Exotic Spheres and Kervaire

It seems to me that Kervaire comes from mixing vector-type and spinor-type structures such as a vector-type S7 = Spin(8)/Spin(7) and a spinor-type Clifford S7 = Spin(7)/G2 for Kervaire in 14. (The terminology "Clifford" comes from Ian Porteous, in his book "Clifford Algebras and the Classical Groups" (Cambridge 1995)).My current understanding (possibly somewhat in error) about the history of Kervaire is: Browder (1969) showed: Kervaire Invariant can only exist for dimensions 2n where n = 2^j - 1 where j = 1 , 2 , 3 , 4 , 5 , 6 , 7 , 8 ... give dimension = 2 , 6 , 14 , 30 , 62 , 126 , 254 ... Kervaire exists for 2,6,14,30,62 and does not exist for 254 ... As to 126, it is now open. As Mike Hopkins said in email to John Baez: "... in all other dimensions [than 2,6,14,30,62, and maybe 126] of the form (4k + 2) every framed manifold is framed cobordant to a sphere ...". Note that the case of dimension 4, and its exotic structure, is not relevant to the Kervaire invariant.

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        2       = S1xS1 with framing not carrying to S2
              1...S1
  1...............S1

        6       = S3xS3 with framing not carrying to S6
              3...S3
  3...............S3

       14       = S7xS7 with framing not carrying to S14
              7...S7 = Spin(8)/Spin(7)
                  and
  7...............Clifford embedding S7 = Spin(7)/G2

       30       = S15xS15 with framing not carrying to S30
             15...S15 = Spin(16)/Spin(15)
                  and
 15...............Clifford S15 = Spin(9)/Spin(7)

       62         = S31xS31 with framing not carrying to S62
             31...S31 = Spin(32)/Spin(31)
                  and
 31...............Clifford S31 containing C30
                  where C30 = Spin(10)/Spin(6) = A21xS9
                  where A21 = Spin(9)/Spin(6) = Spin(10)/SU(5)
                  and the embedding of Spin(6) in Spin(9) is Clifford
                  and the SU(5) = B24 = Spin(10)/Spin(7)
                  and the embedding of Spin(7) in Spin(10) is Clifford
                  (see page 276 and related pages of Porteous's book)

      126       = S63xM63
            63....S63 = Spin(64)/Spin(63)
                  and
 63...............M63 = SU(8) = even part of E7 under
                        grading 7 + 35 + U(7) + 35 + 7

      254       = S127xS127 with framing carrying over to S254
            127...S127 = Spin(128)/Spin(127)
                         Spin(128) acts on even subalgebra of Cl(8)
                  and
127...............S127 = Spin(128)/Spin(127)
                         Spin(128) acts on odd part of Cl(8)
                  Both S127 act on Cl(8) so that Cl(8) acts as
                  a common framework so that the framing carries over.
                  Since the total S127xS127 action acts on all of Cl(8),
                  and comes from Cl(128+128) = Cl(256) = Cl(Cl(8))
                         and
Since Cl(n) has periodicity 8 with Cl(8) being maximal,
Cl(Cl(n)) has periodicity 256 with Cl(Cl(8)) = Cl(256) being maximal.
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