return to Peace

Nuclear War:

USA Finance Capitalists refuse to recognize $300 Trillion Ponzi Scheme Loss
and use Conventional Military to impose the cost on the Rest of the World.

Russia Nukes USA in response.
USA and Russia go to Global Nuclear War.
Earth fragments into many small countries trying to rebuild.

Return to Peace a la Supernova Precession Period of Peace:

USA Finance Capitalists recognize $300 Trillion Ponzi Scheme Loss
forcing USA to revert to Industrial Capitalism and sale of Resources

China Finance and Remnant of USA Finance merge on equal terms
Finance Capitalism activity subject to Veto Power of China or Russia or USA

Llullian Art gives Unified Natural Philosophy of both Science and Religion.

Appendix I:
Ramon Llull’s Art and E8 Physics

Ramon Llull described the Structure of E8 Physics

(for details see viXra 1305.0060)

(images in this section from "The Art and Logic of Ramon Llull" (2007) by Anthony Bonner
and for Universal Figure from quisestlullus.narpan.net)

Correspondences of the initial Quarternary Phase of Llull's Art with E8 Physics:

Figures Y and Z

are a Binary Pair - the basis of **Real Clifford Algebras**.
Real Clifford Algebras have 8-Periodicity so that

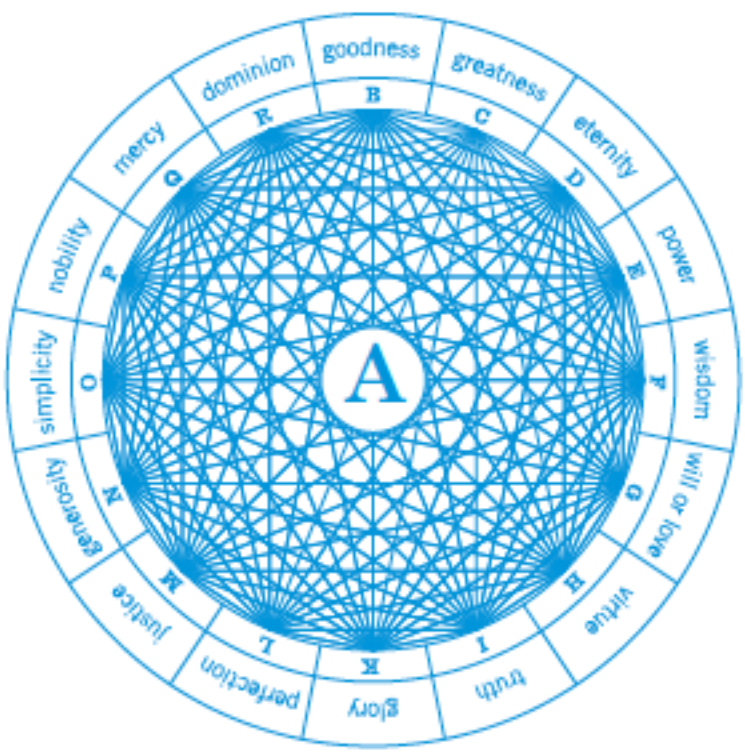
Cl(8N) = Cl(8) x ...(N times tensor product) ... x Cl(8)

and

Cl(8) x Cl(8) = Cl(16)

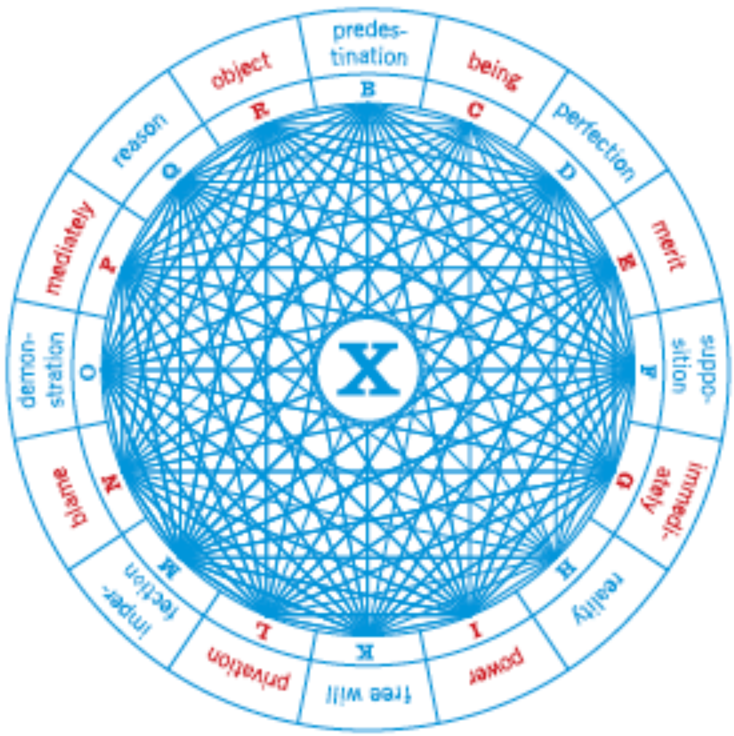
248-dim E8 = 120-dim D8 Bivectors of Cl(16) + 128-dim D8 Half-Spinors of Cl(16)

Figure A



has 16 vertices and 120 connecting lines
corresponding to Cl(16) and its 120 D8 BiVectors.

Figure X



divides the 16 vertices into two groups of 8 vertices
thus dividing the 120 connecting lines into 3 sets:

28 that connect vertices of one group of 8 with each other
28 that connect vertices of the other group of 8 with each other
64 that connect vertices of one group of 8 with vertices of the other group of 8

Elemental Figure

The Figure of Fire (heat)

fire	air	water	earth
air	fire	earth	water
water	earth	fire	air
earth	water	air	fire

The Figure of Air (moisture)

air	fire	water	earth
fire	air	earth	water
water	earth	air	fire
earth	water	fire	air

The Figure of Water (cold)

water	earth	air	fire
earth	water	fire	air
air	fire	water	earth
fire	air	earth	water

The Figure of Earth (dryness)

earth	water	air	fire
water	earth	fire	air
air	fire	earth	water
fire	air	water	earth

Elemental Figure

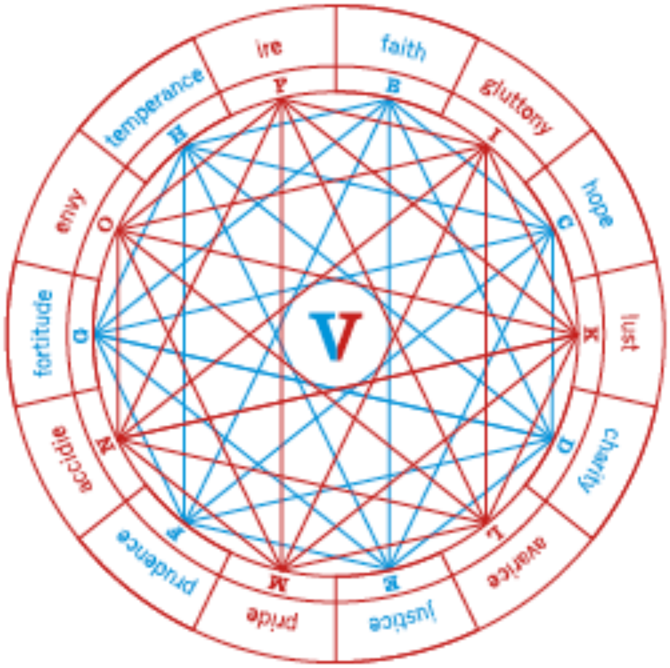
has 4x4x4 = 64 elements corresponding to 64-dim D8 / D4xD4
that is contained in the 120-dim D8 BiVectors

and to
the 64 Figure X lines that connect vertices of one group of 8 with vertices of the other
physically representing 8-dim SpaceTime Position x Momentum

and therefore by Triality also to
64-dim +half of 128-dim D8 Half-Spinors of Cl(16)
physically representing 8 components of 8 First-Generation Fermion Particles

and to
64-dim -half of 128-dim D8 Half-Spinors of Cl(16)
physically representng 8 components of 8 First-Generation Fermion AntiParticles.

Figure V describes the two 28-dim D4 of the 120-dim D8 of 248-dim E8
by which 64-dim D8 / D4xD4 is formed. Figure V has 14 vertices
divided into two groups of 7 vertices



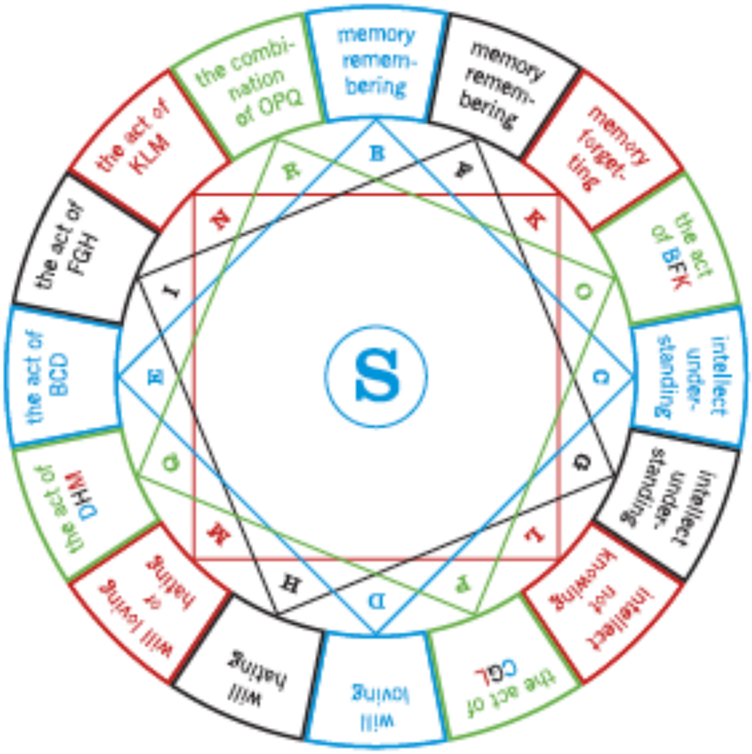
each with 21 lines connecting vertices of one group of 7 with each other.

Each set of 7 vertices + 21 lines corresponds to one of the 28-dim D4.

Mathematically, each set of 7 vertices represents 7 Imaginary Octonions = 7-sphere S7.
The Non-Associativity of Octonions prevents the 7-sphere S7 from forming a Lie Group.
If you try to make a Lie Algebra out of the 7 generators of S7
you find that the products do NOT form a closed 7-dim Lie Algebra
but
they generate: a 14-dim G2 Lie Algebra of the Automorphisms of the Octonions
and a second 7-sphere S7
so
if you combine the initial 7 with the newly generated 14 + 7
you get a 28-dim D4 Lie Algebra.

Each of the 7-vertex 21-line systems
is related to the 168-element group PSL(2,7) = SL(2,3)
and to the 480 Octonion Multiplications.

Figure S contains 4 squares

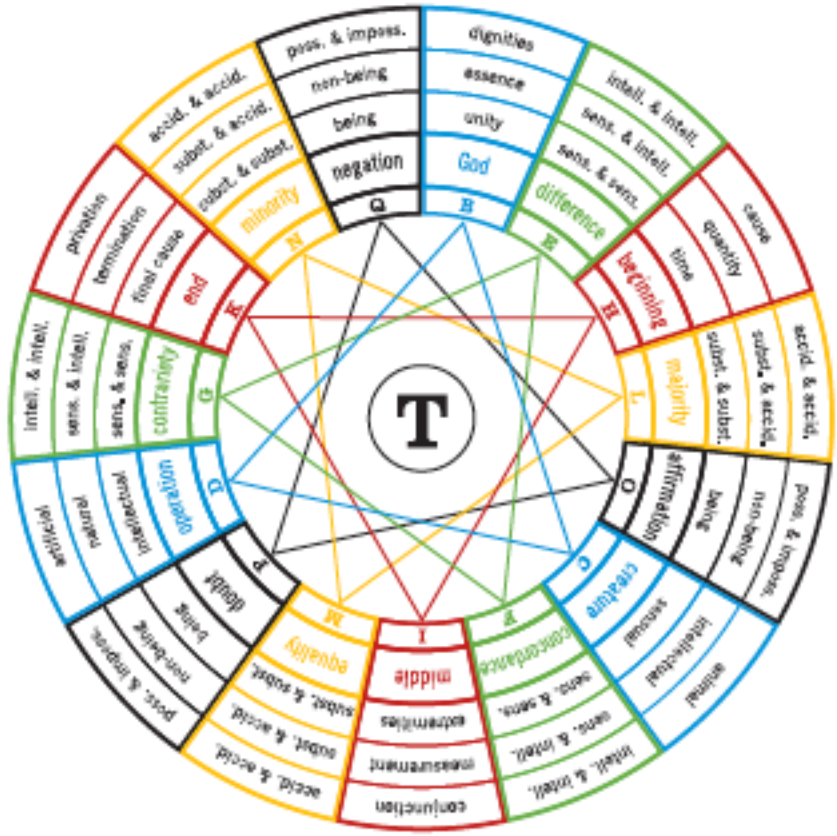


The 4 vertices of the central square correspond to the 4-dim Quaternionic subspaces
that emerge below the Planck energy to break Octonionic 8-dim Spacetime
into (4+4)-dim Kaluza-Klein spacetime with 4-dim Minkowski Physical Spacetime plus
4-dim Internal Symmetry Space CP2 = SU(3) / SU(2)xU(1)
The remaining 3x4 = 12 vertices correspond to 12 of the 28 Figure X lines
that connect vertices of one group of 8 with each other
and represent the 12 Standard Model gauge group generators of SU(3)xSU(2)xU(1)
Two of the remaining 3 squares of Llull's S-wheel form the vertices of a cube

```
x-----y                          x
|\     |\
| o-----x                   y           y
| |    | |                       o o
y-|---o |                   x           x
 \|     \|
  x-----y                        y
```

Looking at the cube along a diagonal axis and projecting all 8 vertices onto a
perpendicular plane shows the Root Vector Diagram of SU(3) and its 8 Gluons.
Since each Gluon links 4-dim Physical SpaceTime to color Internal Symmetry Space
the gauge group SU(3) acts globally on Internal Symmetry Space C2 = SU(3) / U(2)
The third of the remaining squares, that is the final square, corresponds to
the 3 SU(2) weak bosons and the U(1) electromagnetic photon.
Since SU(2)xU(1) = U(2), and since CP2 = SU(3) / U(2),
they act locally on CP2 Internal Symmetry Space.

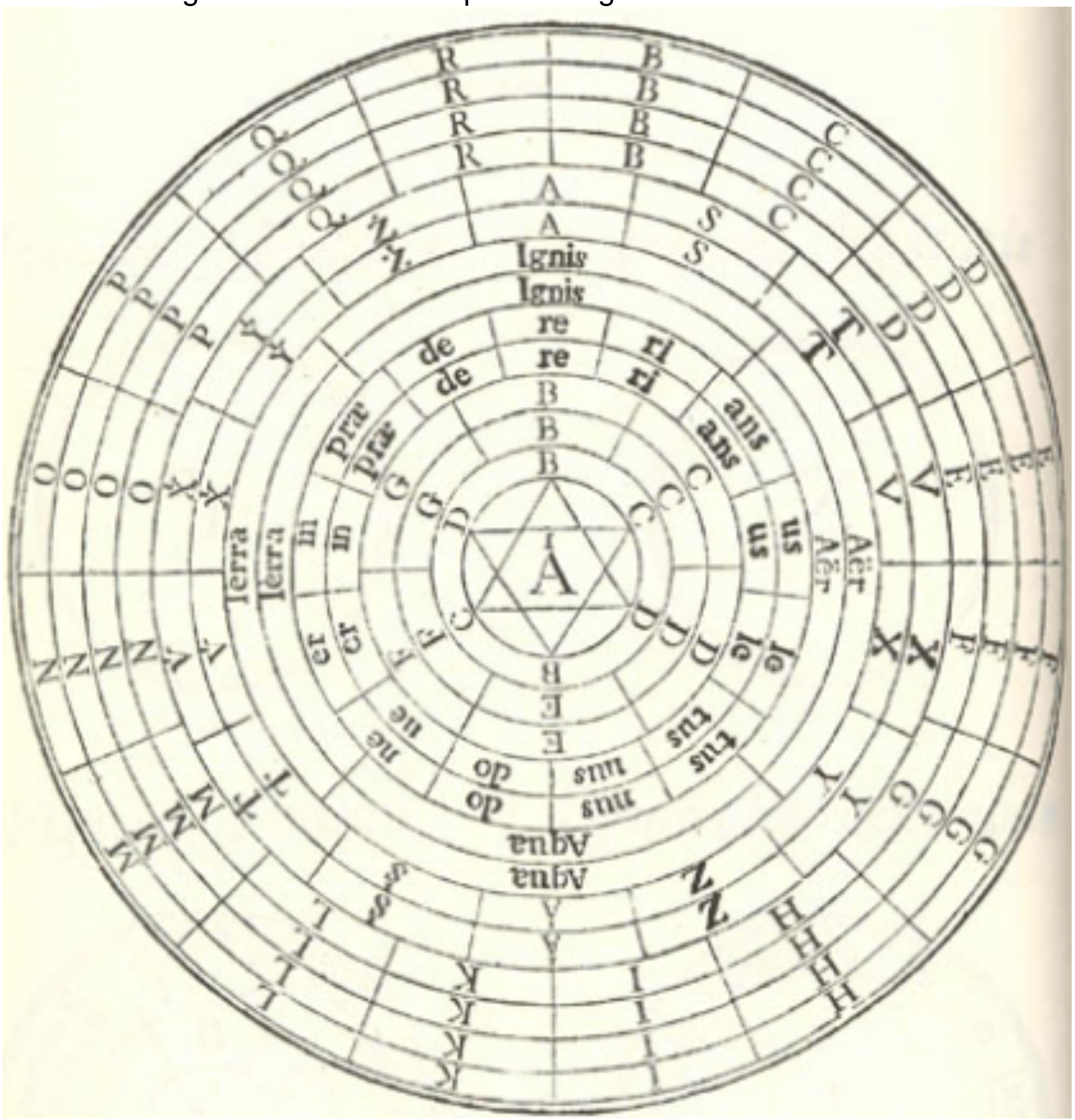
Figure T contains 5 triangles



The 5x3 = 15 vertices correspond to 15 of the 28 Figure X lines
that connect vertices of the other group of 8 with each other
and represent the 15-dimensional Conformal Group SU(2,2) = Spin(2,4)
that describes Gravity and Dark Energy
and lives in 16-dimensional U(2,2) as U(2,2) = U(1) x SU(2,2).

The U(1) of U(2,2) corresponds to the 16th of the 28 Figure X lines
and represents the Complex Phase of Propagators.

Llull's Universal Figure shows how the parts fit together



The outer 4 rings of 16 elements represent 64-Element-Things. There are 3 of these, related by Triality: 8 Position x 8 Momentum, 64 components of Fermion Particles, and 64 components of Fermion Antiparticles,
The next (going in toward the center) 2 rings of 14 elements each represent 28-dim D4 and its Lie subalgebras. There are two of these D4, producing Gauge Groups of Gravity and the Standard Model, respectively.
The next 2 rings of 4 elements each represent the 4+4 = 8 dimensions of Spacetime.
The next 2 rings of 13 elements each represent the 26-dimensional String Theory with Strings seen as World-Lines of Fermions, producing a Bohm-type Quantum Theory.
At the center are 3 rings of 6 elements each surrounding a 6-vertex Star of David whose 3x6 + 6 = 24 elements correspond to the 24-dim Leech Lattice whose Monster Group symmetry defines the size (about 10^(-24) cm of Physical Particles as (viXra 1311.0088) Schwinger Source Kerr-Newman Black Hole clouds of virtual particle-antiparticle pairs.

Appendix II: Gods, Demigods, Spirits

Frank Dodd (Tony) Smith, Jr.

Nicholas Wade, in a 7 December 1999 article in the New York Times, said:
"... Dr. Richard G. Klein of Stanford University and others believe that

some major genetically based neurological change,
like the development of language, occurred about 50,000 years ago.

... A ... genetic survey... drawing on DNA data from 50 ethnic groups around the world, concludes that the ancestral population from which the first emigrants came may have numbered as few as 2,000 people. ... Another new genetic study, by Dr. Marcus Feldman of Stanford University and others ... looked at segments of the Y chromosome ... they concluded that the most recent common ancestor of all these Y's was carried by a man who lived only 40,000 years or so ago. ... The 40,000-year date, which has a large range of uncertainty, is much more recent than others, in part because the earlier estimates were forced to assume, quite unrealistically, that the size of the human population remained constant throughout prehistory. Dr. Feldman assumed an exponentially expanding population, which yields a more recent date of origin....".

About 36,000 years ago the Geminga shock wave hit Earth;
the Late Wisconsin Glaciation of Earth began; and,
according to the Human History Chronology of Manetho (Egyptian historian ca. 343 BC):

about 36,525 years ago - Gods on Earth began to rule Earth,
ruling until

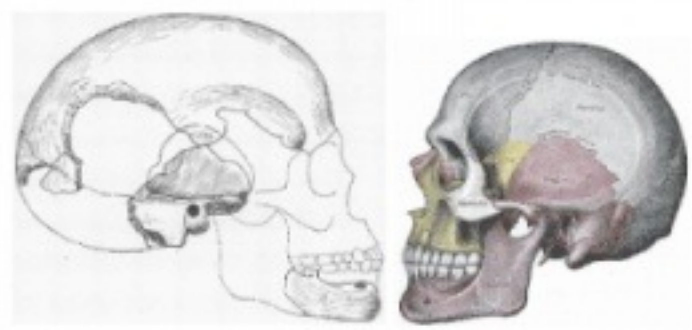
about 22,625 years ago - Demigods and Spirits of the Dead (followers of Horus)
succeeded them, ruling until

about 11,600 years ago - Mortal Humans began to rule Earth,
ruling up to the present day.

The time that the Earth was ruled by Gods, Demigods and Spirits of the Dead,
about 36,525 - 11,600 = 24,925 years, is roughly the time of one Earth precession period,
which is also roughly the travel time of a light beam from the center of our Galaxy to our Sun.

Who were the Gods - Demigods who ruled Earth during the 24,925 years
from 36,525 years ago to the beginning of the Holocence about 11,600 years ago ?

Gary Lynch and Richard Granger, in their book Big Brain (Macmillan 2008), said:
the Boskop "... walked the plains of southern Africa ... 30,000 ... to ... 10,000 years ago ...
they were about our size ... but their brains were far larger than our own. ...".



Comparison of Restoration of Boskop skull
next to a Modern Human Skull.

(image is from a 29 April 2008 ebcak.com post by adminebcak)

Richard Poe, in his book "Black Spark, White Fire" (Prima 1997),
quoted Diodorus of Sicily as saying:
"Now the Ethiopians ... were the first of all men. ... the Egyptians are colonists sent out by the Ethiopians, Osiris having been the leader of the colony ... Osiris ... gathered together a great army, with the intention of visiting all the inhabited earth and teaching the race of men how to cultivate ... for he supposed that if he made men give up their savagery and adopt a gentle manner of life he would receive immortal honors. ...".

Greek Hermes and Arabic Idris and Sufi Khidr = Khizar (compare Khazar),
and the teaching aspects of the IFA Orisha Ogun and Egyptian Thoth and Osiris,
have been identified with Enoch.

1 Enoch (Ethiopic Enoch) Chapters 1, 28, 33, 41, 43, 44, 65:
"... Enoch ... saw the vision of the Holy One in heaven ... and understood ...
Enoch ... saw a wilderness ... full of trees and plants. And water ...
and ... large beasts ... and birds ... and ... the stars of heaven ...
Enoch ... saw all the secrets of the heavens ... the secrets of the lightning and of the thunder
... of the winds ... of the clouds and dew ... the sun and moon ... the stars ...
some of the stars arise and become lightning ...
how silver is produced from the dust of the earth, and how soft metal originates in the earth ...".

How could the mind of Boskop Enoch have comprehended the Secrets of Heaven ?

Endogenous DMT enhanced perception might have given Enoch/Boskops mental/spiritual abilities described by Terence McKenna in his May 1993 OMNI magazine interview:
"... MCKENNA: ... From 75,000 to about 15,000 years ago,
there was a kind of human paradise on Earth. ...

Nobody went more than three or four weeks before they were redissolved into pure feeling and boundary dissolution. ... DMT drops you into a place where the stress is on a transcending language. ... You burst into a space. ...

The world is not a single, one-dimensional, forward-moving, causal, connected thing,
but some kind of interdimensional nexus. ... entities are there ... They are teaching something.

Theirs is a higher dimensional language that condenses as a visible syntax. ...
they ... offer you something ... so beautiful, so intricately wrought, so something else that cannot be said in English, that just gazing on this thing, you realize such an object is impossible. ...
The object generates other objects, and it's all happening in a scene of wild merriment and confusion. ... [compare El Aleph, written in 1945 by Jorge Luis Borges]

Something in an unseen dimension is acting as an attractor for our forward movement in understanding. ... It's a point in the future that affects us in the present. ...

Our model that everything is pushed by the past into the future,
by the necessity of causality, is wrong.

There are actual attractors ahead of us in time -- like the gravitational field of a planet.
Once you fall under an attractor's influence, your trajectory is diverted. ...

I think [that the attractor has a kind of intelligence]...".

Around 11,600 years ago, about when the Vela Pulsar supernova was seen in Earth, a very sudden (50 years or so) warming event ended the Ice Age and marked the start of the Holocene Age of warm climate and glacial retreat, and rising sea levels flooded much productive land creating shortages of food and consequently competition for available food, and what Terence McKenna called "tooth-and-claw dominance". As Gary Lynch and Richard Granger said in their 28 December 2009 Discover Magazine article: "... human history has often been a history of savagery. ... Perhaps the preternaturally civilized Boskops had no chance against our barbarous ancestors ...".

Not only might the "preternaturally civilized" Boskops have been at a physical disadvantage in fighting smaller-brain humans, but the social organizations useful in forming successful armies to seize resources might tend to create evolutionary pressure in favor of humans with less creativity and more blind-faith obedience to authority. Further, as John Hawks said on his weblog on 30 March 2008, "... brains are [energetically] expensive ... brains take a long time to mature ... brains require high protein and fat consumption ... there has been a reduction in the average brain size in South Africa during the last 10,000 years, and there have been parallel reductions in Europe and China -- pretty much everywhere we have decent samples of skeletons, it looks like brains have been shrinking ...".

All this is consistent with the Vedic view (see nersp.nerdc.ufl.edu/~ghi/vcchap.html)
of what happened after about 11,600 years ago,
into the Bronze and Iron Ages and on through the present Industrial Age:
"... prior to the beginning of Kali-yuga ...
all living beings were on a higher average level than they are at present, a
nd advanced beings such as demigods and great sages regularly visited the earth. ...
Once the Kali-yuga began,
demigods and higher beings greatly curtailed communications with people on the earth,
and the general sensory level of human beings also declined. ...
due to the lack of feedback from higher sources
and the natural cheating propensity of human beings,
the traditions ... became more and more garbled ...
the present stage of civilization was reached,
in which old traditions are widely viewed as useless mythology,
and people seek knowledge entirely through the use of their current, limited senses.".

It is also consistent with 1 Enoch (Ethiopic Enoch) Chapters 106, 107, 100:
"... after that (flood) there shall be more unrighteousness ...
generation after generation shall transgress,
until a generation of righteousness arises ...
wise men will seek the truth and ... will understand ...
and transgression is destroyed ... and all manner of good comes on ... the earth ...".

What were the unrighteous transgressions ?

Note - the English term "giants" in books of Enoch and in the Torah is a mistranslation
of the Hebrew word "Nephilim" which, in my opinion, should be translated
as "those who kill and ruin", or, for a short equivalent singular term, "Army"
(compare USA Marines whose goal is to kill people and break things),
so I will use the term "Army" in Enochian quotations herein.
Also, the "size" of Nephilim = Army means the numbers/firepower of the Army,
not the physical size of the individual soldiers, and I will modify quotations accordingly
as may be necessary or convenient for clear understanding.

1 Enoch (Ethiopic Enoch) Chapters 6, 7, 10, 12:
"... two hundred ... angels ... the Watchers ... descended ... and bound themselves by mutual curses ... And ... took wives ... and ... taught ... charms and spells ...
And the women ... bare large Armies ...
The Armies consumed all the work and toil of men ... when the men could no longer sustain them, the Armies turned against them and devoured mankind ...
And they began to sin against birds, and beasts, and reptiles, and fish ... and one another...
Then the earth laid accusation against the lawless ones. ...
Then said ... the Great and Holy One ... "Go to Noah and tell him in my name
"Hide yourself!" and reveal to him the end that is approaching:
that ... a flood is about to come on the whole earth ..." ...
Then Enoch disappeared and no children of men knew where he was hidden ...".

3 Enoch (Hebrew Book of Enoch) Chapters 4, 6, 10, 15, 17, 19, 21, 33:
"... Metatron ... said ... I am Enoch ... When the generation of the of the flood sinned
and twisted and contorted in their deeds, saying unto God:
"Department from us! We do not want the knowledge of your ways,"
... then the Holy One ... removed me from their midst ...
he lifted me up to heavens together with the Shekina ...
and the announcement went forth ... "This is Metatron ...
Metatron['s] ... flesh was changed into flames ... muscles into flaming fire ...
bones into coals of juniper wood ... light of ... eye-lids into hot flames ...
all ... limbs into wings of burning fire ... body into flowing fire ...
the Holy One ... caused 72 wings to grow on ... Metatron ... and attached to Metatron 365 eyes ...
the Holy One ... took ... Metatron ... to attend the Throne of Glory and the Wheels ... of the Merkaba ... and the service of Shekina ...
the wheels of Merkaba ... are ... Eight, two in each direction ...
the feet of the ... Four Chayoth ... are resting on the wheels ...
Each Chaya ... has four faces ... The size of the faces is 248 faces ...
Each one has four wings ... the size of the wings is 365 wings ...
seven ... princes ... are assigned over the seven heavens ...
each one of them is accompanied by 496 thousand groups of ten-thousand ... angels ...

Metatron ... said ... At the time that the Holy One ... is sitting on the Throne of Judgment ...
The Holy Chayoth carry the Throne of Glory ...
And underneath the feet of the Chayoth there are seven rivers of fire running and flowing.
And the distance across is 365 thousand parasangs
and its depth is 248 thousand times ten-thousand parasangs. ...
And each river turns around in a bow in the four directions of Araboth ... of Raquia ...
and from there it falls down to Maon ... to Zebul ... to Shechaqim ... to Raquia ... to Shamayim ...
from Shamayim it flows on the head of the wicked who are in Gehenna ...".

7 = Imaginary Octonions = Number of E8 Lattices
72 = root vectors of 78-dimensional E6 Lie algebra
78 = Tarot cards
365 = central number of 9x9x9 Magic Cube and 27x27 Magic Square
(compare days/year and degrees/circle and 27-dim Jordan algebra J3(O))
8 = 4+4 = dimension of Kaluza-Klein spacetime
248 = dimension of E8 Lie algebra = dimension of SL(8)xH92 Quantum Algebra
(each of which is contained in Clifford Algebra Cl(8))
(SL(8)xH92 = contraction of E8)
496 = 248 + 248 = E8 + SL(8)xH92 contained in Clifford Algebra Cl(16) = Cl(8)xCl(8)
(4 Chaya x 4 faces) x (4 Chaya x 4 wings) = 16x16 = 256-dim Cl(8) and IFA

E8 has 240 root vectors (Fermionic from D8 half-spinor; Bosonic from D8 adjoint):
Fermionic Anticommutators: 64 = 8x8 particle components; 64 = 8x8 antiparticle components;
Bosonic Commutators: 64 spacetime (8 position x 8 momentum); 24 + 24 of two D4

E6 has 72 root vectors (Fermionic from D5 spinor; Bosonic from D5 adjoint):
Fermionic Anticommutators: 16 = 8x2 particles (complex); 16 = 8x2 antiparticles (8 complex);
Bosonic Commutators: 16 = 8x2 spacetime (8 complex position); 24 of one D4

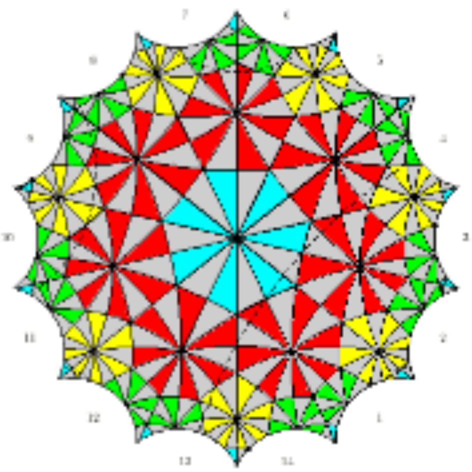
E8 - E6 has 48 = 8x6 particle components; 48 = 8x6 antiparticle components (Fermionic);
48 = 8x6 spacetime; 24 of another D4 (Bosonic)
for a total of 24x7 = 168 root vectors.
168 = 2^3 x 3 x 7 is the order of PSL(2,7) = SL(3,2) which is a simple group of Lie type
and can be thought of as the group of linear fractional transformations of the vertices of
a heptagon or the 7 imaginary Octonions or the 7 independent E8 lattices.

The book The Eightfold Way: The Beauty of Klein's Quartic Curve, edited by Silvio Levy (MSRI Publications -- Volume 35, Cambridge University Press, Cambridge, 1999) describes the roots of the Klein Quartic Equation

$$x^3 y + y^3 z + z^3 x = 0$$

which has the symmetry group PSL(2,7).

To represent the roots, Felix Klein constructed the Klein Configuration:



$$Cl(8N) = Cl(8) \times \ldots(\text{N times tensor product})\ldots \times Cl(8)$$
$$Cl(16) = C(8) \times Cl(8)$$
$$Cl(8) = 2^8 = 256 = 16 \times 16$$

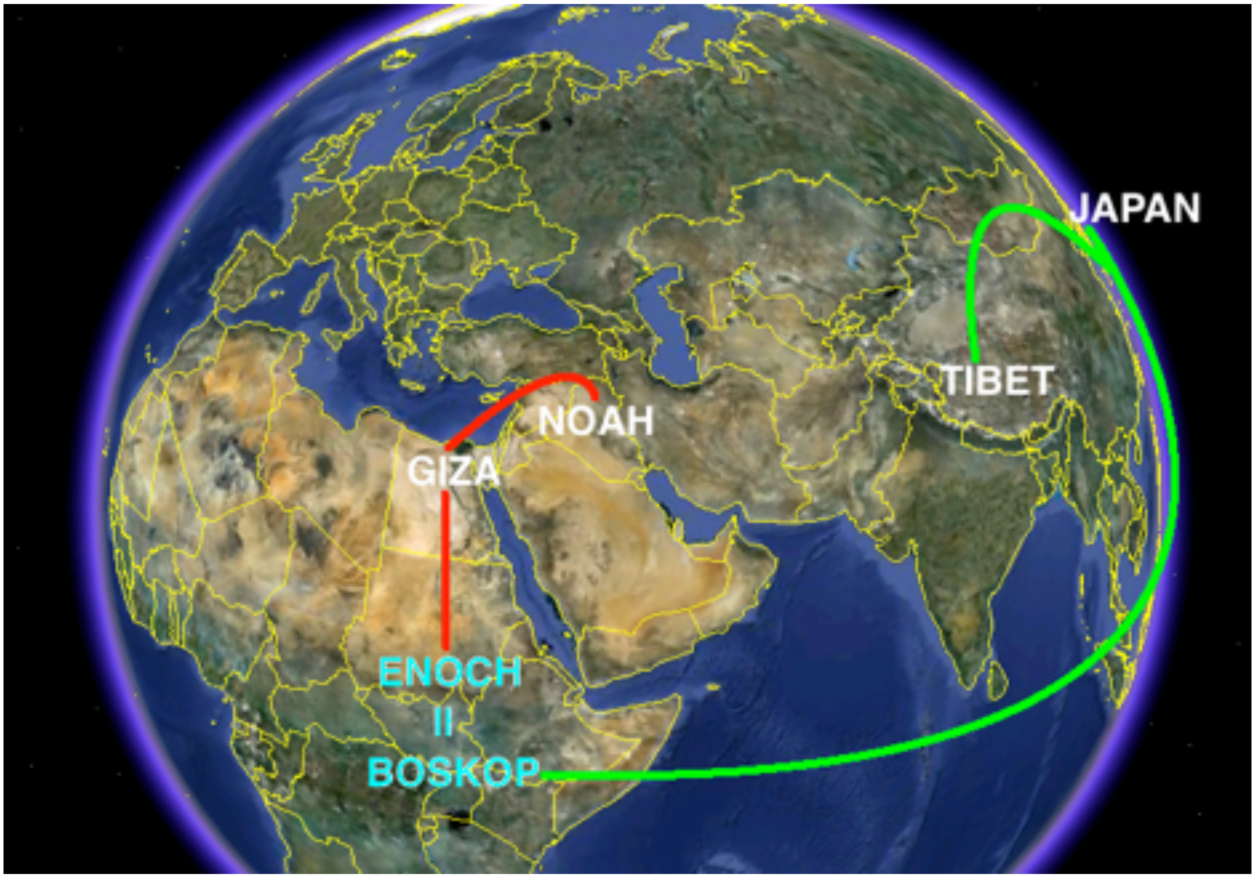
E8 = 248 = 120 + 128
D8 = 120 with spinor 128+128
E7 = 66 + 64 + 3
B7 = 105 with spinor 128
D7 = 91 with spinor 64+64
E6 = 78 = 45 + 32 + 1
B6 = 78 with spinor 64
D6 = 66 with spinor 32+32

B5 = 55 with spinor 32
D5 = 45 with spinor 16+16
F4 = 36 + 16
B4 = 36 with spinor 16
D4 = 28 with spinor 8+8
B3 = 21 with spinor 8
D3 = 15 with spinor 4+4 = A3
G2 = 14 = A2 + S6

As Terence McKenna said in his May 1993 OMNI magazine interview:
"... For 10,000 years ... we've pursued an agenda of beasts and demons ...
If history goes off endlessly into the future,
it will be about scarcity, preservation of privilege, forced control of populations,
the ever-more-sophisticated use of ideology to enchain and delude people.
We are at the breakpoint.
It's like when a woman comes to term ...
if the child is not severed from the mother and launched into its own separate existence,
toxemia will set in and create a huge medical crisis.
The mushrooms said clearly,
"When a species prepares to depart for the stars, the planet will be shaken to its core."
All evolution has pushed for this moment, and there is no going back.
What lies ahead is a dimension of such freedom and transcendence,
that once in place, the idea of returning to the womb will be preposterous.
We will ... expand infinitely into pleasure, caring, attention, and connectedness. ...".

50,000 years ago - Africa Emigration/Trade to Japan and Tibet

36,000 to 12,000 years ago - Boskop/Enoch emerged, built Giza Pyramids, went away

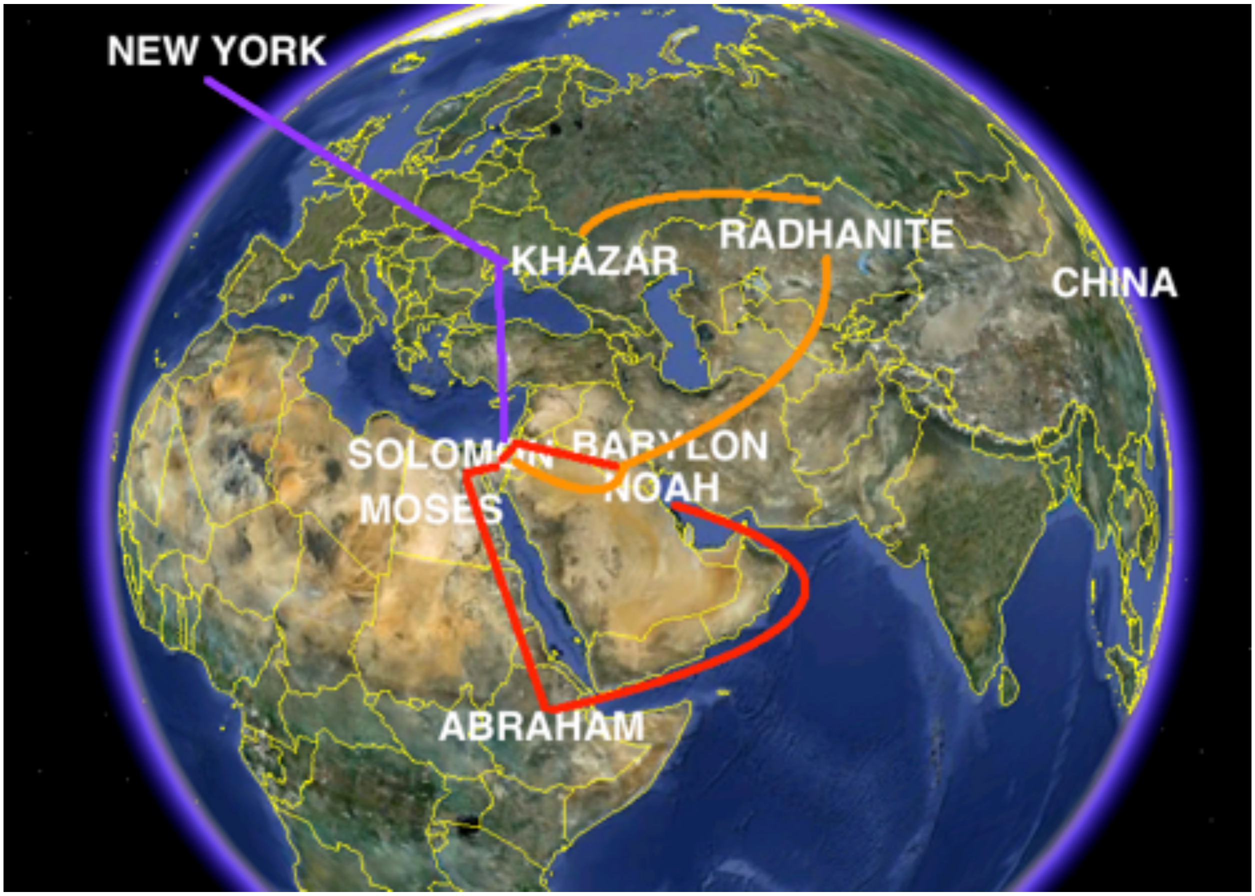


12,000 years ago -

1 Enoch (Ethiopic Enoch) Chapter 21 v. 2-3:
"... I saw neither a heaven above nor a firmly founded earth,
but a place chaotic and horrible ... seven stars of heaven
bound together in it, like great mountaions and burning with fire ...".
Comet Encke (parent of the Taurid meteors) may have been
about the size of Comets Hale-Bopp and Sarabat (about 100 km).
Its initial breakup may may been about 12,000 years ago,
roughly coincident with the Vela Pulsar Supernova and the end of the Ice Age.
Observers on Earth see the Taurids as appearing to come from a radiant point
in Taurus near the seven stars bound together in the Pleiades.

- Noah (normal descendant of Enoch) and the Flood

12,000 years ago - Flood takes Noah to Africa
Nile takes Moses from Abraham to Giza
Moses from Giza to Sinai
Solomon builds Temple
Temple lost to Babylon



After Babylon:
Return to Temple or Expand to Radhanite/Khazar
(www.valdostamuseum.com/hamsmith/QintoNow.html)

After Hitler:
Return to Temple or Expand to New York

Giambattista Vico (1668-1744) wrote of New Science
and saw
the Ages of Gods, Demigods, Spirits as Theocratic, Heroic, Human

Samuel Beckett in “Dante - Bruno - Vico - Joyce” said:
“... **Giordano Bruno**’s treatment of identified contraries ...
There is no difference, says Bruno
between the smallest possible chord and the smallest possible arc,
no difference between the infinite circle and the straight line.
The maxima and minima of particular contraries are one and indifferent.
Minimal heat equals minimal cold.
Consequently transmutations are circular. Therefore ...
the minima ... coincide ... with the maxima in the succession of transmutations.
... And all things are ultimately identified with God, the universal monad
...
From these considerations ... **Giambattista Vico** ...
evolved a Science and Philosophy of History ...
the passage from Scipio to Caesar is as inevitable
as the passage from Caesar to Tiberius,
since the flowers of corruption in Scipio and Caesar
are the seeds of vitality in Caesar and Tiberius.
Thus ... human progression ... depends for its movement on individuals,
and ... at the same time is independent of individuals
in virtue of what appears to be a preordained cyclicism. ...
It follows that History ... is not the result of Fate or Chance ...
but the result of a ... force he called Divine Providence
...
His division of the development of human society into three ages:

Theocratic,
Heroic,
Human (civilized)

with a corresponding classification of language:

Hieroglyphic (sacred)
Metaphorical (poetic),
Philosophical (capable of abstraction and generalisation),

was by no means new, although it must have appeared so to his contemporaries.
He derived this convenient classification from the Egyptians, via Herodotus. ...”.
(see viXra 1404.0057)

Donald Phillip Verene in “Knowledge of Things Human and Divine: **Vico’s New Science** ...” said:

“... **Fas [compare IFA] is divine law**, as opposed to ius or human law ...
natural law ...[was]... communicated by God to Adam and by Adam to Noah.
Noah ... passed these down to his descendants ...
from their common creator the nations are the same in their form of origin ...

law is present in language from the beginning ...
all nations originally think in terms of poetic characters ...
from this type of unreflective thought, governed by fantasia,
there develops reflective thought and the human society that is based on it ...
there was a time when
life was a mute yet communal affair of feelings, sensings, and expressions.
This mute language is ... the ... basis of all human community and communication ...
Vico speaks of an “eternal point of the perfect state of the nations”.
This state of perfection would come about when all the sciences, disciplines, and arts
have been fully directed toward the perfection of man’s nature
and have reached ... acme ...
when all the sciences, disciplines, and arts have grasped their own natures
as developed from the religions and laws of humanity
and have ... come to serve them ...
The state of perfection of the nations is not the heroic age
but the recapturing of a ...
heroic moment of the **integration of human and divine things in the third age**
...
James Joyce ... made ... **Vico** ... the central figure of ... **Finnegans Wake** ...”.

Bill Cole Cliett in his 2011 book “Riverrun to Livvy” said:

“... **the last piece of the Wake’s puzzle will fall into place**
on the same day
physicists find the formula that unifies all the forces of nature ...”.

Since the
Cl(16) - E8 Lagrangian - AQFT model “unifies all the forces of nature”
(see viXra 1405.0030)

here is a pattern of pieces of the Wake’s puzzle:

“Finnegans Wake” contains (my comments in red):

Book I of 8 Episodes
8 of E8

Books II and III of 4 + 4 Episodes
8 + 4 + 4 = 16 of Cl(16)

Book IV of a Final Episode 0

0 =  = Ouroboros Recursive Periodicity AQFT

Finnegans Wake

Finnegan = Fin + Again = End + ReWaken

I = 8 = RP1xS7 => M4 x CP2

I.1
“... riverrrun, past Eve and Adam’s, from swerve of shore to bend of bay,
brings us by a commodius vicus of recirculation ...
nor had topsawyer’s rocks by the stream Oconee exaggerated themselse
to Laurens County’s gorgios while they went doublin their mumber all the time ...”

Genesis River Oconee = Ocho = 8 = Octonions of E8
doublin = Powers of 2 = Clifford Algebra Cl(8) = 2^8 and Cl(16) = Cl(8)xCl(8)

1132 A. D. ... Blubby wares ...

The Southern Song court was the first to create a large, permanent standing naval institution for China in 1132.

566 A.D. On Baalfire’s night of this year after deluge ...

"... a great catastrophe struck the Earth in AD 540. ... Not only in Northern Ireland and Britain, but right across northern Siberia, North and South America - it is a global event of some kind. ...", according to an 8 September 2000 BBC article by Johnathan Amos, that also says "... cometary fragments smashed into the atmosphere throwing up dust and gas that blocked out the Sun. This, in turn, led to crop failures, famine and even plague among the weakened peoples of the world. ... some archaeologists and historians are beginning to come round to the opinion that this was the date when the Dark Ages began in Northern Europe. ..."

566 A. D. At this time it fell out that a brazenlockt damsel grieved ... Bloody wars ...

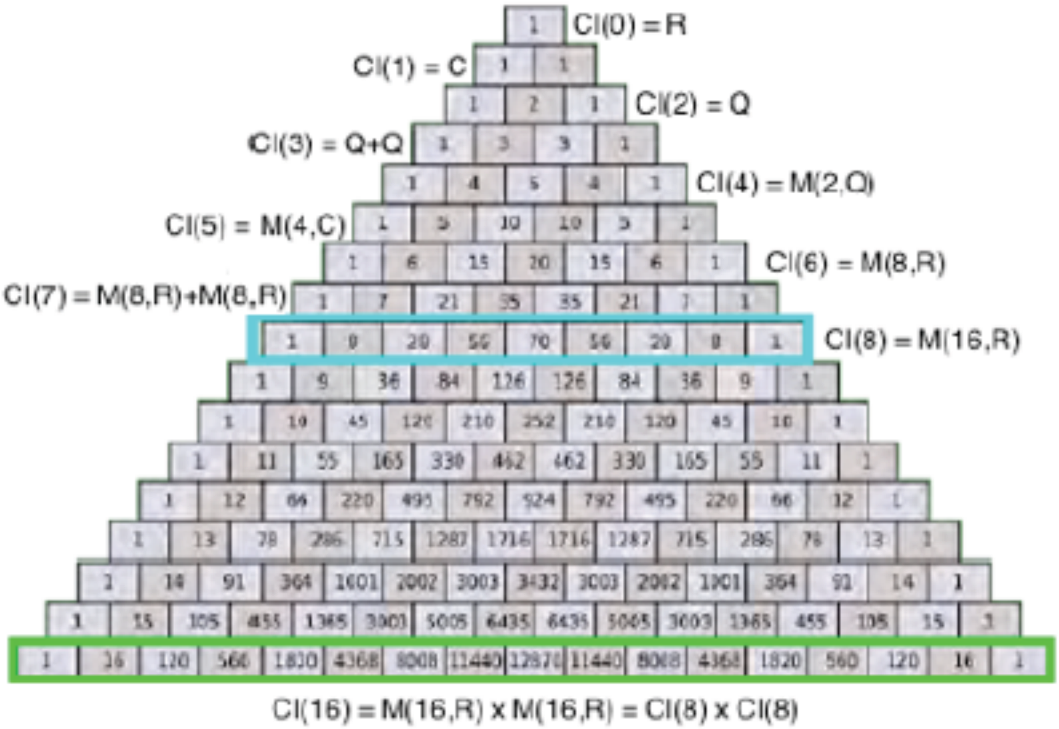
Muhammad was born around 570 AD. His father died before his birth.
He lived from birth to age 3 with foster parent, then with his mother.
His mother died when he was 6.
From 622 A. D. there were Bloody Wars.

1132 A. D. ... Blotty words ...

European Crusaders took Jerusalem in 1099.
Saladin took Jerusalem from European Crusaders in 1187.

I.3
(what has come over the face of wholebroader E?),
and (shrine of Mount Mu save us!) ...

E8 lives in the Clifford Algebra Cl(16) = Cl(8)xCl(8) as seen in Meru Prastara



I.6
the choicest and the cheapest from Atlanta to Oconee ...
what would that fargazer seem to seemself to seem seeming of, dimm it all?
Answer: A collideorscape! ...

collideorscape = Fermilab + LHC = Truth Quark + Higgs

I.8
El Negro ... La Plate ... neuphraties ... Neva ... Mersey ...Waal ... chattahoochee ...
wabbash ...jordan ... niester ... Saint Lawrence ... Laagen ... Niger ... O'Delawarr ...
Susquehanna ... hudson ... marne ... chinook's ... Floss ... Alt-muehler ... Hoangho ...
Shannons ... Orbe ... Oronoko ... limpopo ... Scamander ... Seints ... Kishtna ... Indes ...

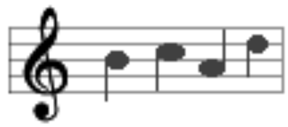
Many Rivers represent Many Possible Quantum Worlds

Hadn't he seven dams to wive him?
And every dam had her seven crutches.
And every crutch its seven hues. ...

7 dams = 7-Sphere of Vector of Cl(8) = SpaceTime
7 crutches = 7-Spheres of +half-spinor of Cl(8) = Fermion Particles
7 hues = 7-Sphere of -half-spinor of Cl(8) = Fermion AntiParticles
7 = 7 = 7 Related by Triality

II = 4 = CP2

II.2
by ribbon development ... 2,280,960 radiolumin lines

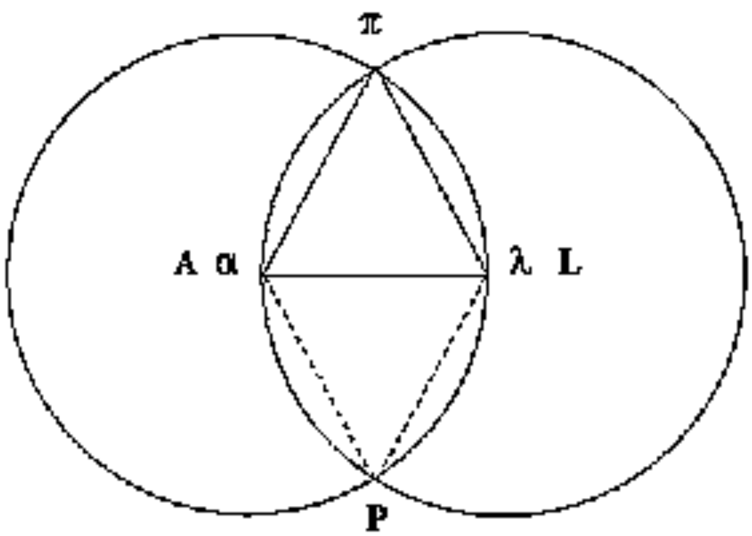
2280960 = 11 x 5 x 3^4 x 2^9 = 11 x 5 x 9^2 x 8^3 = 11 x 5 x 81 x 512

A brat ... a pipe clerk ... perfect little cad ...

B C A D

Sunday King. His sevencoloured's soot (IOchone! Ochonal!) ... Heptagrammaton ...

Ocho = 8 Octonions Hepta = 7 Imaginary Octonions = 7-Sphere

... construct ann aquilittoral dryankle ... Concoct an equo-angular trillitter ...



..."

Two Circles = M4 x CP2 of 4 + 4

Two Triangles ALP + alpha lambda pi =

= 3 Imaginary Quaternions of M4 + 3 Imaginary Quaternions of CP2

II.3
Coach With The Six Insides ...

6 Star of David Root Vector Vertices of SU(3) of CP2 = SU(3) / U(2)

B E N K ... B I N K ... B U N K ...
B E N K B A N K B O N K ...

3 Generations of Quarks, each Quark with 3 colors (red, green, blue)

Up/Down = BENK/BENK

Charm/Strange = BINK/BANK

Truth/Beauty = BUNK/BONK

II.4
- Three quarks for Muster Mark! ...

III = 4 = M4

III.1
Hark! Tolv two elf kater ten (it can't be) sax.
Hork! Pedwar pemp foify tray (it must be) twelve. ...
12 Cuboctahedral Root Vector Vertices of Conformal Spin(2,4) = SU(2,2) Gravity
mailman ... Letter, carried ... Ex. Ex. Ex. Ex. ...
Message carried across 4-dimensional SpaceTime = M4
root language ... hundredlettered name ... last word of perfect lan-guage
Language = Quantum Information = Clifford Algebra
... seven senses ... threestar monothong ...
7 = Imaginary Octonions   3star + 1mono = M4 Space + Time
III. 2
X. X. X. X.
4-dimensional SpaceTime coordinates of M4
the 4.32 ...
4 x 32 = 128 = E8 / D8

Oasis ... Oisis ...
Oasis ... Oisis ...
Oasis ... Oisis ...
Pipetto, Pipetta ...
8 = 6+2 = Vector Space of Cl(2,6) = 8x8 Quaternionic Matrix Algebra

Frida! Freda! Paza! Paisy! Irine! Areinette! Bradomay! Bentamai! Soso-sopky! Bebekka! Bababadkessy! Ghugugoothoyou! Dama! Damadomina! Takiya! Tokaya! Scioccara! Siuccherillina! Peoc-chia! Peucchia! Ho Mi Hoping! Ha Me Happinice! MIrra! My-rha! Solyma! Salemita! Sainta! Sianta!
O Peace!
O Peace = Octonion Rotations Spin(8) = 28-dimensional
III. 3
Are you roman cawthrick 432?
- Quadrigue my yoke.
Triple my tryst.
Tandem my sire.
4 x 3 x 2 = 24 Root Vector Vertices of D4 = Spin(8)
and dimension of Leech Lattice = E8 + E8 + E8

bold O'Conee weds on Alta Mahar ...

III.4
Tiers, tiers and tiers. Rounds.
Tiers of E8 Lattices and Leech Lattice
Rounds = Quantum Superpositions

IV = 

IV.0
“.. A way a lone a last a loved a long the”

(image from Wikipedia)

I.1
“... riverrrun, past Eve and Adam’s, from swerve of shore to bend of bay,
brings us by a commodius vicus of recirculation ...

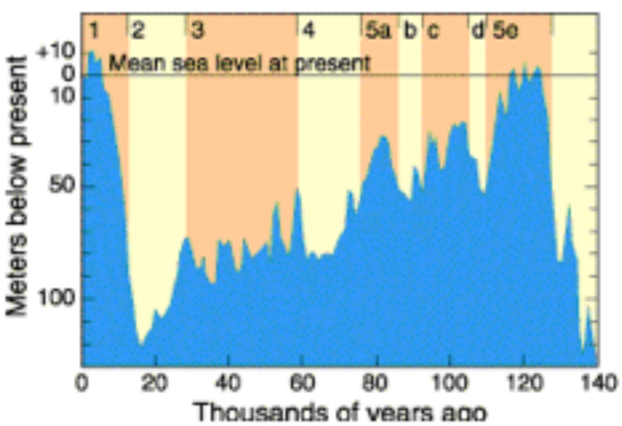
Appendix III:
Geminga and Toba

The SuperNova Precession Period of Peace began 36,525 years ago
when the Shock Wave of the Geminga SuperNova arrived at Earth.

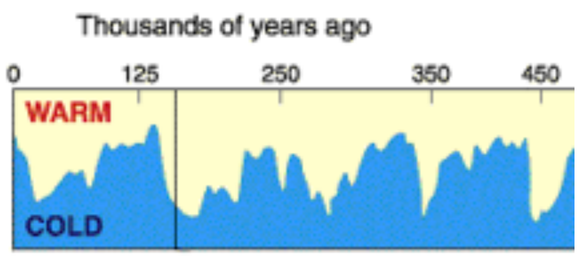
“... Geminga ... exploded as a supernova about 300,000 years ago.
This nearby ... 815 lyr ... explosion may be responsible for the low density
of the interstellar medium in the immediate vicinity of the Solar System.
This low-density area is known as the Local Bubble. ...” (Wikipedia).

Geminga Explosion Cosmic Rays of 300,000 years ago could have caused Mutations
in Earth Life producing the first Modern Human Homo sapiens in Africa,
leading to Ethiopian fossil remains. According to National Geographic Genographic:
“... Our species is an African one: Africa is where we first evolved, and where we have
spent the majority of our time on Earth. The earliest fossils of recognizably modern
Homo sapiens appear in the fossil record at Omo Kibish in Ethiopia, around 200,000
years ago. Although earlier fossils may be found over the coming years, this is our best
understanding of when and approximately where we originated. ...”.

Walter et al, in Nature 405 (4 May 2000) 65-69, said:
"... the 'out of Africa' hypothesis contends that modern humans evolved in Africa
between 200 and 100 kyr ago, migrating to Eurasia at some later time ... the discovery
of early Middle Stone Age artefacts in an emerged reef terrace on the Red Sea coast of
Ertitrea, which we date to the last interglacial (about 125 kyr ago) ... this is the earliest
well-dated evidence for human adaptation to a coastal marine environment ...”.
This was the last time that the Sea Level was as high as it is now (image from Wikipedia):



Late Quaternary sea-level history



Global climate history

Mellars, Gori, Carr, Soares, and Richards in pnas.1306043110 said:
“... initial dispersal of anatomically modern humans from Africa ...[prior to]... the volcanic “supereruption” of the Mount Toba volcano ... (the largest volcanic eruption of the past 2 million y) at ~74,000 y before present (B.P.) .. is in serious conflict with both the most recent genetic evidence from both Africa and Asia and the archaeological evidence from South Asian sites. ... a combination of genetic analyses and recent archaeological evidence from South Asia andAfrica ... support a coastally oriented dispersal of modern humans from eastern Africa to southern Asia ~60–50 thousand years ago (ka). ...

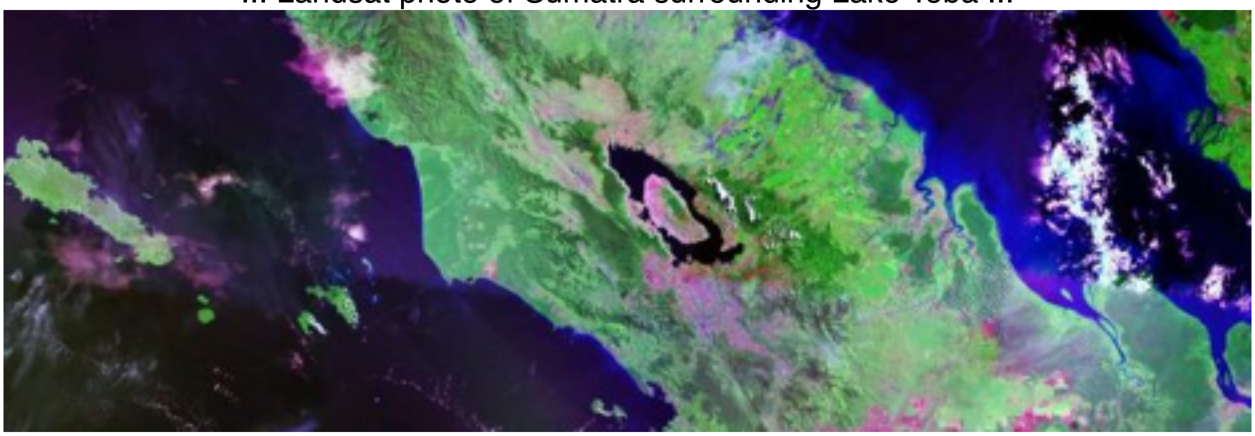
Fig. 2. Map of sites referred to in the main text. The zone of “high marine productivity” is inferred from Google Earth satellite images of chlorophyll concentrations in coastal waters. Graphic by Dora Kemp.

... This was associated with distinctively African microlithic and “backed-segment” technologies analogous to the African “Howiesons Poort” and related technologies, together with a range of distinctively “modern” cultural and symbolic features (highly shaped bone tools, personal ornaments, abstract artistic motifs, microblade technology, etc.), similar to those that accompanied the replacement of “archaic” Neanderthal by anatomically modern human populations in other regions of western Eurasia at a broadly similar date ...”.

According to Wikipedia articles:
“... The Toba eruption (the Toba event) occurred at what is now Lake Toba about 67,500 to 75,500 years ago. It was the last in a series of at least three caldera-forming eruptions at this location, with earlier calderas having formed around 700,000 and 840,000 years ago. This last eruption ...[was]... possibly the largest explosive volcanic eruption within the last 25 million years. ... the total amount of material released in the eruption was about 2,800 km3 (670 cu mi) -

about 2,000 km3 (480 cu mi) of ignimbrite that flowed over the ground, and approximately 800 km3 (190 cu mi) that fell as ash mostly to the west.
The pyroclastic flows of the eruption destroyed an area of 20,000 km2 (7,722 sq mi), with ash deposits as thick as 600 m (1,969 ft) by the main vent.
The eruption was large enough to have deposited an ash layer approximately 15 cm (5.9 in) thick over all of South Asia; at one site in central India, the Toba ash layer today is up to 6 m (20 ft) thick[13] and parts of Malaysia were covered with 9 m (30 ft) of ash fall. In addition it has been variously calculated that 10,000 million tonnes (1.1×1010 short tons) of sulfurous acid or 6,000 million tonnes (6.6×109 short tons) of sulfur dioxide were ejected into the atmosphere by the event.
The subsequent collapse formed a caldera that, after filling with water, created Lake Toba. The island in the center of the lake is formed by a resurgent dome.

... Landsat photo of Sumatra surrounding Lake Toba ...



The exact year of the eruption is unknown, but the pattern of ash deposits suggests that it occurred during the northern summer because only the summer monsoon could have deposited Toba ashfall in the South China Sea. The eruption lasted perhaps two weeks, and the ensuing "volcanic winter" resulted in a decrease in average global temperatures by 3.0 to 3.5 °C (5 to 6 °F) for several years. Greenland ice cores record a pulse of starkly reduced levels of organic carbon sequestration. Very few plants or animals in southeast Asia would have survived, and it is possible that the eruption caused a planet-wide die-off. ... evidence from pollen analysis has suggested prolonged deforestation in South Asia ... The Toba eruption has been linked to a genetic bottleneck in human evolution about 50,000 years ago ... between 50,000 and 100,000 years ago, human populations sharply decreased to 3,000–10,000 surviving individuals. It is supported by genetic evidence suggesting that today's humans are descended from a very small population of between 1,000 to 10,000 breeding pairs that existed about 70,000 years ago. ... The theory is based on ... coalescence of some genes (including mitochondrial DNA, Y-chromosome and some nuclear genes) as well as the relatively low level of genetic variation among present-day humans. ... human mitochondrial DNA (which is maternally inherited) and Y chromosome DNA (paternally inherited) coalesce at around 140,000 and 60,000 years ago, respectively. This suggests that
the female line ancestry of all present-day humans traces back to
a single female (Mitochondrial Eve) at around 140,000 years ago,
and the male line to a single male (Y-chromosomal Adam) at 60,000 ... years ago. ...".

Tetrahedra and Physics

Frank Dodd (Tony) Smith, Jr. - viXra 1501.0078v4

Astract

E8 Physics (viXra 1405.0030) at high energies has Octonionic 8-dim Spacetime
that is fundamentally a superposition of E8 Lattices
each of which has vertices surrounded by the 240-vertex E8 Root Vector Polytope.

At lower energies Octonionic symmetry is broken to Quaternionic symmetry
in accord with E8 = H4 + H4 so that the 240-vertex E8 Polytope is decomposed
into two copies of the Quaternionic 4-dim 120-vertex 600-cell whose relative
size is the Golden Ratio. If you give one copy a rational number size,
then the size of the other will be in a Golden Ratio Algebraic Extension space.

Let the Rational Number 600-cell be
the Vertex Polytope for 4-dim M4 Physical Spacetime of M4 x CP2 Kaluza-Klein
and
the Algebraic Extension 600-cell be
the Vertex Polytope for 4-dim CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein

Look at the 4-dim Physical Spacetime 600-cell. It has 120 vertices and 600 tetrahedra.
20 x 24 = 480 of the 600 tetrahedra are in 24 icosahedra within the 600-cell.
5 x 24 = 120 of the 600 tetrahedra are, 5 in each,
connected to each of the 24 icosahedra to form 24 octahedra.
The 24 octahedra form a 4-dim 24-cell,
the Vertex Polytope of the 4-dim Feynman Checkerboard.
24 of the 120 vertices correspond to vertices of the 24-cell and
96 of the 120 vertices correspond to Golden Ratio points, arranged in one of the two
possible consistent ways, on the 96 edges of the 24-cell dual to the original 24-cell.

Even though 3-dim simplex tetrahedra cannot tile flat 3-dim space,
they can combine to form curved 3-dim subspaces in 4-dim space, so that
3-dim simplex tetrahedra can be used as building blocks to construct E8 Physics
by taking 1200 of them to make two 600-cells, each in its own 4-dim space,
and then combining the two 600-cells and their two 4-dim spaces
to make 8-dim E8 Root Vector Polytopes, and then to make E8 Lattices
whose E8 Lie Algebra lives in Cl(16) Clifford Algebras whose completion of union of all
tensor products form a generalized hyperfinite II1 von Neumann factor AQFT
(Algebraic Quantum Field Theory) based on the realistic E8 Physics Lagrangian
and corresponding to a realistic 4-dim Feynman Checkerboard.

Table of Contents

8-dim E8 and 4-dim 600-cell

E8 Physics (viXra 1405.0030) at high energies has Octonionic 8-dim Spacetime
that is fundamentally a superposition of E8 Lattices
each of which has vertices surrounded by the 240-vertex E8 Root Vector Polytope.

E8 Lattice = D8 Lattice + ([1] + D8 Lattice)

There are 7 independent E8 Integral Domain Lattices.

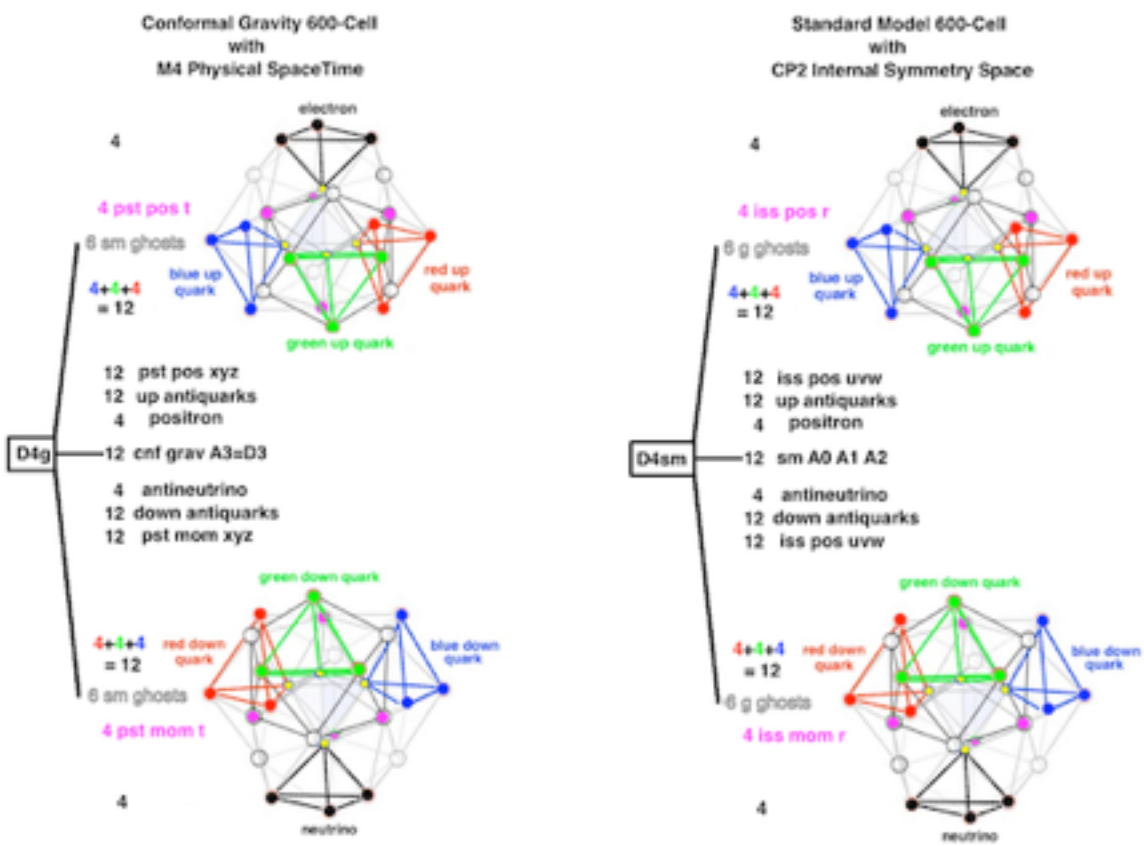
Physically, the D8 Lattice represents SpaceTime and Gauge Bosons
while the ([1] + D8 Lattice) represents Fermions.
At high energies (for example, during Inflation) E8 Physics is Octonionic and
there is only one generation of fermions, so the first generation is the only generation.
Therefore, each charged Dirac fermion particle, and its antiparticle, correspond to one
imaginary Octonion, to one associative triangle, and to one E8 lattice
so each Fermion propagates in its own **E8 8D Feynman Checkerboard Lattice**:

```
     red Down Quark                 red Up Quark              .
         green Down Quark    Electron     green Up Quark
                 blue Down Quark                 blue Up Quark

           rD     gD     bD       E      rU     gU     bU

           I      J      K       E      i      j      k

                                  j
                                 / \
                                i---k

           J      j      J             I      J      K
          / \    / \    / \           / \    / \    / \
         i---K  I---K  I---k         E---i  E---j  E---k

          3E8    6E8    4E8     7E8    1E8    2E8    5E8
```

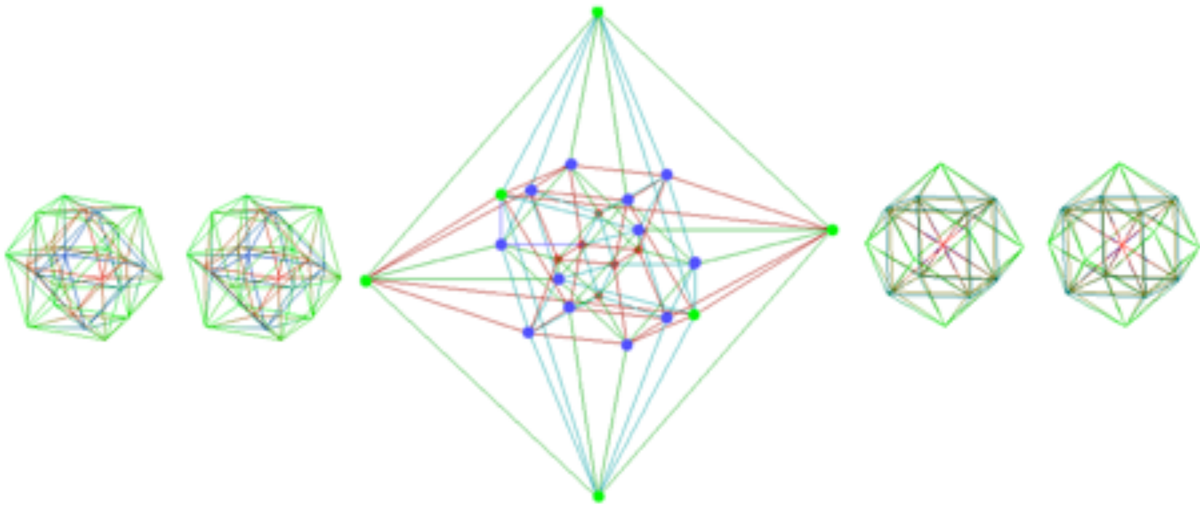
Since all the E8 lattices have in common the vertices { ±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke }, all the charged Dirac fermions can interact with each other. Composite particles, such as Quark-AntiQuark mesons and 3-Quark hadrons, propagate on the common parts of the E8 lattices involved. The uncharged neutrino fermion, which corresponds to the Octonion real axis with basis {1}, propagates on the 8th Kirmse E8 Lattice that is not an independent Octonion Integral Domain.

At lower energies Octonionic symmetry is broken to Quaternionic symmetry in accord with E8 = H4 + H4 so that the 240-vertex E8 Polytope is decomposed into two copies of the Quaternionic 4-dim 120-vertex 600-cell whose relative size is the Golden Ratio. If you give one copy a rational number size, then the size of the other will be in a Golden Ratio Algebraic Extension space.

Let the Rational Number 600-cell be
the Vertex Polytope for 4-dim M4 Physical Spacetime of M4 x CP2 Kaluza-Klein
and the Algebraic Extension 600-cell be
the Vertex Polytope for 4-dim CP2 Internal Symmetry Space of M4 x CP2 Kaluza-Klein



Look at the 4-dim Physical Spacetime 600-cell. It has 120 vertices and 600 tetrahedra.
20 x 24 = 480 of the 600 tetrahedra are in 24 icosahedra within the 600-cell.
5 x 24 = 120 of the 600 tetrahedra are, 5 in each,
connected to each of the 24 icosahedra to form 24 octahedra.
The 24 octahedra form a 4-dim 24-cell (center image from Frans Marcelis web site)



the Vertex Polytope of the 4-dim Feynman Checkerboard.
24 of the 120 vertices correspond to vertices of the 24-cell and
96 of the 120 vertices correspond to Golden Ratio points, arranged in one of the two possible consistent ways, on the 96 edges of the 24-cell dual to the original 24-cell.

Each of the 24 Octahedra that fill up the volume of the 24-cell contains an Icosahedron

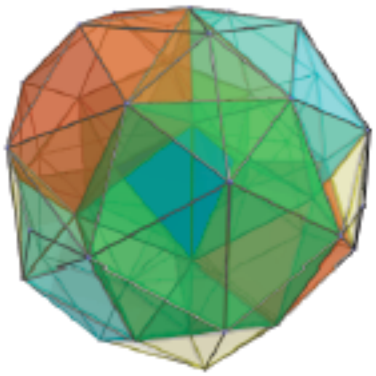
(image from wolfram mathworld)

plus some extra volume in each Octahedron.

The extra volume for all 24 Octahedra is made up of
24 vertex Tetrahedra (6 Octahedra meet at a 24-cell vertex, 6 x 24/6 = 24)
+
96 edge Tetrahedra (3 Octahedra meet at a 24-cell edge, 12 x 24/3 = 96)

so

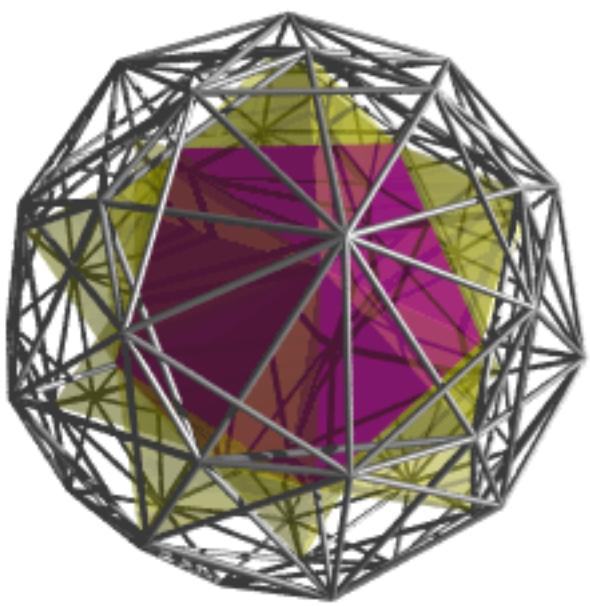
the 24-cell becomes a **Snub 24-cell = 24 Icosahedral and 120 Tetrahedral cells**



(image from eusebia.dyndns.org)

Each of the 24 Icosahedra contains 20 Tetrahedra for a total of 480 Tetrahedra which when added to the 24+96 = 120 Tetrahedra outside the Icosahedra give you the **480+120 = 600 Tetrahedra of the 600-cel**l.

According to Wikipedia: "...



... This image shows a vertex-first perspective projection of the 600-cell into 3D. The 600-cell is scaled to a vertex-center radius of 1, and the 4D viewpoint is placed 5 units away. Then the following enhancements are applied: The 20 tetrahedra meeting at the vertex closest to the 4D viewpoint are rendered in solid color. Their icosahedral arrangement is clearly shown. The tetrahedra immediately adjoining these 20 cells are rendered in transparent yellow. The remaining cells are rendered in edge-outline. Cells facing away from the 4D viewpoint (those lying on the "far side" of the 600-cell) have been culled, to reduce visual clutter in the final image. ...".

Sections of 600-cell

Sadoc and Mosseri in their book “Geometrical Frustration” (Cambridge 1999, 2006), say: “...

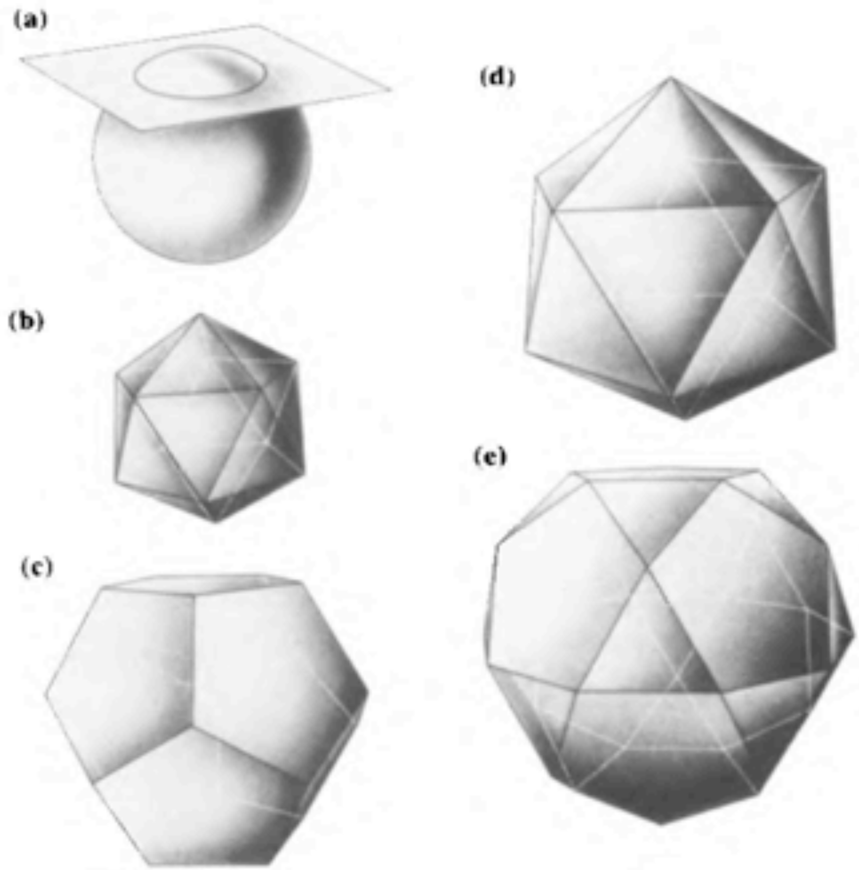


Fig. A5.1. The $\{3, 3, 5\}$ polytope. Different flat sections in S^3 (with one site on top) give the following successive shells; (a) an icosahedral shell formed by the first 12 neighbours, (b) a dodecahedral shell, (c) a second and larger icosahedral shell, (d) an icosidodecahedral shell on the equatorial sphere. Then other shells are symmetrically disposed in the second ‘south’ hemi-hypersphere, relative to the equatorial sphere (e).

$\omega = \pi/2$: the ‘equatorial’ sphere is tiled by 30 vertices which form a regular icosidodecahedron. For larger values of ω, the situation is then symmetrical with respect to the equatorial sphere.
$\omega = 3\pi/5$: an icosahedron.
$\omega = 2\pi/3$: a dodecahedron.
$\omega = 4\pi/5$: an icosahedron.
$\omega = \pi$: one vertex at the south pole $x_0 = -R$, $x_1 = x_2 = x_3 = 0$.

...

...

Table A5.1. *Sections of the {3, 3, 5} polytope (with an edge length equal to $2\tau^{-1}$) beginning with a vertex*

Section	x_0	(x_1, x_2, x_3)[1]	Vertex number	Shape
0	2	$(0, 0, 0)$	1	point
1	τ	$(1, 0, \tau^{-1})$	12	icosahedron
2	1	$(1, 1, 1)$ $(\tau, \tau^{-1}, 0)$	20	dodecahedron
3	τ^{-1}	$(\tau, 0, 1)$	12	icosahedron
4	0	$(2, 0, 0)$ $(\tau, 1, \tau^{-1})$	30	icosidodecahedron
5	$-\tau^{-1}$	$(\tau, 0, 1)$	12	icosahedron
6	-1	$(1, 1, 1)$ $(\tau, \tau^{-1}, 0)$	20	dodecahedron
7	$-\tau$	$(1, 0, \tau^{-1})$	12	icosahedron
8	-2	$(0, 0, 0)$	1	point

[1]Cyclic permutation with all possible changes of signs. $\tau = (1 + \sqrt{5})/2$.

... **Another ... description ... fixing a polytope cell center at the north pole** ...

Table A5.2. *Section of the {3, 3, 5} polytope (edge length $2\tau^{-1}\sqrt{2}$) beginning with a cell*

Section	x_0	(x_1, x_2, x_3)	Vertex number	Shape
0	τ^2	$(\tau^{-1}, \tau - 1, \tau^{-1})$[1]	4	tetrahedron
1	$\sqrt{5}$	$(-1, 1, 1)$	4	tetrahedron
2	2	$(2, 0, 0)$	6	octahedron
3	τ	(τ, τ, τ^{-2})	12	distorted
4	1	$(\sqrt{5}, 1, 1)$	12	cubo-octahedron
5	τ^{-1}	$(\tau^2, \tau^{-1}, \tau^{-1})$	12	
6	τ^{-2}	(τ, τ, τ)	4	tetrahedron
7	0	$(2, 2, 0)$	12	cubo-octahedron
8	$-\tau^{-2}$	$(-\tau, \tau, \tau)$	4	tetrahedron
—	—	—	—	
14	$-\tau^2$	$(-\tau^{-1}, \tau^{-1}, \tau^{-1})$	4	tetrahedron

[1]Permutation with an even number of sign changes. $\tau = (1 + \sqrt{5})/2$. Distorted cubo-octahedra are such that their square faces are changed into golden rectangles.

... ". **At the north pole and its antipodal south pole are Maximal Contact Groupings (57G) with 4+4+6+12 = 26 vertices**.

The Wikipedia entry on the 600-cell says:

“... the **600-cell** ... is the convex regular polytope ...]{3,3,5}. Its boundary is composed of 600 tetrahedral cells with 20 meeting at each vertex ... they form 1200 triangular faces, 720 edges, and 120 vertices. The edges form 72 flat regular decagons. Each vertex of the 600-cell is a vertex of six such decagons. ... Its vertex figure is an icosahedron ... It has a dihedral angle of 164.48 degrees. ... **Each cell touches, in some manner, 56 other cell**s.

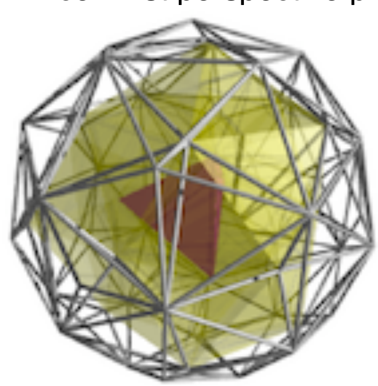
[4+1 = 5] One cell contacts each of the four faces;

[2x6 +5 = 17] two cells contact each of the six edges, but not a face;

[10x4 +17 = 57] and ten cells contact each of the four vertices, but not a face or edge.

[] ...

This image shows the 600-cell in cell-first perspective projection into 3D. ...



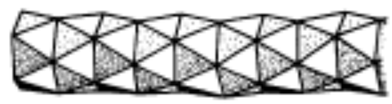
... The nearest cell to the 4d viewpoint is rendered in solid color, lying at the center of the projection image. The cells surrounding it (sharing at least 1 vertex) are rendered in transparent yellow. [**They are a 57G Maximal Contact Grouping**]

The remaining cells are rendered in edge-outline.

Cells facing away from the 4D viewpoint have been culled for clarity. ...”.

240 vertices of Two 600-cells and E8

Sadoc and Mosseri in that book also say:
"... {3,3,5} vertices ... as a set of 120 unit quaternions, form the binary icosahedral group ... the 120 polytope vertices can be grouped into four symmetry related sets of 30 sites, whose local order is linear arrangement of tetrahedra resembling Coxeter's 'simplicial helix' ...  ... In R3, the simplicial helix has pseudo-periods ... every 30 tetrahedra, the structure almost repeats itself ... In the polytope ... {3,3,5} ...the set of 30 tetrahedra perfectly closes on itself on a great circle ...
It is possible to describe the {3,3,5} polytope using the ... spherical torus which is a two-dimensional surface embedded in the spherical space S3 ...[that]... can be built from a square sheet, whose opposite sides are joined together. ...
Any line parallel to a diagonal of the square corresponds to a great circle of the 3-sphere ... For the spherical torus ... the two 'axes' of the torus ... are great circles ... the {3,3,5} polytope has two sets of 10 vertices ... on the two opposite axes of the torus foliation. The remaining 100 vertices belong to two sets of 50 vertices, forming triangular tilings on two tori ... placed symmetrically ... to the spherical torus.

We can represent one such torus by a cylinder ...  ...
if we decompose further the sets of 50 vertices on each torus into five sets of 10 vertices, we ... get ... 12 sets of 10 vertices belonging to 12 great circles,
which is .. the discretized Hopf fibration ... The Hopf mapping of this discrete set onto S2 gives 12 points which form a regular icosahedron on the base
...
The E8 lattice is ... the densest sphere packing in eight dimensions ...
[its] first ... shell is a 240-vertex ... Gosset polytope ...
split its 240 vertices into ten ... subsets ... each ... belonging ... to a sphere S3 ...
This is ... a discrete version of the Hopf fibration of S7 with S3 fibres and a S4 base
...
On each fibre, the 24 points form a [24-cell] polytope {3,4,3} ...
each fibre ... generates a four-dimensional sublattice {3,3,4,3} of the E8 lattice.
There are ten ... sublattices through the origin, associated with the ten points on the base S4 ... a 'cross' polytope on S4 ... images of the fibres under the Hopf map.
Let P be ... (1/sqrt(5))(1,1,1,1,1) ... It ... defines a four-dimensional space E.
The mapping of the Gossett polytope onto E produces two sets of five (3,4,3) ...
form[ing] ... two concentric {3,3,5} on E ... which differ by a factor [of the Golden Ratio] ...".

E8 Physics and 8D Feynman Checkerboard

E8 Physics is described in http://vixra.org/pdf/1405.0030vG.pdf
in which the 240 vertices of the Gosset polytope are given physical interpretations that produce a Local Classical Lagrangian for Gravity and the Standard Model. Embedding E8 in the Real Clifford Algebra Cl(16) = Cl(8)xCl(8) and taking the completion of the union of all tensor products of Cl(16) gives a realistic Algebraic Quantum Field Theory (AQFT).

An equivalent Quantum Field Theory can be constructed using
Tetrahedra, 600-cells, and the E8 Gossett polytope
along with a generalized Feynman Checkerboard in 4 SpaceTime dimensions.

Conway and Sloane, in their book Sphere Packings, Lattices, and Groups (3rd edition, Springer, 1999), in chapter 4, section 7.3, pages 119-120) define a packing [where the glue vector [1] = (1/2, ... , 1/2)]

D+n = Dn u ([1] + Dn)

and say:

"... D+n is a lattice packing if and only if n is even.
D+3 is the tetrahedral or diamond packing ... and
D+4 = Z4.
When n = 8 this construction is especially important,
the lattice **D+8 being known as E8** ...".

Therefore

E8 Lattice = D8 Lattice + ([1] + D8 Lattice)

There are 7 independent E8 Integral Domain Lattices.

Physically, the D8 Lattice represents SpaceTime and Gauge Bosons
while the ([1] + D8 Lattice) represents Fermions.

At high energies (for example, during Inflation) E8 Physics is Octonionic and there is only one generation of fermions, so the first generation is the only generation.

Therefore, each charged Dirac fermion particle, and its antiparticle, correspond to one imaginary Octonion, to one associative triangle, and to one E8 lattice
so each Fermion propagates in its own **E8 8D Feynman Checkerboard Lattice**:

```
red Down Quark                 red Up Quark          .
    green Down Quark   Electron    green Up Quark
            blue Down Quark              blue Up Quark

       rD     gD     bD      E     rU     gU     bU

       I      J      K      E      i      j      k

                            j
                           / \
                          i---k

       J      j      J             I      J      K
      / \    / \    / \           / \    / \    / \
     i---K  I---K  I---k         E---i  E---j  E---k

      3E8    6E8    4E8    7E8    1E8    2E8    5E8
```

Since all the E8 lattices have in common the vertices { ±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke }, all the charged Dirac fermions can interact with each other. Composite particles, such as Quark-AntiQuark mesons and 3-Quark hadrons, propagate on the common parts of the E8 lattices involved. The uncharged neutrino fermion, which corresponds to the Octonion real axis with basis {1}, propagates on the 8th Kirmse E8 Lattice that is not an independent Octonion Integral Domain.

If a preferred Quaternionic Structure is introduced into an Octonionic E8 Lattice
then the Octonionic E8 Lattice is transformed into Quaternionic Lattice structure.
The Quaternionic Integral Domain Lattice is the D4 Lattice.

D8 Lattice is transformed to D4g + D4sm

([1] + D8 Lattice) is transformed to ([1] + D4g) + ([1] + D4sm)

so

E8 is transformed to { D4g + ([1] + D4g) } + { D4 sm + ([1] + D4sm) }

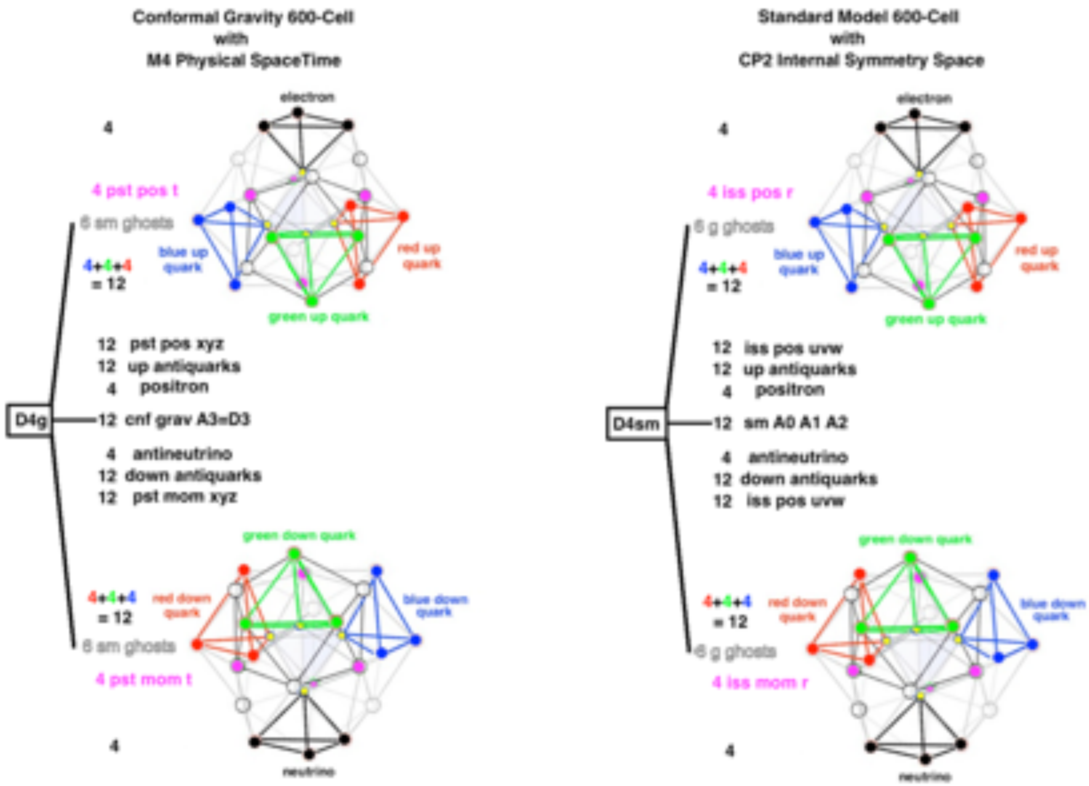
E8 = D+4g + D+4sm

D+4g corresponds to the 600-cell containing D4g

D+4sm corresponds to the 600-cell containing D4sm

To begin, consider
the two 600-cells underlying the Gosset polytope at each vertex of an E8 Lattice.

Split 8-dim Kaluza-Klein E8 SpaceTime into its two 4-dimensional components:
M4 Physical SpaceTime and CP2 = SU(3 / SU(2)xU(1) Internal Symmetry Space
Let one 600-cell represent Gravity and physics of Physical SpaceTime.
Let the other 600-cell represent the Standard Model and its Internal Symmetry Space.



The 120 vertices of the D4g 600-cell and the 120 vertices of the D4sm 600-cell combined form the 240 vertices of the E8 Root Vectors of E8 Physics:

E8 lives inside the Real Clifford Algebra Cl(16) as E8 = D8 + Cl(16) half-spinors so

240 E8 Root Vectors = 112 D8 Root Vectors + 128 Cl(16) half-spinors

E8 Lattice = D8 Lattice + ([1] + D8 Lattice)

where the lattice shifting glue vector [1] = (1/2, ... , 1/2)

4D Feynman Checkerboard Quantum Theory

Conway and Sloane, in their book Sphere Packings, Lattices, and Groups (3rd edition, Springer, 1999), in chapter 4, section 7.3, pages 119-120) define a packing [where the glue vector [1] = (1/2, ... , 1/2)]

D+n = Dn u ([1] + Dn)

and say:

"... D+n is a lattice packing if and only if n is even.
D+3 is the tetrahedral or diamond packing ... and
D+4 = Z4.
When n = 8 this construction is especially important,
the lattice **D+8 being known as E8** ...".

Therefore

E8 Lattice = D8 Lattice + ([1] + D8 Lattice)

There are 7 independent E8 Integral Domain Lattices.

Physically, the D8 Lattice represents SpaceTime and Gauge Bosons
while the ([1] + D8 Lattice) represents Fermions.

At high energies (for example, during Inflation) E8 Physics is Octonionic and there is only one generation of fermions, so the first generation is the only generation. Therefore, each charged Dirac fermion particle, and its antiparticle, correspond to one imaginary Octonion, to one associative triangle, and to one E8 lattice so each Fermion propagates in its own **E8 8D Feynman Checkerboard Lattice**:

```
red Down Quark                   red Up Quark          .
    green Down Quark    Electron      green Up Quark
            blue Down Quark                 blue Up Quark


          rD     gD     bD       E      rU     gU     bU


          I      J      K       E      i      j      k


                                j
                               / \
                              i---k

          J      j      J              I      J      K
         / \    / \    / \            / \    / \    / \
        i---K  I---K  I---k          E---i  E---j  E---k


         3E8    6E8    4E8     7E8     1E8    2E8    5E8
```

Since all the E8 lattices have in common the vertices { ±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke }, all the charged Dirac fermions can interact with each other. Composite particles, such as Quark-AntiQuark mesons and 3-Quark hadrons, propagate on the common parts of the E8 lattices involved. The uncharged neutrino fermion, which corresponds to the Octonion real axis with basis {1}, propagates on the 8th Kirmse E8 Lattice that is not an independent Octonion Integral Domain.
If a preferred Quaternionic Structure is introduced into an Octonionic E8 Lattice then the Octonionic E8 Lattice is transformed into Quaternionic Lattice structure.
The Quaternionic Integral Domain Lattice is the D4 Lattice.
D8 Lattice is transformed to D4g + D4sm
([1] + D8 Lattice) is transformed to ([1] + D4g) + ([1] + D4sm)
so
E8 is transformed to { D4g + ([1] + D4g) } + { D4 sm + ([1] + D4sm) }

E8 = D+4g + D+4sm

D+4g corresponds to the 600-cell containing D4g
D+4sm corresponds to the 600-cell containing D4sm

Conway and Sloane (Sphere Packings, Lattices, and Groups - Springer) (Chapter 4, eq. 49) give equations for the number of vertices N(m) in the m-th layer of the D+4 HyperDiamond lattice where d is a divisor (including 1 and m) of m:

for m odd: N(m) = 8 SUM(d|m) d   for m even: N(m) = 24 SUM(d|m, d odd) d

Here are the numbers of vertices in some of the layers of the D4+ lattice.
The even-numbered layers correspond to the even D4 sublattice:

m=norm of layer	N(m)=no. vert.		
0	1		
1	8	=	1 x 8
2	24	=	1 x 24
3	32	=	(1 + 3) x 8
4	24	=	1 x 24
5	48	=	(1 + 5) x 8
6	96	=	(1 + 3) x 24
7	64	=	(1 + 7) x 8
8	24	=	1 x 24
9	104	=	(1 + 3 + 9) x 8
10	144	=	(1 + 5) x 24
11	96	=	(1 + 11) x 8
12	96	=	(1 + 3) x 24
13	112	=	(1 + 13) x 8
14	192	=	(1 + 7) x 24
15	192	=	(1 + 3 + 5 + 15) x 8
16	24	=	1 x 24
17	144	=	(1 + 17) x 8

First Stage of 4D Feynman Checkerboard:

D+4g vertices have HyperOctahedron 8 nearest-neighbors {+/-1,+/-i,+/-j,+/-k}
where 4-dim 1,i,j,k are descendants of 8-dim 1,i,j,k
to be used as 4D Feynman Checkerboard Primary Links representing the 4-dim M4 Physical SpaceTime of the Kaluza-Klein of E8 Physics whose 4 basis elements are {1,i,j,k} each of which has 8 momentum components with respect to 8-dim SpaceTime to represent 4x8 = 32 of 600-cell vertices.

D+4g vertices have 24-cell 24 next-nearest neighbors representing the
12 Conformal Gravitons (Root Vectors of U(2,2) and
12 Ghosts of Standard Model Gauge Bosons
that live on the nearest-neighbor links and represent 24 of 600-cell vertices.

D+4g vertices have 6-semi-HyperCube 32 next-next-nearest neighbors representing 4 M4 Physical SpaceTime components of 8 First-Generation Fermion Particles. Fermion AntiParticles are represented by Particles moving backward in Time for representation of 2x32 = 64 of 600-cell vertices.

D+4g odd (1 and 3) layers correspond to Vectors and Fermion Spinors which are related by Triality.
D+4g even (2) layers correspond to BiVectors.

From each vertex of the 4D Feynman Checkerboard the First Stage uses a Triad of Quantum Choice Vectors.

Second Stage of 4D Feynman Checkerboard:

D+4sm vertices have HyperOctahedron 8 nearest-neighbors {+/-1,+/-i,+/-j,+/-k}
where 4-dim 1,i,j,k are descendants of 8-dim E,I,J,K
to be used as 4D Feynman Checkerboard Secondary Links representing the 4-dim CP2 Internal Symmetry Space of the Kaluza-Klein of E8 Physics whose 4 basis elements are {1,i,j,k} each of which has 8 momentum components with respect to 8-dim SpaceTime to represent 4x8 = 32 of 600-cell vertices.

D+4sm vertices have 24-cell 24 next-nearest neighbors representing the
12 Standard Model Gauge Bosons and
12 Ghosts of Conformal Gravitons (Root Vectors of U(2,2)
that live on the nearest-neighbor links and represent 24 of 600-cell vertices.

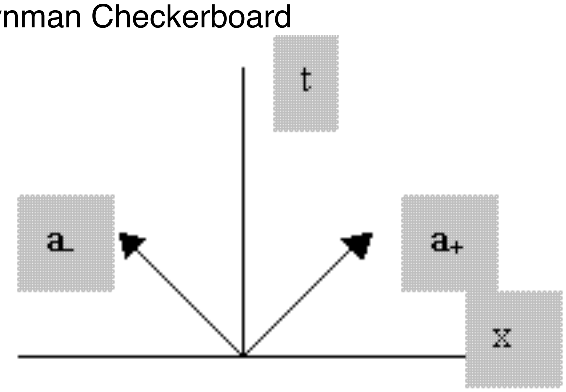
D+4sm vertices have 6-semi-HyperCube 32 next-next-nearest neighbors representing 4 CP2 Internal Symmetry Space components of 8 First-Generation Fermion Particles. Fermion AntiParticles are represented by Particles moving backward in Time for representation of 2x32 = 64 of 600-cell vertices.

D+4g odd (1 and 3) layers correspond to Vectors and Fermion Spinors which are related by Triality.
D+4g even (2) layers correspond to BiVectors.

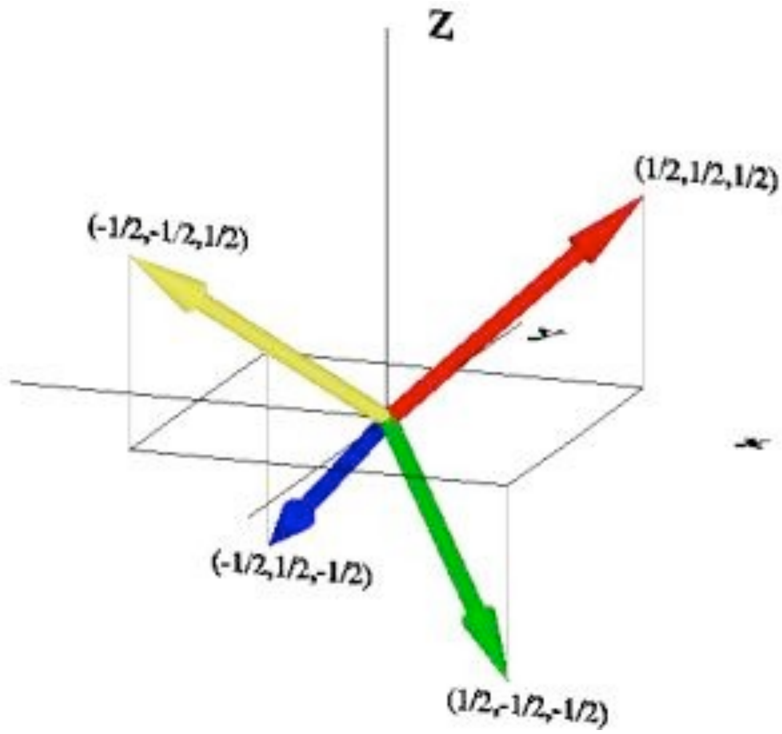
From each vertex of the 4D Feynman Checkerboard the Second Stage uses a second Triad of Quantum Choice Vectors.

A significant consequence of using two Triads of Quantum Choice Vectors is the emergence of Second and Third Generation Fermions.

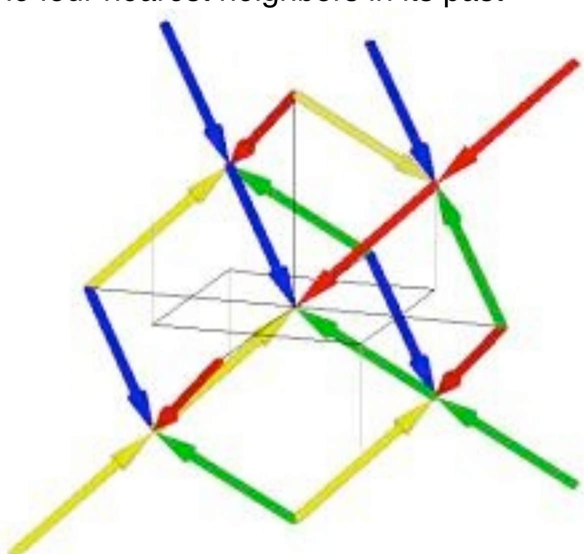
In my earlier paper (arXiv quant-ph/9503015) I used **a simpler version of 4D Feynman Checkerboard which is useful for showing consistency with the Dirac equation** using the following approach: The Feynman Checkerboard in 1+3 SpaceTime dimensions reproduces the Dirac equation, using work of Urs Schreiber and George Raetz. (See my paper at CERN-CDS-EXT-2004-030) A very nice feature of the George Raetz web site is its illustrations, which include an image of a vertex of a 1+1 dimensional Feynman Checkerboard



and an image of a projection into three dimensions of a vertex of a 1+3 dimensional Feynman Checkerboard



and an image of flow contributions to a vertex in a HyperDiamond Random Walk from the four nearest neighbors in its past



Urs Schreiber wrote on the subject:

Re: Physically understanding the Dirac equation and 4D

in the newsgroup sci.physics.research on 2002-04-03 19:44:31 PST (including an appended forwarded copy of an earlier post)
and again on 2002-04-10 19:03:09 PST as found on the web page
http://www-stud.uni-essen.de/~sb0264/spinors-Dirac-checkerboard.html

and the following are excerpts from those posts:

"... I know ... the ... lanl paper ...[http://xxx.lanl.gov/abs/quant-ph/9503015]...
and
I know that Tony Smith does give
a generalization of Feynman's summing prescription from 1+1 to 1+3 dimensions.

But I have to say that I fail to see that this generalization reproduces the Dirac propagator in 1+3 dimensions,
and that I did not find any proof that it does.

Actually, I seem to have convinced myself that it does not,
but
I may of course be quite wrong.

I therefore take this opportunity to state my understanding of these matters.

First, I very briefly summarize (my understanding of) Tony Smith's construction: The starting point is the observation that the left |-> and right |+> going states of the 1+1 dim checkerboard model can be labeled by complex numbers

|-> ---> (1 + i)
|+> ---> (1 - i)

(up to a factor) so that multiplication by the negative imaginary unit swaps components:

(-i) (1 + i)/2 = (1 - i)/2
(-i) (1 - i)/2 = (1 + i)/2 .

Since the path-sum of the 1+1 dim model reads
phi = sum over all possible paths of (-i eps m)^(number of bends of path) =
= sum over all possible paths of product over all steps of one path of -i eps m
(if change of direction after this step generated by i) 1 (otherwise)
this makes it look very natural to identify the imaginary unit appearing in the sum over paths with the "generator" of kinks in the path.
To generalize this to higher dimensions, more square roots of -1 are added,
which gives the quaternion algebra in 1+3 dimensions.
The two states I+> and I-> from above, which were identified with complex numbers, are now generalized to four states identified with the following quaternions
(which can be identified with vectors in M^4 indicating the direction in which a given path is heading at one instant of time):
(1 + i + j + k) (1 + i - j - k) (1 - i + j - k) (1 - i - j + k) ,
which again constitute a (minimal) left ideal of the algebra
(meaning that applying i,j, or k from the left on any linear combination of these four states gives another linear combination of these four states).
Hence,
now i,j,k are considered as "generators" of kinks in three spatial dimensions
and the above summing prescription naturally generalizes to
phi = sum over all possible paths of product over all steps of one path of
-i eps m (if change of direction after this step generated by i)
-j eps m (if change of direction after this step generated by j)
-k eps m (if change of direction after this step generated by k)
1 (otherwise)
The physical amplitude is taken to be
A * e^(i alpha)
where A is the norm of phi and alpha the angle it makes with the x0 axis.

As I said, this is merely my paraphrase of Tony Smith's proposal as I understand it.

I fully appreciate that the above construction is a nice (very "natural") generalization of the summing prescription of the 1+1 dim checkerboard model.

But if it is to describe real fermions propagating in physical spacetime, this generalized path-sum has to reproduce the propagator obtained from the Dirac equation in 1+3 dimensions, which we know to correctly describe these fermions. Does it do that?
...
Hence I have taken a look at the material [that] ... George Raetz ... present[s] ... titled "The HyperDiamond Random Walk", found at
http://www.pcisys.net/~bestwork.1/QRW/the_flow_quaternions.htm ,
which is mostly new to me. ...

I am posting this in order to make a suggestion for a more radical modification
...
[The]... equation ... DQ = (iE)Q ... is not covariant.
That is because of that quaternion E sitting on the left of the spinor Q
in the rhs of [the] equation ... .
The Dirac operator D is covariant,
but the unit quaternion E on the rhs refers to a specific frame.
Under a Lorentz transformation L one finds
L DQ = iE LQ = L E' Q <=> DQ = E'Q now with E' = L~ E L instead of E.
This problem disappears
when the unit quaternion E is brought to the *right* of the spinor Q.
What we would want is an equation of the form DQ = Q(iE) .
In fact, demanding that the spinor Q be an element of the minimal left ideal
generated by the primitive projector P = (1+y0)(1+E)/4 ,
so that Q = Q' P ,
one sees that DQ = Q(iE) almost looks like the the *Dirac-Lanczos equation*.
(See hep-ph/0112317, equation (5) or ... equation (9.36) [of]... W. Baylis, Clifford (Geometric) Algebras, Birkhaeuser (1996) ...).
To be equivalent to the Dirac-Lanczos equation, and hence to be correct,
we need to require that D = y0 @0 + y1 @1 + y2 @2 + y3 @3
instead of ... = @0 + e1 @1 + e2 @2 + e3 @3 .
All this amounts to sorting out
in which particular representation we are actually working here.
In an attempt to address these issues, I now redo the steps presented on
http://www.pcisys.net/~bestwork.1/QRW/the_flow_quaternions.htm
with some suitable modifications to arrive at the correct Dirac-Lanczos equation
(this is supposed to be a suggestion subjected to discussion):

So consider a lattice in Minkoswki space
generated by a unit cell spanned by the four (Clifford) vectors

r = (y0 + y1 + y2 + y3)/2 g = (y0 + y1 - y2 - y3)/2 b =
= (y0 - y1 + y2 - y3)/2 y = (y0 - y1 - y2 + y3)/2 .

(yi are the generators of the Dirac algebra {yi,yj} = diag(+1,-1,-1,-1)_ij.)

This is Tony Smith's "hyper diamond".

(Note that I use Clifford vectors instead of quaternions.)
Now consider a "Clifford algebra-weighted" random walk along the edges of this lattice,
which is described by four Clifford valued "amplitudes": Kr, Kg, Kb, Ky and
such that
@r Kr = k (Kg y2 y3 + Kb y3 y1 + Ky y1 y2)
@b Kb = k (Ky y2 y3 + Kr y3 y1 + Kg y1 y2) @g Kg = k (Kr y2 y3 + Ky y3 y1 + Kb y1 y2) @y Ky = k (Kb y2 y3 + Kg y3 y1 + Kr y1 y2) .

(This is geometrically motivated. The generators on the rhs are those that rotate the unit vectors corresponding to the amplitudes into each other. "k" is some constant.)

Note that I multiply the amplitudes from the *right* by the generators of rotation, instead of multiplying them from the left.

Next, assume that this coupled system of differential equations is solved by a spinor Q
Q = Q' (1+y0)(1+iE)/4
E = (y2 y3 + y3 y1 + y1 y2)/sqrt(3) with
Kr = r Q Kg = g Q Kb = b Q Ky = y Q .

This ansatz for solving the above system by means of a single spinor Q is,
as I understand it, the central idea.
But note that I have here modified it on the technical side:

Q is explicitly an algebraic Clifford spinor in a definite minial left ideal,

E squares to -1, not to +1,

and the Ki are obtained from Q by premultiplying with the Clifford basis vectors defined above.

Substituting this ansatz into the above coupled system of differential equations one can form one covariant expression by summing up all four equations:

(r @r + g @g + b @b + y @y) Q = k sqrt(3) Q E
The left hand side is immediate.
To see that the right hand side comes out as indicated
simply note that r + g + b + y = y0 and that Q y0 = Q by construction.

The above equation is the Dirac-Lanczos-Hestenes-Guersey equation,
the algebraic version of the equation describing the free relativistic electron.

The left hand side is the flat Dirac operator r @r + g @g + b @b + y @y = ym @m and
the right hand side, with k = mc / (hbar sqrt(3)) ,
is equal to the mass term i mc / hbar Q.
As usual, there are a multitude of ways to rewrite this.
If one wants to emphasize biquaternions then
premultiplying everything with y0 and
splitting off the projector P on the right of Q to express everything in terms of the,
then also biquaternionic, Q' (compare the definitions given above)
gives Lanczos' version (also used by Baylis and others).

I think this presentation improves a little on that given on George Raetz's web site:

The factor E on the right hand side of the equation is no longer a nuisance
but a necessity.

Everything is manifestly covariant (if one recalls that algebraic spinors are manifestly covariant when nothing non-covariand stands on their *left* side). The role of the quaternionic structure is clarified,
the construction itself does not depend on it.
Also, it is obvious how to generalize to arbitrary dimensions.
In fact, one may easily check that for 1+1 dimensions the above scheme reproduces the Feynman model.

While I enjoy this, there is still some scepticism in order
as long as a central questions remains to be clarified:

How much of the Ansatz K(r,g,b,y) = (r,g,b,y) Q is whishful thinking?

For sure,
every Q that solves the system of coupled differential equations that describe the amplitude of the random walk on the hyper diamond lattice
also solves the Dirac equation.

But what about the other way round?
Does every Q that solves the Dirac equation also describe such a random walk. ...".

My proposal to answer the question raised by Urs Schreiber

Does every solution of the Dirac equation
also describe a HyperDiamond Feynman Checkerboard random walk?

uses symmetry.

The hyperdiamond random walk transformations
include the transformations of the Conformal Group:

rotations and boosts (to the accuracy of lattice spacing);
translations (to the accuracy of lattice spacing);
scale dilatations (to the accuracy of lattice spacing): and
special conformal transformations (to the accuracy of lattice spacing).

Therefore, to the accuracy of lattice spacing,
the hyperdiamond random walks give you all the conformal group Dirac solutions,
and since the full symmetry group of the Dirac equation is the conformal group,
the answer to the question is "Yes".

Thanks to the work of Urs Schreiber:

The HyperDiamond Feynman Checkerboard in 1+3 dimensions does reproduce the correct Dirac equation.

Here are some references to the **conformal symmetry of the Dirac equation**:

R. S. Krausshar and John Ryan in their paper Some Conformally Flat Spin Manifolds, Dirac Operators and Automorphic Forms at math.AP/022086 say:
"... In this paper we study Clifford and harmonic analysis on some conformal flat spin manifolds. ... manifolds treated here include RPn and S1 x S(n-1).
Special kinds of Clifford-analytic automorphic forms associated to the different choices of are used to construct Cauchy kernels, Cauchy Integral formulas, Green's kernels and formulas together with Hardy spaces and Plemelj projection operators for Lp spaces of hypersurfaces lying in these manifolds. ...
Solutions to the Dirac equation are called Clifford holomorphic functions or monogenic functions.
Such functions are covariant under ... conformal or .... Mobius transformations acting over Rn u {oo}. ...".

Barut and Raczka, in their book Theory of Group Representations and Applications (World 1986), say, in section 21.3.E, at pages 616-617:
"... E. The Dynamical Group Interpretation of Wave Equations.
... Example 1. Let G = O(4,2).
Take U to be the 4-dimensional non-unitary representation in which the generators of G are given in terms of the 16 elements of the algebra of Dirac matrices as in exercise 13.6.4.1.
Because (1/2)L_56 = gamma_0 has eigenvalues n = +/-1,
taking the simplest mass relation mn = K, we can write
(m gamma_0 - K) PSI(dotp) = 0, where K is a fixed constant.
Transforming this equation with the Lorentz transformation of parameter E
PSI(p) = exp(i E N) PSI(p)
N = (1/2) gamma_0 gamma
gives
(gamma^u p_u - K) PSI(p) = 0
which is the Dirac equation ...".

P. A. M. Dirac, in his paper Wave Equations in Conformal Space, Ann. Math. 37 (1936) 429-442, reprinted in The Collected Works of P. A. M. Dirac: Volume 1: 1924-1948, by P. A. M. Dirac (author), Richard Henry Dalitz (editor), Cambridge University Press (1995), at pages 823-836, said:
"... by passing to a four-dimensional conformal space ...
a ... greater symmetry of ... equations of physics ... is shown up, and their invariance under a wider group is demonstrated. ...
The spin wave equation ... seems to be the only
simple conformally invariant wave equation involving the spin matrices. ... This equation is equivalent to the usual wave equation for the electron, except ...[that it is multiplied by]... the factor (1 + alpha_5) ,
which introduces a degeneracy. ...".

Here are some comments on **Lorentz Invariance based on D4 Lattice** properties:

The D4 lattice nearest neighbor vertex figure, the 24-cell,
is the 4HD HyperDiamond lattice next-to-nearest neighbor vertex figure.
Fermions move from vertex to vertex along links.
Gauge bosons are on links between two vertices, and so can also be considered as moving from vertex to vertex along links.
The only way a translation or rotation can be physically defined is by a series of movements of a particle along links.
A TRANSLATION is defined as a series of movements of a particle along links, each of which is
the CONTINUATION of the immediately preceding link IN THE SAME DIRECTION.
An APPROXIMATE rotation, within an APPROXIMATION LEVEL D,
is defined with respect to a given origin as a series of movements of a particle along links among vertices ALL of which
are in the SET OF LAYERS LYING WITHIN D of norm (distance^2) R from the origin, that is,
the SET OF LAYERS LYING BETWEEN norm R-D and norm R+D from the origin.
Conway and Sloane (Sphere Packings, Lattices, and Groups - Springer) pp. 118-119 and 108, is the reference that I have most used for studying lattices in detail.
(Conway and Sloane define the norm of a vector x to be its squared length xx.)
In the D4 lattice of integral quaternions,
layer 2 has the same number of vertices as layer 1, N(1) = N(2) = 24.
Also (this only holds for real, complex, quaternionic, or octonionic lattices),
K(m) = N(m)/24 is multiplicative,
meaning that, if p and q are relatively prime, K(pq) = K(p)K(q).
The multiplicative property implies that:
K(2^a) = K(2) = 1 (for a greater than 0) and
K(p^a) = 1 + p + p^2 + ... + p^a (for a greater than or equal to 0).
So,
for the D4 lattice,
there is always an arbitrarily large layer (norm xx = 2^a, for some large a)
with exactly 24 vertices, and
there is always an arbitrarily large layer(norm xx = P, for some large prime P)
with 24(P+1) vertices (note that Mersenne primes are adjacent to powers of 2), and
given a prime number P whose layer is within D of the origin,
which layer has N vertices,
there is a layer kP with at least N vertices within D of any other given layer in D4.
Some examples I have used are chosen so that
the 2^a layer adjoins the prime 2^a +/- 1 layer.

The notation in the following table is based on the minimal norm of the D4 Lattice being 1, in which case **the D4 lattice is the lattice of integral quaternions**.
This is the second definition (equation 90) of the D4 Lattice in
Chapter 4 of Sphere Packings, Lattices, and Groups, 3rd ed., by Conway and Sloane (Springer 1999)
who note that the Dn lattice is the checkerboard lattice in n dimensions.

m=norm of layer	N(m)=no. vert.	K(m)=N(m)/24
1	24	1
2	24	1
3	96	4
4	24	1
5	144	6
6	96	4
7	192	8
8	24	1
9	312	13
10	144	6
11	288	12
12	96	4
13	336	14
14	192	8
15	576	24
16	24	1
17	432	18
18	312	13
19	480	20
20	144	6
127	3,072	128
128	24	1
65,536=2^16	24	1
65,537	1,572,912	65,538
2,147,483,647	51,539,607,552	2,147,483,648
2,147,483,648=2^31	24	1

What about 3D ?

Single Tetrahedron:

Tetrahedra can be used as Josephson Junctions.

A very useful reference is the 2003 dissertation of Christopher Bell at St. John's College Cambridge entitled "Nanoscale Josephson devices", on the web at http://www.dspace.cam.ac.uk/bitstream/1810/34607/1/chris_bell_thesis.pdf

Feigelman, Ioffe, Geshkenbein, Dayal, and Blatter in cond-mat/0407663 say: "... Superconducting tetrahedral quantum bits ...

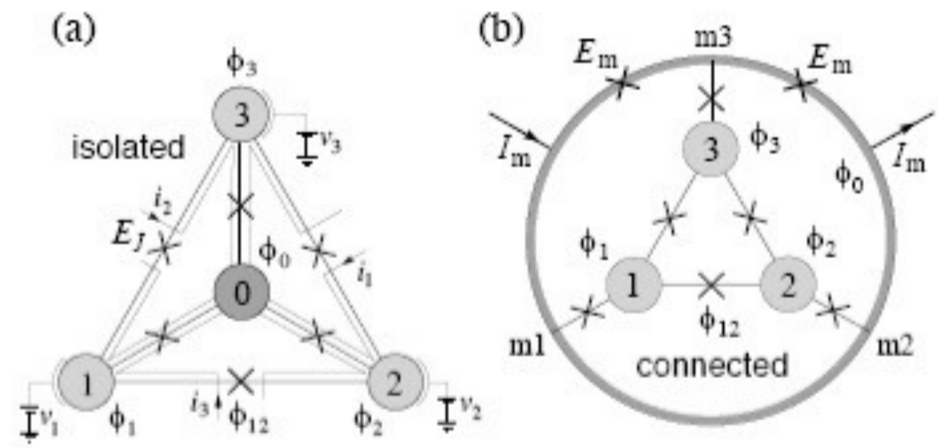


FIG. 1: (a) Tetrahedral superconducting qubit involving four islands and six junctions (with Josephson coupling E_J and charging energy E_C); all islands and junctions are assumed to be equal and arranged in a symmetric way. The islands are attributed phases ϕ_i, $i = 0, \ldots, 3$. The qubit is manipulated via bias voltages v_i and bias currents i_i. In order to measure the qubit's state it is convenient to invert the tetrahedron as shown in (b) — we refer to this version as the 'connected' tetrahedron with the inner dark-grey island in (a) transformed into the outer ring in (b). The measurement involves additional measurement junctions with couplings $E_m \gg E_J$ on the outer ring which are driven by external currents I_m (schematic, see Fig. 6 for details); the large coupling E_m effectively binds the ring segments into one island.

... The novel tetrahedral qubit design we propose below operates in the phase-dominated regime and exhibits two remarkable physical properties:

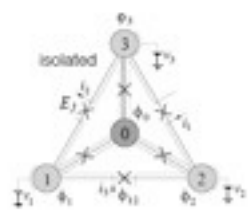
first, its non-Abelian symmetry group (the tetrahedral group Td) leads to the natural appearance of degenerate states and appropriate tuning of parameters provides us with a doubly degenerate groundstate. Our tetrahedral qubit then emulates a spin-1/2 system in a vanishing magnetic field, the ideal starting point for the construction of a qubit.
Manipulation of the tetrahedral qubit through external bias signals translates into application of magnetic fields on the spin;
the application of the bias to different elements of the tetrahedral qubit corresponds to rotated operations in spin space.
Furthermore, geometric quantum computation via Berry phases ... might be implemented through adiabatic change of external variables.
Going one step further, one may hope to make use of this type of systems in the future physical realization of non-Abelian anyons, thereby aiming at a new generation of topological devices ... which keep their protection even during operation ...

The second property we wish to exploit is geometric frustration:
In our tetrahedral qubit ... it appears in an extreme way by rendering the classical minimal states continuously degenerate along a line in parameter space. Semi-classical states then appear only through a fluctuation-induced potential, reminiscent of the Casimir effect ... and the concept of inducing
'order from disorder'...
The quantum-tunneling between these semi-classical states defines the operational energy scale of the qubit, which turns out to be unusually large due to the weakness of the fluctuation-induced potential. Hence the geometric frustration present in our tetrahedral qubit provides a natural boost for the quantum fluctuations without the stringent requirements on the smallness of the junction capacitances, thus avoiding the disadvantages of both the charge- and the phase- device:
The larger junctions reduce the demands on the fabrication process and the susceptibility to charge noise and mesoscopic effects, while the large operational energy scale due to the soft fluctuation-induced potential reduces the effects of flux noise. Both types of electromagnetic noise, charge- and flux noise, appear only in second order ...

in order to benefit from a protected degenerate ground state doublet, the qubit design requires a certain minimal complexity; it seems to us that the tetrahedron exhibits the minimal symmetry requirements necessary for this type of protection and thus the minimal complexity necessary for its implementation. ...".

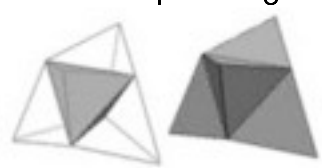
Multiple Tetrahedra - try to build in flat 3D just a 57G part of the 600-cell:

Eric A. Lord, Alan L. Mackay, and S. Ranganathan in their book “New Geometries for New Materials” (Cambridge 2006) said:
“... start ... from a single tetrahedron

[]

Place four spheres in contact.

Then place a sphere over each face of the tetrahedral cluster.
The centres and bonds then form a stella quadrangula



built from five regular tetrahedra ...[a total of 1+4 = 5 tetrahedra]...

Six more spheres [vertices] placed over the edges of the original tetrahedron form an octagonal shell. In terms of the network of centres and bonds we now have added 12 [= 2x6] more tetrahedra ...

There are now five tetrahedra around each edge of the original tetrahedron. ...

...[we now have 1+4+12 = 17 tetrahedra]...

[The 12 newly added tetrahedra]... are not quite regular ...

[i.e., nonzero Fuller unzipping angles appear as described by Thomas Banchoff in his book “Beyond the Third Dimension” (Scientific American Library 1990) where he said:
“... in three-space .... we can fit five tetrahedra around an edge ...

[image from Conway and Torquato PNAS 103 (2006) 10612-10617

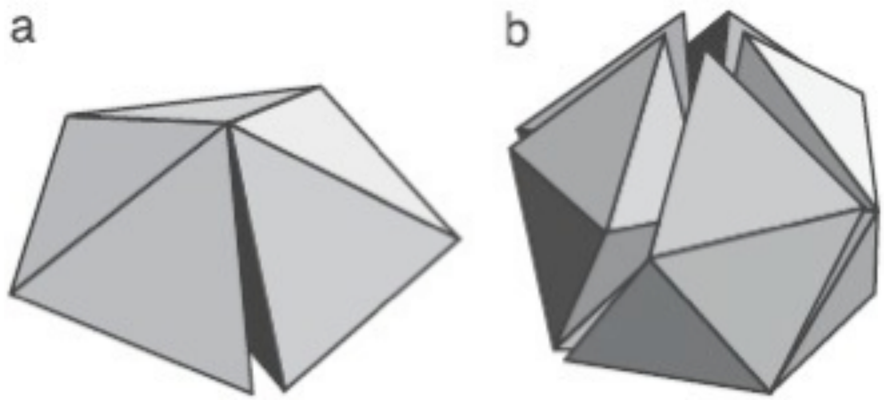


Fig. 1. Certain arrangements of tetrahedra. (*a*) Five regular tetrahedra about a shared edge. The angle of the gap is 7.36°. (*b*) Twenty regular tetrahedra about a shared vertex. The gaps amount to 1.54 steradians.]

... with a ... small amount of room to spare,
which allows folding into 4-space ...[where the fit can be made exact]...”.

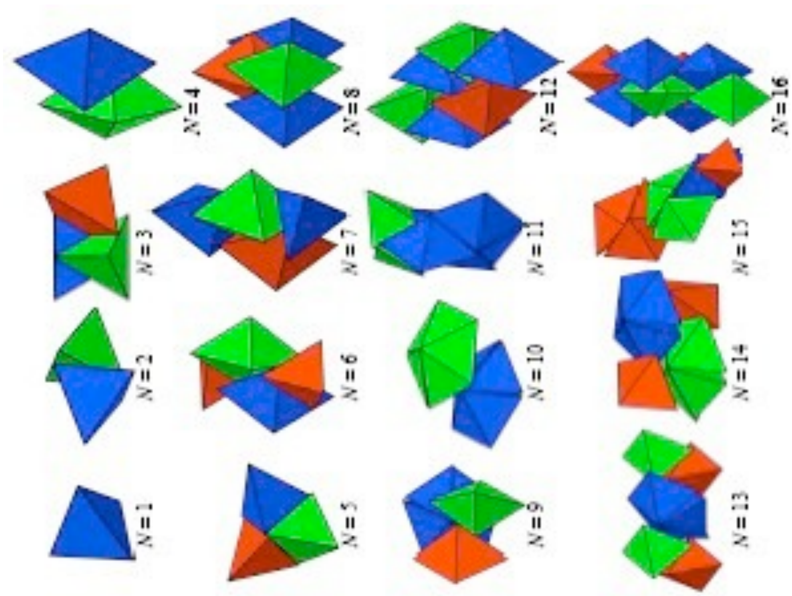
Add 4 half-Icosahedra (10 Tetrahedra each) to form a 40-Tetrahedron Outer Shell around the 17 Tetrahedra and so form a 57G

Like the 12 of 17, the Outer 40 do not exactly fit together in flat 3-dim space.

If you could force all 57 Tetrahedra to fit together exactly, you would be curving 3-dim space by a Dark Energy Conformal Transformation.

What happens if you require the 3-dim space to remain flat ?

If you construct with (exactly regular) tetrahedra in 3-dim space that remains flat that is like making a tetrahedral dense packing of flat 3-dim space.
The densest such packing now known is described by Chen, Engel, and Glotzer in arXiv 1001.0586 :
"... We present the densest known packing of regular tetrahedra with density Phi = 4000 / 4671 = 0.856347 ...



... The dimer structures are remarkable in the relative simplicity of the 4-tetrahedron unit cell as compared to the 82-tetrahedron unit cell of the quasicrystal approximant, whose density is only slightly less than that of the densest dimer packing.
The dodecagonal quasicrystal is the only ordered phase observed to form from random initial configurations of large collections of tetrahedra at moderate densities.
It is thus interesting to note that for some certain values of N, when the small systems do not form the dimer lattice packing, they instead prefer clusters (motifs) present in the quasicrystal and its approximant, predominantly pentagonal dipyramids. This suggests that the two types of packings - the dimer crystal and the quasicrystal/ approximant - may compete, raising interesting questions about
the relative stability of the two very different structures at finite pressure. ...".

If you regard a Tetrahedron as a pair of Binary Dipoles

then the Chen - Engel - Glotzer high (0.85+) density configurations have the same 8-periodicity property as the Real Clifford Algebras:

#Binary Dipoles M	Maximum Density Numerical, $\hat{\phi}$	Analytical, ϕ	Success Rate	Motifs, Structural Description
2	0.367346	18/49	100%	1 monomer [11]
4	0.719486	ϕ_2	100%	2 monomers, transitive [22]
6	0.666665	2/3	21%	3 monomers, three-fold symmetric
8	0.856347	4000/4671	80%	2 dimers (positive + negative)
10	0.748096	ϕ_5	22%	1 pentamer, asymmetric
12	0.764058	ϕ_6	11%	2 dimers + 2 monomers
14	0.749304	3500/4671	15%	2×2 dimers minus 1 monomer
16	0.856347	4000/4671	44%	2×2 dimers, identical to $N = 4$
18	0.766081		—	1 pentagonal dipyramid + 2 dimers
20	0.829282	ϕ_{10}	2%	2 pentagonal dipyramids
22	0.794604		—	1 nonamer + 2 monomers
24	0.856347	4000/4671	3%	3×2 dimers, identical to $N = 4$
26	0.788728		4%	1 pentagonal dipyramid + 4 dimers
28	0.816834		3%	2 pentagonal dipyramids + 2 dimers
30	0.788693		—	Disordered, non-optimal
32	0.856342	4000/4671	< 1%	4×2 dimers, identical to $N = 4$
⋮	⋮			⋮
164x8	0.850267			Quasicrystal approximant [21]

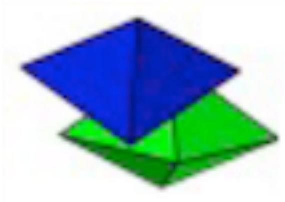
The Binary Pair of one Tetrahedron corresponds to the Cl(2) Real Clifford Algebra, isomorphic to the Quaternions, with graded strucure 1+2+1.
The 4 Binary Pairs of 4 Tetrahedra (2 Dimers) correspond to Cl(2x4) = Cl(8).

The Large N Limit of 4N Tetra Clusters =

= **Completion of Union of All 4N Tetra Clusters** would correspond to

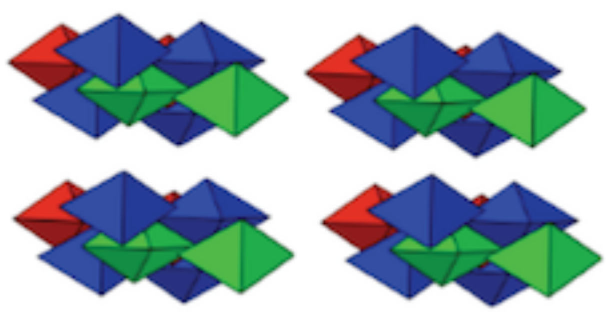
the same generalized Hyperfinite II1 von Neumann factor of Cl(16)-E8 Physics that gives a natural Algebraic Quantum Field Theory structure.

Geometrically:



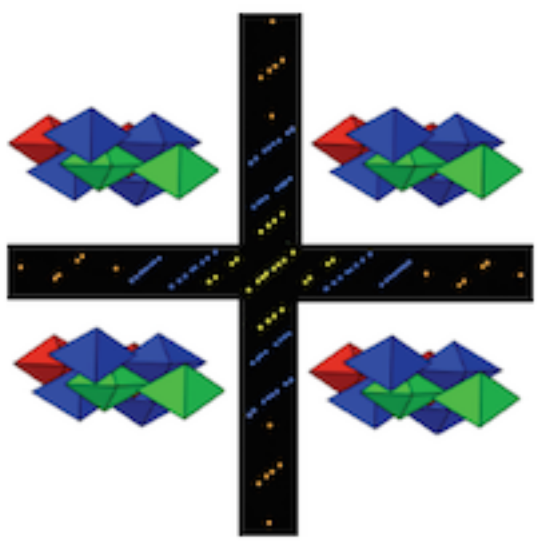
represents Cl(8) Clifford Algebra Vectors and Half-Spinors

represents Cl(16) Vectors



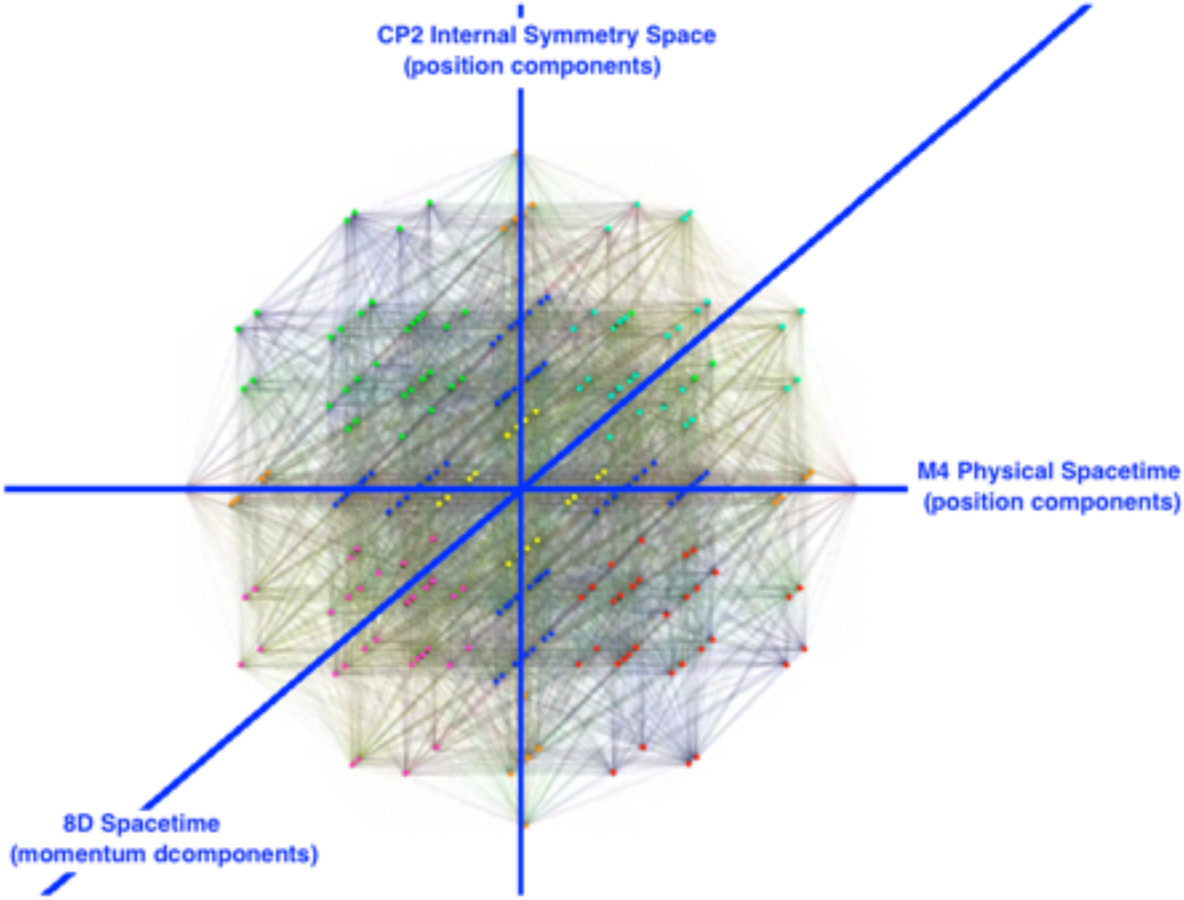
represents Cl(16) half-spinors

E8 = Cl(16) half-spinors + Cl(16) BiVectors



where the axes (central cross) corresponds to Cl(16) = Cl(8) x Cl(8) tensor product:
Cl(8) Vectors x Cl(8) Vectors (blue dots)
Cl(8) BiVectors x Cl(8) Scalar (yellow dots)
Cl(8) Scalar x Cl(8) Bivectors (orange dots)

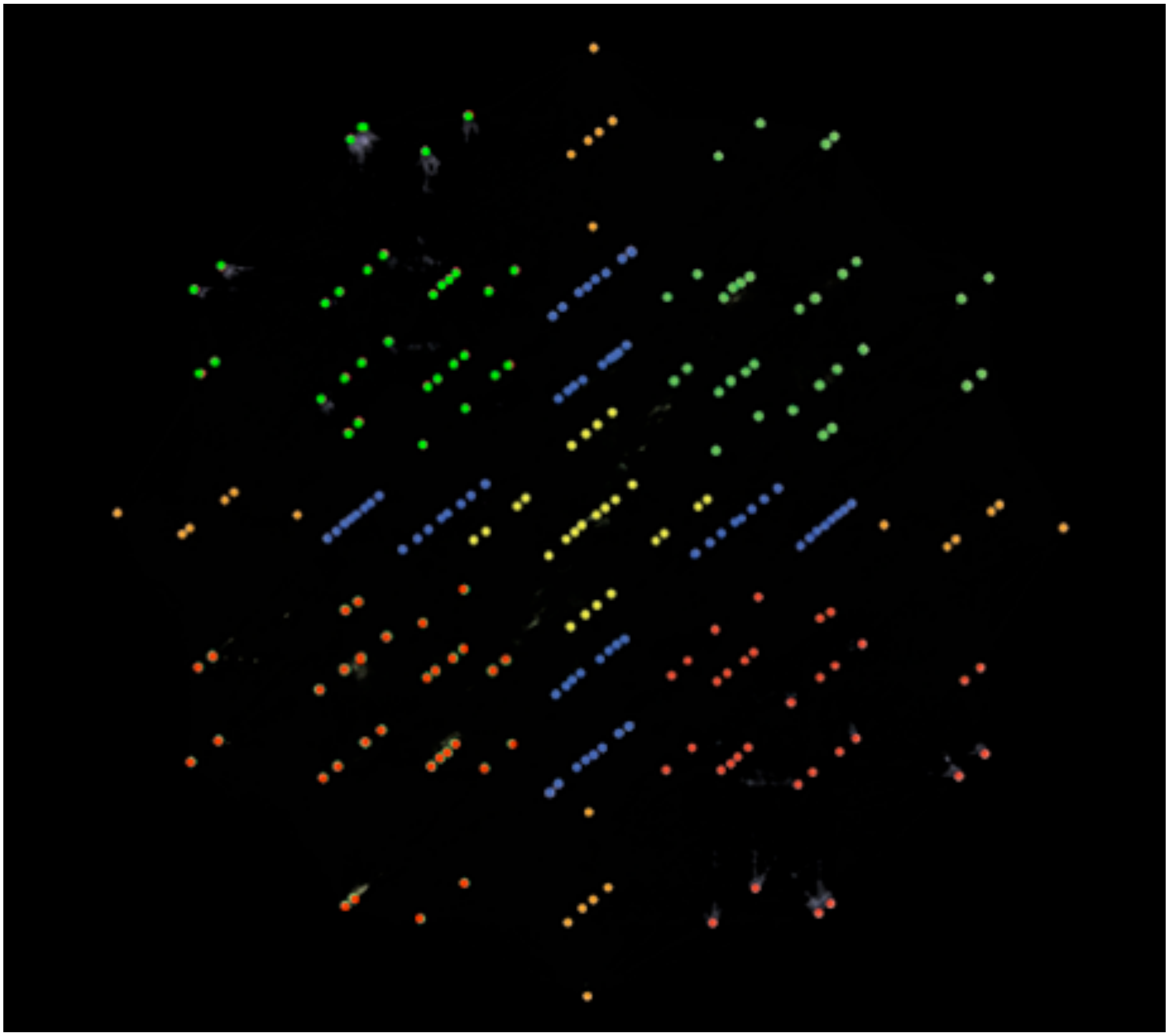
so
Extension of Local Lagrangian E8 Root Vector Physics over large Spacetime regions can be described in terms of Flat 2D and 3D projections of E8 Root Vectors by tiling 3D Space using Regular Cubic Tiling



can be extended to fully and exactly tile 3D space
with
the two axes representing the two parts of (4+4)D M4xCP2 Kaluza-Klein Spacetime
and the third axis representing the momentum components of 8D Spacetime
and
Gauge Bosons of Gravity and the Standard Model living on the two K-K position axes
and
Fermion Particles and AntiParticles living in 2^3 = 8 off-axis octants at each vertex.

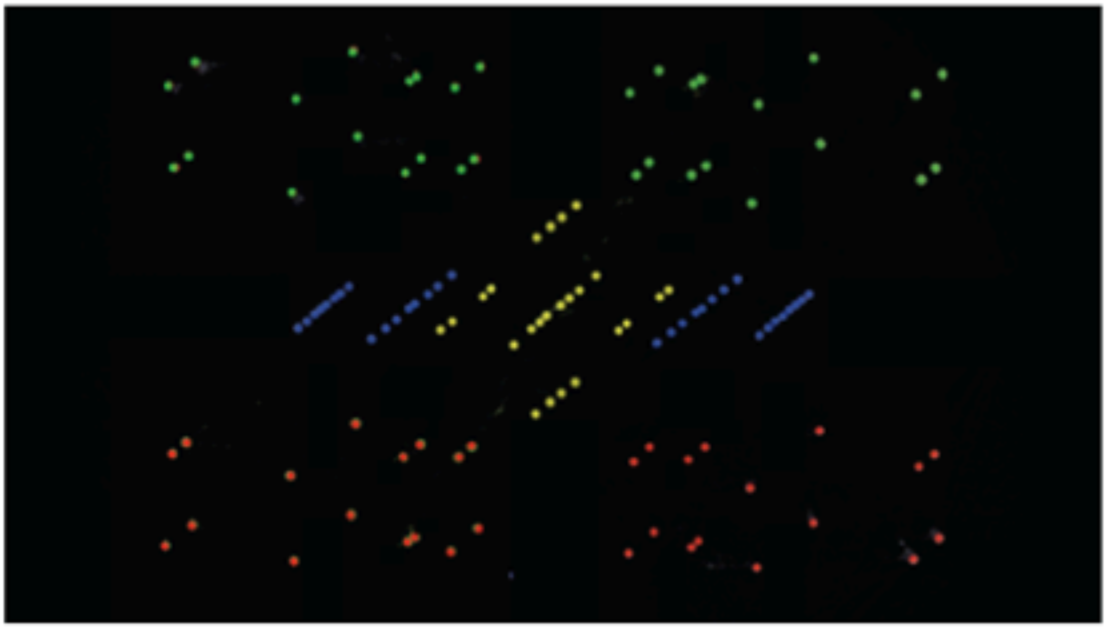
The physical interpretations of the 240 E8 Root Vectors are clear:
green / cyan and red / magenta for fermion particles and antiparticles (E8 / D8)
blue for M4 x CP2 Kaluza-Klein SpaceTime (D8 / D4xD4)
yellow for Gravity + Dark Energy (one of the D4)
orange for Standard Model SU(3) (the other D4)

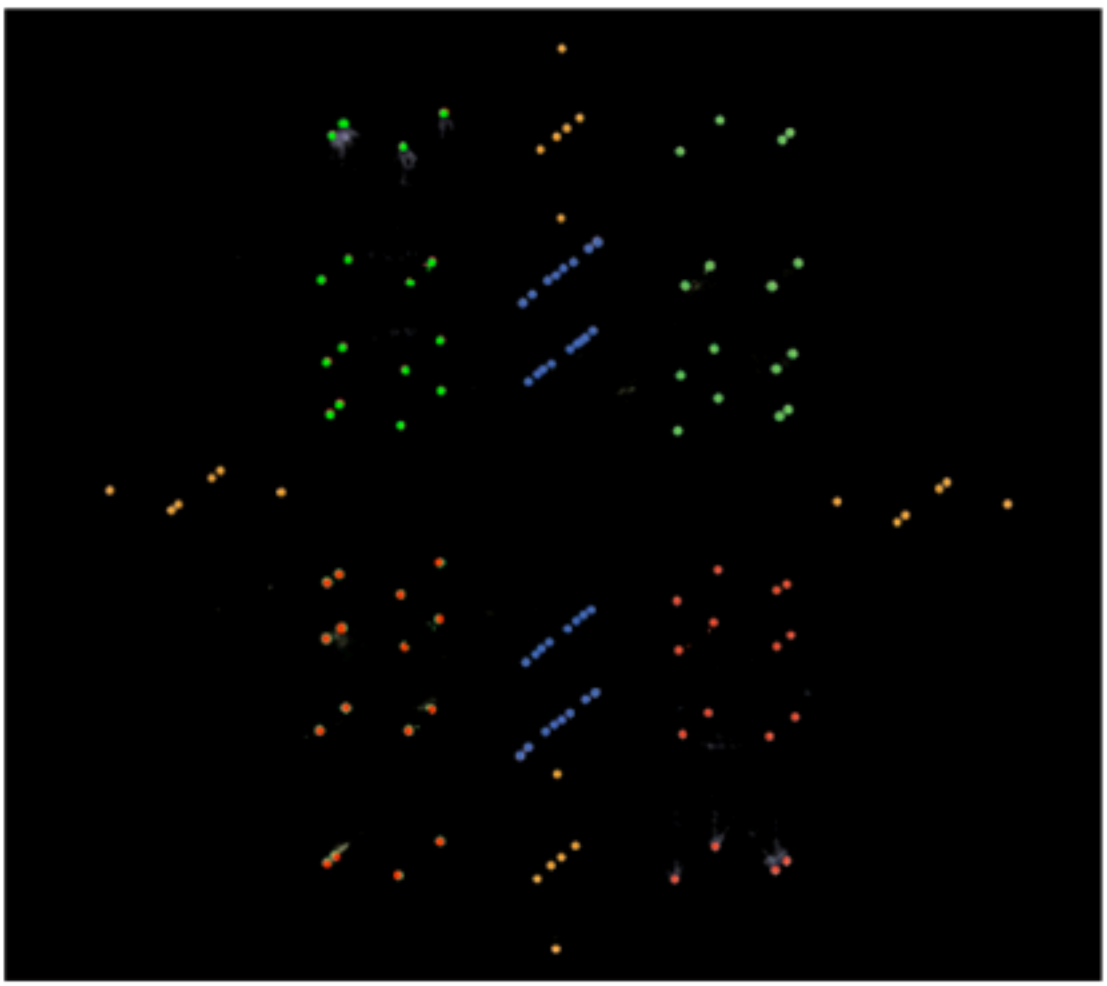
In this Cube-type 3D projection the 240 Root Vectors of E8



can be seen as

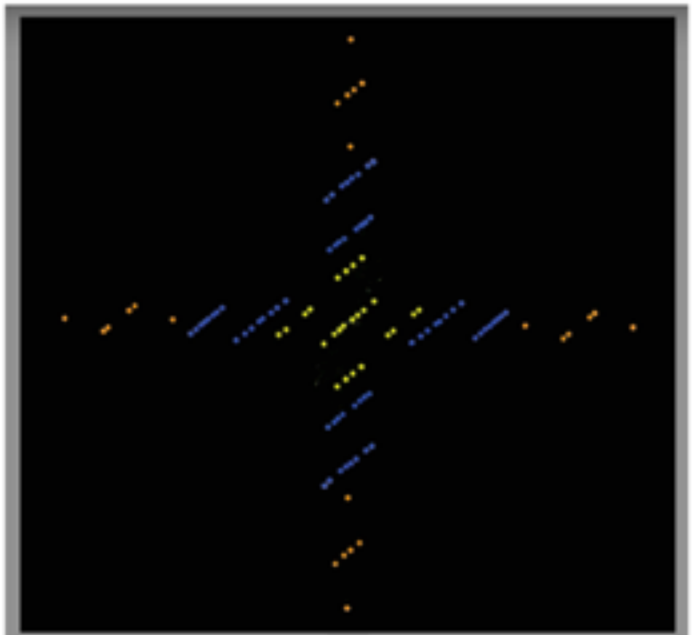
120 vertices of the 600-cell containing the D4 of Conformal Gravity and M4
and
120 vertices of the 600-cell containing the D4 of the Standard Model and CP2



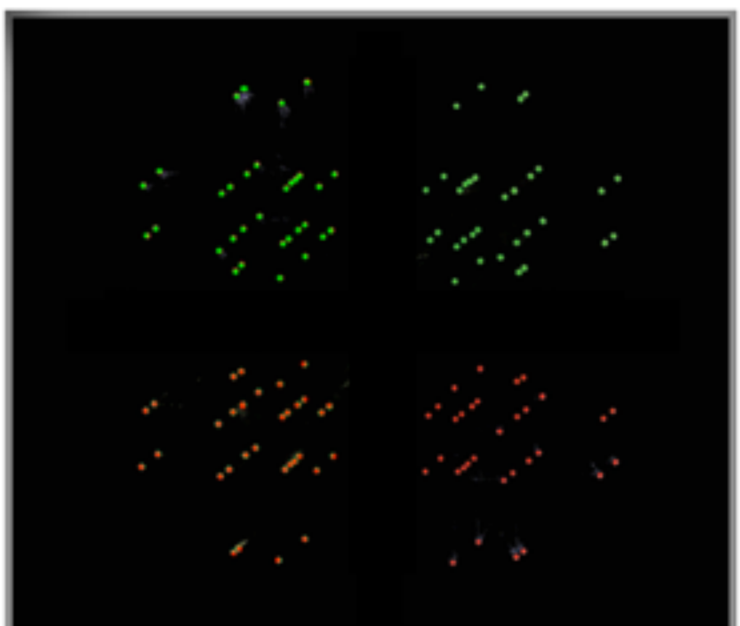


or, since D8 = Cl(16) bivectors,

240 E8 Root Vectors = 112 D8 Root Vectors + 128 Cl(16) half-spinors
E8 Lattice = D8 Lattice + ([1] + D8 Lattice)
where the lattice shifting glue vector [1] = (1/2, ... , 1/2)



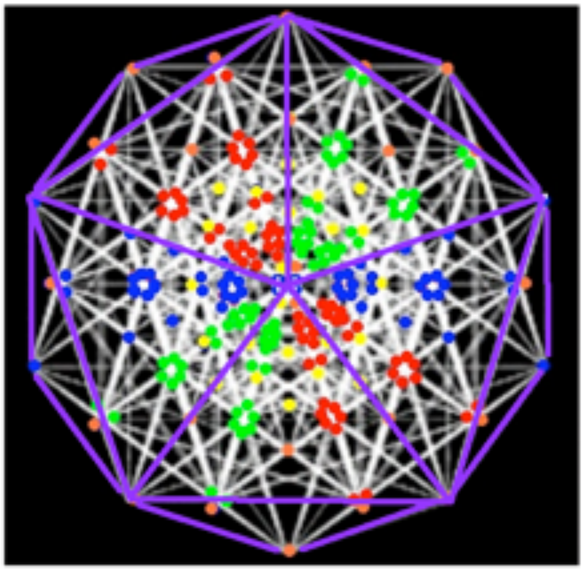
D8

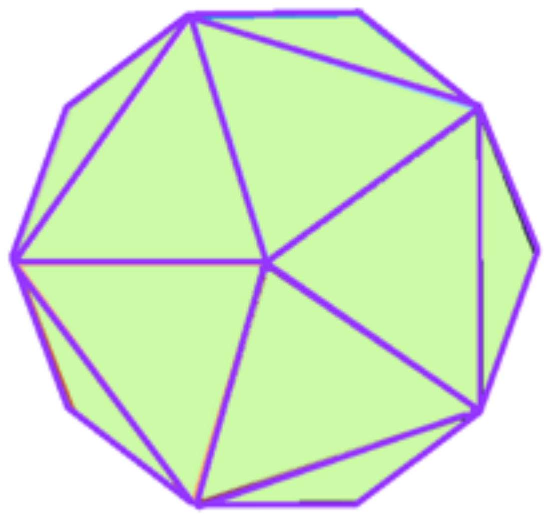


E8 / D8

What about the QuasiCrystal / approximant in flat 3-dim space ?

Icosahedral 3D Projection





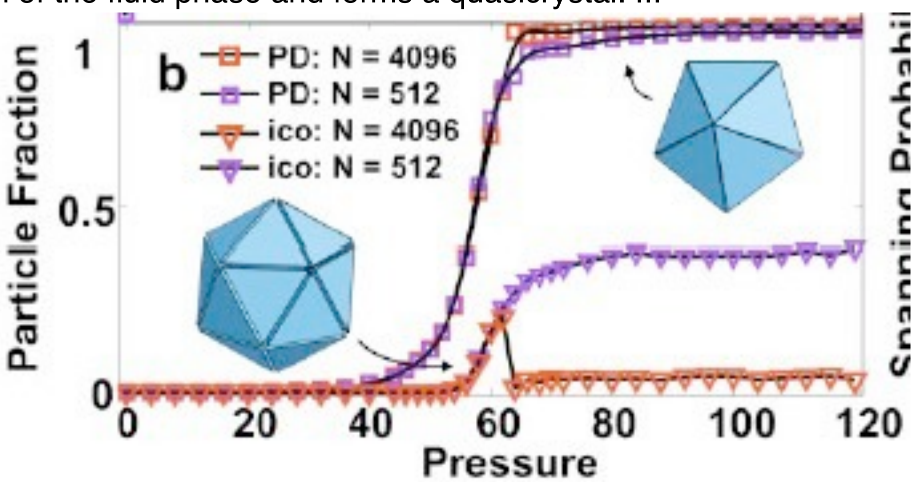
does NOT produce a Regular Tiling
but
gives an Icosahedron (purple) that can produce 3D Quasi-Crystals
in which
the root vector correspondence to E8 Physics
is not as easy to see as with the Cube-type 3D projection.

Also, if you make an Icosahedral Quasi-Crystal,
some of the Quasi-Crystal vertices will not be at the center of a full Icosahedron
so some E8 Physics information will be lost from the Quasi-Crystal.

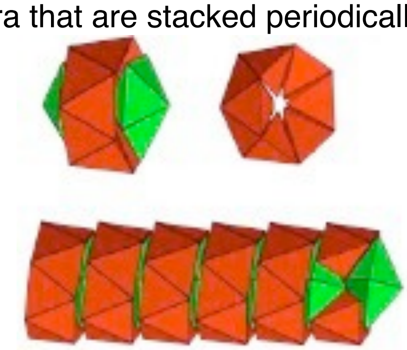
Haji-Akbari1, Engel, Keys, Zheng, Petschek, Palffy-Muhoray, and Glotzer in arXiv 1012.5138 say: "... a fluid of hard tetrahedra undergoes a first-order phase transition to a dodecagonal quasicrystal,
which can be compressed to a packing fraction of $\phi = 0.8324$. By compressing a crystalline approximant of the quasicrystal, the highest packing fraction we obtain is $\phi = 0.8503$.

...

To obtain dense packings of hard regular tetrahedra, we carry out Monte-Carlo (MC) simulations ... of a small system with 512 tetrahedra and a large system with 4096 tetrahedra. ... The large system undergoes a first order transition on compression of the fluid phase and forms a quasicrystal. ...

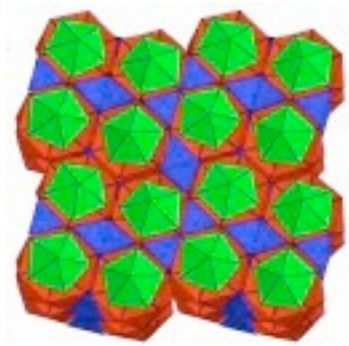


... the quasicrystal consists of a periodic stack of corrugated layers ... Recurring motifs are rings of twelve tetrahedra that are stacked periodically to form "logs"...

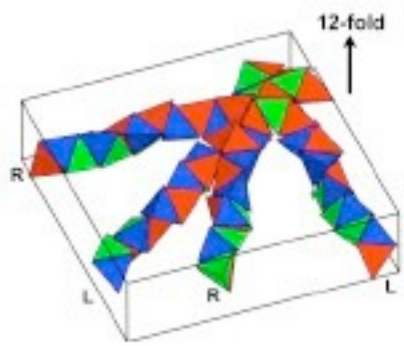


... Perfect quasicrystals are aperiodic while extending to infinity; they therefore cannot be realized in experiments or simulations, which are, by necessity, finite. ... Quasicrystal approximants are periodic crystals with local tiling structure identical to that in the quasicrystal. Since they are closely related, and they are often observed in experiments, we consider them as candidates for dense packings.

The dodecagonal approximant with the smallest unit cell (space group) has 82 tetrahedra ...



... At each vertex we see the logs of twelve-member rings (shown in red) capped by single PDs (green). The logs pack well into squares and triangles with additional, intermediary tetrahedra (blue). The vertex configuration of the tiling is ...



...".

The QuasiCrystal approximant is not as dense as the 4N Tetra Cluster packing, so I do not regard it as being as useful for fundamental physics as the 4N Tetra packing.

The true QuasiCrystal is less dense than the QuasiCrystal approximant, so I regard it as being less useful for fundamental physics. However,
as Sadoc and Mosseri say in their book "Geometrical Frustration" (Cambridge 2005)
"... quasiperiodic structures [can be] derived from the eight-dimensional lattice E8. ...
... using the cut and project method, it is possible to generate
a four-dimensional quasicrystal having the symmetry of the [600-cell] polytope {3,3,5} ... a shell-by-shell analysis ...

Table A9.1. *Number of vertices on shells surrounding the origin in the E8 lattice. The first shell is a Gosset polytope in eight dimensions*

N	Squared radius r^2	Vertices on $E8$ shell
1	1/2	240
2	1	2160
3	3/2	6720

... recalls in some respects ... the Fibonacci chain ...

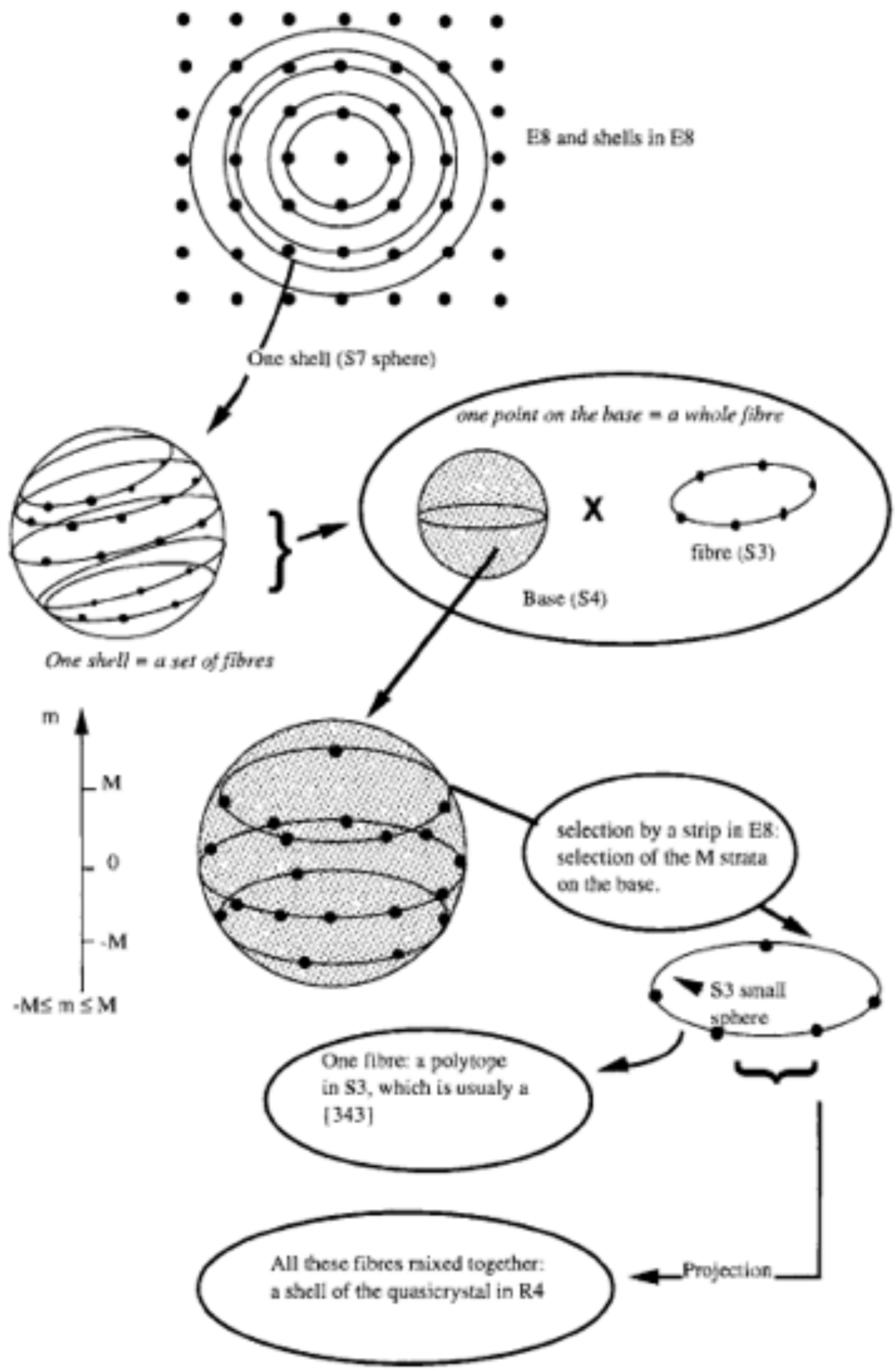


Fig. A9.1. Scheme summarizing the four-dimensional construction method: take an $E8$ shell, considered as a discrete fibration of S^7, select the fibres which map (H-map) onto a stratum M of the base of the fibration, and finally orthogonally map (O-map) the selected sites onto R^4.

The relationship between QuasiCrystals and QuasiCrystal approximants is discussed by An Pang Tsai in an IOP review “Icosahedral clusters, icosahedral order and stability of quasicrystals - a view of metallurgy”:
“... we overview the stability of quasicrystals ... in relation to phason disorder ...
the phonon variable leads to long wavelength and low energy distortion of crystals, the phason variable in quasicrystals leads to a ... type of distortion ...
Let a two-dimensional lattice points sit at the corners of squares in a grid.
... a strip with a slope of an irrational number ... golden mean ... is ... a Fibonacci sequence and is exactly a one-dimensional quasicrystal ...
... [if] the slope of the strip is ... a rational number ...[it]... is a periodic sequence ... [and]... is called an approximant ...
in the approximant where the sequence changes by a flip ... This flip is called phason flip ... a flipping of tiles in two-dimensions or three-dimensions ...

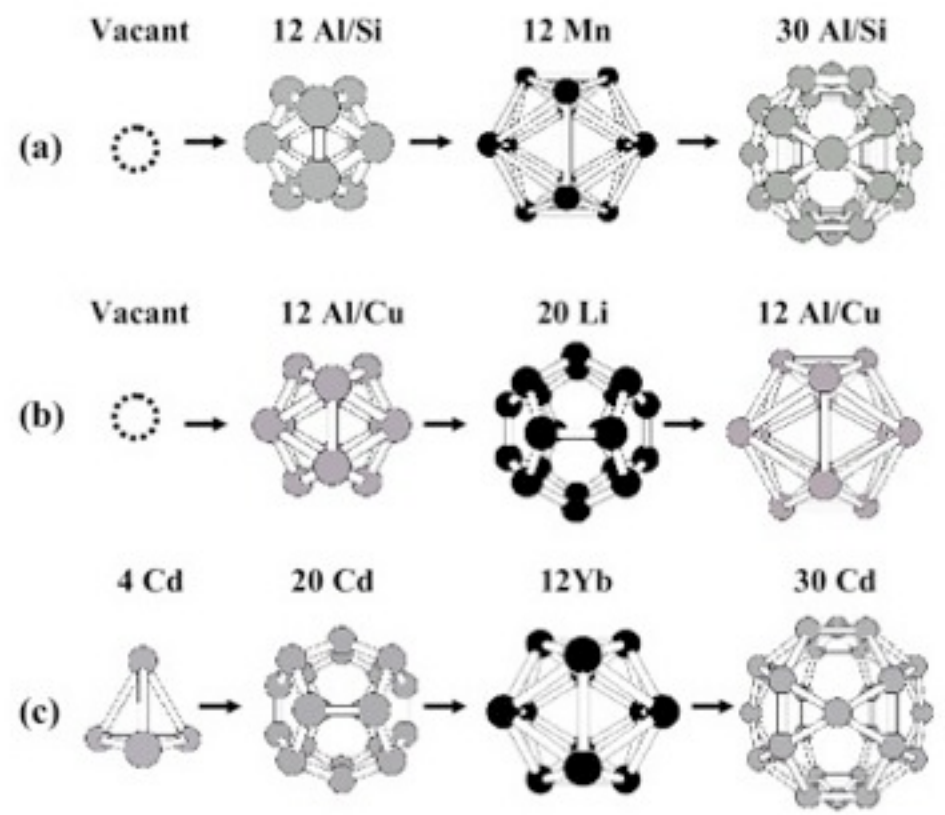


Figure 3. Concentric structures of three types of icosahedral clusters derived from three 1/1 approximants of quasicrystals. (a) The Al–Mn–Si class or Mackay icosahedral cluster: the center is vacant, the 1st shell is an Al/Si icosahedron, the 2nd shell is a Mn icosahedron, and the 3rd shell is an Al/Si icosidodecahedron. (b) The Zn–Mg–Al class or Bergman cluster: an example is R-AlLiCu: the center is vacant, the 1st shell is an Al/Cu icosahedron, the 2nd shell is a Li dodecahedron, the 3rd shell is a larger Al/Cu icosahedron. (c) The Cd–Yb class: the center is a Cd tetrahedron, the 1st shell is a Cd dodecahedron, the 2nd shell is a Yb icosahedron, and the 3rd shell is a Cd icosidodecahedron.

... ‘phason strain’ ... is the characteristic disorder for quasicrystals
but does not exist in crystals ...
a fully annealed stable iQc [icosahedral quasicrystal]...
is almost free of phason disorder ...”.

High Energy Particle Physics

E8 Root Vectors - Physical Interpretation

Authors: Frank Dodd Tony Smith Jr

Physical interpretations of the 240 Root Vectors of E8 are given by Garrett Lisi (arXiv 1506.08073) and by Tony Smith (viXra 1405.0030) using, respectively, an 8-circle x 30 vertex projection to 2-dim and a square/cube vertex projection to 2-dim. This paper compares the two interpretations and their projections and gives a graphical view of the physical interpretations of viXra 1405.0030 using the square/cube projection.

E8 Root Vectors - Physical Interpretation

Frank Dodd (Tony) Smith, Jr. - 2015 - viXra 1508.0065

Garrett LIsi has proposed (arXiv 1506.08073) that the 240 Root Vectors of E8 have a Physical Interpretation, sayng "... Spacetime is ... part of the Lie group ... When all fields and particles of General Relativity and the Standard Model, including three generations of fermions, are described ... as excitations of the ... Lie group E8(-24), having 248 dimensions ...

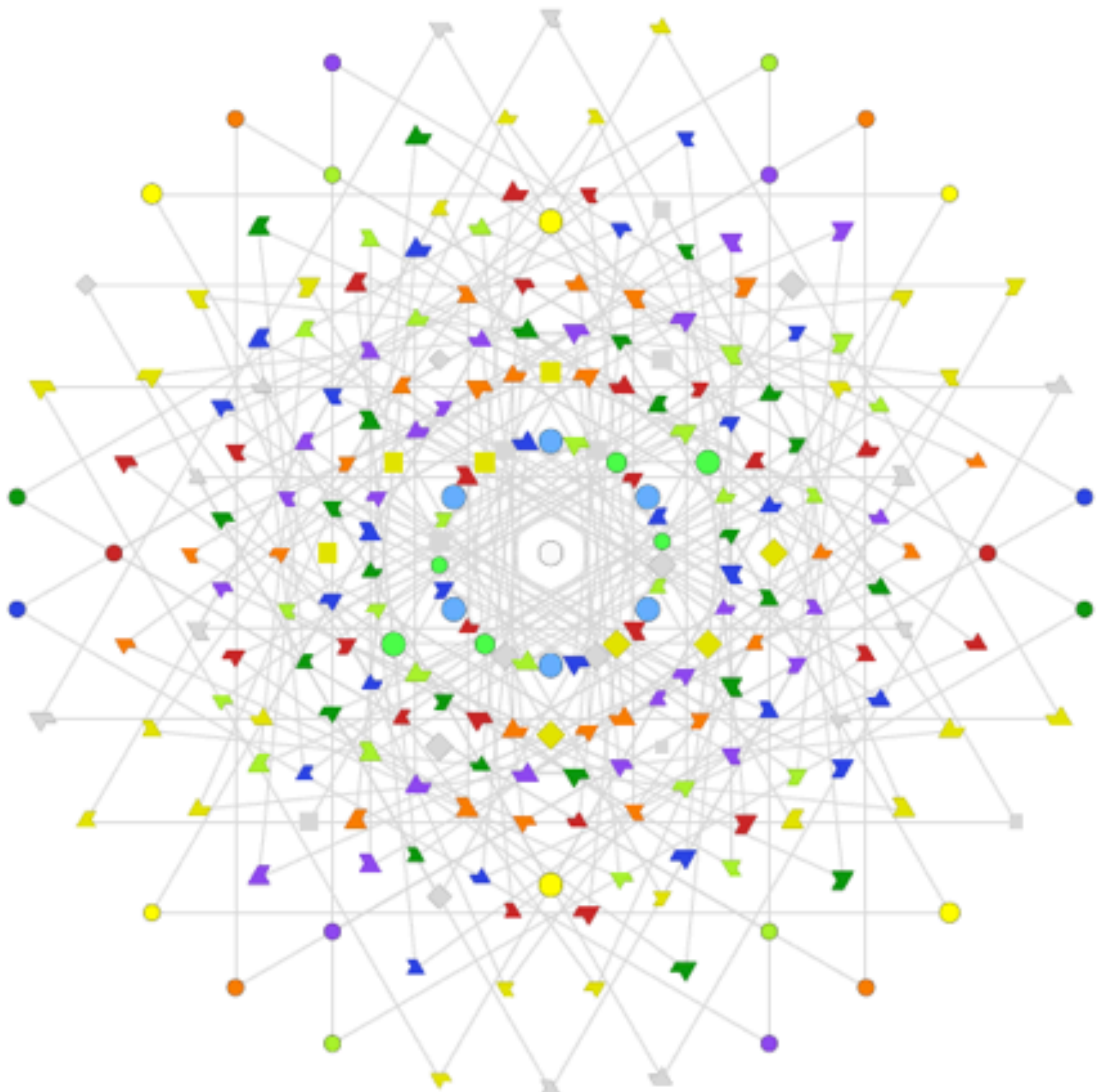
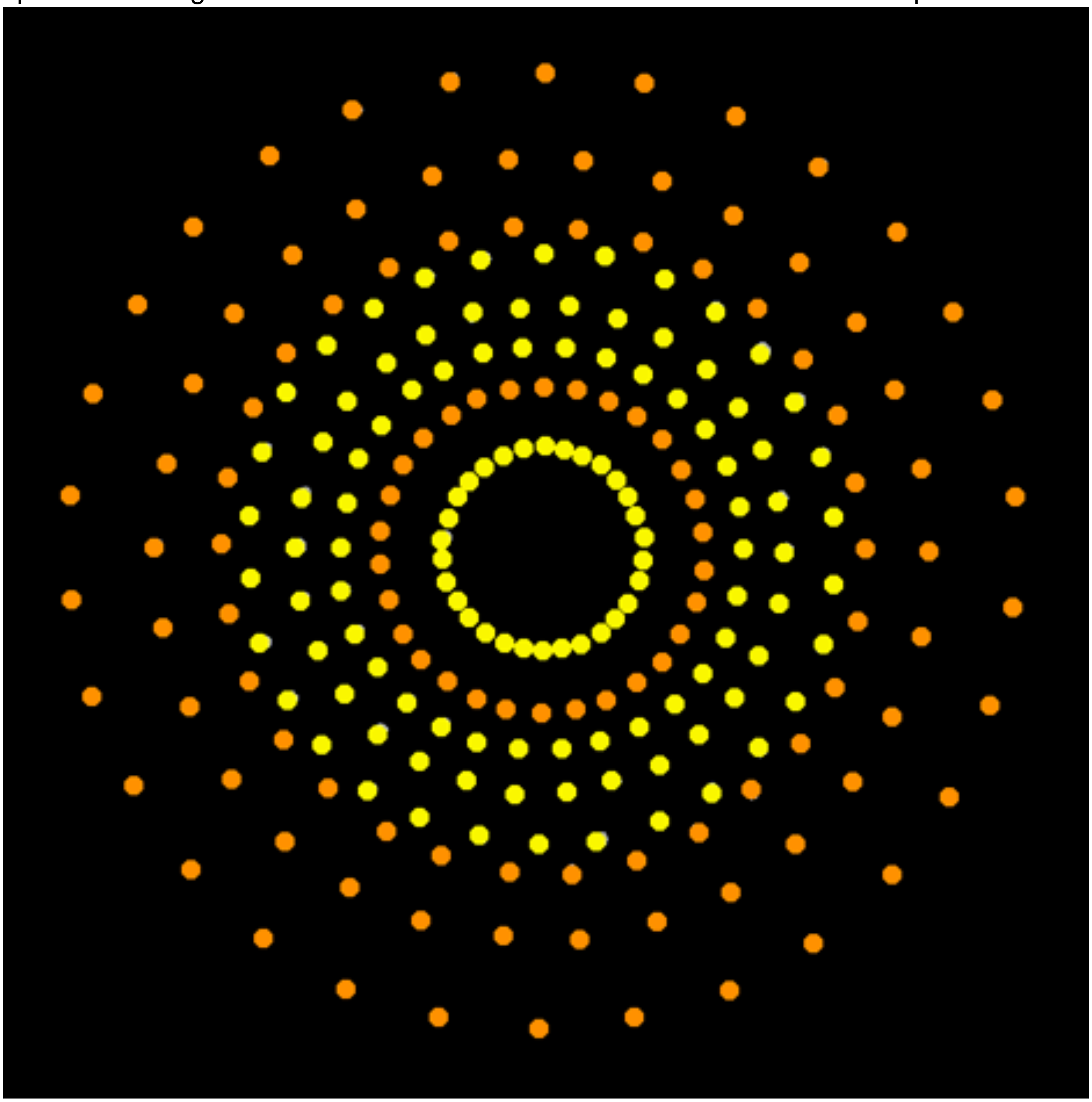


Figure 1. The E_8 root system, with three generations of particles related by triality. These particle states are meant to be suggestive rather than definitive. The detailed assignments of elementary particle states to E_8 roots, views of other rotations, and other unification models, are available at the Elementary Particle Explorer: http://deferentialgeometry.org/epe/

... there is one new, colored gauge boson and its antiparticle ...".
Garrett Lisi represents the 240 Root Vectors of E8 as projected from 8-dim to 2-dim in the form of 8 concentric circles of 30 Root Vectors each, and attempts to interpret the 240 Root Vectors as directly representing all 3 generations of Fermions.

I prefer a different interpretation of the 240 Root Vectors, based on 8-dim Octonionic spacetime being seen as 4+4 -dim Quaternionic M4 x CP2 Kaluza-Klein Spacetime:

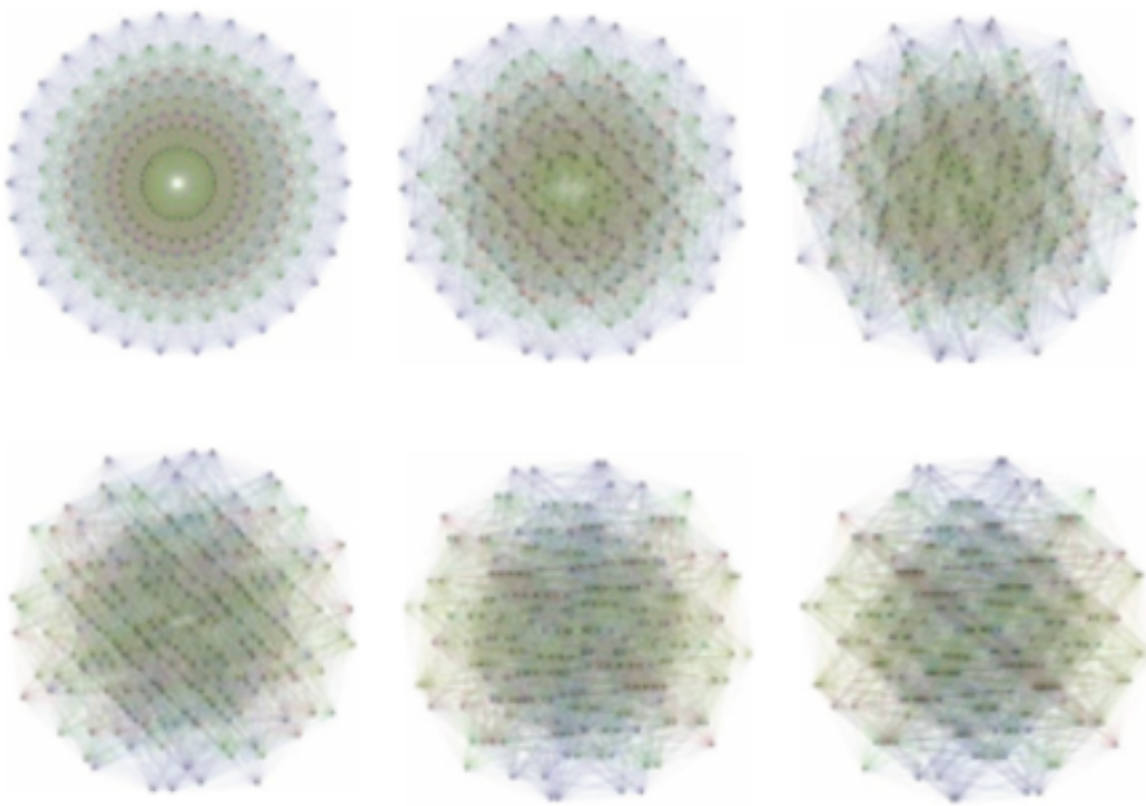


120 of the 240 (yellow dots) represent aspects of First-Generation Fermions, Gauge Bosons and Ghosts, and Position and Momentum related to M4 Physical Spacetime.
120 of the 240 (orange dots) represent aspects of First-Generation Fermions, Gauge Bosons and Ghosts, and Position and Momentum related to CP2 = SU(3) / SU(2)xU(1) Internal Symmetry Space. (for details see viXra 1405.0030)

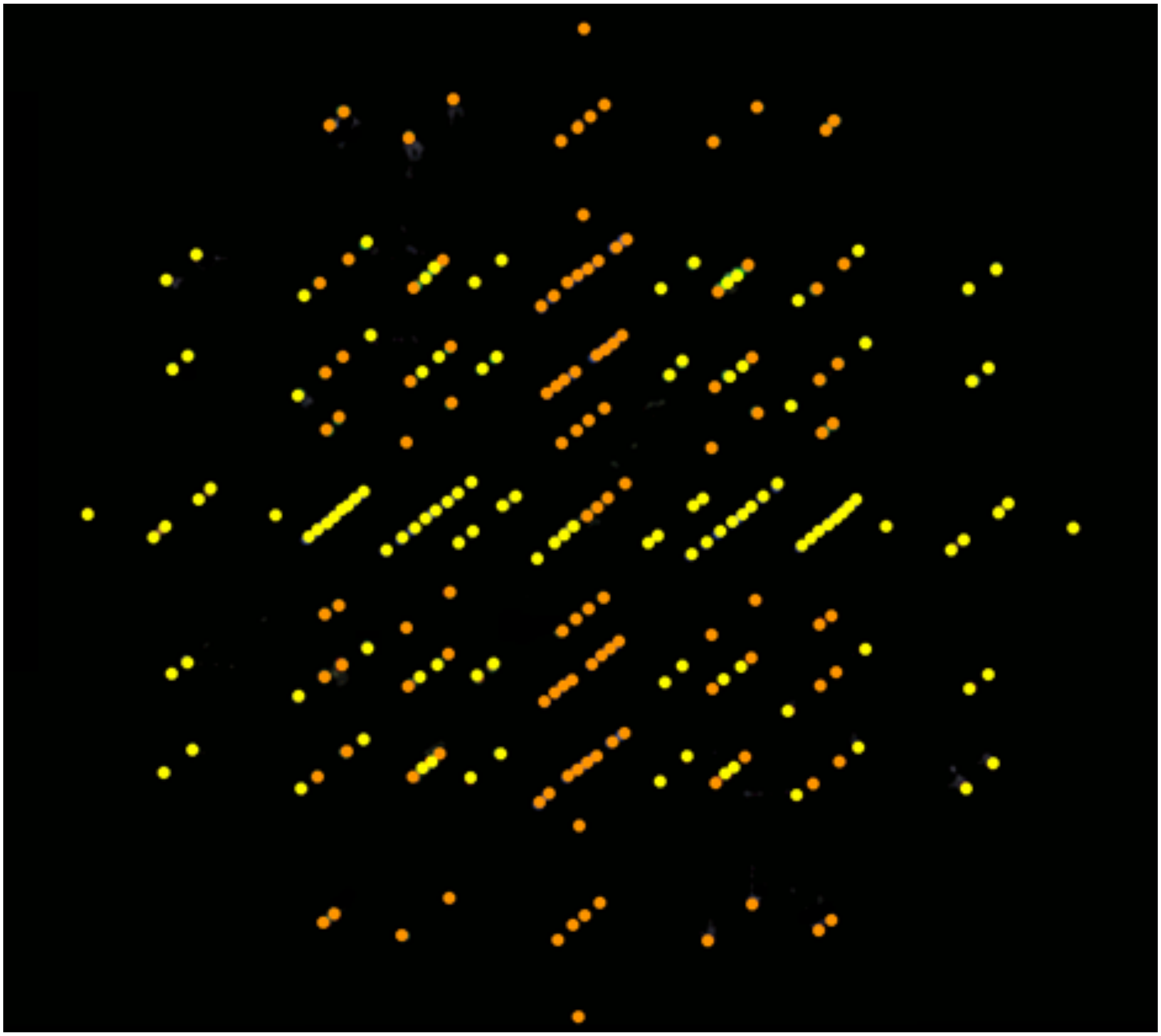
The 120 Root Vectors (yellow dots) related to M4 Physical Spacetime form 4 circles of 30 Root Vectors each, corresponding 120 vertices of a Quaternionic 600-cell.

The 120 Root Vectors (orange dots) related to CP2 Internal Symmetry Space form the remaining 4 circles of 30 Root Vectors each, corresponding 120 vertices of a second Quaternionic 600-cell whose radii are larger than those of the 4 M4 circles by the Golden Ratio (1 + sqrt(5)) / 2.

Garrett LIsi (around 2007) produced a video from mathematica code that shows a transformation from the 2-dim projection 8 Circles of 30 Root Vectors
to another 2-dim projection with Square Geometry related to Cube Geometry.
Here is a sequence of images from that video:



Here is a Square/Cube Geometry version of the 4 + 4 circles of 120 + 120 Root Vectors:



At this stage, it looks more confusing and complicated than the 4+4 Circle Geometry,

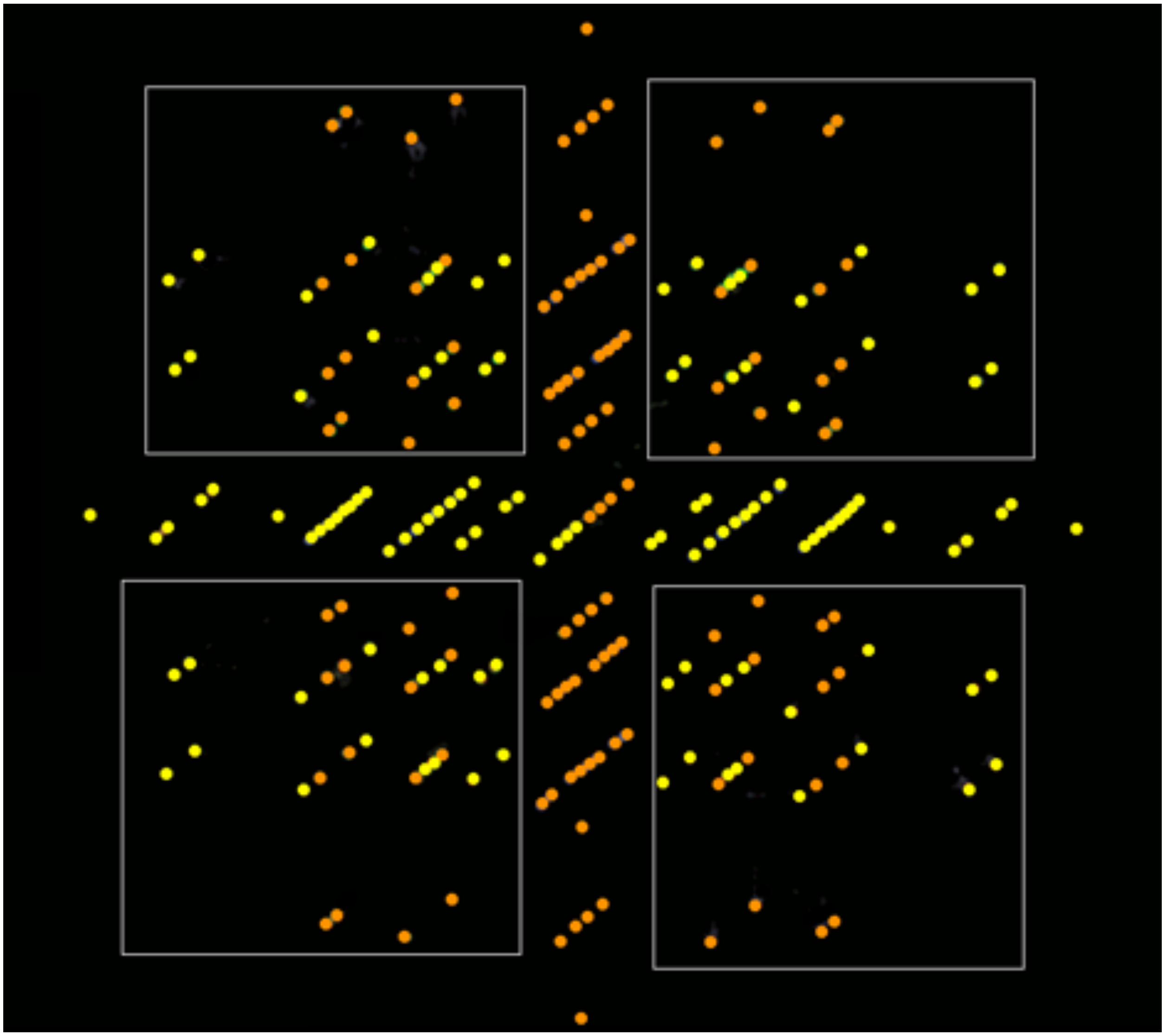
but

if you separate

the Horizontal Axis and Vertical Axis Root Vectors (56 + 56 = 112 of them)

from

the 4 Off-Axis Quadrant Root Vectors (4 x 32 = 128 of them)



you see that the 112 Horizontal Axis and Vertical Axis Root Vectors represent
the Root Vectors of a 120-dim D8 subalgebra of the 248-dim E8 Lie algebra
and
the 128 Off-Axis Quadrant Root Vectors represent the Symmetric Space

E8 / D8 = (OxO)P2 representing

64 = 8 Octonionic Components of 8 First-Generation Fermion Particles

and

64 = 8 Octonionic Components of 8 First-Generation Fermion Anti-Particles

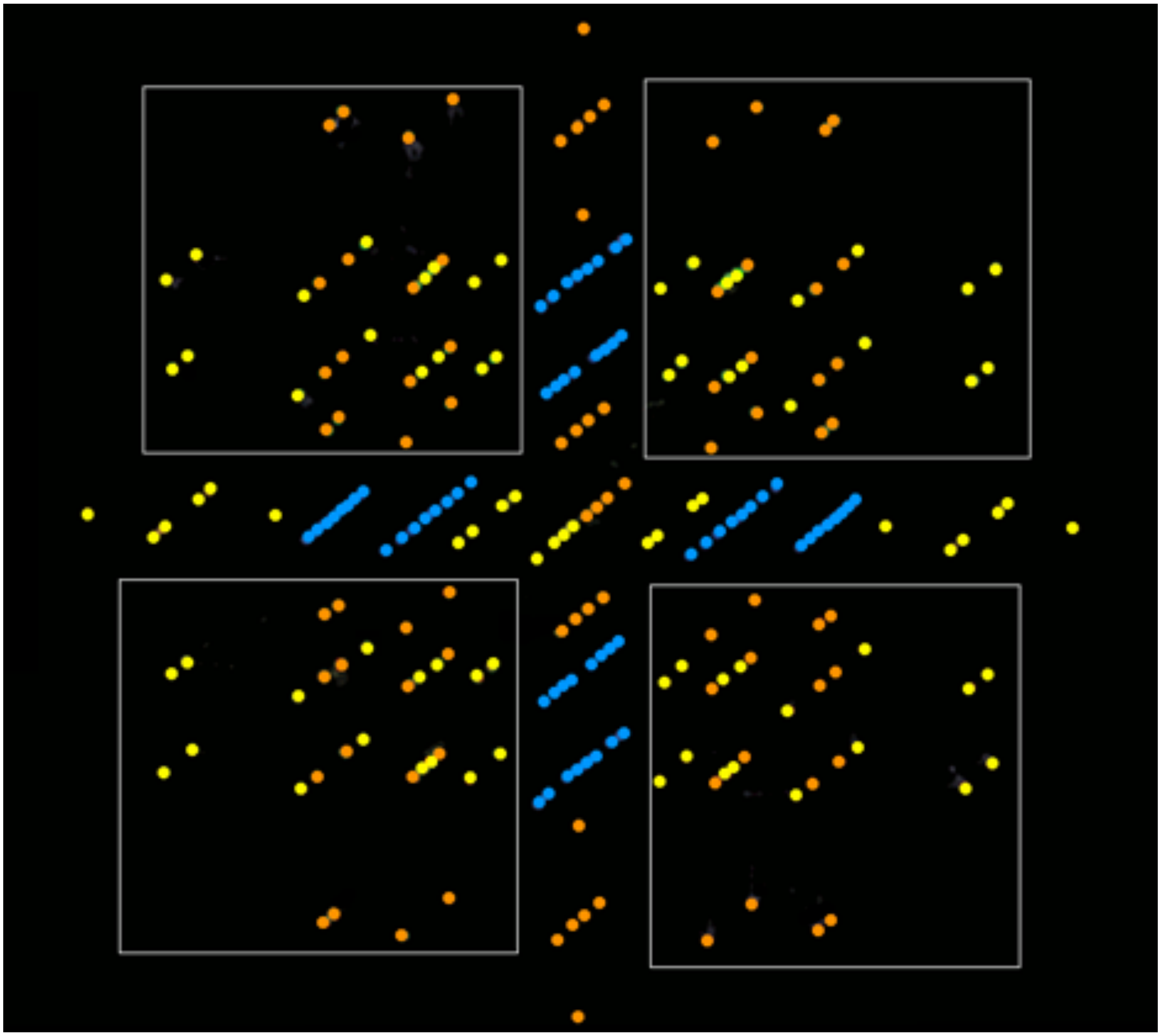
D8 contains a 28+28 = 56-dim D4xD4 subalgebra and the Symmetric Space

D8 / D4 x D4 = Gr(8,16) = 64-dim Octonionic Subspaces of R16

(Gr = Grassmanian and R16 = Vectors of Clifford Cl(16) Matrix Algebra for D8)

(8-dim Octonionic spacetime => Quaternionic 4+4 Kaluza-Klein M4 x CP2 spacetime
which symmetry breaking produces second and third generation fermions and Higgs)

The 64 (blue dots) represent 8 Position x 8 Momentum of Octonionic Spacetime



The 56-dim D4xD4 on the Horizontal Axis (orange dots) and Vertical Axis (yellow dots) represents Gauge Bosons and Ghosts.

Each 28-dim D4 is represented by 24 Root Vectors + 4 E8 Cartan Subalgebra Elements

The yellow dot D4 represents Confromal Gravity + Dark Energy

D4 / D3 x U(1) = 12 Standard Model Gauge Boson Ghosts (8 Root Vector + 4 Cartan)

U(1) = 1 Cartan Element

D3 = A3 = Spin(2,4) = SU(2,2) Conformal Gravity + Dark Energy with 12 Root Vectors and 3 Cartan Elements

The orange dot D4 represents Standard Model Gauge Bosons

D4 / A3 x U(1) = 12 Gravity+DE Root Vector Ghosts

U(1) = 1 Gravity+DE Cartan Ghost

A3 / A2 x U(1) = 6 = 4 SU(2) x U(1) Gauge Bosons + 2 Gravity+DE Cartan Ghosts

U(1) = 1 Gravity+DE Cartan Ghost

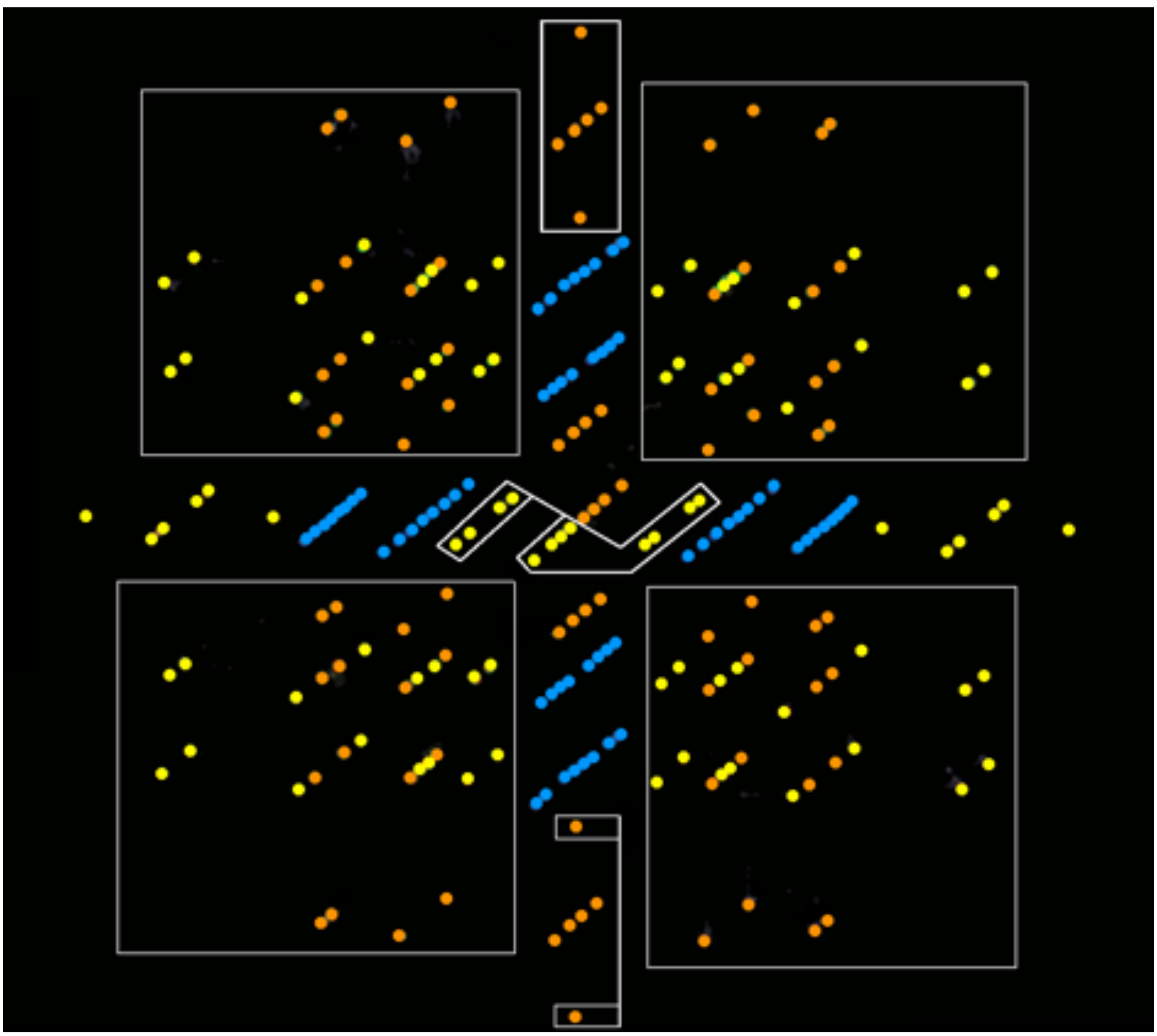
A2 = 8 SU(3) Color Gauge Bosons with 6 Root Vectors and 2 Cartan Elements

SU(2) = 3 Weak Gauge Bosons with 2 Root Vectors and 1 Cartan Element

U(1) = Photon with 1 Cartan Element

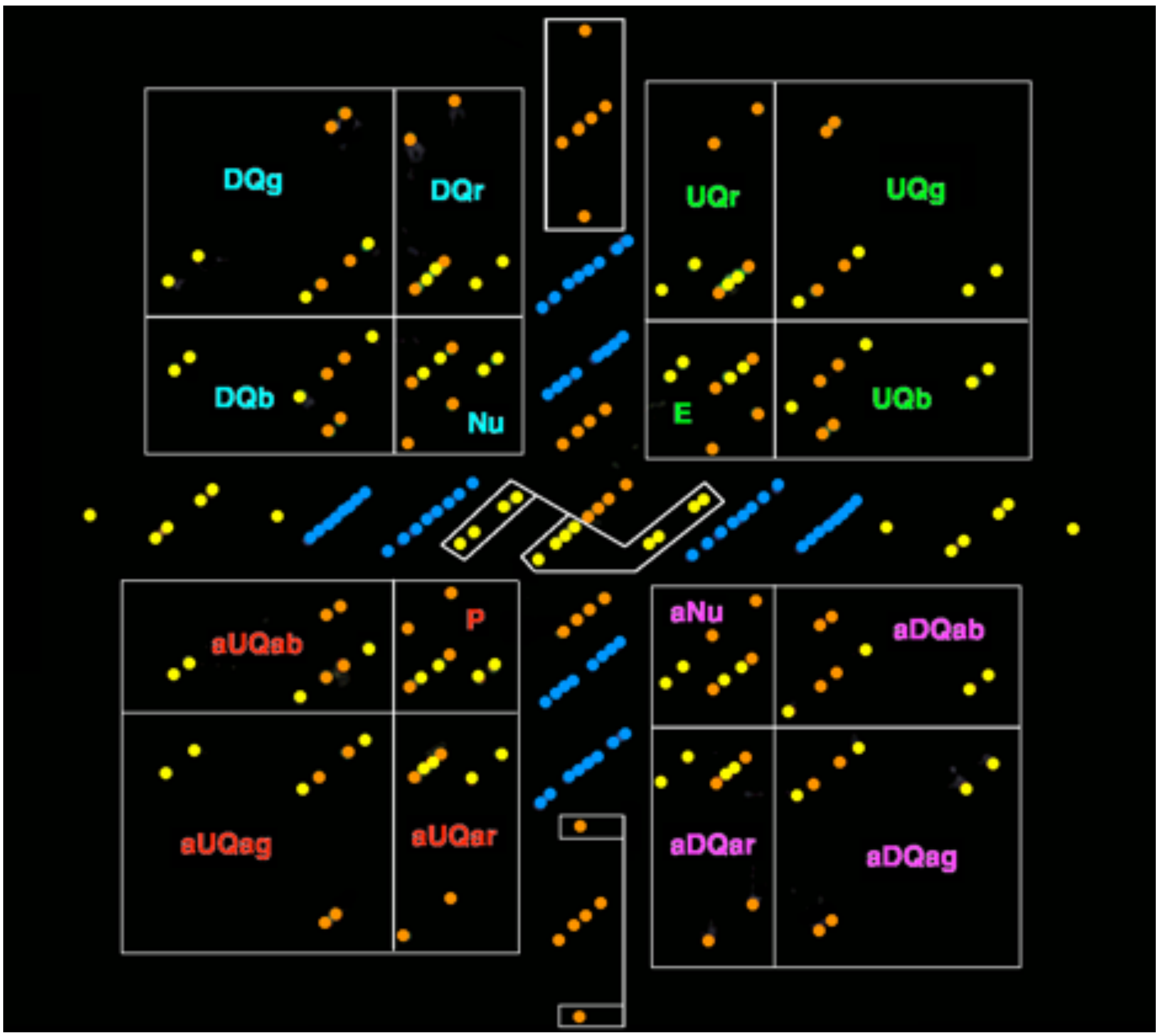
Gauge Bosons and Fermions are enclosed in white boundaries.

Ghosts and Spacetime are not so enclosed.



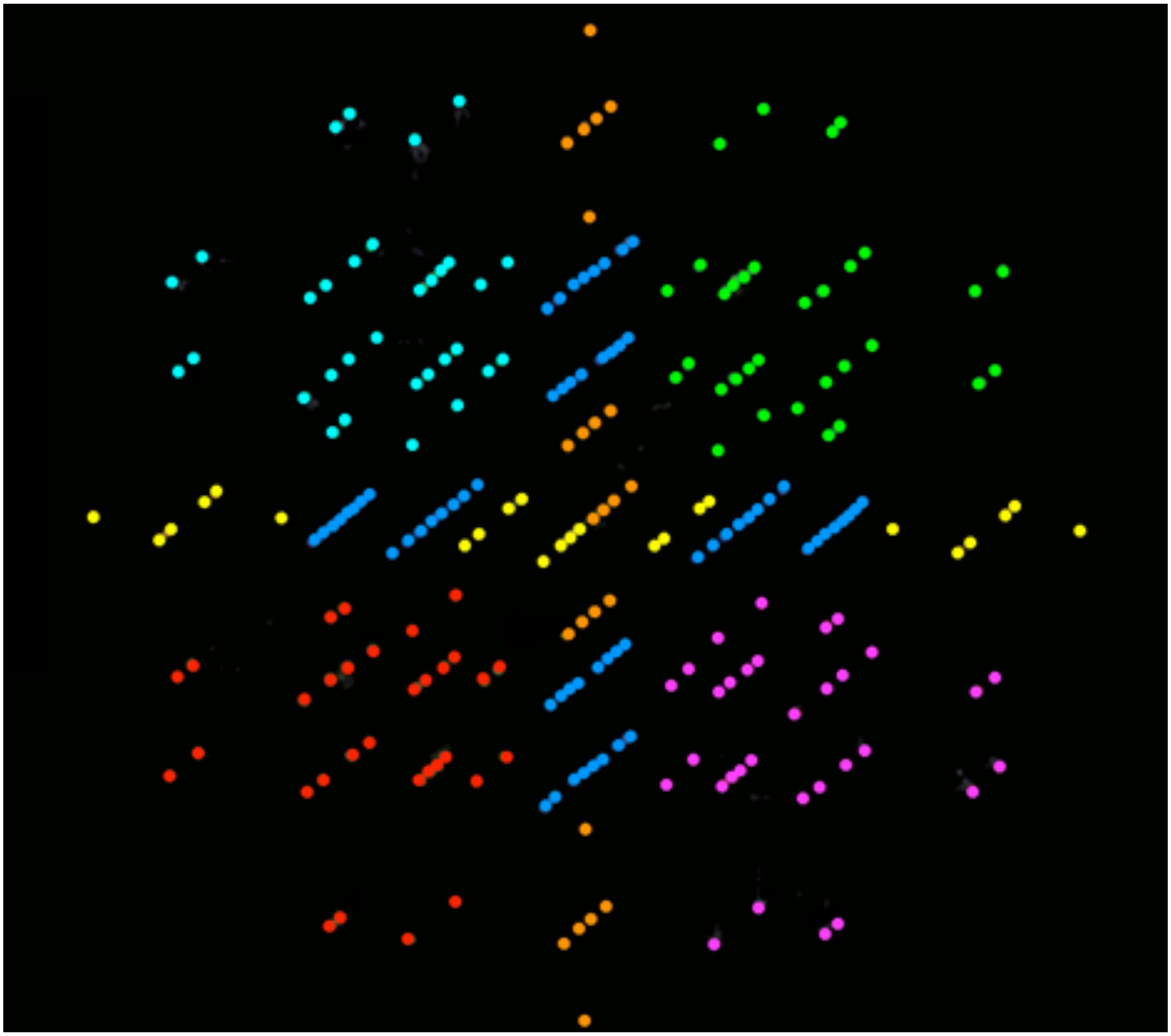
The 128 Off-Axis Quadrant Root Vectors represent the Symmetric Space
E8 / D8 = (OxO)P2 representing
64 = 8 Octonionic Components of 8 First-Generation Fermion Particles
and
64 = 8 Octonionic Components of 8 First-Generation Fermion Anti-Particles

Leptons and Quarks are:
Electron, U Quark red, U Quark green, U Quark blue
Neutrino, D Quark red, D Quark green, D Quark blue
Positron, U antiQuark antired, U antiQuark antigreen, U antiQuark antiblue
antiNeutrino, D antiQuark antired, D antiQuark antigreen, D antiQuark antiblue



Each Fermion has 8 Octonionic Components,
4 for M4 (yellow dots) and 4 for CP2 (orange dots)
of Quaternionic M4 x CP2 Kaluza-Klein

If you color the Fermions geen, cyan, red, magenta for types E, Nu, P, and anti-Nu
then
the Square/Cube Geometry of the 240 E8 Root Vectors is

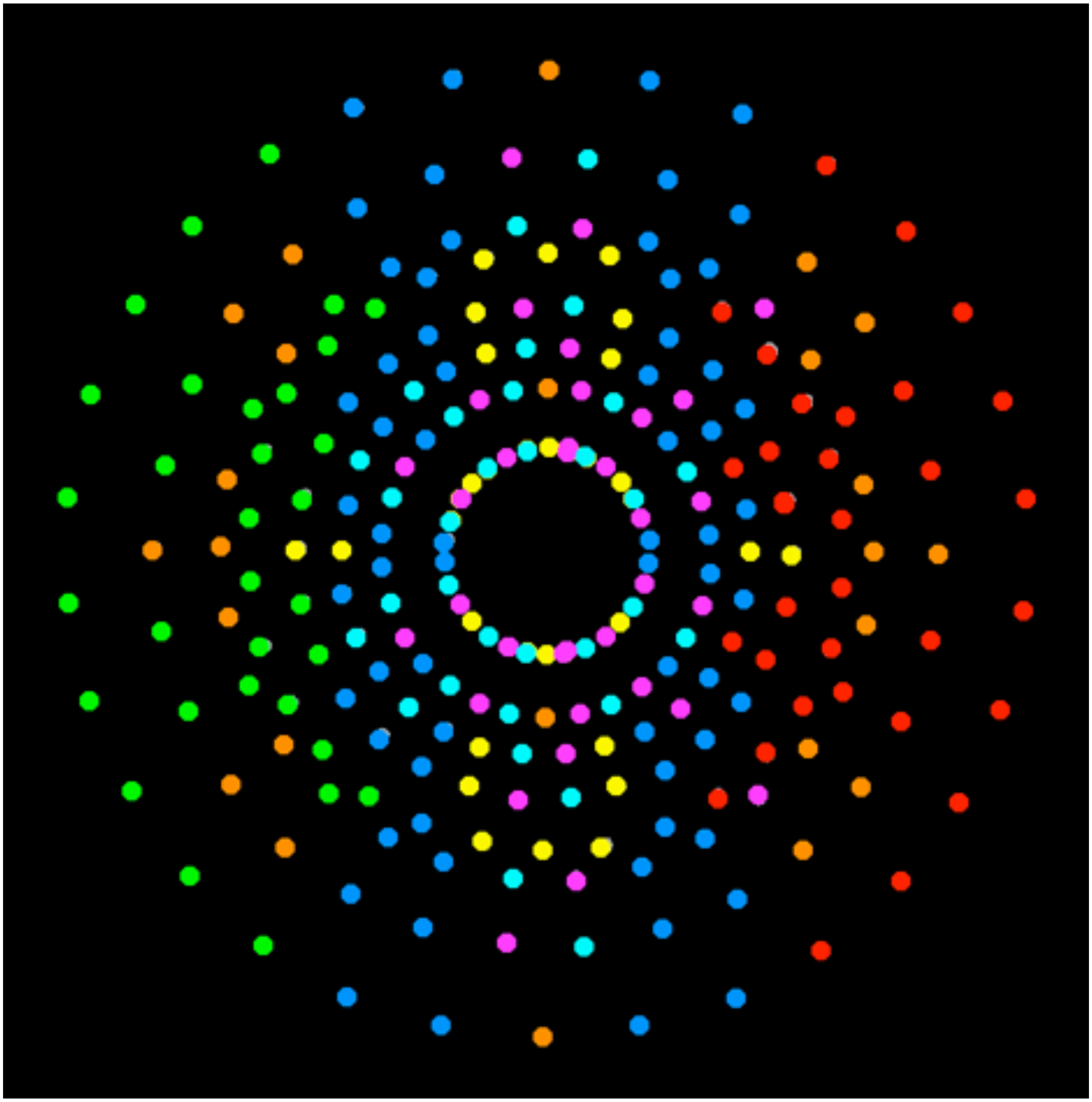


This is consistent with the full unprojected 8-dim picture of the 240 Root Vectors:

1 at North Pole
56 nearest neighbors of North Pole
126 next-to-nearest neighbors of North Pole
56 next-to-next-to-nearest neighbors of North Pole (nearest neighbors to South Pole)
1 at South Pole

If the 4+4 Cartan Subalgebra elements of E8 are added to the 56 and 56 you get a 1 + 60 + 126 + 60 + 1 grading of 248-dim E8 in which
60 + 60 = D8 and 1 + 126 + 1 = E8 / D8 = (OxO)P2

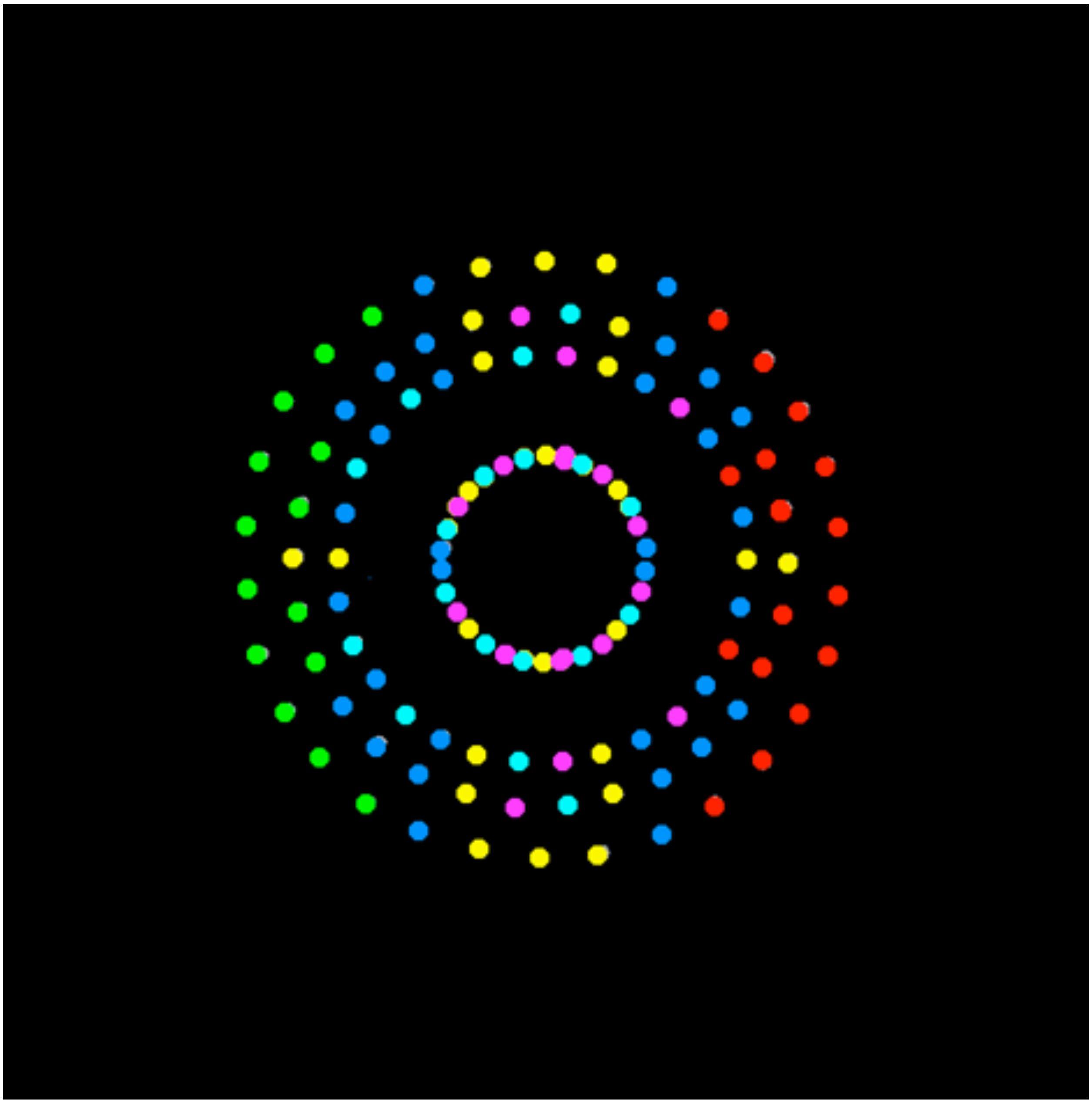
In the 8 Circles of 30 Geometry, the 240 E8 Root Vectors with Fermion Color-Coding
geen, cyan, red, magenta for types E, Nu, P, and anti-Nu
look more confusing and complicated



which is why I prefer the Square/Cube Geometry for visualizing E8 Physics.

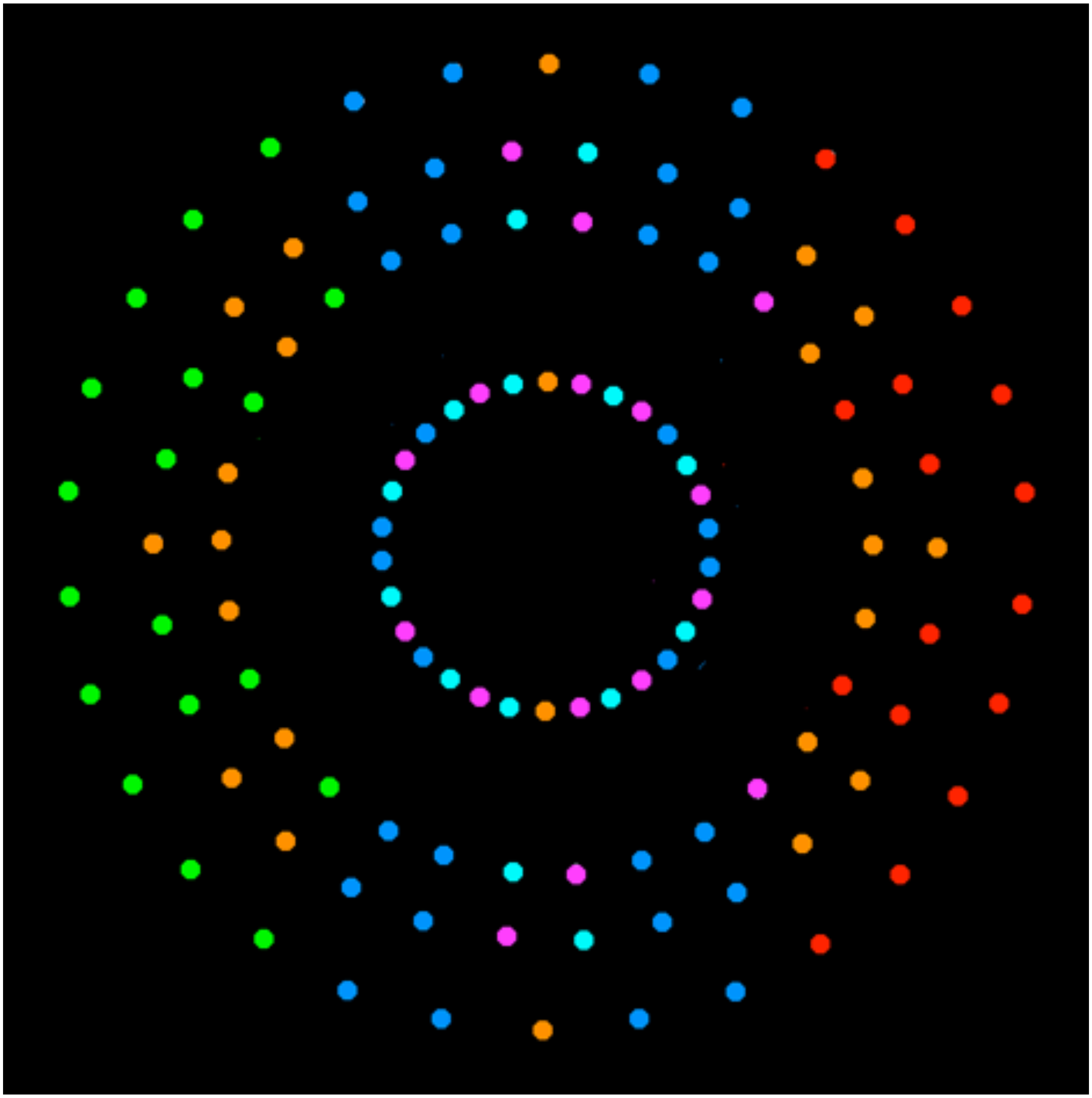
However, the 8 Circles of 30 Geometry is equivalent to the Square/Cube Geometry so that it gives the same E8 Physics, and is useful for some visualizations, such as

the breakdown of 240 E8 Root Vectors into 120 for Gravity+Dark Energy represented by the 120 vertices of a 600-cell



and

120 for the Standard Model
represented by the 120 vertices of a 600-cell (larger by factor of the Golden Ratio)



Connes NCG Physics and E8

Frank Dodd (Tony) Smith, Jr. - viXra 1511.0098

Connes has constructed a realistic physics model in 4-dim spacetime
based on **NonCommutative Geometry (NCG) of M x F**
where **M = 4-dim spacetime and F = C x H x M3(C)**
and C = Complex Numbers, H = Quaternions, and M3(C) = 3x3 Complex Matrices.

E8 has been used as a basis for physics models such as those
by Lisi (arXiv 1506.08073) and Smith (viXra 1508.0157) so the purpose
of this paper is to show **a connection between Connes NCG Physics and E8**.

Connes NCG is described by van den Dungen and van Suijlekom in arXiv 1204.0328
where they say: "... this review article is to present
the applications of Connes' noncommutative geometry to elementary particle physics.
...
the noncommutative description of the Standard Model does not require the
introduction of extra spacetime dimensions,
its construction is very much like the original Kaluza-Klein theories.
In fact, **one starts with a product M x F of ordinary four-dimensional spacetime M with an internal space F** which is to describe the gauge content of the theory.
Of course, spacetime itself still describesmthe gravitational part.
The main difference with Kaluza-Klein theories is that the additional space is a
discrete ... space whose structure is described by a ... noncommutative algebra ...
This is very much like the description of spacetime M
by its coordinate functions as usual in General Relativity,
which form an algebra under pointwise multiplication:

(x^mu x^nu)(p) = x^mu(p) x^nu(p)

Such commutative relations are secretly used in any physics textbook.
However, for a discrete space, ... propose to describe F by matrices ...
yielding a much richer internal (algebraic) structure ...
one can also describe a metric on F in terms of algebraic data,
so that we can fully describe the geometrical structure of M x F .
This type of noncommutative manifolds are called almost-commutative (AC)
...
Given an AC manifold M x F ... the group of diffeomorphisms ... generalized to such
noncommutative spaces combines ordinary diffeomorphisms of M with gauge
symmetries ... we obtain a combination of general coordinate transformations on M
with the respective groups ... U(1) x SU(2) x SU(3) ...[whose]... finite space is ...

internal space F ...[=]... C x H x M3(C)

... to construct a Lagrangian from the geometry of M x F . This is accomplished by ...
a simple counting of the eigenvalues of a Dirac operator on M x F
which are lower than a cutoff /\ ... we derive local formulas (integrals of Lagrangians)
... using heat kernel methods ...

The fermionic action is given as usual by an inner product.
The Lagrangians that one obtains in this way ... are the right ones,
and in addition minimally coupled to gravity.
This is unification with gravity of ... the full Standard Model. ...
We study conformal invariance ... with particular emphasis on the Higgs mechanism coupled to the gravitational background
...
the Lagrangian derived ... from the relevant noncommutative space is not just the Standard Model Lagrangian, but it implies that there are relations between some of the Standard Model couplings and masses
...
If we would assume that the mass of the top quark is much larger than all other fermion masses, we may neglect the other fermion masses. In that case ...

m_top ≤ sqrt(8/3) Mw [= sqrt(8/3) 80 = 130 GeV]
...
we shall evaluate the renormalization group equations (RGEs) for the Standard Model from ordinary energies up to the ... GUT ... unification scale ...
The scale Λ12 ... is given by ... 1.03 x 10^13 GeV ...
The [scale] Λ23 is given by ... 9.92 x 10^16 GeV ...
we have ... included the simple case wherewe ignore the Yukawa coupling of the tau-neutrino
[as is realistic with no neutrino see-saw mechanism] ... Numerical results [are]...

Λgut (10^16 GeV) ... m_top (GeV) 186.0 ... m_h (GeV) 188.1 ...

Λgut (10^13 GeV) ... m_top (GeV) 183.2 ... m_h (GeV) 188.3 ...".

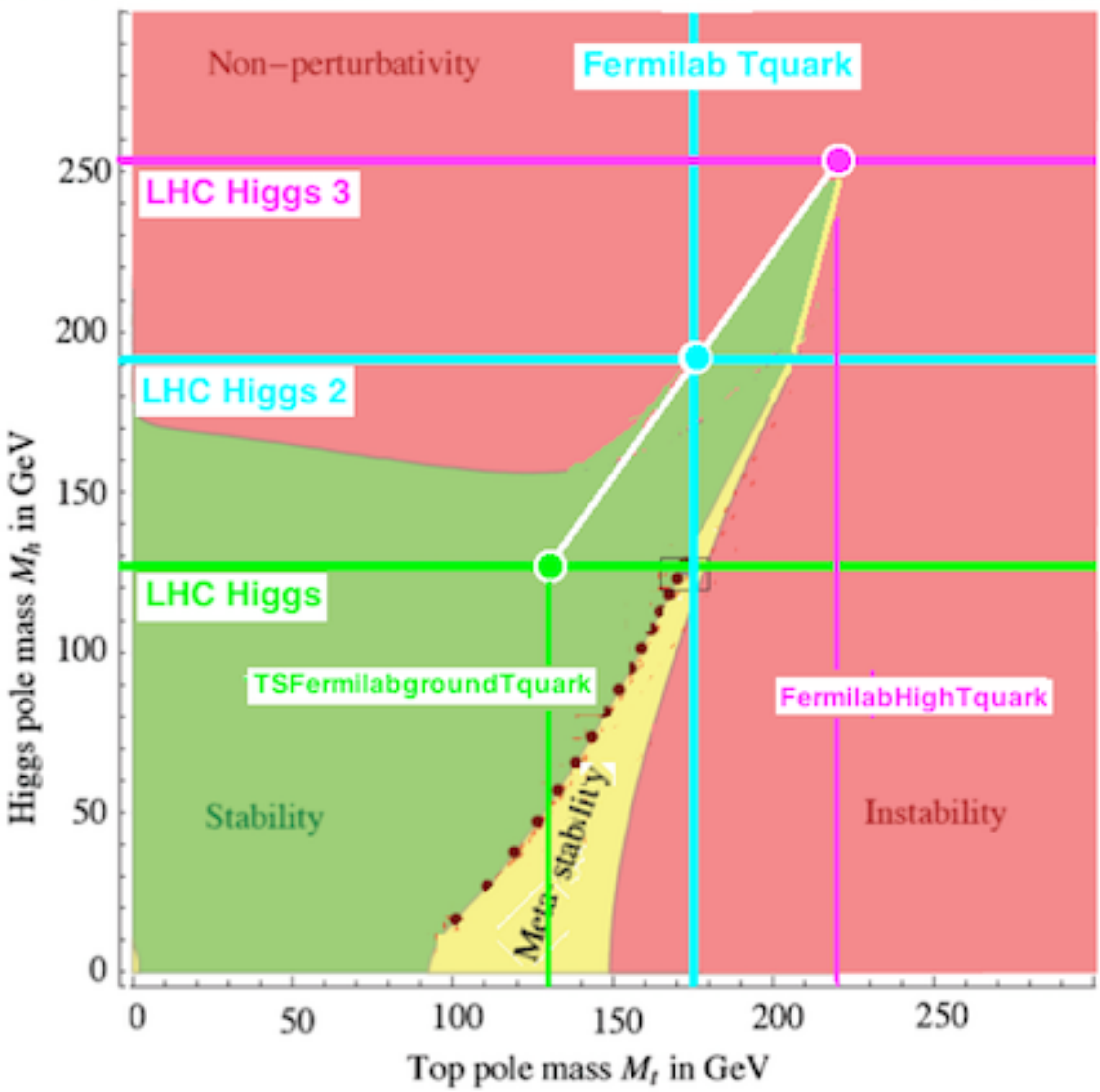
If you do a naive extrapolation down to the Higgs VeV 250 GeV energy scale where the compositeness of a Higgs as Tquark condensate system might become evident (the Non-perturbativity Boundary)

Λcomp (250 GeV) ... m_top (GeV) 173.2 ... m_h (GeV) 189

so the naively extrapolated
NCG masses for the Tquark-Higgs Middle Mass States are consistent with those of the E8 model of Smith (viXra 1508.0157)

Further,
the Basic Ground State NCG Tquark mass of 130 GeV is consistent with that of the E8 model of Smith (viXra 1508.0157)

Here is a chart showing the 3 Mass States of the Smith E8 model (viXra 1508.0157): the green dot in the Stable region (green) has the 130 GeV Tquark mass state that is also calculated by NCG; the cyan dot on the Non-perturbativity Boundary has the 173 GeV Tquark and 189 GeV Higgs mass states that are also calculated by NCG; I have not seen where NCG may or may not calculate High-Mass (220 and 250 GeV) Tquark and Higgs mass states indicated by the magenta dot at the Critical Point.



Structure of M and F of NCG

The **M of NCG** is 4-dim Spacetime, a discrete version of which is
the Integral Domain of Integral Quaternions whose vertex figure (nearest neighbors to the origin) is the 24-cell Root Vector Polytope of the 28-dim D4 Lie Algebra which contains as a subalgebra the 15-dim D3 Lie Algebra of the Conformal Group Spin(2,4) = SU(2,2) for MacDowell-Mansouri Gravity plus Conformal Dark Energy.

4-dim Riemannian Spacetime can be Wick Rotated to 4-dim Euclidean Space which can be compactified to the 4-sphere S4 which can be discretized as the 600-cell

so **the M of NCG can be locally represented as a 600-cell which has 120 vertices**.

F of NCG is the 24-dim algebra C + H + M3(C).
Identify the 24 generators of F with the 24 elements of the Binary Tetrahedral Group and therefore **identify F with the Tetrahedron** of which it is the symmetry group.
NCG, by using M x F as its basic structure, puts a copy of F at each point of M.

Consider a flat 2-dim subspace of M, and add to it F Tetrahedra following this construction recipe from a Don Davis 8 Sep 1999 sci math post:
“... build ... a hollow torus of 300 cells ... as follows:
lay out a 5x10 grid of unit edges. omit the lefthand and lower boundaries' edges,
because we're going to roll this grid into a torus later.
thus, the grid contains 100 edges: 50 running N-S, and 50 running EW.
attach one tetrahedron to each edge from above the grid.
the opposite edges of these tetrahedra will form a new 5x10 grid,
whose vertices overlie the centers of the squares in the lower grid.
thus, these 100 tetrahedra now form an egg-carton shape,
with 50 squarepyramid cups on each side.
divide each cup into two non-unit tetrahedra,
by erecting a right-triangular wall across the cup, corner-to-corner.
make the upper cups' dividers run NE/SW,
and make the upside-down lower cups' dividers run NW/SE.
note that the egg-carton is now a solid flat layer, one tetrahedron deep,
containing 100 unit tetra- hedra and 200 non-unit tetrahedra.

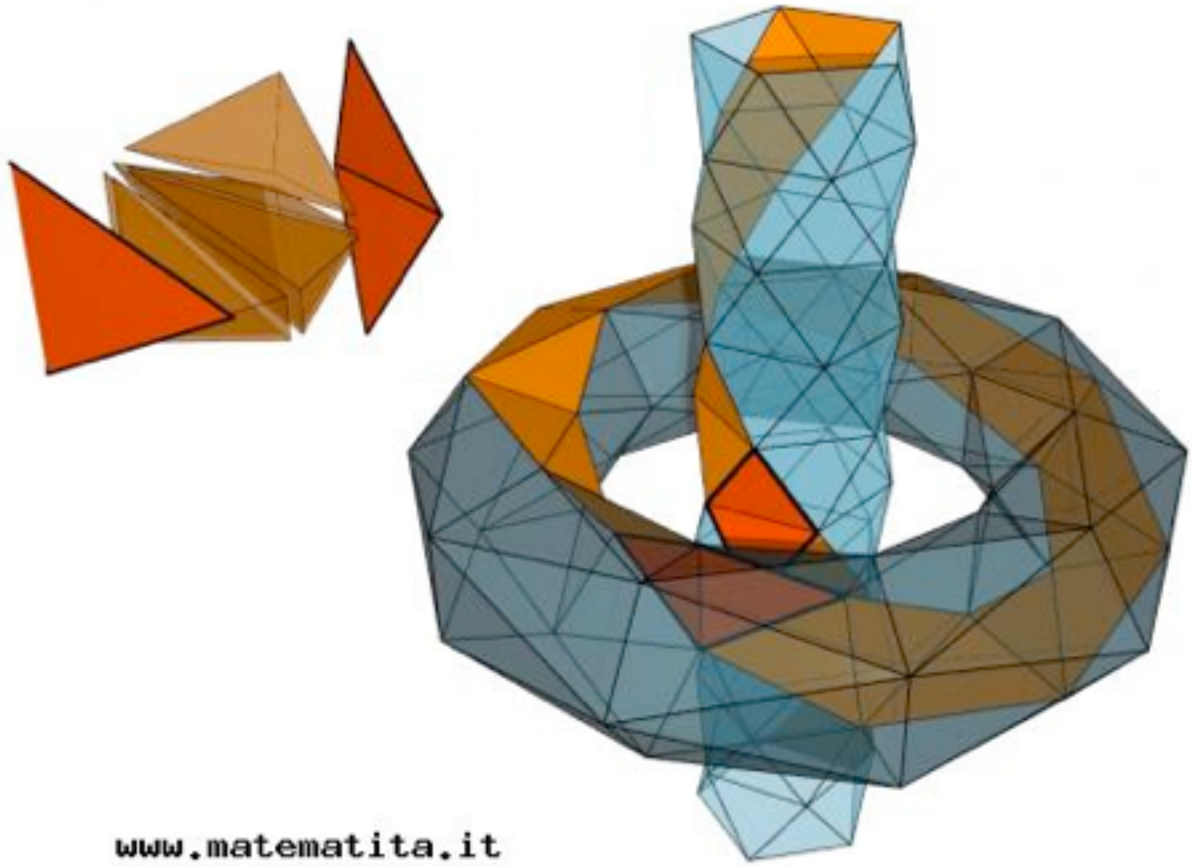
when we shrink the right-triangular dividing walls into equilateral triangles,
we distort each egg-cup into a pair of unit-tetrahedra.

at the same time,
the opening of each egg-cup changes from a square to a bent rhombus.
as the square openings bend,
the flat sheet of 300 tethrahedra is forced to wrap around into a hollow torus
with a one-unit- thick shell.

surprisingly,
this bends each 5x10 grid into a toroidal sheet of 100 equilateral triangles.
each grid's short edge is now a pentagon that threads through the donut hole.
the grid's long edge is now a decagon that wraps around both holes in its donut.
the two grids' long edges are now linked decagons.

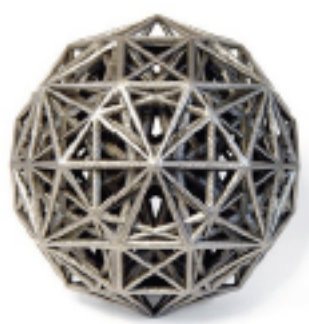
this wrapping cannot occur in R3, but it works fine in R4.
I admit that this part of my presentation is not easy to visualize.
perhaps a localized visualization image will help:
as an upper egg-cup is squeezed in one direction,
the edge-tetrahedra around it rotate,
squeezing the nearby lower egg-cups in the other direction.
this forces the flat sheet into a saddle-shape.
in R4, when this saddle-bending happens across the whole egg-carton at once,
the carton's edges can meet to make the toroidal sheet.
...
build each solid torus ....[of]... two solid tori of 150 cells each ... as follows:
using 100 tetrahedra, assemble 5 solid icosahedra (this is possible in R4).
daisy-chain five such icosahedra pole-to-pole ... between every pair of adjacent
icosahedra, surround the common vertex with 10 tetrahedra.
each solid torus has a decagonal "axis" running through the centers
and poles of the icosahedra. each solid torus contains 5*20 + 5*10 = 150
tetrahedra, and its surface is tiled with 100 equilateral triangles.
on this surface, six triangles meet at every vertex.
...

we will link these solid tori, like two links of a chain. with the hollow torus acting as a glue layer between them ...[



]...

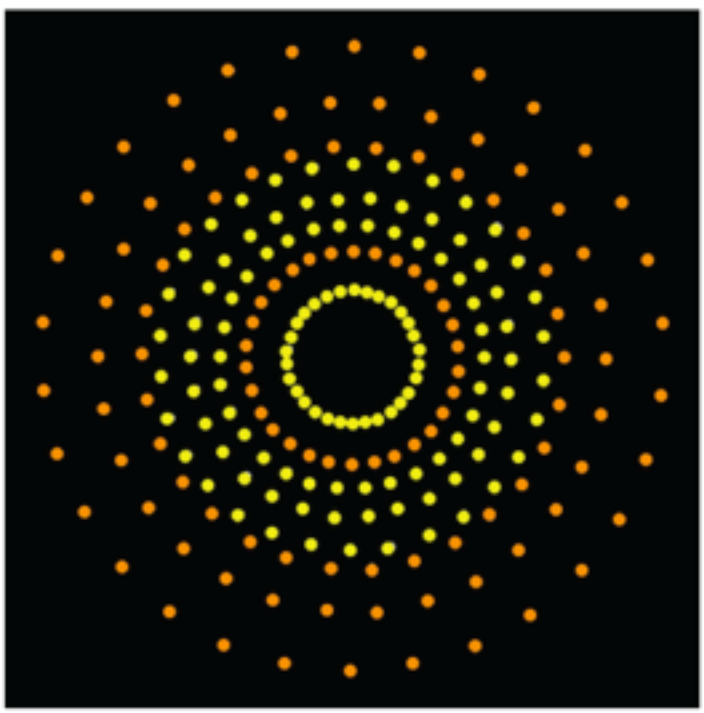
finally,
put one solid torus inside the hollow toroidal sheet,
attaching the 100 triangular faces of the solid
to the 100 triangles of the sheet's inner surface.
this gives us a fat solid torus,
10 units around and 4 units thick, containing 450 tetrahedral cells.
nevertheless, its surface has only 100 triangular faces.
thread the second 150-cell solid torus through this fat torus,
and attach the two solids' triangular faces. **this is the 600-cell polytope** ...”.



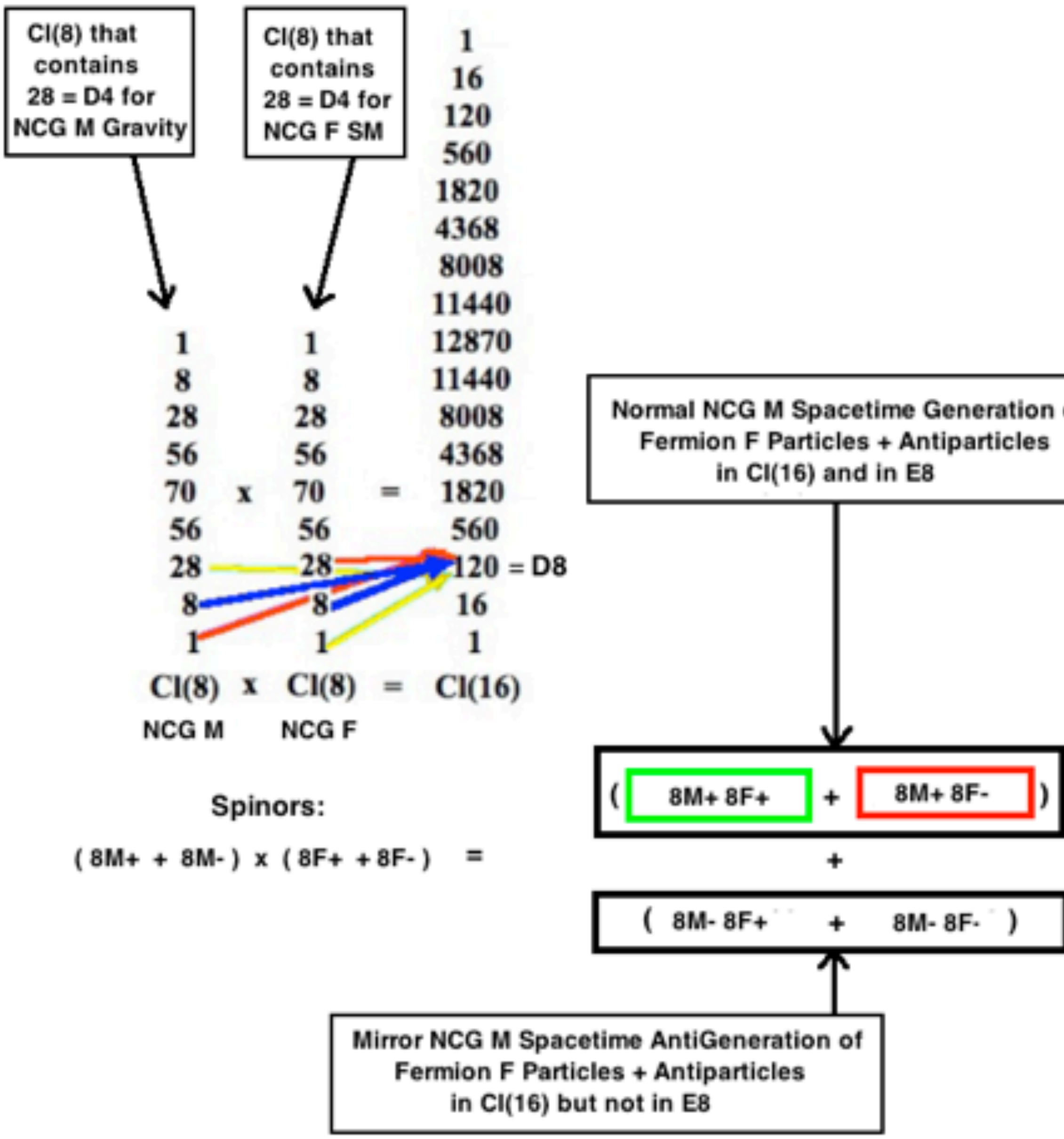
Combine the M 600-cell (yellow)
with the F 600-cell expanded by the Golden Ratio (orange)

to get the 120+120 = 240-vertex 8-dim E8 polytope
which is the Root Vector Polytope of the Lie Algebra E8

In this way the 8-dim space of E8 Root Vectors is seen as
being made up of two independent 4-dim spaces:
a Rational Number 4-dim space of yellow M dots
and
an Algebraic Extension by the Golden Ratio 4-dim space of orange F dots



The Lie Algebra E8 lives in the Clifford Algebra Cl(16) = Cl(8) x Cl(8)



This is the basic structure of the E8 = Cl(16) Physics Model

described in viXra 1508.0157

E8 Cosets and 4+4 Kaluza-Klein Lagrangian

Frank Dodd (Tony) Smith, Jr. - 2015 - viXra 1507.0069

The Coset structure of E8 represents the structure of a 4+4 Kaluza-Klein Lagrangian.
As Steven Weinberg said in the 1986 Dirac Memorial Lectures:
"... Let's examine the following equation:

$$\mathscr{L} = -\bar{\psi}\left(\gamma^{\mu}\frac{\partial}{\partial x^{\mu}} + m\right)\psi$$

$$-\frac{1}{4}\left(\frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}}\right)^2$$

... L stands for Lagrangian density ...
the first term involves the [fermion] field ...
the next term involves the [gauge boson] field ...
the Lagrangian density [is] integrated over spacetime ...".

Examine the Coset structure of E8 and see how it corresponds to Lagrangian Physics:

E8 / D8 = (OxO)P2 = 64-dim fermion particles + 64-dim fermion antiparticles
(O = Octonions)
(64 = 8 Kaluza-Klein spacetime components of 8 first-generation fermion types)

D8 / D4 x D4 = Gr(8,16) = 64-dim Octonionic Subspaces of R16
(Gr = Grassmanian and R16 = Vectors of Clifford Cl(16) Matrix Algebra for D8)
(8-dim Octonionic spacetime => Quaternionic 4+4 Kaluza-Klein M4 x CP2 spacetime
which symmetry breaking produces second and third generation fermions and Higgs)
(M4 = Physical Minkowski Spacetime)
(CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space)

D4 / D3 x U(1) = 12 Standard Model Gauge Boson Ghosts

D3 = A3 = Spin(2,4) = SU(2,2) Conformal Gravity + Dark Energy

D4 / A3 x U(1) = 12 Gravity+DE Root Vector Ghosts

A3 / A2 x U(1) = 6 = 4 SU(2) x U(1) Gauge Bosons + 2 of 3 Gravity+DE Cartan Ghosts
(Electroweak SU(2) x U(1) are in CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space)

A2 = 8 SU(3) Color Gauge Bosons

for details see viXra 1405.0030

World-Line String Bohm Quantum Potential, E8, and Consciousness

Frank Dodd (Tony) Smith, Jr. - 2015 - viXra 1512.0300

Abstract

Penrose-Hameroff type Quantum Consciousness is described in terms of E8 Physics (see viXra 1508.0157) and 26D String Theory with Strings seen as World-Lines and Bohm Quantum Potential and Sarfatti Thought Decoherence. Tubulin Dimer information content of Microtubules is seen to correspond to the Cl(16) Clifford Algebra in which the E8 Lie Algebra is naturally contained. Creation-Annihilation Operators and Algebraic Quantum Field Theory and AQFT Quantum Code for Bohm Quantum Theory are also described.

Table of Contents

E8 Root Vectors and 26D String Theory

The Cl(16)-E8 AQFT inherits structure from the Cl(16)-E8 Local Lagrangian

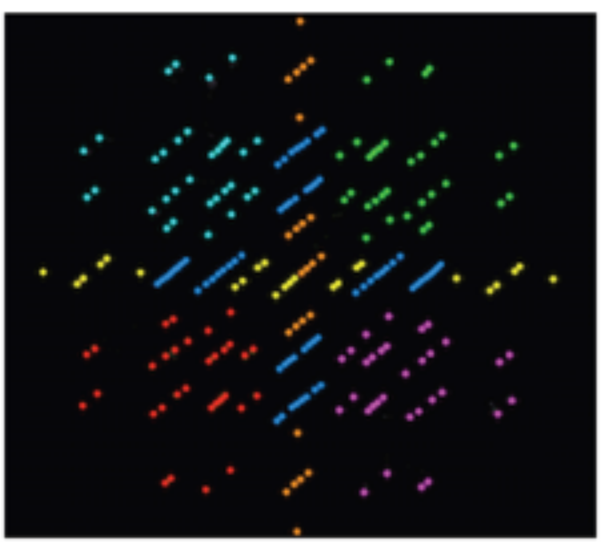
$$\int_{\text{8-dim SpaceTime}} \text{Gauge Gravity} + \text{Standard Model} + \text{Fermion Particle-AntiParticle} .$$

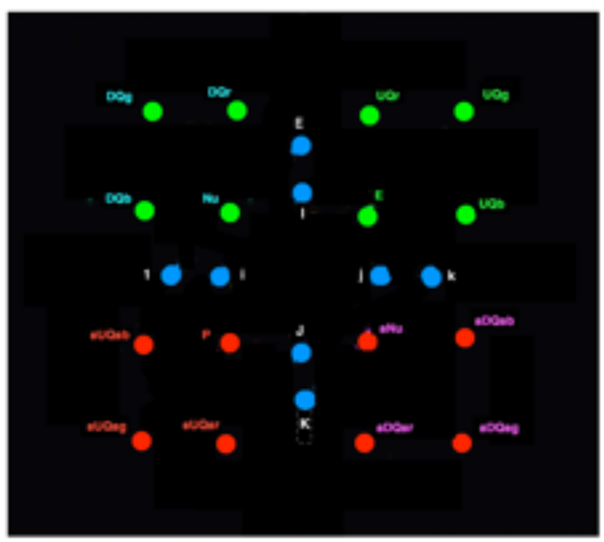
whereby World-Lines of Particles are represented by Strings moving in a space whose dimensionality includes 8v = 8-dim SpaceTime Dimensions +
+ 8s+ = 8 Fermion Particle Types + 8s- = 8 Fermion AntiParticle Types
combined in the traceless part J(3,O)o of the 3x3 Octonion Hermitian Jordan Algebra

a	8s+	8v
8s+*	b	8s-
8v*	8s-*	-a-b

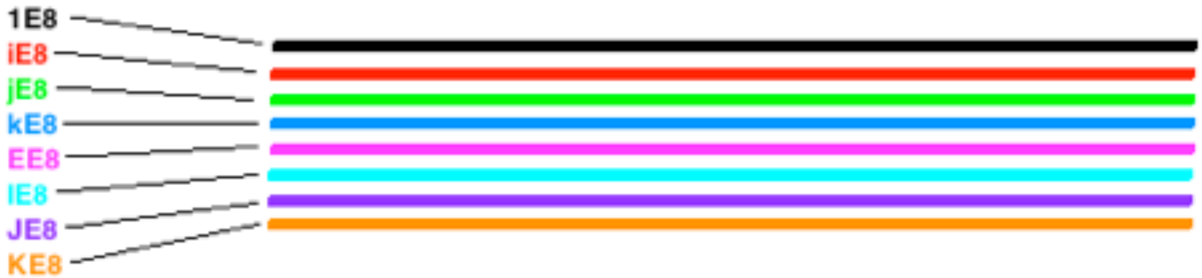
which has total dimension 8v + 8s+ + 8s- + 2 = 26 and is the space of a 26D String Theory with Strings seen as World-Lines.

24 = 8v + 8s+ + 8s- of the 26 dimensions of 26D String Theory correspond to 24x8 = 192 of the 240 E8 Root Vectors by representing the 8v + 8s+ + 8s- as superpositions of their respective 8 components

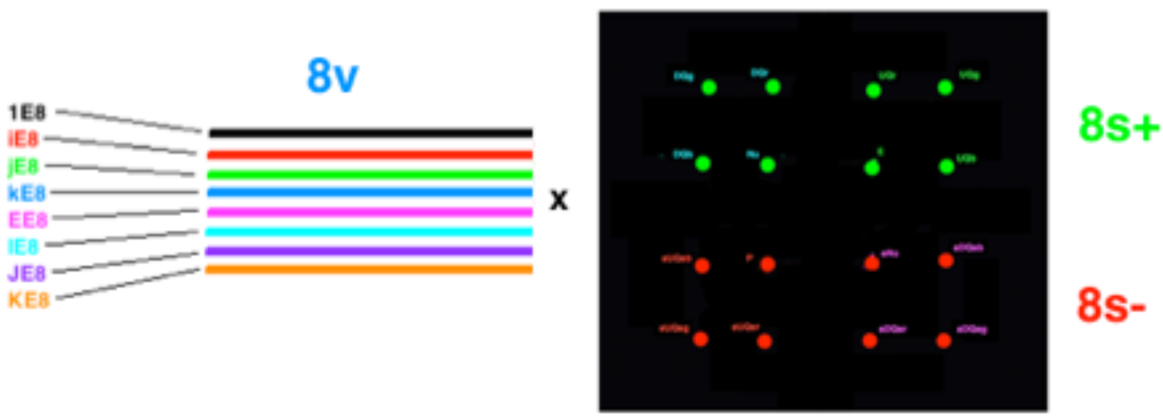




8v SpaceTime is represented by D8 branes. A D8 brane has
Planck-Scale Lattice Structure superpositions of 8 types of E8 Lattice
denoted by 1E8, iE8, jE8, kE8, EE8, IE8, JE8, KE8

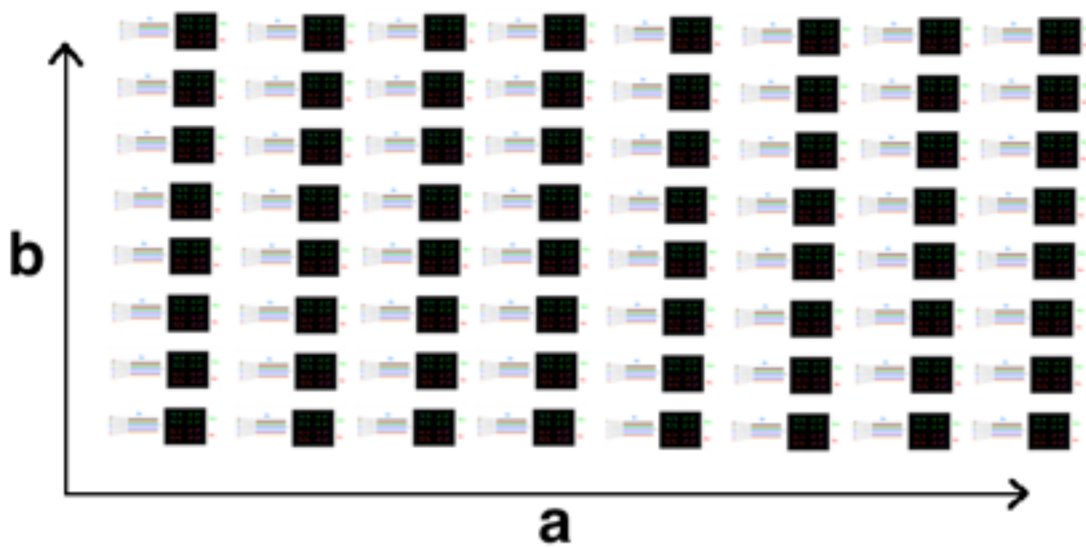


A single Snapshot of SpaceTime is represented by a D8 brane at each point of which is placed Fermion Particles or AntiParticles represented by 8+8 = 16 orbifolded dimensions of the 26 dimensions of 26D String Theory.



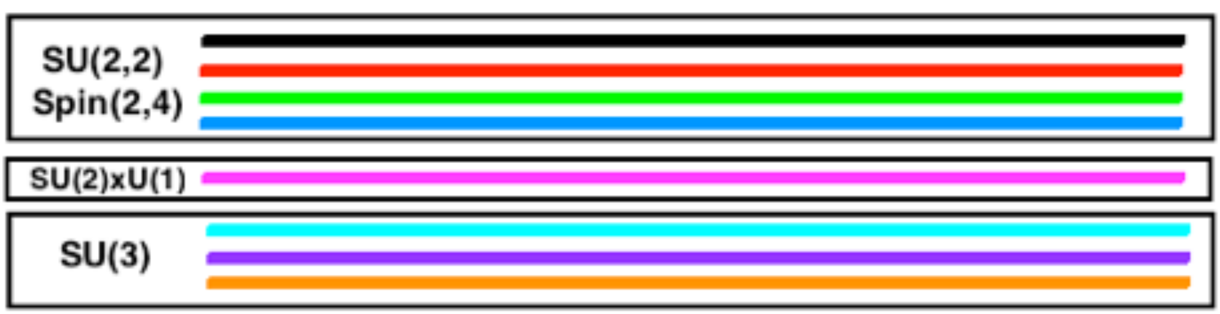
It is necessary to patch together SpaceTime Snapshots to form a Global Structure describing a Many-Worlds Global Algebraic Quantum Field Theory (AQFT)
whose overall structure is described by Deutsch in "The Fabric of Reality" (Penguin 1997 pp. 276-283):
"… there is no fundamental demarcation between snapshots of other times and snapshots of other universes ... Other times are just special cases of other universes ...
Suppose ... we toss a coin ... Each point in the diagram represents one snapshot
... in the multiverse there are far too many snapshots for clock readings alone to locate a snapshot relative to the others. To do that, we need to consider the intricate detail of which snapshots determine which others. …
in some regions of the multiverse, and in some places in space,
the snapshots of some physical objects do fall, for a period, into chains,
each of whose members determines all the others to a good approximation …".

The Many-Worlds Snapshots are structured as a 26-dim Lorentz Leech Lattice
of 26D String Theory parameterized by the a and b of J(3,O)o
as indicated in this 64-element subset of Snapshots

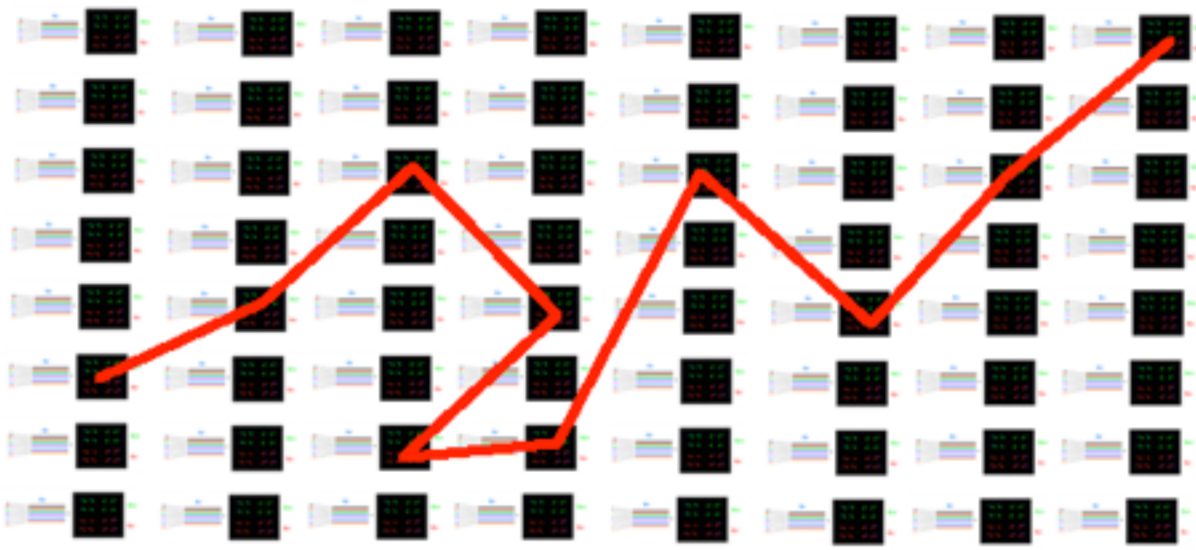


The 240 - 192 = 48 = 24+24 Root Vector Vertices of E8 that do not represent
the 8-dim D8 brane or the 8+8 = 16 dim of Orbifolds for Fermions
do represent the **Gauge Bosons (and their Ghosts) of E8 Physics**:

Gauge Bosons from 1E8, iE8, jE8, and kE8 parts of a D8 give U(2,2) Conformal Gravity
Gauge Bosons from EE8 part of a D8 give U(2) Electroweak Force
Gauge Bosons from IE8, JE8, and KE8 parts of a D8 give SU(3) Color Force



Each Deutsch chain of determination represents a World-Line of Particles / AntiParticles corresponding to a String of 26D String Theory
such as the red line in this 64-element subset of Snapshots



26D String Theory is the Theory of Interactions of Strings = World-Lines.

Interactions of World-Lines can describe Quantum Theory
according to Andrew Gray (arXiv quant-ph/9712037):
"... probabilites are ... assigned to entire fine-grained histories ...
base[d] ... on the Feynman path integral formulation ...
The formulation is fully relativistic and applicable to multi-particle systems.
It ... makes the same experimental predictions as quantum field theory …".
Green, Schwartz, and Witten say in their book "Superstring Theory" vol. 1 (Cambridge 1986)
"... For the ... closed ... bosonic string [**26D String Theory**] .... The first excited level ... consists of ... the ground state ... tachyon ... and ... a scalar ... 'dilaton' ... and ... SO(24) ... little group of a ...[26-dim]... massless particle ... and ...
a ... massless ... spin two state ...".

Closed string tachyons localized at orbifolds of fermions produce virtual clouds of particles / antiparticles that dress fermions.

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analagous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The SO(24) little group is related to the Monster automorphism group that is the symmetry of each cell of Planck-scale local lattice structure.

The massless spin 2 state = Bohmion = Carrier of the Bohm Force of the Bohm Quantum Potential.

“... **Bohm’s Quantum Potential can be viewed as an internal energy** of a quantum system ...” according to Dennis, de Gosson, and Hiley (arXiv 1412.5133) and
Peter R. Holland says in "The Quantum Theory of Motion" (Cambridge 1993): "... **the total force ... from the quantum potential ... does not ... fall off with distance** ...
because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

Penrose-Hameroff-type Quantum Consciousness is due to Resonant Quantum Potential Connections among Quantum State Forms.
The Quantum State Form of a Conscious Brain is determined by the configuration of a subset of its 10^18 to 10^19 Tubulin Dimers with math description in terms of a large Real Clifford Algebra:

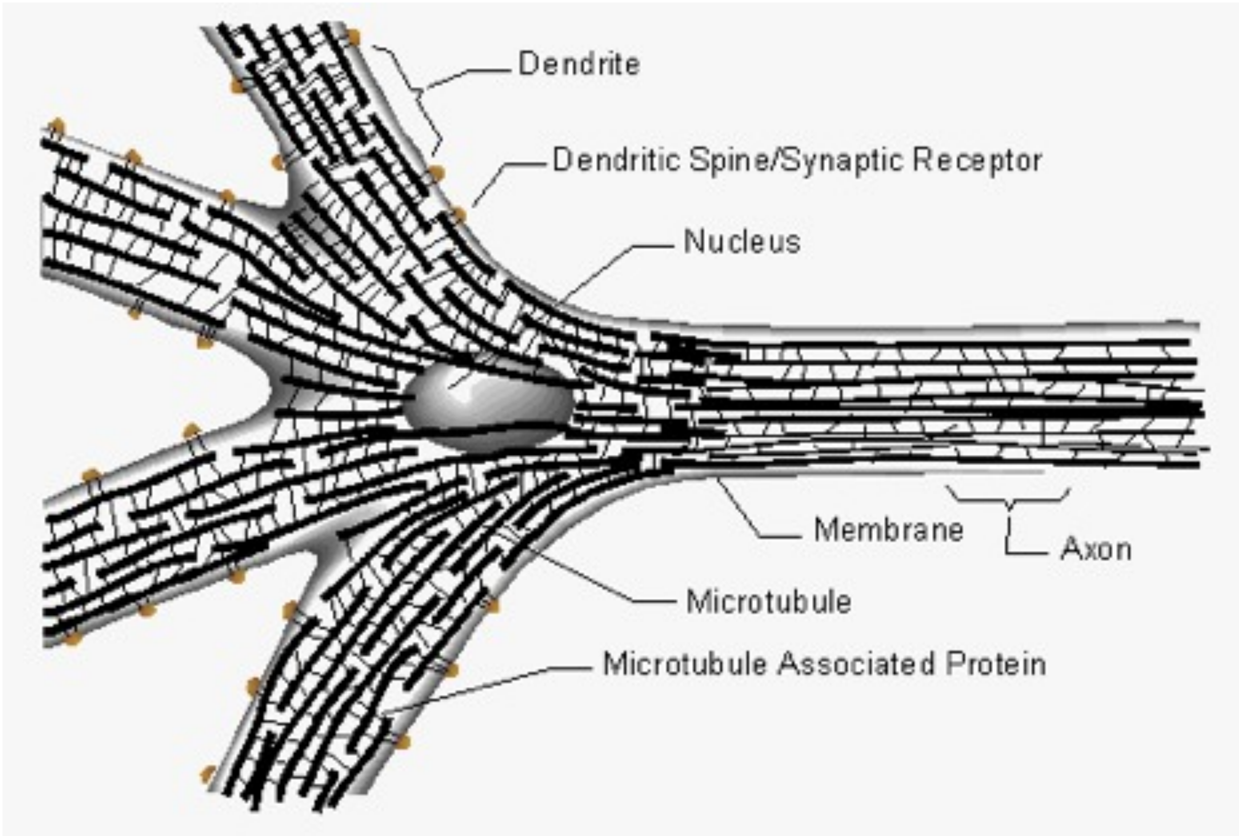
Resonance is discussed by Carver Mead in “Collective Electrodynamics“ (MIT 2000):
"... we can build ... a resonator from ... electric dipole ... configuration[s] ...
[such as  Tubulin Dimers]
Because there are charges at the two ends of the dipole, we can have a contribution to the electric coupling from the scalar potential ... as well [as] from the magnetic coupling ... from the vector potential ... electric dipole coupling is stronger than magnetic dipole coupling ... the coupling of ... two ... configurations ... is the same, whether retarded or advanced potentials are used. Any ... configuration ... couples to any other on its light cone, whether past or future. ... The total phase accumulation in a ... configuration ... is the sum of that due to its own current, and that due to currents in other ... configurations ... far away ...
The energy in a single resonator alternates between the kinetic energy of the electrons (inductance), and the potential energy of the electrons (capacitance). With the two resonators coupled, the energy shifts back and forth between the two resonators in such a way that the total energy is constant ... The conservation of energy holds despite an arbitrary separation between the resonators ... Instead of scaling linearly with the number of charges that take part in the motion, the momentum of a collective system scales as the square of the number of charges! ... The inertia of a collective system, however, is a manifestation of the interaction, and cannot be assigned to the elements separately. ... Thus, it is clear that collective quantum systems do not have a classical correspondence limit. ...”.

For **the 10^18 Tubulin Dimers of the human brain**,
the resonant frequencies are the same and exchanges of energy among them
act to keep them **locked in a Quantum Protectorate collective coherent state**.

Philip W. Anderson in cond-mat/0007287 and cond-mat/007185 said:
"... Laughlin and Pines have introduced the term "Quantum protectorate" as a general descriptor of the fact that certain states of quantum many-body systems exhibit properties which are unaffected by imperfections, impurities and thermal fluctuations. They instance ... flux quantization in superconductors, equivalent to the Josephson frequency relation which again has mensuration accuracy and is independent of imperfections and scattering. ...
... the source of quantum protection is a collective state of the quantum field involved such that the individual particles are sufficiently tightly coupled that elementary excitations no longer involve a few particles but are collective excitations of the whole system, and therefore, macroscopic behavior is mostly determined by overall conservation laws ... **a "quantum protectorate" ...[is]... a state in which the many-body correlations are so strong that the dynamics can no longer be described in terms of individual particles, and therefore perturbations which scatter individual particles are not effective** ...".
Mershin, Sanabria, Miller, Nawarathna, Skoulakis, Mavromatos, Kolomenskii, Scheussler, Ludena, and Nanopoulos in physics/0505080 "Towards Experimental Tests of Quantum Effects in Cytoskeletal Proteins" said:

Classically, the various dimers can only be in the ...[]... conformations. Each dimer is influenced by the neighboring dimers resulting in the possibility of a transition. This is the basis for classical information processing, which constitutes the picture of a (classical) cellular automaton.
If we assume ... that each dimer can find itself in a QM superposition of ...[those]... states, a quantum nature results. Tubulin can then be viewed as a typical two-state quantum mechanical system, where the dimers couple to conformational changes with 10^(-9) - 10^(-11) sec transitions, corresponding to an angular frequency ~ 10^10 - 10^12 Hz. In this approximation, the upper bound of this frequency range is assumed to represent (in order of magnitude) the characteristic frequency of the dimers, viewed as a two-state quantum-mechanical system ...[
The Energy Gap of our Universe as superconductor condensate spacetime is from 3 x 10^(-18) Hz (radius of universe) to 3 x 10^43 Hz (Planck length). Its RMS amplitude is 10^13 Hz = 10 THz = energy of neutrino masses = critical temperature Tc of BSCCO superconducting crystal Josephson Junctions]...
large-scale quantum coherence ...[has been observed]... at temperatures within a factor of three of biological temperatures. MRI magnets contain hundreds of miles of superconducting wire and routinely carry a persistent current. There is no distance limit - the macroscopic wave function of the superfluid condensate of electron pairs, or Cooper pairs, in a sufficiently long cable could maintain its quantum phase coherence for many thousands of miles ... there is no limit to the total mass of the electrons participating in the superfluid state. The condensate is "protected" from thermal fluctuations by the BCS energy gap at the Fermi surface ... The term "quantum protectorate" ... describe[s] this and related many-body systems ...".

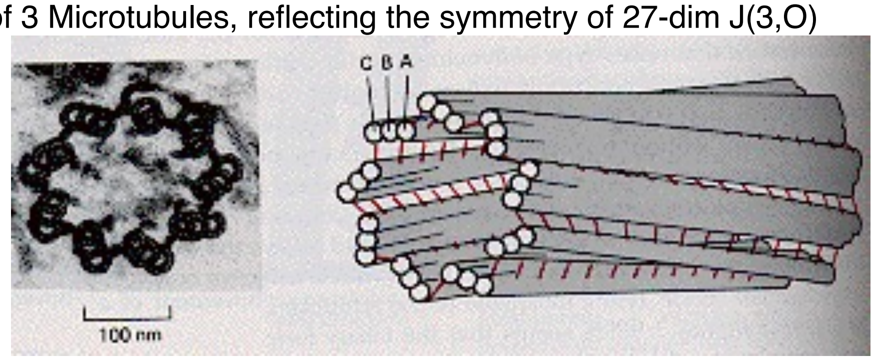
The Human Brain has about 10^11 Neuron cells, each about 1,000 nm in size.
The cytoskeleton of cells, including neurons of the brain, is made up of Microtubules



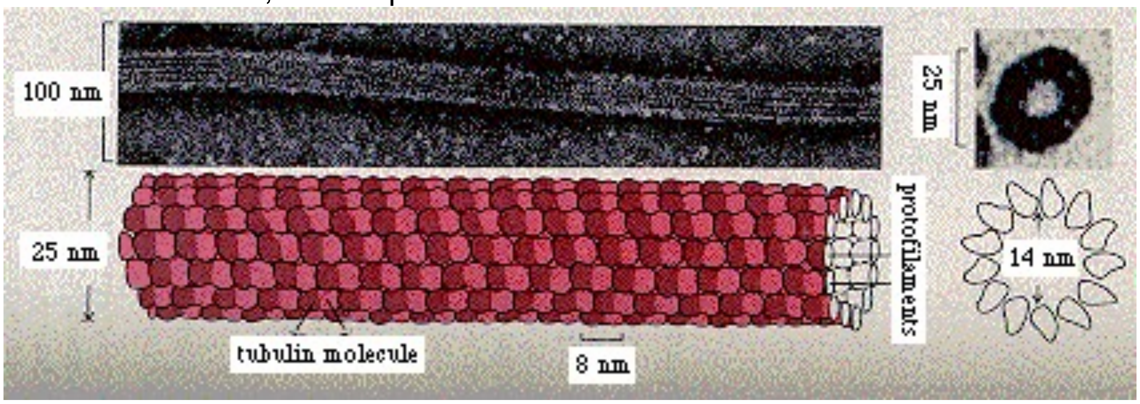
(image from “Orchestrated Objective Reduction of Quantum Coherence in Brain Microtubules: The “Orch OR” Model for Consciousness” by Penrose and Hameroff)

Each Neuron contains about 10^9 Tubulin Dimers, organized into Microtubules some of which are organized by a Centrosome. Centrosomes contain a pair of Centrioles.

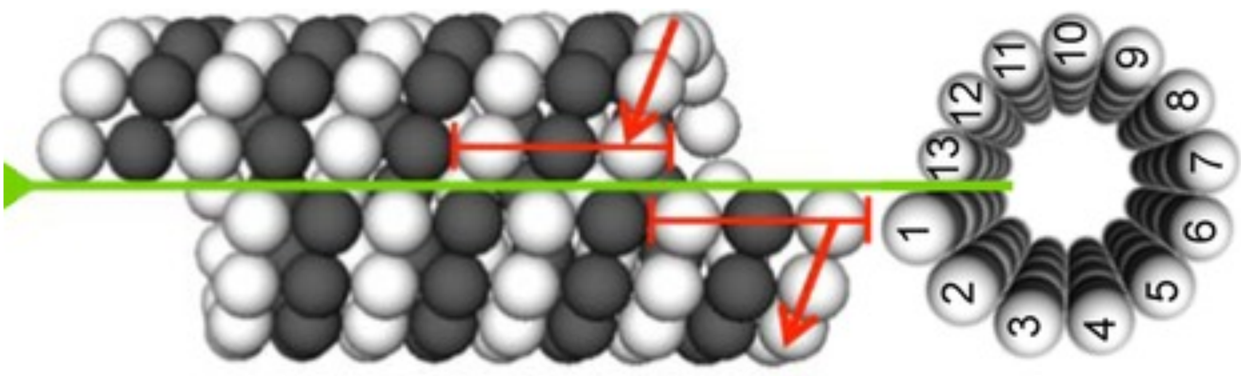
A Centriole is about 200 nm wide and 400 nm long. Its wall is made up of 9 groups of 3 Microtubules, reflecting the symmetry of 27-dim J(3,O)



Each Microtubule is a hollow cylindrical tube with about 25 nm outside diameter and 14 nm inside diameter, made up of 13 columns of Tubulin Dimers



(illustrations and information about cells, microtubules, and centrioles are from Molecular Biology of the Cell, 2nd ed, by Alberts, Bray, Lewis, Raff, Roberts, and Watson (Garland 1989))



(image from Wikipedia on Microtubule)

Each Tubulin Dimer is about 8 nm x 4 nm x 4 nm, consists of two parts, alpha-tubulin and beta-tubulin (each made up of about 450 Amino Acids, each containing roughly 20 Atoms)
A Microtubule 40 microns = 40,000 nm long contains 13 x 40,000 / 8 = 65,000 Dimers

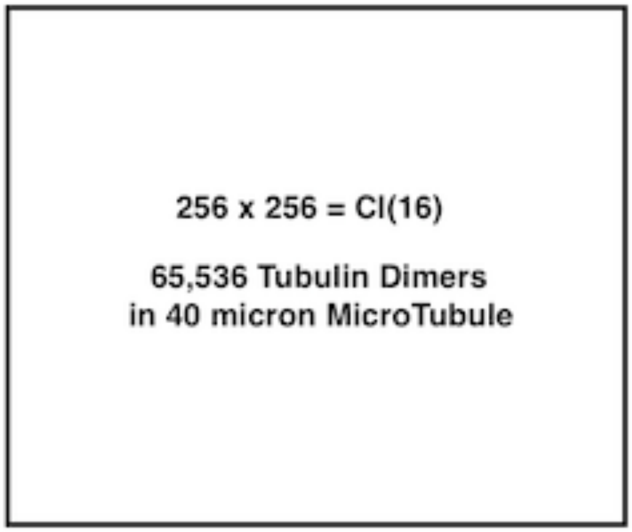
(images adapted from nonlocal.com/hbar/microtubules.html by Rhett Savage)

The black dots indicate the position of the Conformation Electrons.
There are two energetically distinct configurations for the Tubulin Dimers:

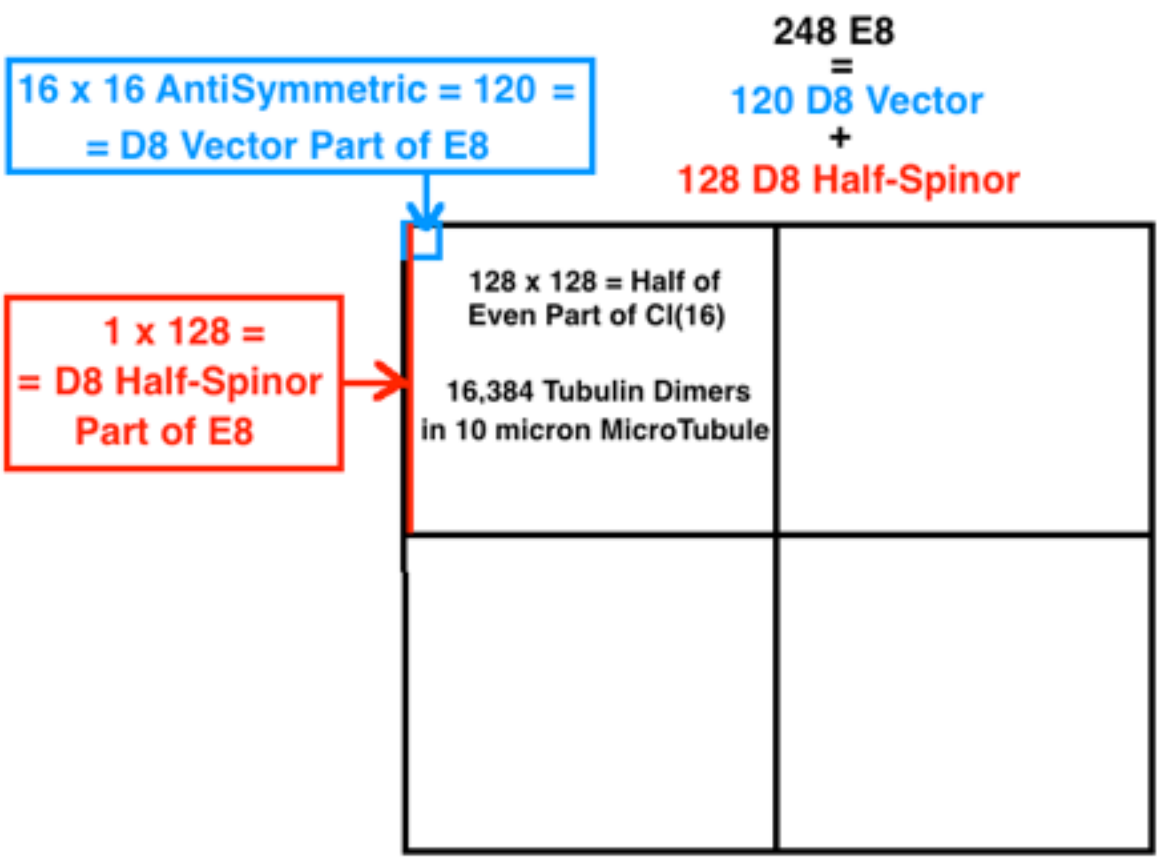
Conformation Electrons Similarly Aligned (left image) - State 0
Conformation Electrons Maximally Separated (right image) - State 1

The two structures - State 0 ground state and State 1 higher energy state - make Tubulin Dimers the basis for a Microtubule binary math / code system.

Microtubule binary math / code system corresponds to Clifford Algebras Cl(8) and Cl(8)xCl(8) = Cl(16) containing E8

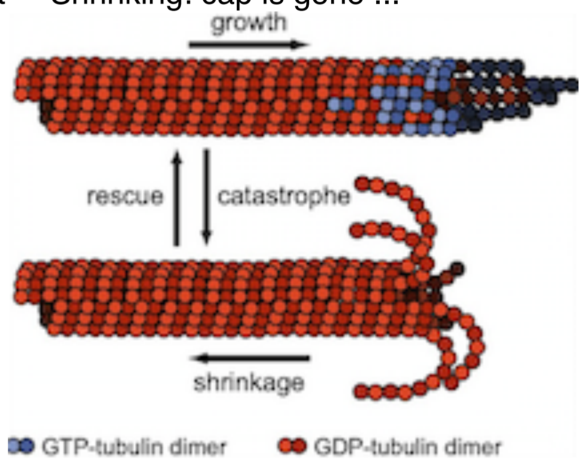


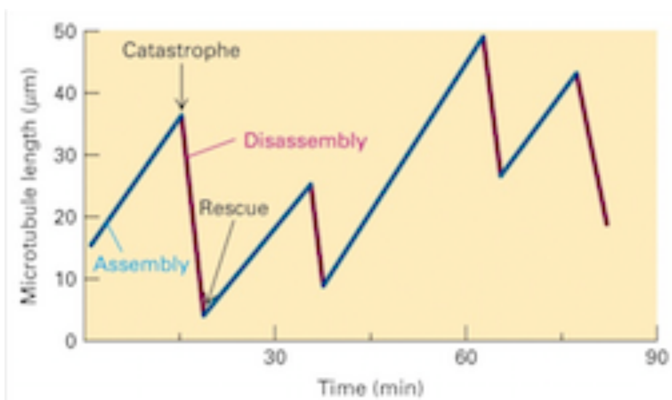
A 40 micron Microtubule contains Dimers representing the 65,536 elements of Cl(16) which contains the 248 elements of Lie Algebra E8 that defines E8 Physics Lagrangian.



E8 lives in only half of the block diagonal Even Part half of Cl(16) so that E8 of E8 Physics can be represented by the 16,384 Dimers of a 10 micron Microtubule.

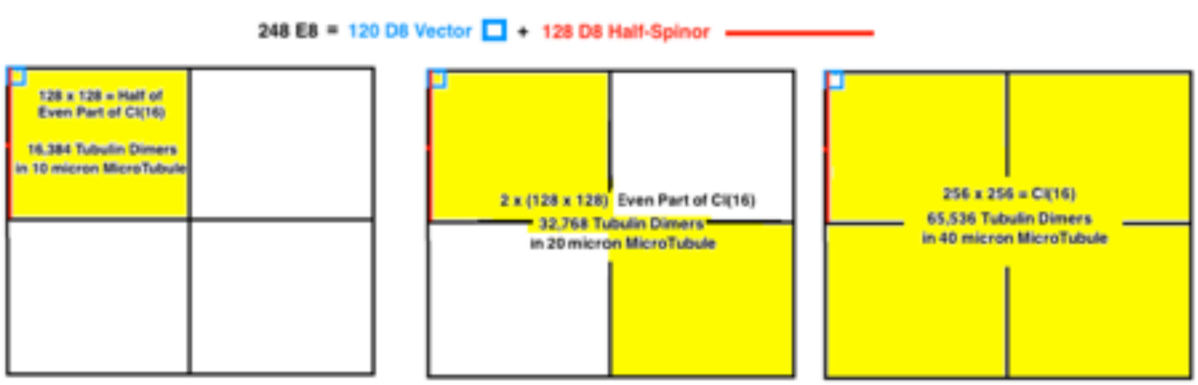
According to 12biophys.blogspot.com Lecture 11 Microtubule structure is dynamic:
“... One end of the microtubule is composed of stable (GTP) monomers
while the rest of the tubule is made up of unstable (GDP) monomers.
The GTP end comprises a cap of stable monomers.
Random fluctuations either increase or decrease the size of the cap.
This results in 2 different dynamic states for the microtubule.
Growing: cap is present   Shrinking: cap is gone ...





...”.

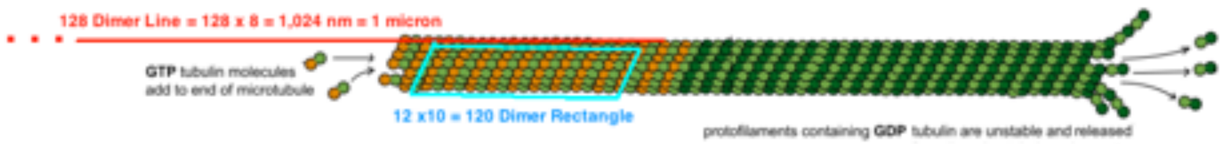
Microtubules spend most of their lives between 10 microns and 40 microns,
sizes that can represent E8 as half of the Even Part (half) of Cl(16) (10 microns)



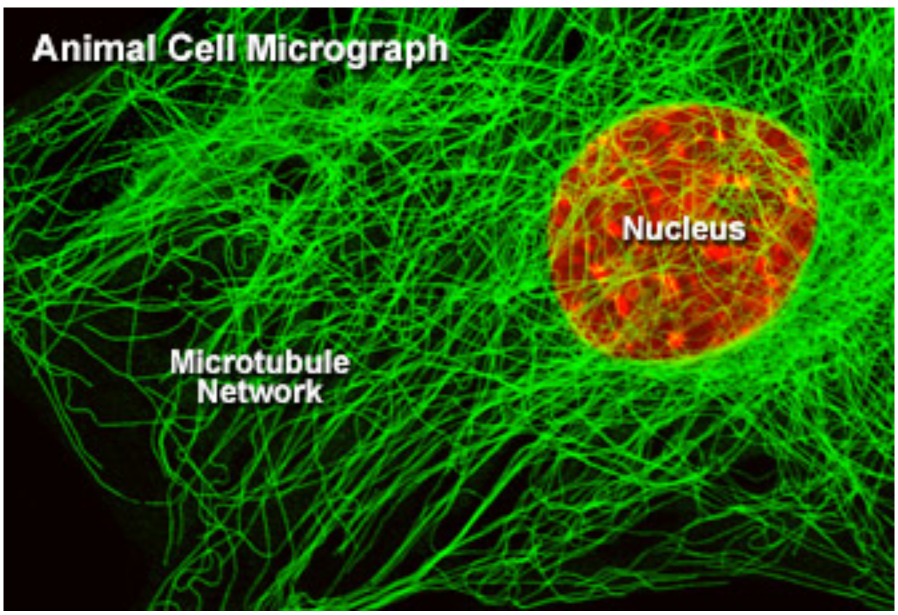
or as the Even Part (half) of Cl(16) (20 microns) or as full Cl(16) (40 microns).

In a given Microtubule
the 128 D8 Half-Spinor part is represented by a line of 128 Dimers in its stable GTP region
and

the 120 D8 Vector part by a 12 x 10 block of Dimers in its stable GTP region
(image adapted from 12biophys.blogspot.com Lecture 11)



The image immediately above does not show how thin is the Microtubule.
The following image (from micro.magnet.fsu.edu) shows overall Microtubule shape



How do the Microtubules communicate with each other ?

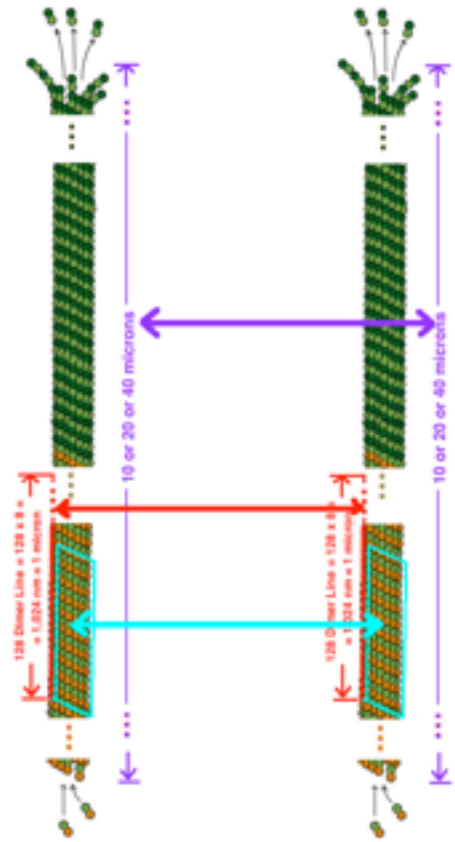
Consider the Superposition of States State 0 and State 1 involving one Tubulin Dimer with Conformation Electron mass m and State1 / State 0 position separation a .

The Superposition Separation Energy Difference is the internal energy

E_ssediff = G m^2 / a

that can be seen as either the **energy of 26D String Theory spin two gravitons** or the **Bohm Quantum Potential internal energy**, equivalently.

Communication between two Microtubules is by the Bohm Quantum Potential
between their respective corresponding Dimers (purple arrow)
with the correspondence being based on connection between respective E8 subsets,
the 128 D8 Half-Spinors (red arrow) and the 120 D8 BiVectors (cyan arrow)



How is information encoded in the Microtubules ?

Each Microtubule contains E8, allowing Microtubules to be corrrelated with each other.
The parts of the Microtubule beyond E8 are in Cl(16) for 40 micron Microtubules,
or the Even Subalgebra of Cl(16) for 20 micron Microtubules,
or half of the Even Subalgebra of Cl(16) for 10 micron Microtubules
so since by 8-Periodicity of Real Clifford Algebras Cl(16) = Cl(8) x Cl(8) and
since Cl(8) information is described by the Quantum Reed-Muller code [[256 , 0 , 24]]
the information content of Cl(16) and its Subalgebras **is described by the Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]**

For a 40-micron Microtubule there are, outside the 248-E8 part, about 65,000 TD Qubits available to describe one Quantum Thought State among about 2^65,000 possibilities, analagous to the Book of Genesis of (22+5)^78,064 Hebrew Letter/Final possibilities.

What about information in a large number of Microtubules ?

Since the information in one Microtubule is based on Cl(16)
and
since by 8-Periodicity Cl(16) x...(8N times tensor product)...x Cl(16) = Cl(16 N)
information of a large number of Microtubules is described by
Tensor Products of [[256 , 0 , 24]] x [[256 , 0 , 24]] Quantum Reed-Muller codes

How does all this give rise to Penrose-Hameroff Quantum Consciousness ?

Consider the Superposition of States State 0 and State 1 involving one Tubulin Dimer with Conformation Electron mass m and State1 / State 0 position separation a .
The Superposition Separation Energy Difference is the internal energy

E_ssediff = G m^2 / a

that can be seen as either the **energy of 26D String Theory spin two gravitons** or the **Bohm Quantum Potential internal energy**, equivalently.

For a given Tubulin Dimer a = 1 nanometer = 10^(-7) cm so that
T = h / E_electron = (Compton / Schwarzschild) (a / c) = 10^26 sec = 10^19 years

Now consider the case of N Tubulin Dimers in Coherent Superposition connected by the Bohm Quantum Potential Force that does not fall off with distance.
Jack Sarfatti defines coherence length L by L^3 = N a^3 so that
the Superposition Energy E_N of N superposed Conformation Electrons is

E_N = G M^2 / L = N^(5/3) E_ssediff

The decoherence time for the system of N Tubulin Electrons is

T_N = h / E_N = h / N^(5/3) E_ssediff = N^(-5/3) 10^26 sec

so we have the following rough approximate Decoherence Times T_N

Number of Involved Tubulin Dimers	Time T_N	
10^(11+9) = 10^20	10^(-33 + 26) = 10^(-7) sec	10^11 neurons x 10^9 TD / neuron 10^20 Tubuin Dimers in Human Brain
10^16	10^(-27 + 26) = 10^(-1) sec - 10 Hz -	

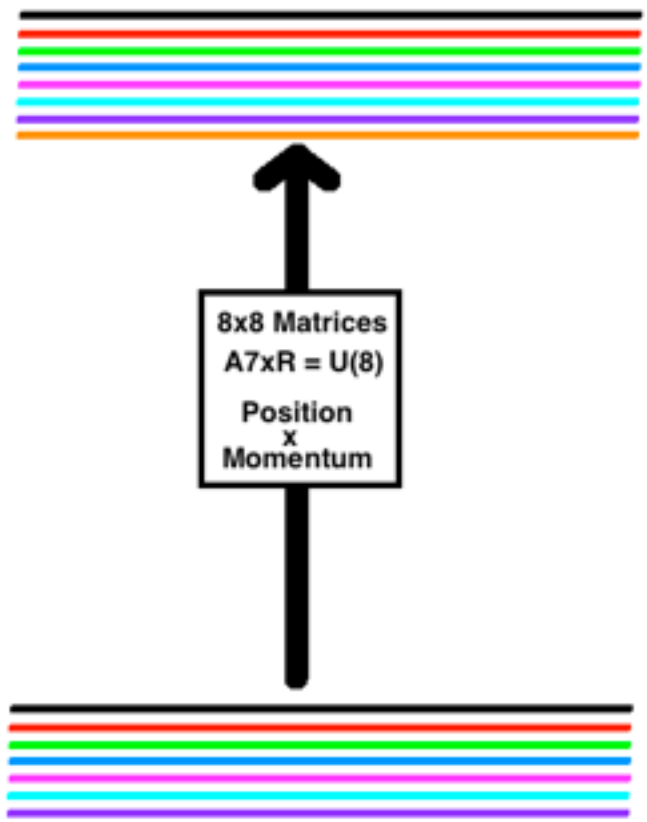
Human Alpha EEG is 8 to 13 Hz -
Fundamental Schumann Resonance is 7.8 Hz -
Time of Traverse by a String World-Line Quantum Bohmion of a Quantum Consciousness Hamiltonian Circuit of 10^16 TD separated from nearest neighbors by 10 nm is 10^16 x 10 nm / c = (10^16 x 10^(-6)) cm / c = 10^10 cm / c = 0.3 sec -

The Creation-Annihilation Operator structure of the Bohm Quantum Potential of 26D String Theory is given by the

Maximal Contraction of E8 = semidirect product A7 x h92
where h92 = 92+1+92 = 185-dim Heisenberg algebra and A7 = 63-dim SL(8)

The Maximal E8 Contraction A7 x h92 can be written as a 5-Graded Lie Algebra
28 + 64 + (SL(8,R) + 1) + 64 + 28
Central Even Grade 0 = SL(8,R) + 1
The 1 is a scalar and SL(8,R) = Spin(8) + Traceless Symmetric 8x8 Matrices,
so SL(8,R) represents a local 8-dim SpaceTime in Polar Coordinates.
Odd Grades -1 and +1 = 64 + 64
Each = 64 = 8x8 = Creation/Annihilation Operators for 8 components of 8 Fundamental Fermions.
Even Grades -2 and +2 = 28 + 28
Each = Creation/Annihilation Operators for 28 Gauge Bosons of Gravity + Standard Model.

The 8x8 matrices linking one D8 to the next D8 of a World-Line String
give A7xR = U(8) representing Position x Momentum



The Algebraic Quantum Field Theory (AQFT) structure of the Bohm Quantum Potential of 26D String Theory is given by the Cl(16)-E8 Local Lagrangian

$$\int_{\text{8-dim SpaceTime}} \text{Gauge Gravity} + \text{Standard Model} + \text{Fermion Particle-AntiParticle} \; .$$

living in Cl(16) and by 8-Periodicity of Real Clifford Algebras,
as the Completion of the Union of all Tensor Products of the form

Cl(16) x ...(N times tensor product)... x Cl(16) = Cl(16N)

For N = 2^8 = 256 the copies of Cl(16) are on the 256 vertices of the 8-dim HyperCube

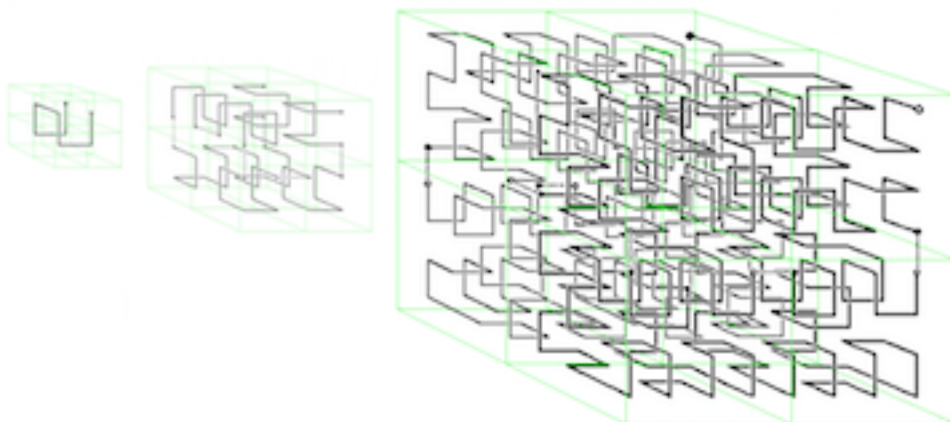
For N = 2^16 = 65,536 = 4^8 the copies of Cl(16) fill in the 8-dim HyperCube as described by William Gilbert's web page: "... **The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube**. ...".

The vertices of the Hilbert curve are at the centers of the 2^8 sub-8-HyperCubes whose edge lengths are 1/2 of the edge lengths of the original 8-dim HyperCube

As N grows, the copies of Cl(16) continue to fill the 8-dim HyperCube of E8 SpaceTime using higher Hilbert curve stages from the 8-bit reflected binary Gray code subdividing the initial 8-dim HyperCube into more and more sub-HyperCubes.

If edges of sub-HyperCubes, equal to the distance between adjacent copies of Cl(16), remain constantly at the Planck Length, then the

full 8-dim HyperCube of our Universe expands as N grows to 2^16 and beyond
similarly to the way shown by this 3-HyperCube example for N = 2^3, 4^3, 8^3
from Wiliam Gilbert's web page:



The Union of all Cl(16) tensor products is
the Union of all subdivided 8-HyperCubes
and
their Completion is a huge superposition of 8-HyperCube Continuous Volumes
which Completion belongs to the Third Grothendieck Universe.

AQFT Quantum Code

Cerf and Adami in quantum-ph/9512022 describe virtual qubit-anti-qubit pairs (they call them ebit-anti-ebitpairs) that are related to negative conditional entropies for quantum entangled systems and are similar to fermion particle-antiparticle pairs. Therefore quantum information processes can be described by particle-antiparticle diagrams much like particle physics diagrams and
the Algebraic Quantum Field Theory of the E8 - Cl(16) = Cl(8)xCl(8) Physics Model should be equivalent to a Quantum Code Information System.

Quantum Reed-Muller code [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8)

Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
Real Clifford Algebra Cl(8) x Cl(8) = Cl(16) containing E8

Completion of the Union of All Tensor Products of [[256 , 0 , 24]] x [[256 , 0 , 24]]
corresponds to
AQFT (Algebraic Quantum Field Theory) hyperfinite von Neumann factor algebra that is Completion of the Union of All Tensor Products of Cl(16)

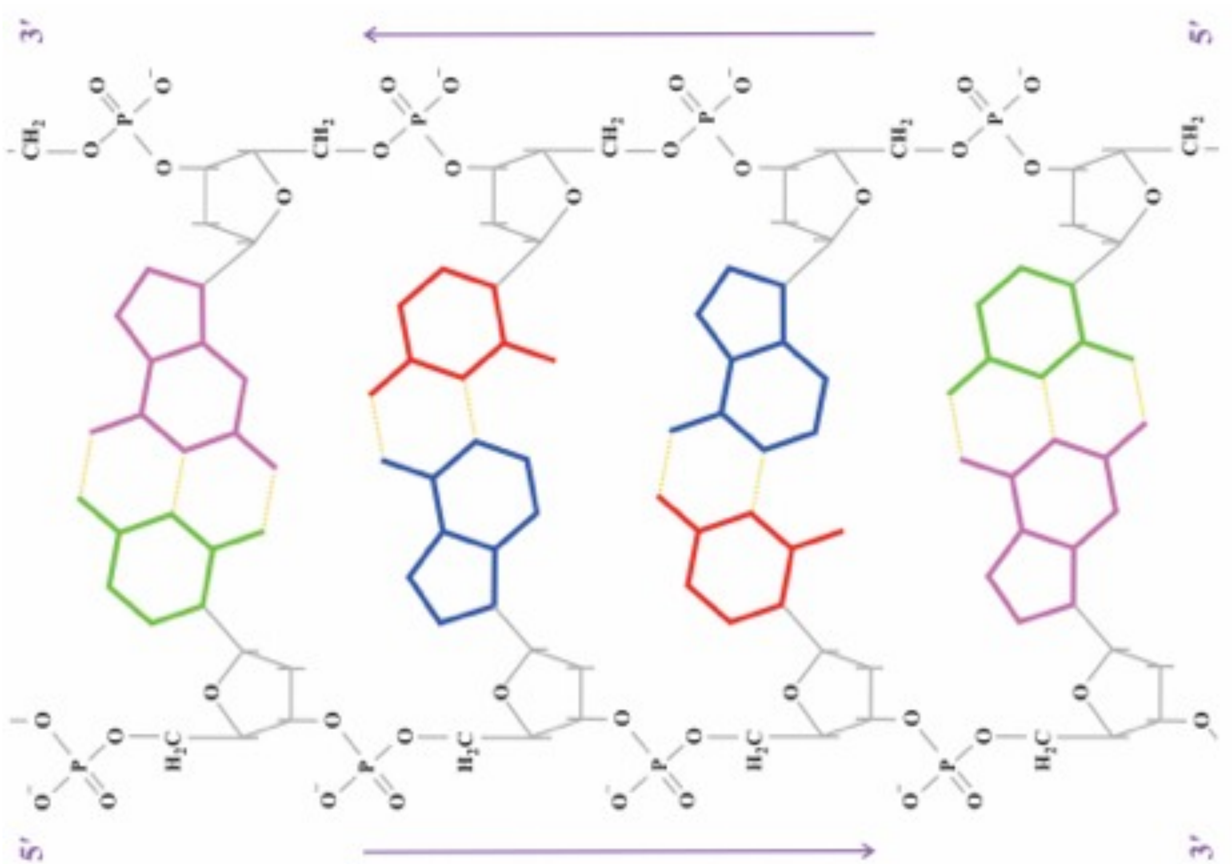
DNA-RNA and Cl(16) Clifford Algebra of E8 Physics

Frank Dodd (Tony) Smith, Jr. - January 2016 viXra 1601.0177

65,536-dimensional Cl(16) not only contains the E8 of E8 Physics (viXra 1508.0157) but also corresponds to the information content of Microtubules that are the basis of Penrose-Hameroff Quantum Consciousness (viXra 1512.0300) and to information content of DNA chromosome condensation and to information content of mRNA triple - amino acid transformations.

In “Living Matter: Algebra of Molecules” (CRC Press 2016) Valery V. Stcherbic and Leonid P. Buchatsky say: “... DNA structure contains four nucleotides:

adenine A, guanine G, cytosine C and thymine T. ...



... The Sugar-phosphate group consists of 2-deoxyribose and phosphoric acid residues. DNA chain orientation is identified by carbon atoms of 2-deoxyribose: (5′)CH2 and (3′)COH. The biological function of DNA and storage and transfer of genetic information to daughter cells is based on specific, complimentary pairing of nucleotides:

A is paired with T, and G with C.

...

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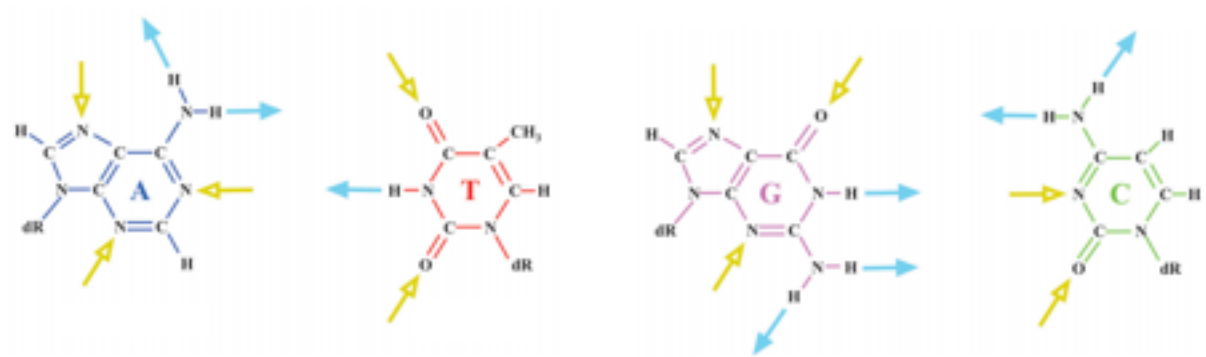


Figure 1.4 Potential vectors of hydrogen bond of DNA nucleotides. Yellow arrows—acceptors, blue arrows—donors of hydrogen.

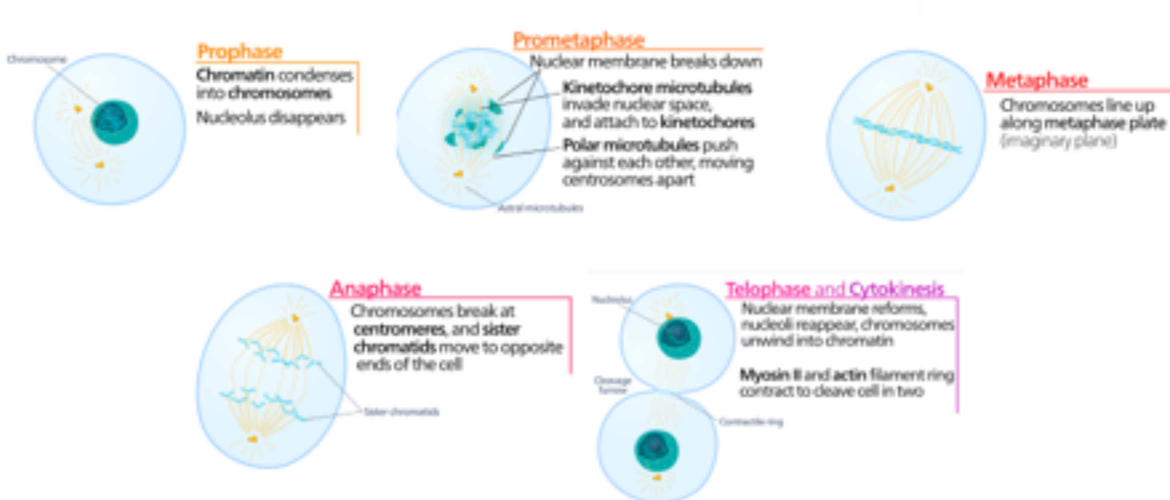
The space of DNA nucleotide states contains T2^3 ⊗ C2^4 ⊗ A2^5 ⊗ G2^6 = 2^18 elements of Clifford algebras. This space reduction to four nucleotides means compression of DNA information by a factor of 2^18 / 4 = 65536.
Reduction of the nucleotide state space leads
to DNA compactization and chromosome condensation. ...”.

In “Chromosome Condensation and Cohesion” (eLS December 2010) Laura Angelica Diaz-Martinez and Hongtau Yu say: “... The diploid human genome consists of 46 chromosomes, which collectively contain about 2 m of deoxyribonucleic acid (DNA). During mitosis, the genome is packaged into 46 pairs of sister chromatids, each less than 10 μm long. ...”.

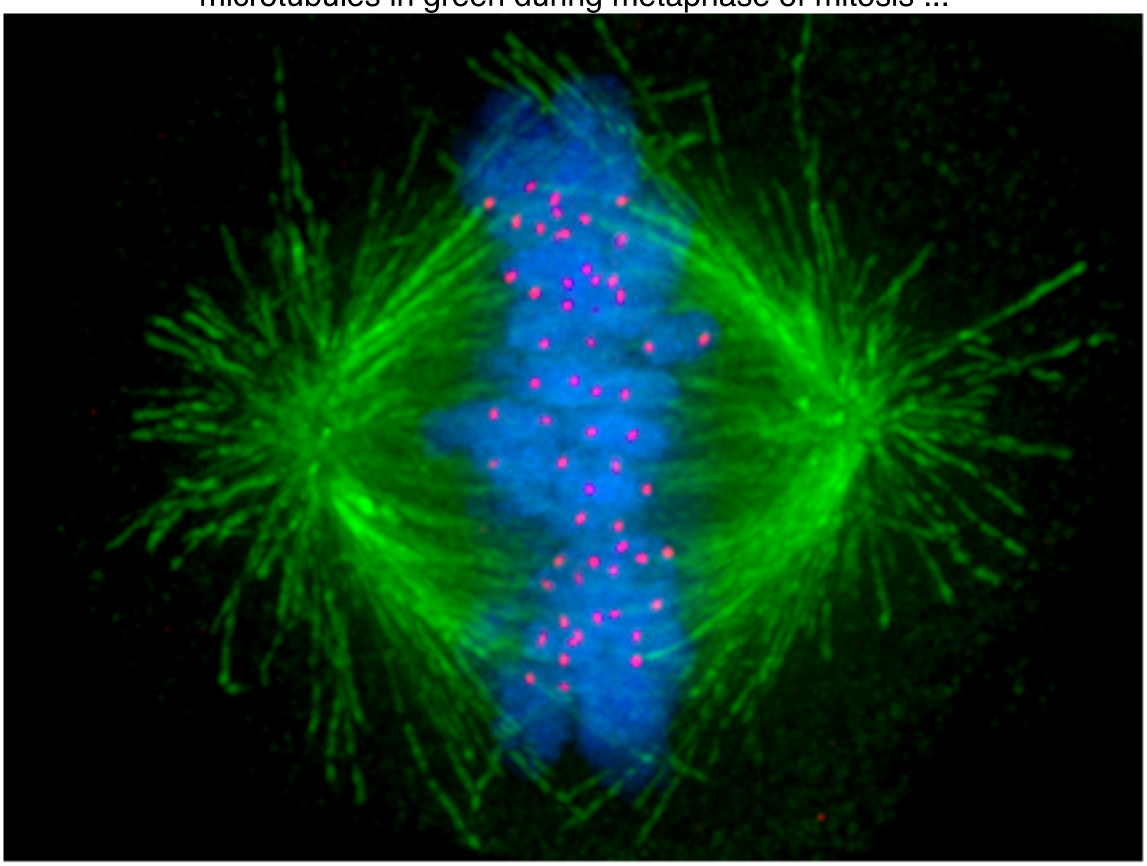
The DNA information condensation factor of 65,536 is the dimension of Cl(16) which is
the Real Clifford Algebra containing 248-dim E8 of viXra 1508.0157 E8 Physics as 120-dim bivector D8 plus 128-dim D8 half-spinor
and is also the Clifford Algebra
of Microtubule information in viXra 1512.0300 Quantum Consciousness.

Microtubule information = 65,536 = CI(16) = DNA condensation information

Wikipedia describes interaction of Microtubules with DNA in mitosis condensation: “...



... Micrograph showing condensed chromosomes in blue, kinetochores in pink, and microtubules in green during metaphase of mitosis ...



...”. **Information lost by condensing DNA is stored in Microtubules through Anaphase after which it has been restored to the new Duplicated DNA.**

Stcherbic and Buchatsky also say: “... Ribonucleic acid (RNA) can also store genetic information. A single RNA helix is seldom used as a carrier of genetic information (only in some viruses); its main role is storing DNA sites as copies of individual proteincoding genes (mRNA) or in formation of large structural complexes, e.g., ribosomes and spliceosomes. At self-splicing, RNA may perform the function of an enzyme. RNA also performs an important role during DNA replication. So called RNA-primers are necessary to synthesize DNA complementary chains, although this fact is not obvious. RNA contains sugar, ribose, which hydroxyl groups make more reactive than DNA. Besides, RNA contains uracil U, which is somewhat lighter than thymine.

...

At translation of mRNA triplets into genetic code amino acids,
the dynamics of triplets to amino acids transformation should be taken into account.

...

At transition ... functional volume is equal to 3^5 = 243.
To this volume there should be added the volume of auxiliary spaces,
equal to 13 = 5 + 4 + 3 + 1.
Accordingly, we get
256 functions of mRNA triplet transformation into amino acids of the genetic code.
Reverse transition ... from amino acids ... to triplet ... needs 5^3 + 3^1 = 128 functions.
In addition, 128 triplets of mRNA-tRNA pairing should be added to this number. ...”.

The 256 of mRNA triplet to amino acids is represented by Cl(8) Clifford algebra
and
the 128+128 = 256 of amino acids to mRNA triplets is representd by another Cl(8)
so
that the mRNA triple - amino acid connection is represented by the tensor product Cl(8) x Cl(8) which by 8-Periodicity of Real Clifford Algebras is the Real Clifford Algebra Cl(16)
which also contains 248-dim E8 of viXra 1508.0157 E8 Physics
and is also the Clifford Algebra
of Microtubule information in viXra 1512.0300 Quantum Consciousness.

E8 AQFT and Sarfatti-Bohm Free Will

Frank Dodd (Tony) Smith, Jr. - 2016 - viXra 1602.0056

E8 Physics AQFT is constructed from an E8 Physics Lagrangian (viXra 1508.0157)
by embedding E8 into the Real Clifford Algebra Cl(16) = Cl(8)xCl(8)
and taking the completion of the union of all tensor products of copies of Cl(16)
which forms a generalized hyperfinite II1 von Neumann factor algebra AQFT
(Algebraic Quantum Field Theory) that by Periodicity retains underlying E8 symmetry.

The World-Line of a Particle in E8 Physics is a String connecting the Cl(16) copies
that make up points / events in the History of the Particle.

Interaction among those History World-Line Strings by String Theory
produces a Force / Potential that is similar to Gravity
but
the Local Lagrangian of E8 Physics in each copy of Cl(16) in the AQFT
already contains Gravity (as well as the Standard Model).
The purpose of this paper is to describe

the Physical Interpretation of the E8 AQFT String Gravity-like Force / Potential

as the Sarfatti-Bohm Quantum Potential with Back-Reaction that permits Free Will
and is the fundamental Force of Quantum Consciousness that is described as Gravity
by Penrose and Hameroff (Physics of Life Reviews 11 (March 2014) 39-78).

The Quantum Potential of this paper is a Bohm internal energy of a quantum system
whose total force does not fall off with distance since it depends on the form of
the quantum state rather than its magnitude.

The form is described in terms of Cl(16) which is related (see viXra 1512.0300)
to the Tensor Product Quantum Reed-Muller code [[256 , 0 , 24]] x [[256 , 0 , 24]]
and which contains not only 248-dim E8 but a total of 65,536 elements.

E8 AQFT in the Bohm picture would have a Bohm Quantum Potential that would guide

(analogous to Gravity Curvature of Spacetime guiding particles within 4-dim Spacetime)

particles and their forces within Spacetime (including CP2 Internal Symmetry Space) that are represented in 248-dim E8 as these 128+64+28+28 = 248-dim structures:

Fermion Particle Types by 8+8 = 16-dim OP2 Octonion Projective Plane
If you count each of the 8 E8 Spacetime components of each type separately, then by E8 / D8 = 64+64 = 128-dim (OxO)P2 Octo-Octonionic Projective Plane

Spacetime by 4+4 = 8-dim Octonions
If you see both position and momentum, then by 8x8 = 64-dim D8 / D4 x D4 = Gr(8,16)

Conformal Gravity gauge bosons + Standard Model ghosts = 16+12 = 28-dim D4

Standard Model gauge bosons + Gravity ghosts = 12+16 = 28-dim D4

So,

the Bohm Quantum Potential of E8 AQFT acts as curvature of E8 in terms of those structures.

Jack Sarfatti has noticed that General Relativistic Gravity not only
has an action by Curvature of Spacetime that guides particles
but also
has a back-reaction of particles by their mass distribution on Curvature of Spacetime
so he has generalized the Bohm Quantum Potential to form

the Sarfatti-Bohm Quantum Potential that also acts as back-reaction of particle / force distribution to modify curvature of E8 structures.

In terms of Quantum Creation and Annihilation Operators, E8 AQFT is described by the maximal contraction of E8 which is a realistic generalized Heisenberg Algebra

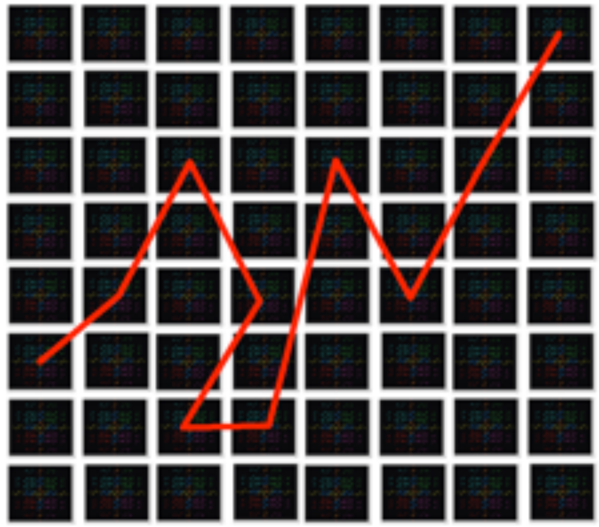
h92 x A7 = 5-graded 28 + 64 + ((SL(8,R)+1) + 64 + 28

(see viXra 1507.0069 and 1405.0030)

but from the Heisenberg Algebra Operator point of view the Geometric Structure analogy with General Relativity is not so clear so this paper will use a Lie Algebra / Symmetric Space view of E8 AQFT Geometric Structure.

(The two approaches are analogous to two views of Gravitation described by Steven Weinberg in his book "Gravitation and Cosmology" (Wiley 1972) as
"... the theory of elementary particles ..."
and
"... the ... Riemannian geometry ...geometrical approach ...".)

In terms of **E8 structure whose curvature by action and back-reaction guides and in turn is influenced by E8 particle / force distribution**
consider the World-Lines of Particles moving in the E8 structure
such as the red line in this 64-element subset of Deutsch Many-World Snapshots



Andrew Gray (quant-ph/9712037) described Quantum Interactions of World-Lines:
"... probabilites are ... assigned to entire fine-grained histories ...
base[d] ... on the Feynman path integral formulation ...
The formulation is fully relativistic and applicable to multi-particle systems.
It ... makes the same experimental predictions as quantum field theory …".

David Tong (String Theory University of Cambridge Part III Mathematical Tripos 2009)
said: "... if the dimension of spacetime is **D = 26 ... bosonic string** ...
the critical dimension ... our theory contains a bunch of massless particles ...
massless particles are interesting because they give rise to long range forces ...
The states transform in the 24 x 24 representation of SO(24) ...
the symmetric traceless representation of SO(24) is ... a massless spin 2 particle ...".
Joseph Polchinski ("String Theory" vol. 1 (Cambridge 1998)) said:
"... **Closed plus open unoriented bosonic string** ... have ...
the tachyon ...
and ... a [24x24 trace term] scalar ... dilaton ...
and ... **a symmetric ... 24x24 ... tracelesss tensor ... spin-2 graviton** ...".

Closed string tachyons localized at orbifolds of fermions produce virtual particles / antiparticles for Schwinger Sources (viXra 1311.0088, 1507.0173, 1508.0157).

Dilatons are Goldstone bosons of spontaneously broken scale invariance that (analogous to Higgs) go from mediating a long-range scalar gravity-type force to the nonlocality of the Bohm-Sarfatti Quantum Potential.

The symmetric traceless spin 2 particle = Bohmion = Carrier of the Bohm Force of the Bohm Quantum Potential.

“... Bohm’s Quantum Potential can be viewed as an internal energy of a quantum system ...” according to Dennis, de Gosson, and Hiley (arXiv 1412.5133) and Peter R. Holland says in "The Quantum Theory of Motion" (Cambridge 1993): "... the **total force ... from the quantum potential ... does not ... fall off with distance** ... because ... the quantum potential ... depends on the form of ...[the quantum state]... rather than ... its ... magnitude ...".

There is redundancy in the E8 description of Quantum States:

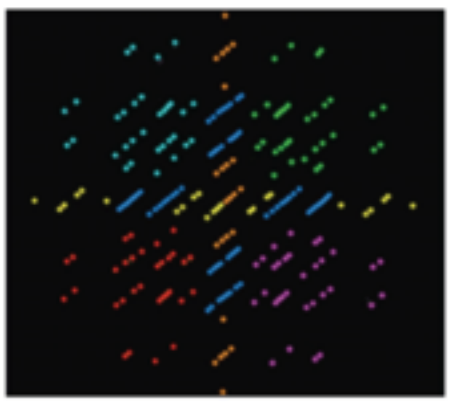
Fermion components carry Spacetime information
so E8 / D8 = 8x8 + 8x8 -> 8 + 8

Spacetime position and momentum are redundant
so D8 / D4 x D4 = 8x8 -> 8

Gauge Bosons correspond to Emission / Absorption Fermion Pairs
so D4 x D4 = 28 + 28 -> 0

How to reduce redundancy to get efficient Geometric Structure ?

Look at the 240 Root Vectors of the E8 Root Vector Polytope.



Each E8 Root Vector has

1 Root Vector Itself - North Pole

56 Nearest Neighbors

126 Next-Nearest Neighbors - Equator

56 Next-Next-Nearest Neighbors

1 Antipodal Opposite - South Pole

The 126 Equator Next-Nearest Neighbors are the Root Vectors of 133-dim E7.

The 1 North Pole and 1 South Pole are the Root Vectors of 3-dim SU(2).

The 56 Nearest Neighbors and 56 Next-Next-Nearest Neighbors represent the 112-dim symmetric space

E8 / E7 x SU(2) = Set of (QxO)P2 in (OxO)P2 = E8 / D8
where Q = Quaternions, O = Octonions, and P2 = Projective Plane

E7 is the Automorphism Group of the 112-dimensional Brown Algebra Br3(O)
so identify Br3(O) as the algebra corresponding to 112-dim E8 / E7 x SU(2).

Br3(O) is a Complexification of the 56-dimensional Freudenthal Algebra Fr3(O).
Automorphism Group of Fr3(O) is E6 of 54-dim E7 / E6 x U(1) = (CxO)P2 in (QxO)P2

Fr3(O) is a Complexification of the 27-dim Jordan Algebra J3(O).
Automorphism Group of J3(O) is F4 of 26-dim E6 / F4 = OP2 in (CxO)P2

The Traceless Part of J3(O) is 26-dim J3(O)o = Space of Bosonic String Theory =

$$\begin{matrix} a & S+ & V_{M4} x V_{CP2} \\ S+^* & -a-b & S- \\ V_{M4} x V_{CP2}^* & S-^* & b \end{matrix}$$

where a and b are Real Numbers and V_{M4} and V_{CP2} are Quaternions
and S+ and S- are Octonions and * denotes Octonion conjugation

so

J3(O)o is a 26-dim space describing Fermion Particles and Spacetime in which Strings correspond to World-Lines of Particles moving in Spacetime and String Interactions describe Sarfatti-Bohm Quantum Theory

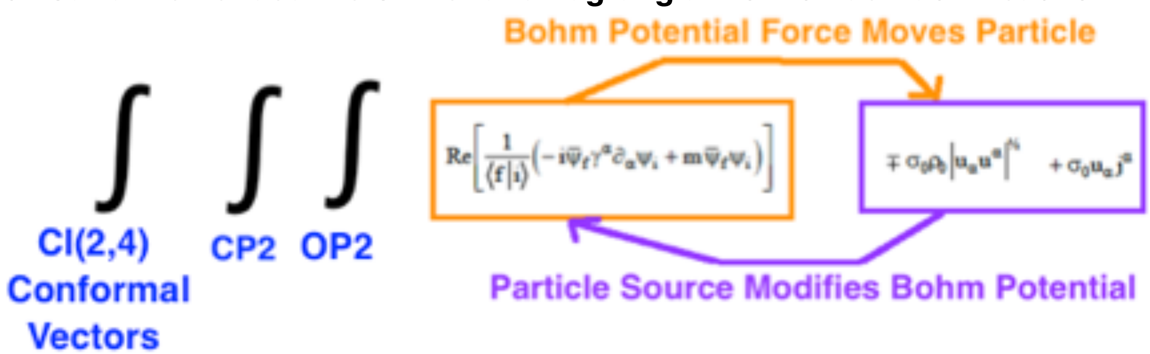
(viXra 1210.0072, 1308.0064)

Roderick Sutherland (arXiv 1509.02442) gave a Lagrangian for the Bohm Potential saying: “... This paper focuses on interpretations of QM in which the underlying reality is taken to consist of particles have definite trajectories at all times ... An example ... is the Bohm model ... This paper ... provid[es]... a Lagrangian ...[for]... the unfolding events ...
... **describing more than one particle while maintaining a relativistic description requires the introduction of final boundary conditions as well as initial, thereby entailing retrocausality** ...
In addition ... the Lagrangian approach pursued here to describe particle trajectories also entails the natural inclusion of an accompanying field to influence the particle’s motion away from classical mechanics and reproduce the correct quantum predictions. In so doing, it is ... providing a physical explanation for why quantum phenomena exist at all ... **the particle is seen to be
the source of a field which alters the particle’s trajectory via self-interaction** ...
The Dirac case ... each particle in an entangled many-particle state will be described by an individual Lagrangian density ... of the form:

$$\mathfrak{L} = \mathrm{Re}\left[\frac{1}{\langle f|i\rangle}\left(-i\bar{\Psi}_f\gamma^\alpha\partial_\alpha\Psi_i + m\bar{\Psi}_f\Psi_i\right)\right] \mp \sigma_0\rho_0\left|u_\alpha u^\alpha\right|^{1/2} + \sigma_0 u_\alpha j^\alpha$$

... the ...[first]... term ...[is]... the ... Lagrangian densities for the PSI field alone ...
... sigma_o is the rest density distribution of the particle through space ... j is the current density ...
... rho_o and u are the rest density and 4-velocity of the probability flow ...”.

Jack Sarfatti extended the Sutherland Lagrangian to include Back-Reaction



where a, b and VM4 form Cl(2,4) vectors and VCP2 forms CP2
and S+ and S- form OP2 so that
26D = 16D orbifolded fermions + 10D and 10D = 6D Conformal Space + 4D CP2 ISS
(ISS = Internal Symmetry Space and 6D Conformal contains 4D M4 of Kaluza-Klein M4xCP2)

saying (linkedin.com Pulse 13 January 2016): “... the reason entanglement cannot be used as a direct messaging channel between subsystems of an entangled complex quantum system, is the lack of direct back-reaction of the classical particles and classical local gauge fields on their shared entangled Bohmian quantum information pilot wave ... Roderick. I. Sutherland ... **using Lagrangian field theory, shows how to make the original 1952 Bohm pilot-wave theory completely relativistic, and how to avoid the need for configuration space for many-particle entanglement**.

The trick is that final boundary conditions on the action
as well as initial boundary conditions influence what happens in the present.
The general theory is "post-quantum" ... and it is non-statistical ...
There is complete two-way action-reaction between quantum pilot waves
and the classical particles and classical local gauge fields ...
orthodox statistical quantum theory, with no-signaling ...[is derived]... in two steps,

first arbitrarily set the back-reaction (of particles and classical gauge field on their pilot waves) to zero. This is analogous to setting the curvature equal to zero in general relativity, or more precisely in setting G to zero.

Second, integrate out the final boundary information, thereby adding the statistical Born rule to the mix. ...
the mathematical condition for zero post-quantum back-reaction of particles and classical fields (aka "beables" J.S. Bell's term) is exactly de Broglie's guidance constraint. That is, in the simplest case, the classical particle velocity is proportional to the gradient of the phase of the quantum pilot wave. It is for this reason, that the independent existence of the classical beables can be ignored in most quantum calculations.
However, orthodox quantum theory assumes that the quantum system is thermodynamically closed between strong von Neumann projection measurements that obey the Born probability rule.
The new post-quantum theory in the equations of Sutherland, prior to taking the limit of orthodox quantum theory, should apply to pumped open dissipative structures. Living matter is the prime example. This is a clue that should not be ignored. ...".

Jack Sarfatti (email 31 January 2016) said: "... Sabine [Hossenfelder]'s argument ...
"... two types of fundamental laws ... appear in contemporary theories.

One type is deterministic, which means that the past entirely predicts the future. There is no free will in such a fundamental law because there is no freedom.

The other type of law we know appears in quantum mechanics and has an indeterministic component which is random. This randomness cannot be influenced by anything, and in particular it cannot be influenced by you, whatever you think "you" are. There is no free will in such a fundamental law because there is no "will" - there is just some randomness sprinkled over the determinism.

In neither case do you have free will in any meaningful way."

... However ...[**There is a Third Way**]...
post-quantum theory with action-reaction between quantum information pilot wave and its be-able is compatible with free will. ...".